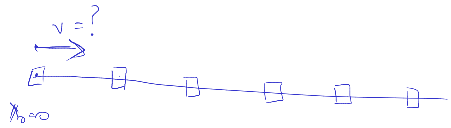
DEPI Seminar 7

(1)



zgomot $\boxed{\mathcal{N}(\mu=0,\ \sigma^2=0.1)}$

$t = [1\quad 2\quad 3\quad 4\quad 5]$

$r = [4.9\quad 9.8\quad 14.3\quad 21.2\quad 25.7]$

$\hat{v}_{ML} = ?$

a) $r(t) = s_\Theta(t) + \text{zgomot}$
$= v\cdot t + \text{zgomot}$

La noi: $s_\Theta(t) = v\cdot t$ = poziția reală
$r(t) = s_\Theta(t) + \text{zgomot}$ = poz. măsurată

$s_\Theta = [v\quad 2v\quad 3v\quad 4v\quad 5v]$

$r = [4.9\quad 9.8\quad 14.3\quad 21.2\quad 25.7]$

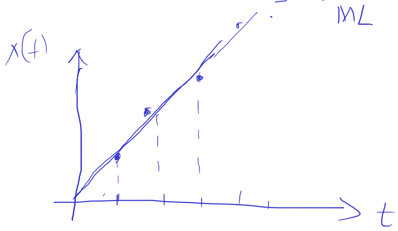
$$\boxed{\hat{v}_{ML} = \underset{v}{\operatorname{argmin}}\ d(r, s_\Theta)^2}$$

$$d(r,s_\Theta)^2 = (v-4.9)^2 + (2v-9.8)^2 + (3v-14.3)^2 + (4v-21.2)^2 + (5v-25.7)^2$$

$$\frac{\partial d(r,s_\Theta)^2}{\partial v} = \cancel{2}(v-4.9)\cdot 1 + \cancel{2}(2v-9.8)\cdot 2 + \cancel{2}(3v-14.3)\cdot 3 + \cancel{2}(4v-21.2)\cdot 4 + \cancel{2}(5v-25.7)\cdot 5 = 0$$

$$\Rightarrow v + 4v + 9v + 16v + 25v = 4.9 + 9.8\cdot 2 + 14.3\cdot 3 + 21.2\cdot 4 + 25.7\cdot 5$$

$$\Rightarrow \hat{v}_{ML} = \frac{4.9 + 9.8\cdot 2 + 14.3\cdot 3 + 21.2\cdot 4 + 25.7\cdot 5}{55} = \frac{280.7}{55} = 5.1$$



b) La mom. 6: $x = v\cdot 6 = 5.1\cdot 6 = 30.6$

c). $x_0 \xrightarrow{v}$ $\quad x(t) = \overset{?}{x_0} + \underbrace{\overset{?}{v}\cdot t}_{\text{deplasarea}} = \Delta x$

$s_\Theta = [x_0+v\quad x_0+2v\quad x_0+3v\quad x_0+4v\quad x_0+5v]$

$r = [4.9\quad 9.8\quad 14.3\quad 21.2\quad 25.7]$

$$\hat{v}_{ML}, \hat{x_0}_{ML} = \underset{v, x_0}{\operatorname{argmin}}\ d(r, s_\Theta)^2$$

$$d(r,s_\Theta)^2 = (x_0+v-4.9)^2 + (x_0+2v-9.8)^2 + (x_0+3v-14.3)^2 + (x_0+4v-21.2)^2 + (x_0+5v-25.7)^2$$

$$\begin{cases} \dfrac{\partial}{\partial v} = 0 \\[2ex] \dfrac{\partial}{\partial x_0} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \dfrac{\partial d(r,s_0)^2}{\partial v} = \cancel{2}(x_0+v-4.9) + \cancel{2}(x_0+2v-9.8)\cdot 2 + \cancel{2}(x_0+3v-14.3)\cdot 3 + \cancel{2}(x_0+4v-21.2)\cdot 4 + \\ \qquad + \cancel{2}(x_0+5v-25.7)\cdot 5 = 0 \\[2ex] \dfrac{\partial d(r,s_0)^2}{\partial x_0} = \cancel{2}(x_0+v-4.9) + \cancel{2}(x_0+2v-9.8) + \cancel{2}(x_0+3v-14.3) + \cancel{2}(x_0+4v-21.2) + \\ \qquad + 2(x_0+5v-25.7) = 0 \end{cases}$$

$$\begin{cases} 5\cdot x_0 + 55\cdot v = 280.7 \\ 5\cdot x_0 + 15\cdot v = 75.9 \end{cases} \Rightarrow 40v = 204.8 \Rightarrow \boxed{\begin{array}{l} \hat{v}_{ML} = 5.12 \\ \hat{x}_{0_{ML}} = -0.18 \end{array}}$$

$$\Rightarrow x_0 = \frac{75.9 - 15\cdot 5.12}{5} = -0.18$$

La momentul 6: $x_0 + v\cdot 6 = -0.18 + 5.12\cdot 6 = \dots$

g

d). $x(t) = \dfrac{a\,t^2}{2} + v_0 t + x_0$? ? ?

$$s_0 = \left[\frac{a}{2} + v_0 + x_0 \quad 2a + 2v_0 + x_0 \quad a\cdot\frac{9}{2} + 3v_0 + x_0 \quad \dots \right]$$

$$r = [\ 4.9 \quad 9.8 \quad 14.3 \quad \dots\ ]$$

$$d(r, s_0)^2 = \left(\frac{a}{2} + v_0 + x_0 - 4.9\right)^2 + \left(2a + 2v_0 + x_0 - 9.8\right)^2 + \dots$$

$$\begin{cases} \dfrac{\partial d}{\partial a} = 0 \\ \dfrac{\partial d}{\partial v_0} = 0 \\ \dfrac{\partial d}{\partial x_0} = 0 \end{cases} \Rightarrow \begin{cases} 2\left(\frac{a}{2}+v_0+x_0-4.9\right)\cdot\frac{1}{2} + 2\left(2a+2v_0+x_0-9.8\right)\cdot 2 + \dots = 0 \\ 2\left(\frac{a}{2}+v_0+x_0-4.9\right)\cdot 1 + 2\left(2a+2v_0+x_0-9.8\right)\cdot 2 + \dots = 0 \\ 2\left(\frac{a}{2}+v_0+x_0-4.9\right) + 2\left(2a+2v_0+x_0-9.8\right) + \dots = 0 \end{cases}$$

2

$y = a\cdot x$

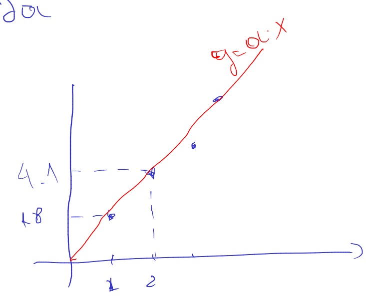
x_i	y_i
1	1.8
2	4.1
2.5	5.1
4	7.9
4.3	8.5

a) $s_0 = [\ a \quad 2a \quad 2.5a \quad 4a \quad 4.3a\ ]$

$r = [\ 1.8 \quad 4.1 \quad 5.1 \quad 7.9 \quad 8.5\ ]$

$$d(r, s_0)^2 = (a-1.8)^2 + (2a-4.1)^2 + (2.5a-5.1)^2 + \dots$$

$$\frac{\partial d}{\partial a} = 2(a-1.8) + 2(2a-4.1)\cdot 2 + 2(2.5a-5.1)\cdot 2.5 + \dots = 0$$



$$\Rightarrow \hat{a}_{ML} = \dots$$

3

$$r(t) = \underbrace{A\cdot t^2 + 2}_{s_\theta(t)} + \text{zgomot}$$

t: 1, 2, 3

$r = [1.2 \quad 3.7 \quad 8.5]$

$$\boxed{N(\mu=0,\ \sigma^2=1)}$$

a) $s_\theta = [A+2 \quad 4A+2 \quad 9A+2]$

$r = [1.2 \quad 3.7 \quad 8.5]$

$A =$

$$d(r,s_0)^2 = (A+2-1.2)^2 + (4A+2-3.7)^2 + (9A+2-8.5)^2$$

$$\frac{\partial d}{\partial A} = \cancel{2}(A+0.8) + \cancel{2}(4A-1.7)\cdot 4 + \cancel{2}(9A-6.5)\cdot 9 = 0$$

$$\hat{A}_{ML} = \frac{0.8 + 1.7\cdot 4 + 6.5\cdot 9}{98} = 0.67$$

b). $w(A) = N(\mu = 1,\ \sigma_2^2 = 2)$

$$\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(A-1)^2}{2\sigma^2}}$$

MAP: $\hat{A}_{MAP} = \underset{A}{\arg\max}\ w(r|A)\cdot w(A)$

$w(A)$: $\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(A-1)^2}{2\sigma_2^2}}$

$$w(r|A) = w(r_1|A)\cdot w(r_2|A)\cdot w(r_3|A) \cdot w(A)$$

$$\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(\overset{1.2}{\cancel{r_1}}-A-2)^2}{2\sigma^2}} \cdot \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(3.7-4A-2)^2}{2\sigma^2}} \cdot \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(8.5-9A-2)^2}{2\sigma^2}}$$

$r_1 = A+2 + \text{zgomot}$

$$w(r|A)\cdot w(A) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^3 \cdot \frac{1}{\sigma_2\sqrt{2\pi}} \cdot e^{-\left(\underbrace{\frac{(1.2-A-2)^2+(3.7-4A-2)^2+(8.5-9A-2)^2}{2\underbrace{\sigma^2}_{=1}} + \frac{(A-1)^2}{2\underbrace{\sigma_2^2}_{=2}}}_{\text{minim}}\right)}$$

Vrem A a.î. să fie maxim $\uparrow$

$(\Rightarrow)$

Vrem A a.î. să fie minim

$$\hat{A}_{MAP} = \underset{A}{\arg\min}\left(\frac{(-0.8-A)^2+(1.7-4A)^2+(6.5-9A)^2}{2} + \frac{(A-1)^2}{4}\right)$$

$$= \underset{A}{\arg\min}\left((A+0.8)^2 + (4A-1.7)^2 + (9A-6.5)^2 + \frac{1}{2}\cdot(A-1)^2\right)$$

$\Rightarrow$ Exact ca la est. M.L, doar că avem 4 termeni, nu 3

$$\frac{\partial}{\partial A} = 0 \Rightarrow \hat{A}_{MAP} = \ldots$$