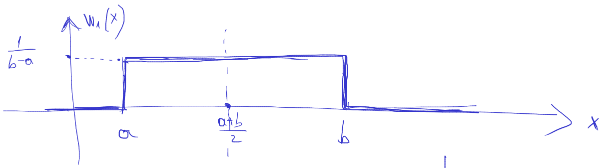
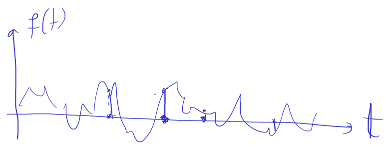
18.10.2021

DEPI Seminar 2

① a) $w_1(x) = U[a, b]$ $\quad a, b \in \mathbb{R}$ $\quad a = -3, \; b = 7$





$$\cdot\; \mu = \overline{X} = \overline{f(t)} = \int_{-\infty}^{\infty} x \cdot w_1(x)\, dx = \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a} \cdot \frac{x^2}{2}\Bigg|_a^b = \frac{1}{2(b-a)} \cdot (b^2 - a^2) = \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \frac{a+b}{2}$$

$a = -3$, $b = 7$ $\Rightarrow 2$

$$\cdot\; \overline{X^2} = \overline{f(t)^2} = \int_{-\infty}^{\infty} x^2 \cdot w_1(x)\, dx = \int_a^b x^2 \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a} \cdot \frac{x^3}{3}\Bigg|_a^b = \frac{b^3 - a^3}{3\cdot(b-a)} = \frac{\cancel{(b-a)}(b^2 + ab + a^2)}{3\cdot\cancel{(b-a)}} =$$

$$= \frac{a^2 + ab + b^2}{3}$$

$a = -3$, $b = 7$ $\Rightarrow \overline{f(t)^2} = \frac{9 - 21 + 49}{3} = \frac{37}{3}$ ≥ 0

$$\cdot\; \sigma_X^2 = \sigma^2_{f(t)} = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot w_1(x) \cdot dx = \int_a^b (x-\mu)^2 \cdot \frac{1}{b-a}\, dx = \frac{1}{(b-a)} \cdot \frac{(x-\mu)^3}{3} = \ldots$$

$$= \frac{1}{(b-a)} \int_a^b (x^2 - 2x\mu + \mu^2)\, dx = \frac{1}{b-a}\left[\frac{x^3}{3}\Bigg|_a^b - \cancel{2}\mu \cdot \frac{x^2}{\cancel{2}}\Bigg|_a^b + \mu^2 \cdot x\Bigg|_a^b\right]$$

$$= \frac{1}{\cancel{b-a}}\left(\frac{b^3 - a^3}{3} - \mu\,(b^2 - a^2) + \mu^2\,(\cancel{b-a})\right) =$$

$(\cancel{b-a})(b^2+ab+a^2)$ $\quad (\cancel{b-a})(b+a)$

$$= \frac{a^2 + ab + b^2}{3} - \mu\cdot(a+b) + \mu^2 = \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{2} + \frac{(a+b)^2}{4} = \ldots$$

$\mu = \frac{a+b}{2}$

$$= \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4}$$

$\overline{X^2} \quad - \quad \mu^2$

SAU:

$$\boxed{\sigma^2 = \overline{X^2} - \mu^2}$$

$$= \frac{37}{3} - 2^2 = \frac{37}{3} - 4 = \frac{25}{3} \geq 0$$

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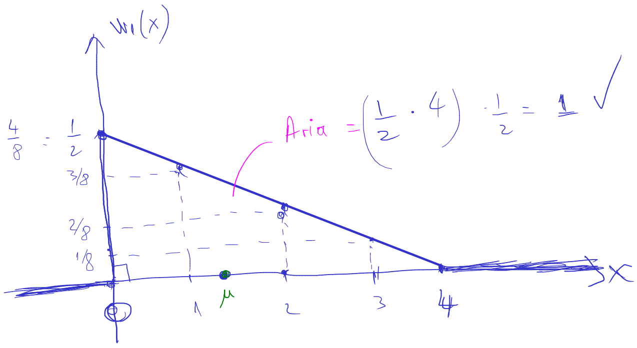
b) $w_1(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}x, & x \in [0, 4] \\ 0, & \text{rest} \end{cases}$



$$\cdot\; \overline{X} = \overline{f(t)} = \int_{-\infty}^{\infty} x \cdot w_1(x)\, dx = \int_0^4 x\cdot\left(\frac{1}{2} - \frac{1}{8}x\right) dx = \frac{1}{2}\int_0^4 x\, dx - \frac{1}{8}\int_0^4 x^2\, dx = \frac{1}{2}\cdot\frac{x^2}{2}\Bigg|_0^4 - \frac{1}{8}\cdot\frac{x^3}{3}\Bigg|_0^4 =$$

$$= \frac{1}{2}\cdot\frac{4^2}{2} - \frac{1}{8}\cdot\frac{4^3}{3} = 4 - \frac{8}{3} = 1.333 \quad \mu = \frac{4}{3}$$

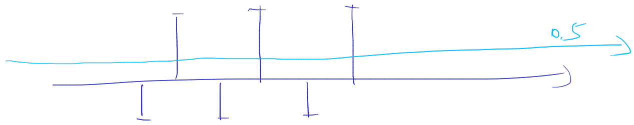
$$\cdot\ \overline{X^2} = \overline{f(t)^2} = \int_{-\infty}^{\infty} x^2 \cdot w_1(x)\, dx = \int_0^4 x^2\left(\frac{1}{2} - \frac{1}{8}x\right) dx = \frac{1}{2}\int_0^4 x^2 dx - \frac{1}{8}\cdot\int_0^4 x^3\, dx = \frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^4 - \frac{1}{8}\cdot\frac{x^4}{4}\Big|_0^4 =$$

$$= \frac{1}{2}\cdot\frac{4^3}{3} - \frac{1}{8}\cdot\frac{4^{\cancel{4}\,3}}{\cancel{4}} = \frac{64}{6} - \frac{64}{8}$$

$$= \frac{4\cdot 64 - 3\cdot 64}{24} = \frac{64}{24}$$

$$\cdot\ \sigma^2 = \sigma^2_{f(t)} = \overline{X^2} - \mu^2$$

$$= \frac{64}{24} - \left(\frac{4}{3}\right)^2 = \underbrace{\frac{64}{24}}_{2.66} - \underbrace{\frac{16}{9}}_{1.77} = \dots$$



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(2) $f^{(k)}[n] = \{-1,\ 2,\ -1,\ 2,\ -1,\ 2,\ -1,\ 2,\ -1,\ 2\}$

t $t+1$

$t-1$ t

Medii temporale:

$$\overline{f} = \mu_t = \overline{f^{(k)}} = \frac{-1+2-1+2-1+2-1+2-1+2}{10} = \frac{5}{10} = 0.5$$

$$\overline{f^2} = \overline{f^{(k)2}} = \frac{(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2}{10} = \frac{25}{10} = 2.5$$

$$\sigma_t^2 = \frac{(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+(2-0.5)^2}{10}$$

$$= \frac{2.25+2.25+\dots+2.25}{10} = 2.25$$

Se respectă: $2.25 = 2.5 - (0.5)^2$

$\sigma^2 = \overline{X^2} - \mu^2$

· Autocorelația

$$R_{ff}(\tau) = \overline{f(t)\cdot f(t+\tau)}$$

$$R_{ff}[0] = \overline{f(t)\cdot f(t)} = \overline{f(t)^2} = \overline{X^2} = \frac{(-1)^2+2^2+(-1)^2+2^2+\dots}{10} = 2.5$$

$$R_{ff}[1] = \overline{f(t)\cdot f(t+1)} = \frac{(-1)\cdot 2+2\cdot(-1)+(-1)\cdot 2+2\cdot(-1)+(-1)\cdot 2+2\cdot(-1)+(-1)\cdot 2+2\cdot(-1)+(-1)\cdot 2}{9} = -2$$

Doar o întâmplare că sunt egale

$$R_{ff}[2] = \overline{f(t)\cdot f(t+2)} = \frac{(-1)\cdot(-1)+2\cdot 2+(-1)\cdot(-1)+2\cdot 2+(-1)(-1)+2\cdot 2+(-1)(-1)+2\cdot 2}{8} = \frac{20}{8} = 2.5$$

$\vdots$

$$R_{ff}[-1] = \overline{f(t)\cdot f(t-1)} = \frac{\overbrace{2\cdot(-1)}^{-2}+\overbrace{(-1)\cdot 2}^{-2}+\overbrace{2\cdot(-1)}^{-2}+\dots}{9} = -2$$

Mereu egale

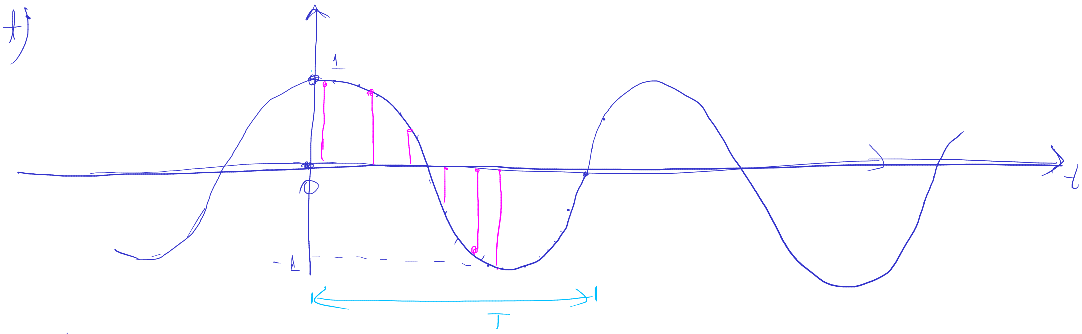
$$R_{ff}[-2] = R_{ff}[2] = 2.5$$

$\vdots$

$$\boxed{R_{ff}(-\tau) = R_{ff}(\tau)}$$

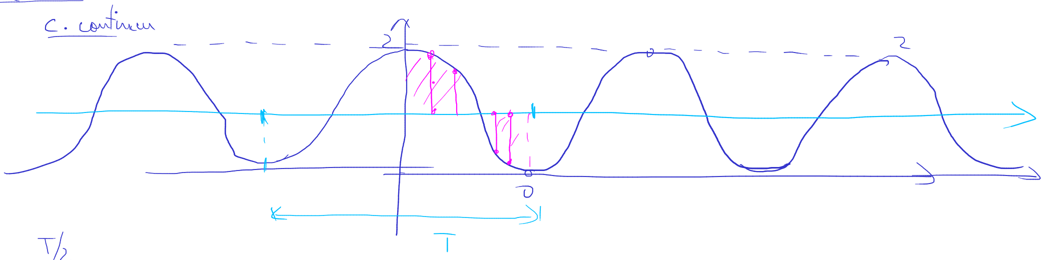
F. de autocorelație este funcție pară

(3) $s(t) = \cos(2\pi f_0 t)$



Media (temporal) = componente de D.C.

c. continua



$$= \lim_{T \to \infty} \frac{\int_{-T/2}^{T/2} s(t)\, dt}{T} = \frac{T}{T} = 1$$

FINISH