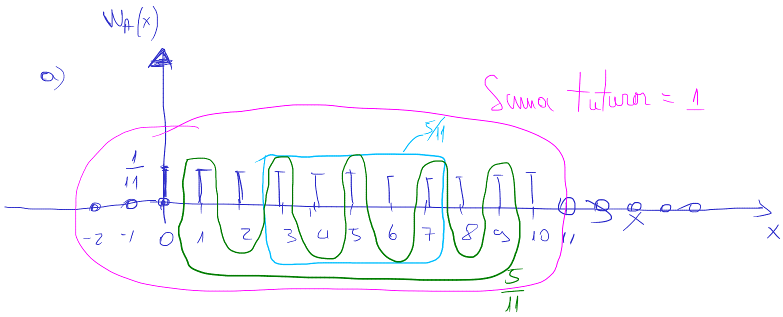
Seminar 1 DEPI

(3)

a)



$\underbrace{0, 1, \dots, 10}_{11 \text{ valori}}$

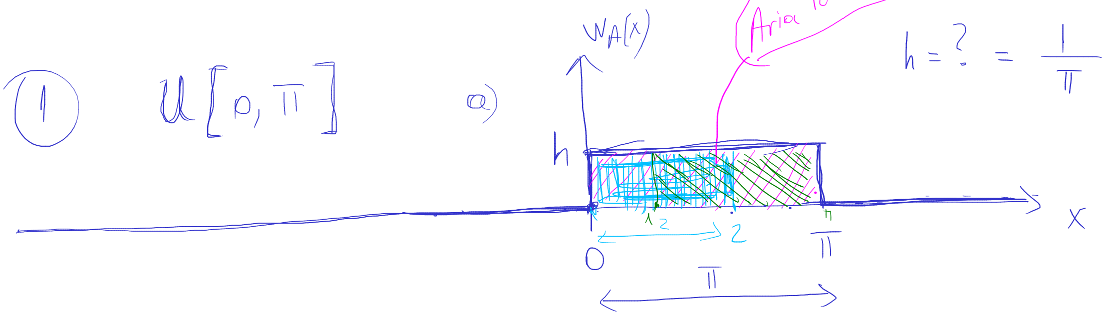
$\frac{1}{11} =$

b). $P(A \in [3,7]) = \frac{5}{11}$

c). $P(A \text{ impar}) = \frac{5}{11}$

(1) $U[0, \pi]$

a)



$h = ? = \frac{1}{\pi}$

$$w_A(x) = \begin{cases} 0, & x \text{ în rest} \\ \frac{1}{\pi}, & x \in [0,\pi] \end{cases}$$

c). $P(A \in (0,2)) = \int_0^2 w_A(x)\,dx = \frac{2}{\pi}$

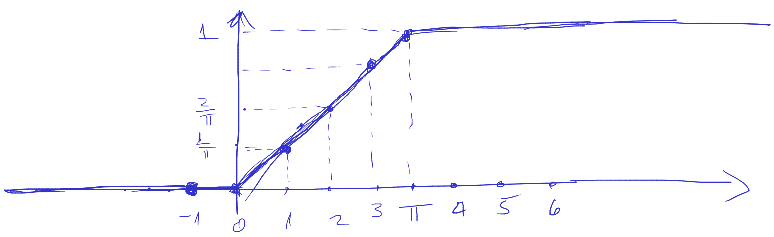
b). $P(A > 1) = \int_1^{\infty} w_A(x)\,dx = \int_1^{\pi} = \frac{(\pi - 1)}{\pi}$

$A \in (1, \infty)$

$\int 1 \cdot dx = x$

$$\int_1^{\pi} w_A(x)\,dx = \int_1^{\pi} \frac{1}{\pi}\,dx = \frac{1}{\pi} \cdot x \Big|_1^{\pi} = \frac{\pi - 1}{\pi}$$

d). $F_A(x) = P(A \le x)$



$$F_A(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{\pi} \cdot x, & x \in [0,\pi] \\ 1, & x \ge \pi \end{cases}$$

$F_A(-1) = P(A \le -1)$

$F_A(0) = P(A \le 0) = \int_{-\infty}^{0} w_A(x)\,dx = 0$

$A \in (-\infty, 0)$

$F_A(1) = P(A \le 1) = \int_{-\infty}^{1} = \frac{1}{\pi}$

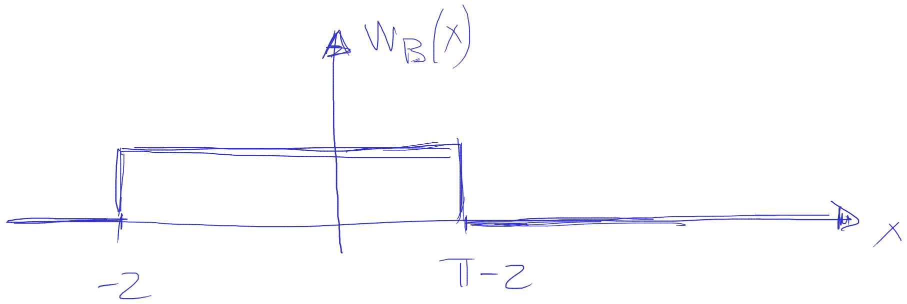
$F_A(2) = P(A \le 2) = \frac{2}{\pi}$

$F_A(3) = P(A \le 3) = 3/\pi$

$F_A(\pi) = P(A \le \pi) = 1$

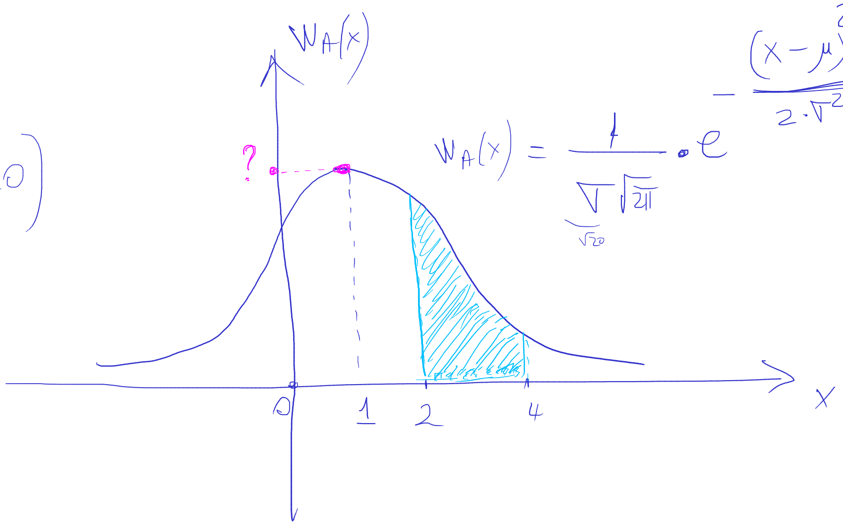
$F_A(4) = P(A \le 4) = 1$

e). $B = A - 2$



$$\int_a^b e^{-x^2} =$$

2 $\quad \mathcal{N}(\mu = 1, \sigma^2 = 20)$



$$W_A(x) = \frac{1}{\underset{\sqrt{20}}{\sigma}\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\cdot\sigma^2}}$$

$$P(A \in [2,4]) = \int_2^4 W_A(x)\,dx$$

$$= \underset{P(A \leq 4)}{F_A(4)} - \underset{P(A \leq 2)}{F_A(2)} = 0.7\cancel{4} - 0.58 = 0.16$$

$$\boxed{F(x) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)}$$

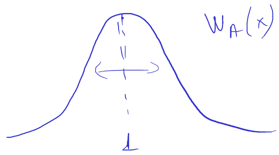
$$F_A(4) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{4-1}{\sqrt{20}\cdot\sqrt{2}}\right)\right) = 0.7\cancel{4}$$

$$F_A(2) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{2-1}{\sqrt{40}}\right)\right) = 0.58$$

$$P(A > 1) = \int_1^\infty W_A(x)\,dx = \underbrace{\underbrace{F_A(\infty)}_{P(A \leq \infty)}}_{1} - F_A(1) = 1 - \ldots =$$

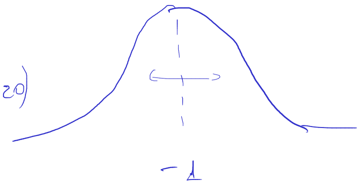
$$P(A < 1) = \int_{-\infty}^{1} = F_A(1) - \underbrace{\underbrace{F_A(-\infty)}_{P(A \leq -\infty)}}_{0} = F_A(1) = \ldots\ldots$$

b). $B = A - 2$



$A : \mathcal{N}(\mu = 1, \sigma^2 = 20)$

$B : ?$ $\underline{\mathcal{N}(\mu = -1, \sigma^2 = 20)}$



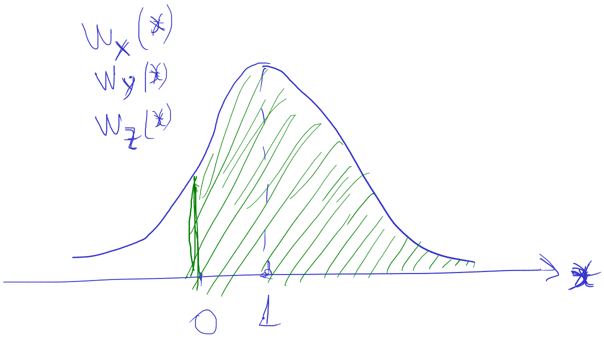
c). max $w_A(x)$ și pt. ce x se atinge ??

Max se atinge pt. $x = \mu = 1$,

și are valoarea $\dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(1-1)^2}{2\sigma^2}} = \dfrac{1}{\sigma\sqrt{2\pi}} = \dfrac{1}{\sqrt{20}\cdot\sqrt{2\pi}}$

(4) $X, Y, Z,$ independente

$\mathcal{N}(\mu = 1, \sigma^2 = 1)$



$P(X > 0 \text{ și } Y > 0 \text{ și } Z > 0) = ?$

$= \underbrace{P(X > 0)}_{0.84} \cdot \underbrace{P(Y > 0)}_{0.84} \cdot \underbrace{P(Z > 0)}_{0.84} = (0.84)^3 = \dots$

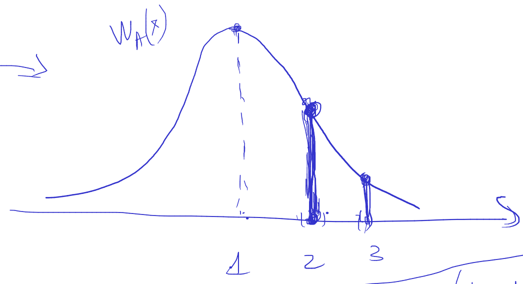
$P(X > 0) = \displaystyle\int_0^\infty w_X(x)\,dx = \underbrace{F(\infty)}_{1} - F(0) = 1 - \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{0-1}{1\cdot\sqrt{2}}\right)\right) =$

$= \cancel{\kappa} = 0.84$

5. $A: \mathcal{N}(\mu = 1, \sigma^2 = 3)$

$B: \mathcal{N}(\mu = -4, \sigma^2 = 3)$

$C: \mathcal{N}(\mu = 5, \sigma^2 = 3)$



$$\boxed{w(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

a) $P((A,B,C) \text{ în jurul lui } (2,-6,3)) =$

$= \underbrace{P(A \text{ în jurul lui } 2)}_{W_A(2)} \cdot \underbrace{P(B \text{ în jurul lui } -6)}_{W_B(-6)} \cdot \underbrace{P(C \text{ în jurul lui } 3)}_{W_C(3)}$

$W_A(2) = \frac{1}{\sqrt{3}\sqrt{2\pi}} \cdot e^{-\frac{1}{6}}$ $\quad W_B(-6) = \frac{1}{\sqrt{6\pi}} \cdot e^{-\frac{4}{6}}$ $\quad W_C(3) = \frac{1}{\sqrt{6\pi}} \cdot e^{-\frac{4}{6}}$

$$= \boxed{\left(\frac{1}{\sqrt{6\pi}}\right)^3 \cdot e^{-\frac{9}{6}}}$$

P_1

$\frac{1}{e^{9/6}}$

$P((A,B,C) \text{ în jurul lui } (-2,-3,2))$

$= \underbrace{W_A(-2)}_{\frac{1}{\sqrt{6\pi}} \cdot e^{-\frac{9}{6}}} \cdot \underbrace{W_B(-3)}_{\frac{1}{\sqrt{6\pi}} \cdot e^{-\frac{1}{6}}} \cdot \underbrace{W_C(2)}_{\frac{1}{\sqrt{6\pi}} \cdot e^{-\frac{9}{6}}} =$

$$= \boxed{\left(\frac{1}{\sqrt{6\pi}}\right)^3 \cdot e^{-\frac{19}{6}}}$$

P_2

$e^{-\frac{19}{6}} = \frac{1}{e^{19/6}}$