Seminar 6 DEPI

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1. Fie următorul set de 10 vectori, compus din 5 vectori din clasa A și 5 vectori din clasa B:

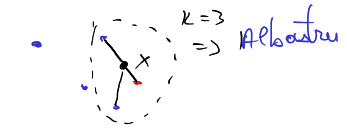
   - Clasa A:

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \quad \vec{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \vec{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

   - Clasa B:

$$\vec{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \vec{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \quad \vec{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \quad \vec{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

   Calculați clasa vectorului $\vec{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ folosind algoritmul k-NN, pentru diverse valori ale lui k: $k = 1$, $k = 3$, $k = 5$, $k = 7$ and $k = 9$



$d(x, v_1) = \sqrt{(-2-2)^2 + (5+4)^2} = \sqrt{97}$

$d(x, v_1) = \sqrt{97}$

$d(x, v_2) = \sqrt{9+100} = \sqrt{109}$

$d(x, v_3) = \sqrt{1}$

$d(x, v_4) = \sqrt{2}$

$d(x, v_5) = \sqrt{116}$

$d(x, v_6) = \sqrt{41}$

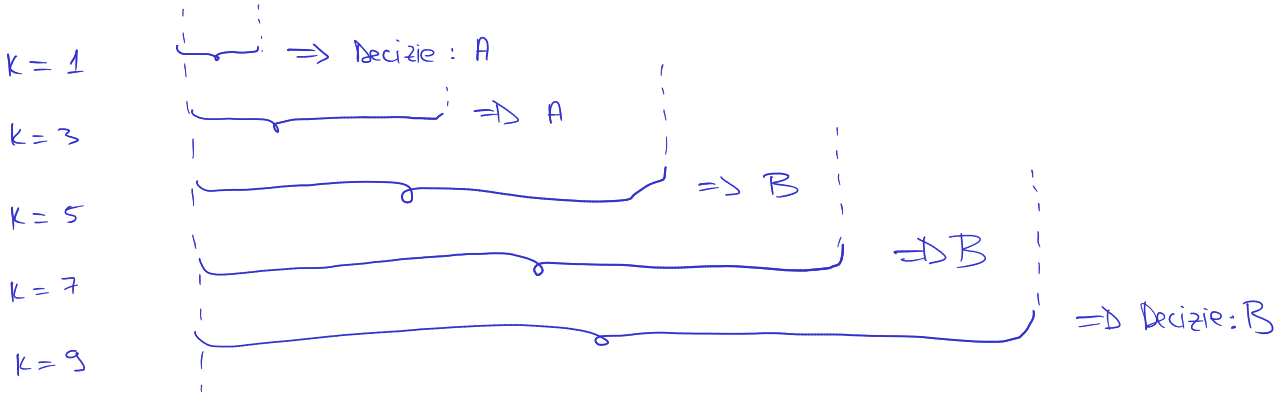
$d(x, v_7) = \sqrt{17}$

$d(x, v_8) = \sqrt{68}$

$d(x, v_9) = \sqrt{26}$

$d(x, v_{10}) = \sqrt{4}$

v_3	v_4	v_{10}	v_7	v_9	v_6	v_8	v_1	v_2	v_5
A	A	B	B	B	B	B	A	A	A



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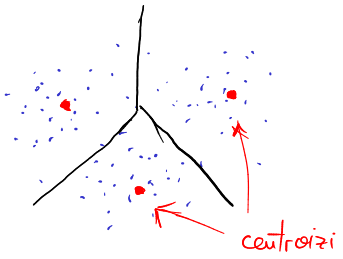
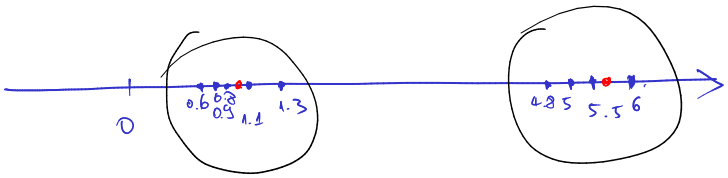
2. Fie următoarele zece valori numerice:

$$\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

Efectuați cinci iterații ale algoritmul k-Means pentru a găsi doi centroizi $\vec{c}_1$ și $\vec{c}_2$, pornind de la două valori aleatoare $\vec{c}_1 = 0.95$ și $\vec{c}_2 = 0.96$.





K-Means:

$C_1 = 0.95$

$C_2 = 0.96$

Iterația 1

1). Clasificare:

C_1 : 0.9 0.6 0.8

C_2 : 1.1 5.5 5 5 1.3 4.8 6

2). Re-estimare:

$C_1 = \frac{0.9 + 0.6 + 0.8}{3} = 0.76$

$C_2 = \frac{1.1 + 5.5 + 5 + 5 + 1.3 + 4.8 + 6}{7} = 4.1$

Iter 2:

1). $\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$

C_1 : 1.1 0.9 0.6 1.3 0.8

C_2 : 5.5 5 6 4.8 6

2). $C_1 = 0.94$

$C_2 = 5.46$

Iter. 3: 1). C_1 : 1.1 0.9 0.6 1.3 0.8 Identic $\Rightarrow$ Stop Iterații

C_2 : 5.5 5 6 4.8 6

Rezultat: Cluster 1: centroid = 0.94, puncte: 1.1, 0.9, 0.6, 1.3, 0.8

Cluster 2: centroid = 5.46, puncte: 5.5, 5, 6, 4.8, 6

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3. Se recepționează un semnal constant de amplitudine necunoscută A, afectat de zgomot gaussian, $r(t) = \underbrace{A}_{s_\Theta(t)} + zgomot$, unde zgomotul este de tip gaussian $\mathcal{N}(\mu = 0, \sigma^2 = 2)$.

Semnalul este eșantionat la momentele $t_i = [0, 1.5, 3, 4]$ și se observă valorile $r_i = [4.6, 5.2, 5.35, 4.8]$.

a. Estimați valoarea lui A folosind estimarea Maximum Likelihood

$\Lambda_\theta(t) = A$

$r(t) = \Lambda_\theta(t) + zgomot = A + zgomot$

$zgomot \quad N(\mu = 0, \sigma^2 = 2)$

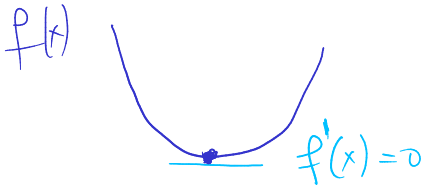
$t_i = [0 \quad 1.5 \quad 3 \quad 4]$

$r_i = [4.6 \quad 5.2 \quad 5.35 \quad 4.8]$

Estimare ML în zgomot gaussian: $\hat{\theta}_{ML} = \arg\min_\theta d(r, \Lambda_\theta)^2$

$r = [4.6 \quad 5.2 \quad 5.35 \quad 4.8]$

$\Lambda_\theta = [A \quad A \quad A \quad A]$



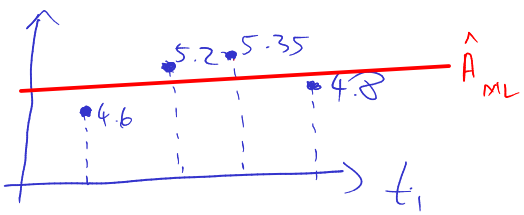
$D = d(r, \Lambda_\theta)^2 = (A-4.6)^2 + (A-5.2)^2 + (A-5.35)^2 + (A-4.8)^2$

Vrem minim!

$\frac{dD}{dA} = 0 \iff 2\cdot(A-4.6)\cdot 1 + 2(A-5.2)\cdot 1 +$

$+ 2(A-5.35)\cdot 1 + 2(A-4.8)\cdot 1 = 0$

$4A = 4.6 + 5.2 + 5.35 + 4.8 \Rightarrow$



$\hat{A}_{ML} = \frac{4.6 + 5.2 + 5.35 + 4.8}{4} = \dots$

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$\Lambda_\theta(t)$

4. Un semnal de forma $r(t) = A \cdot t^2 + 2 + zgomot$ este eșantionat la momentele $t_i = [1, 2, 3, 4, 5]$, și valorile obținute sunt $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. Distribuția zgomotului este $\mathcal{N}(0, \sigma^2 = 1)$.

   a. Estimați parametrul A folosind estimarea ML

zgomot gaussian $\Rightarrow$ $\hat{A}_{ML} = \underset{A}{\text{argmin}}\ d(r, \Lambda_\theta)^2$

$t_i = [1 \quad 2 \quad 3 \quad 4 \quad 5]$

$r = [1.2 \quad 3.7 \quad 8.5 \quad 18 \quad 25.8]$
$\quad r_1 \quad r_2 \quad r_3 \quad r_4 \quad r_5$

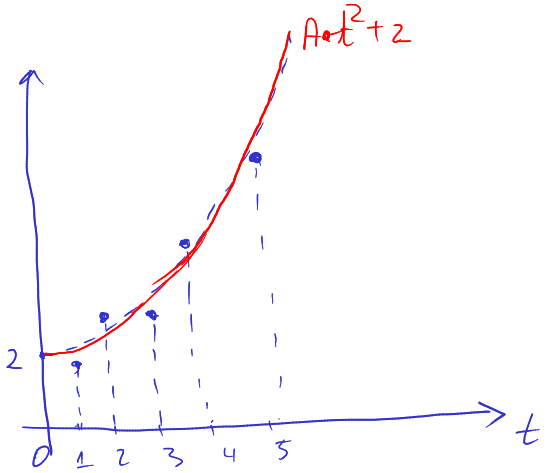
$\Lambda_\theta = [A+2 \quad 4A+2 \quad 9A+2 \quad 16A+2 \quad 25A+2]$
$\quad \Lambda_\theta(t_1) \quad \Lambda_\theta(t_2) \quad \Lambda_\theta(t_3) \quad \Lambda_\theta(t_4) \quad \Lambda_\theta(t_5)$

$d(r, \Lambda_\theta)^2 = \underbrace{(A+2-1.2)^2}_{(A+0.8)^2} + (4A-1.7)^2 + (9A-6.5)^2 + (16A-16)^2 + (25A-23.8)^2$

$\frac{d}{dA} d(r, \Lambda_\theta)^2 = 2(A+0.8)\cdot 1 + 2(4A-1.7)\cdot 4 + 2(9A-6.5)\cdot 9 + 2(16A-16)\cdot 16 +$
$+ 2(25A-23.8)\cdot 25 = 0$

$A + 0.8 + 16A - 1.7 \cdot 4 + 81A - 6.5 \cdot 9 + 256A - 256 + 625A - 23.8 \cdot 25 =$
$= 0$

$\Rightarrow \hat{A}_{ML} = \frac{-0.8 + 1.7 \cdot 4 + 6.5 \cdot 9 + 256 + 23.8 \cdot 25}{1 + 16 + 81 + 256 + 625} = \ldots$



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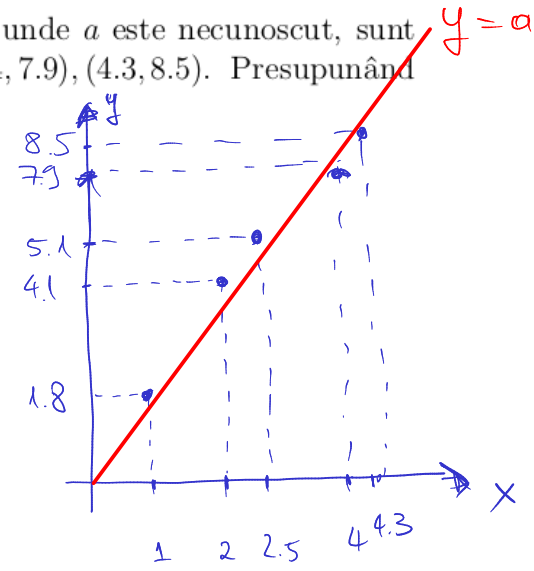
5. Valorile măsurate ale unei funcții liniare $y = ax$, unde a este necunoscut, sunt următoarele: $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$. Presupunând că zgomotul are distribuția $\mathcal{N}(0, \sigma^2 = 1)$

   a. Estimați valoarea lui a folosind estimarea ML



$y = a \cdot x$

x_i	y_i
1	1.8
2	4.1
2.5	5.1
4	7.9
4.3	8.5

t $\qquad$ r

zgomot $\mathcal{N}(\mu=0, \sigma^2=1)$

$\hat{a}_{ML} = ?$

Zg. gaussian $\Rightarrow \hat{a}_{ML} = \arg\min_a d(r, \Delta_a)^2$

$r = [1.8 \quad 4.1 \quad 5.1 \quad 7.9 \quad 8.5]$

$\Delta_a = [a \quad 2a \quad 2.5a \quad 4a \quad 4.3a]$

$$d(r, \Delta_a)^2 = (a-1.8)^2 + (2a-4.1)^2 + (2.5a-5.1)^2 + (4a-7.9)^2 + (4.3a-8.5)^2$$

$$\frac{d}{da} d(r, \Delta_a)^2 = 0 \iff 2(a-1.8) + 4(2a-4.1) + 5(2.5a-5.1) + 8(4a-7.9) + 8.6(4.3a-8.5) = 0$$

$$\Rightarrow \hat{a}_{ML} = \frac{3.6 + 16.4 + 25.5 + 8 \cdot 7.9 + 8.6 \cdot 8.5}{2 + 8 + 12.5 + 32 + 8.6 \cdot 4.3} = \dots$$