Seminar 5 DEPI

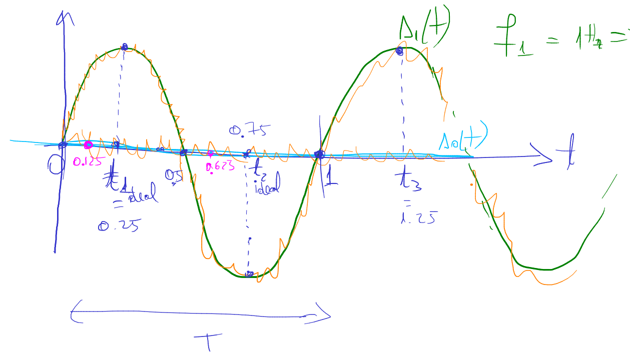
2021-12-04

(1)

$s_0(t) = 0$

$s_1(t) = 3 \sin(2\pi \underbrace{1}_{f_1} t)$

$\mathcal{N}(\mu = 0, \sigma^2 = 1)$



$f_1 = 1Hz \Rightarrow T = \frac{1}{1} = 1s$

a) $\boxed{t_1 = 0.25 \quad t_2 = 0.75}$

$t_3 = 1.25$

Nepotriv.: 0, 0.5, 1,

b). $t_1 = 0.125 \longrightarrow r_1 = 1.1$

$t_2 = 0.625 \longrightarrow r_2 = 4.4$

$s_0 = [\,0 \quad \gtrless \quad 0\,]$

$s_1 = [\,\underbrace{3 \cdot \sin(2\pi \cdot 1 \cdot 0.125)}_{2.12} \quad \gtrless \quad \underbrace{3 \cdot \sin(2\pi \cdot 1 \cdot 0.625)}_{-2.12}\,]$

$r = [\,1.1 \quad r_2 \quad 4.4\,]$

Zgomot gaussian:

$$\boxed{\underbrace{d(r, s_0)^2}_{20.57} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r, s_1)^2}_{43.55} + 2 \cdot \sigma^2 \cdot \ln K}, \quad K = \begin{cases} 1, & ML \\ P(H_0)/P(H_1), & MPE \\ \frac{(R_{10} - C_{00})}{(C_{01} - C_{11})} \cdot \frac{P(H_0)}{P(H_1)}, & MR \end{cases}$$

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_N - b_N)^2} = \sqrt{\sum_i (a_i - b_i)^2}$$

$$d(r, s_0) = \sqrt{(1.1 - 0)^2 + (4.4 - 0)^2} = \sqrt{20.57}$$

$$d(r, s_1) = \sqrt{(1.1 - 2.12)^2 + (4.4 + 2.12)^2} = \sqrt{43.55}$$

b) ML: $\underline{K = 1} \Rightarrow 20.57 \underset{H_0}{\gtrless} 43.55 + 0 \Rightarrow s_0$

c). MPE: $K = \frac{2/3}{1/3} = 2 \Rightarrow 20.57 \underset{H_0}{\gtrless} 43.55 + \underbrace{2 \cdot 1 \cdot \ln(2)}_{1.3} \Rightarrow s_0$

d). MR. $K = \frac{10}{15} \cdot 2 = \frac{20}{15} \Rightarrow 20.57 \underset{H_0}{\gtrless} 43.55 + \underbrace{2 \cdot 1 \cdot \ln\left(\frac{20}{15}\right)}_{mic} \Rightarrow s_0$

e). Nu, pt. că la $t_2 = 0.5$ semnalele sunt egale $s_0(0.5) = s_1(0.5)$

(2) $H_0 : s_0(t) = 0$ $\qquad \mathcal{N}(\mu = 0, \sigma^2 = 1)$

$H_1 : s_1(t) = 6$

$$r = [1.1 \quad 4.4 \quad 3.7 \quad 4.1 \quad 3.8]$$

$$s_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$s_1 = [6 \quad 6 \quad 6 \quad 6 \quad 6]$$

$$\boxed{d(r, s_0)^2 \gtrless d(r, s_1)^2 + 2 \cdot \sigma^2 \cdot \ln(K)}, \quad K = \begin{cases} \dots \\ K = \frac{P(H_0)}{P(H_1)} \\ \dots \end{cases}$$

a) ...

b) ... Aceeași poveste ca la (1), cu alte personaje

c) ...

d). M.P.E.:

$$\underbrace{d(r, s_0)^2}_{A} \underset{H_0}{\gtrless} \underbrace{d(r, s_1)^2}_{B} + 2 \cdot \underbrace{\sigma^2}_{1} \cdot \ln\left(\frac{P(H_0)}{P(H_1)}\right)$$

$$\boxed{P(H_1) + P(H_0) = 1}$$

s_0 : $A < B + 2 \cdot \ln \frac{P(H_0)}{1 - P(H_0)}$?

$$\Leftrightarrow \frac{A - B}{2} < \ln \frac{P(H_0)}{1 - P(H_0)} \quad \Big| \, e^{(\,)}$$

$$\Leftrightarrow e^{\frac{A-B}{2}} < \frac{P(H_0)}{1 - P(H_0)} \quad \Big| \, (1 - P(H_0))$$

$$\Leftrightarrow e^{\frac{A-B}{2}} - P(H_0) \cdot e^{\frac{A-B}{2}} < P(H_0) \Leftrightarrow e^{\frac{A-B}{2}} < P(H_0)\left(1 + e^{\frac{A-B}{2}}\right)$$

$$\Leftrightarrow \boxed{P(H_0) > \frac{e^{\frac{A-B}{2}}}{1 + e^{\frac{A-B}{2}}} = \dots ? (95.73\% \text{ să zicem})}$$

③ $s_0(t)$
$s_1(t)$
$r(t)$

$$\boxed{d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + 2 \cdot \sigma^2 \cdot \ln \frac{P(H_0)}{P(H_1)}}$$

a) $t_1 = 0.5 \quad t_2 = 1.5 \quad t_3 = 3.5$

$$s_0 = [\; 2 \quad 2 \quad -2 \;]$$

$$s_1 = [\; -2 \quad -2 \quad 2 \;]$$

$$r = [\; -1 \quad -1 \quad 1 \;]$$

$$d(r, s_0) = \sqrt{(-1-2)^2 + (-1-2)^2 + (1+2)^2} = \dots$$

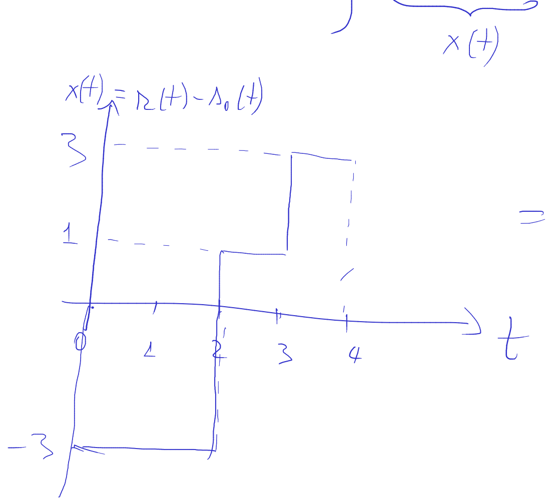
$$d(r, s_1) = \sqrt{(-1+2)^2 + (-1+2)^2 + (1-2)^2} = \dots$$

.... Accogni poveste ...

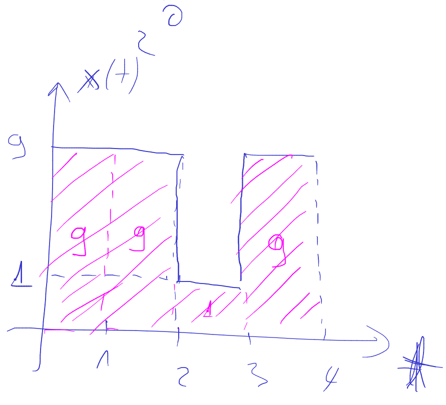
b).

$$d(a(t), b(t)) = \sqrt{\int (a(t) - b(t))^2 \, dt}$$

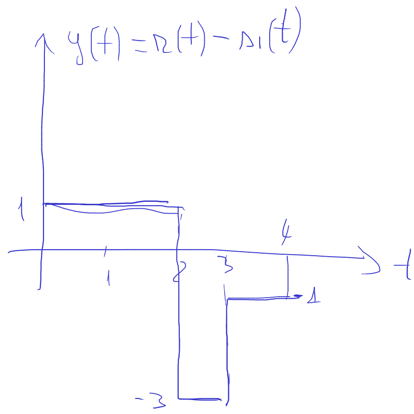
$$d(r, s_0)^2 = \int (\underbrace{r(t) - s_0(t)}_{x(t)})^2 \, dt = \int_0^4 x(t)^2 \, dt = 28$$

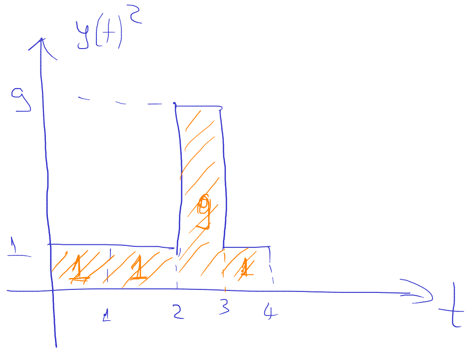


$\Rightarrow$



$$d(r, s_1)^2 = \int \big(\underbrace{r(t) - s_1(t)}_{y(t)}\big)^2 \, dt = 12$$





=> Aceeași poveste :) => . . . .

(4) K − NN

$$\underbrace{v_1, v_2, \dots}_{A} \; \underbrace{\cdot \quad v_{10}}_{B}$$

$d(x, v_1) = \sqrt{(-2-2)^2 + (5+4)^2} = \sqrt{16+81} = \sqrt{97}$

$d(x, v_2) = \sqrt{9 + 100} = \sqrt{109}$

$d(x, v_3) = \sqrt{0 + 1} = \sqrt{1}$ ✗

$d(x, v_4) = \sqrt{1 + 1} = \sqrt{2}$ ✗

$d(x, v_5) = \sqrt{16 + 100} = \sqrt{116}$

$d(x, v_6) = \sqrt{25 + 16} = \sqrt{41}$

$d(x, v_7) = \sqrt{1 + 16} = \sqrt{17}$

$d(x, v_8) = \sqrt{4 + 64} = \sqrt{68}$

$d(x, v_9) = \sqrt{1 + 25} = \sqrt{26}$

$d(x, v_{10}) = \sqrt{4}$ ✗

Vecini : $V_3, V_4, V_{10}, V_7, V_9, V_6, V_8, V_1, V_2, V_5$

V_3	V_4	V_{10}	V_7	V_9	V_6	V_8	V_1	V_2	V_5
A	A	B	B	B	B	B	A	A	A

$K = 1$: $\Rightarrow A$

$K = 3$: $\Rightarrow A$

$K = 5$: $\Rightarrow B$

$K = 7$: $\Rightarrow B$

$K = 9$: $\Rightarrow B$