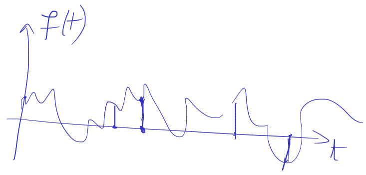
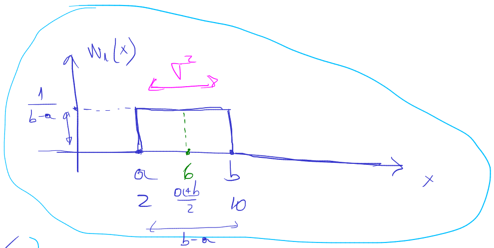
DEPI Seminar 2



1) a) $w_1(x) = U[a,b]$, $a, b \in \mathbb{R}$

$a = 2$

$b = 10$



$$\cdot\ \mu_x = \overline{X} = \overline{f(t)} = \int_{-\infty}^{\infty} x\, w_1(x)\, dx = \int_a^b x \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\int_a^b x\, dx = \frac{1}{b-a}\cdot \frac{x^2}{2}\Big|_a^b = \frac{1}{2\cancel{(b-a)}}\cdot \overbrace{(b^2 - a^2)}^{(b+a)\cancel{(b-a)}} = \frac{a+b}{2} \overset{\substack{a=2\\b=10}}{=} 6$$

$$\cdot\ \overline{X^2} = \overline{f^2(t)} = \int_{-\infty}^{\infty} x^2\, w_1(x)\, dx = \int_a^b x^2 \frac{1}{b-a}\, dx = \frac{1}{b-a}\cdot\int_a^b x^2 dx = \frac{1}{b-a}\cdot\frac{x^3}{3}\Big|_a^b = \frac{\overbrace{b^3 - a^3}^{\cancel{(b-a)}(b^2+ab+a^2)}}{3\cancel{(b-a)}} =$$

$$= \frac{a^2+b^2+ab}{3} \overset{\substack{a=2\\b=10}}{=} \frac{4+100+20}{3} = \frac{124}{3} \geq 0$$

$$\cdot\ \sigma^2 = \sigma^2_{f(t)} = \int_{-\infty}^{\infty} (x-\mu)^2\, w_1(x)\, dx = \int_a^b (x-\mu)^2\cdot\frac{1}{(b-a)}\, dx = \frac{1}{b-a}\cdot\int_a^b (x-\mu)^2 dx = \frac{1}{b-a}\cdot\frac{(x-\mu)^3}{3}\Big|_a^b$$

$$= \frac{1}{(b-a)\cdot 3}\left((b-\mu)^3 - (a-\mu)^3\right) \overset{\substack{a=2\\b=10}}{=} \frac{1}{8\cdot 3}\left((10-6)^3 - \underbrace{(2-6)^3}_{\substack{(-4)^3\\-64}}\right) = \frac{1}{24}(64+64) = \frac{128}{24} = \frac{16}{3}$$

$$\boxed{\sigma^2 = \overline{X^2} - \mu^2} = \frac{124}{3} - 6^2 = \frac{124}{3} - 36 = \frac{124-108}{3} = \frac{16}{3}$$

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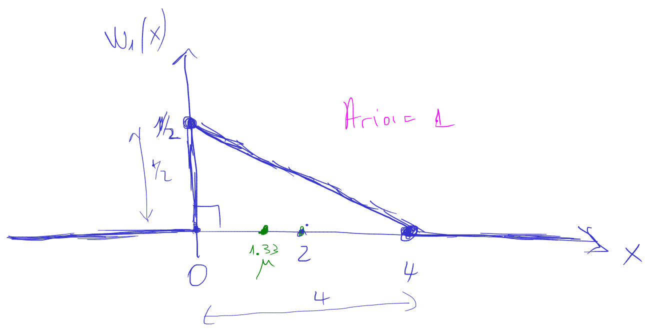
b). $w_1(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}\cdot x, & x \in [0,4] \\ 0, & \text{rest} \end{cases}$



$ax + b$

$$\cdot\ \overline{X} = \overline{f(t)} = \int_{-\infty}^{\infty} x\cdot w_1(x)\, dx = \int_0^4 x\cdot\left(\frac{1}{2} - \frac{1}{8}x\right) dx = \int_0^4 \frac{1}{2}x\, dx - \int_0^4 \frac{1}{8}x^2\, dx =$$

$$= \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^4 - \frac{1}{8}\cdot\frac{x^3}{3}\Big|_0^4 = \frac{1}{2}\cdot\frac{16}{2} - \frac{1}{\cancel{8}_2}\cdot\frac{4^{\cancel{3}\,2}}{3} = 4 - \frac{16}{6} = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$$

$$= 1.33$$

$$\overline{X^2} = \overline{f^2(t)} = \int_{-\infty}^{\infty} x^2 \cdot w_1(x) = \int_0^4 x^2 \cdot \left(\frac{1}{2} - \frac{1}{8}x\right) dx = \frac{1}{2}\int_0^4 x^2 dx - \frac{1}{8}\int_0^4 x^3 dx$$

$$= \frac{1}{2} \cdot \frac{x^3}{3}\Big|_0^4 - \frac{1}{8} \cdot \frac{x^4}{4}\Big|_0^4 = \frac{1}{2} \cdot \frac{4^3}{3} - \frac{1}{8} \cdot \frac{4^{\cancel{4}}}{\cancel{4}} = \frac{64}{6} - \frac{64}{8}$$

$$= \frac{32}{3} - 8 = \frac{32-24}{3} = \frac{8}{3} \geq 0 \checkmark$$

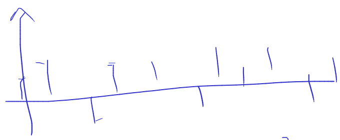
$$\sigma^2 = \overline{X^2} - \mu^2 = \frac{8}{3} - \left(\frac{4}{3}\right)^2 = \frac{8}{3} - \frac{16}{9} = \frac{24-16}{9} = \frac{8}{9} \checkmark$$

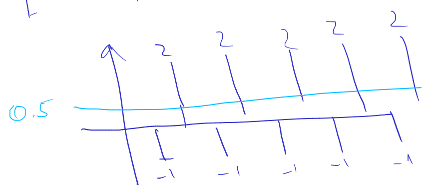
2

$$f = [-1, 2, -1, 2, -1, 2, -1, 2, -1, 2]$$

$$\mu_{temp} = \overline{f(t)}^{(K)}_{temp} = \overline{f}^{(K)}_{temp} = \frac{-1+2-1+2-1+2-1+2-1+2}{10} =$$

$$= 0.5$$





$$\overline{f^2}^{(K)}_{temp} = \frac{(-1)^2+2^2+(-1)^2+2^2+ \dots +2^2}{10} = \frac{25}{10} = 2.5 \geq 0$$

$$\sigma^{2\,(K)}_{temp} = \frac{(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+(2-0.5)^2+ \dots +(2-0.5)^2}{10} =$$

$$= \frac{2.25+2.25+2.25+ \dots +2.25}{10} = 2.25 \geq 0$$

$$\boxed{\sigma^2_{temp} = \overline{f^2_{temp}} - \mu^2_{temp}}$$

$$2.25 = 2.5 - (0.5)^2$$

$R_{ff}(\tau) = \overline{f(t) \cdot f(t+\tau)}$

$$R_{ff}\overset{\tau=}{[0]} = \frac{(-1)(-1) + 2\cdot2 + (-1)(-1) + 2\cdot2 + \dots\ 2\cdot2}{10} = \frac{25}{10} = 2.5 = \overline{f^2_{temp}}$$

$$R_{ff}\overset{\tau=1}{[1]} = \frac{(-1)\cdot2 + 2\cdot(-1) + (-1)\cdot2 + 2\cdot(-1) + \dots + (-1)\cdot2}{9} = \boxed{-2}$$

$$R_{ff}\overset{\tau=2}{[2]} = \frac{(-1)\cdot(-1) + 2\cdot2 + (-1)(-1) + \dots\ (2\cdot2)}{8} = 2.5$$

$\vdots$

$$R_{ff}[\tau=-1] = \frac{2(-1) + (-1)\cdot2 + \dots\ 2\cdot(-1)}{9} = \boxed{-2}$$

putere coincidentă

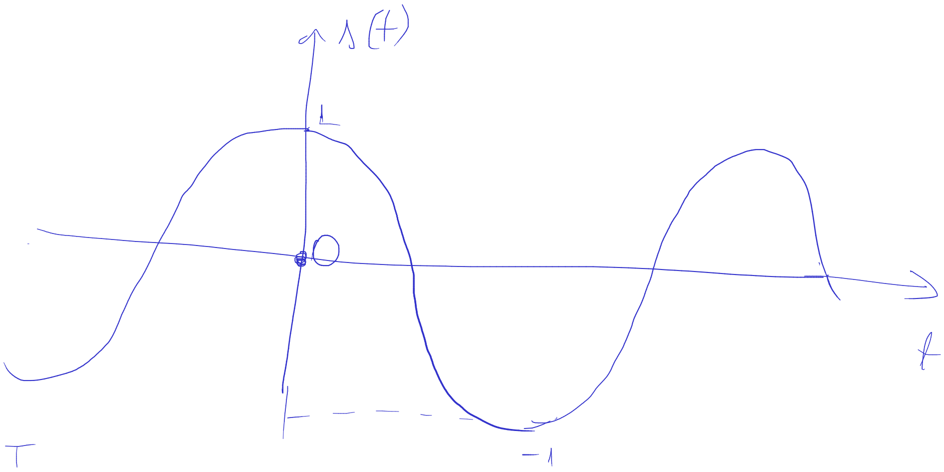
egale

$R_{ff}[-2] = R_{ff}[2] = 2.5$

$$\boxed{R_{ff}(\tau) = R_{ff}(-\tau)}$$

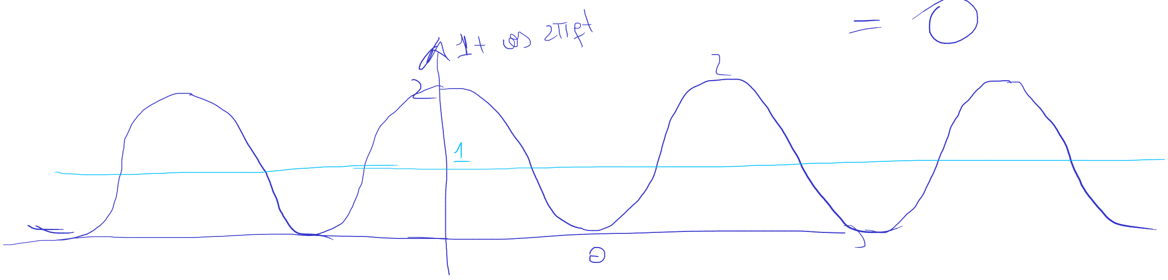
3

$s(t) = \cos(2\pi f t)$



$$\mu_{temp} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} s(t)\,dt = \underline{\text{Componenta de c.c. a semnalului}}$$

$= 0$



$\mu_{temp} = 1$