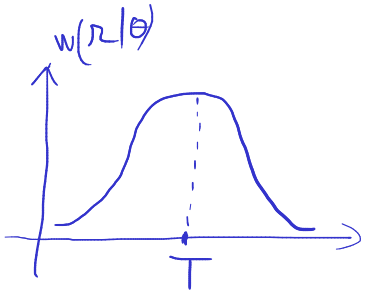
Exemplu Curs 13, estimare M.A.P. (ultimul slide)

$\Lambda_\theta(t) = T$, T = necunoscut

$r(t) = T +$ ~~noise~~ zgomot

① $r = 40 = T +$ zgomot, zg. $\mathcal{N}(\mu = 0, \sigma^2 = 2)$

② $\mathcal{N}(\mu = 35, \sigma^2 = 2) = w(\theta)$ $\quad \theta = T$



Estimare ML: $\hat{\theta}_{ML} = \underset{\theta}{\arg\max}\ w(r|\theta)$

$$w(r|\theta) = \mathcal{N}(\mu = T, \sigma^2 = 2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(40-T)^2}{2\sigma^2}}$$

$$\hat{T}_{ML} = \underset{T}{\arg\max}\ \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(40-T)^2}{2\sigma^2}} \quad \text{minimum}$$

$$= \underset{T}{\arg\min}\ (40-T)^2$$

$$= 40 \quad \Rightarrow \hat{T}_{ML} = 40$$

$c \cdot e^{-\frac{x}{2\sigma^2}} = \frac{1}{e^x}$

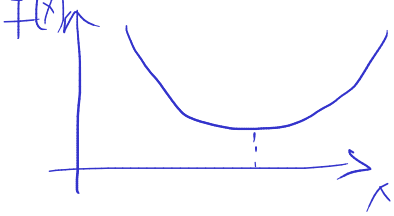
Estimare MAP: $\hat{\theta}_{MAP} = \underset{\theta}{\arg\max}\ w(r|\theta) \cdot w(\theta)$

$$w(r|\theta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(40-T)^2}{2\sigma^2}}$$

$$w(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(T-35)^2}{2\sigma^2}}$$

$$\hat{T}_{MAP} = \underset{T}{\arg\max}\ \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^2 \cdot e^{-\frac{(40-T)^2 + (T-35)^2}{2\sigma^2}} \quad \text{minim}$$

constant



$$= \underset{T}{\arg\min}\ \underbrace{(40-T)^2 + (T-35)^2}_{f(T)}$$

$f'(x) = 0$

$$\frac{df}{dT} = 2(40-T)(-1) + 2(T-35) = 0 \iff$$

$$\iff 2T - 80 + 2T - 70 = 0 \Rightarrow T = \frac{150}{4}$$

$$\hat{T}_{MAP} = 37.5$$