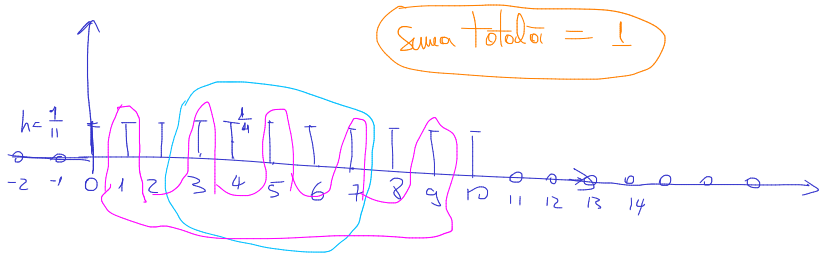
Seminar 1 DEPI

11.10.2021

(3) $A : \{0, 1, 2, \dots 10\}$

Suma totală = 1



b) $P(A \in [3,7]) = ? = \frac{5}{11}$

$\sum_{x=3}^{7} p(x)$

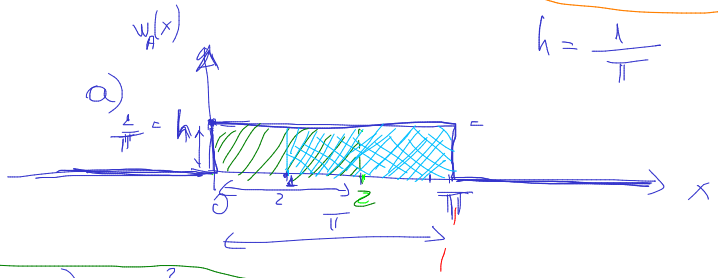
c). $P(A \text{ impar}) = ? = \frac{5}{11}$

(1) $U[0, \pi]$

Aria totală = 1

$h = \frac{1}{\pi}$

a)



$$w(x) = \begin{cases} \frac{1}{\pi}, & \text{pt } x \in [0, \pi] \\ 0, & \text{rest} \end{cases}$$

c). $P(A \in (0,2)) = \int_0^2 w(x)\,dx = \frac{2}{\pi} = 0.6366 = 63.66\%$

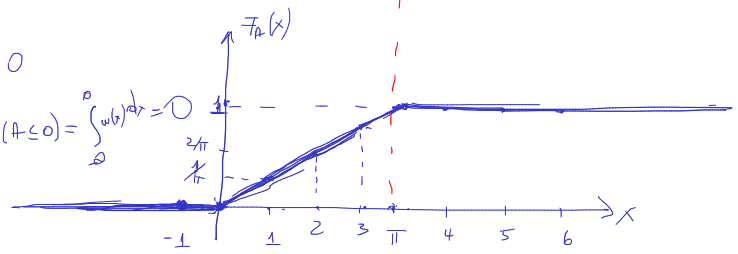
b). $P(A > 1) = \int_1^{\infty} = \int_1^{\pi} w(x)\,dx = \frac{\pi - 1}{\pi} = \dots$

$$\int_1^{\pi} w(x)\,dx = \int_1^{\pi} \frac{1}{\pi}\,dx = \frac{1}{\pi} \cdot x \Big|_1^{\pi} = \frac{\pi - 1}{\pi}$$

d). $F_A(x) = ? = P(A \le x)$

$F_A(-1) = 0$

$F_A(0) = P(A \le 0) = \int_0^0 w(x)\,dx = 0$



$$F_A(x) = \begin{cases} 0, & \text{pt } x \le 0 \\ \frac{1}{\pi} \cdot x, & \text{pt } x \in (0, \pi) \\ 1, & \text{pt } x \ge \pi \end{cases}$$

$F_A(1) = P(A \le 1) = \frac{1}{\pi}$

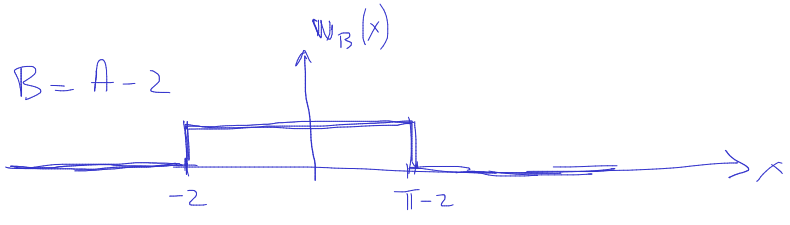
$F_A(2) = P(A \le 2) = \frac{2}{\pi}$

$F_A(3) = P(A \le 3) = \frac{3}{\pi}$

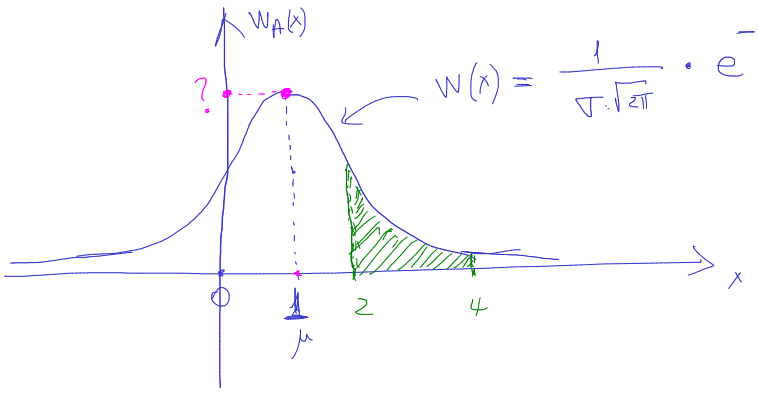
$F_A(\pi) = P(A \le \pi) = 1$

$F_A(4) = P(A \le 4) = 1$

e). $B = A - 2$



(2) $\mathcal{N}\left(\mu = 1,\ \sigma^2 = 20\right)$



a) $P(A \in [2,4]) = ?$

$$= \int_2^4 w(x)\,dx = \int_2^4$$

$$F(x) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$$

$$= \underbrace{F(4)}_{P(A\le 4)} - \underbrace{F(2)}_{P(A\le 2)} = 0.74 - 0.58 = 0.16$$

$$F(4) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{4-1}{\sqrt{20}\cdot\sqrt{2}}\right)\right) = 0.74$$

$$F(2) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{2-1}{\sqrt{40}}\right)\right) = 0.58$$

b) $P(A \ge 1) = ?$ $\quad A \in [1, \infty)$

$$= \int_1^\infty w(x)\,dx = \underbrace{F(\infty)}_{P(A\le\infty)=1} - \underbrace{F(1)}_{} = 1 - \ldots = \ldots$$

$F(1) = \ldots$

$P(A \le 1) = ?$ $\quad A \in [-\infty, 1]$

$$\int_{-\infty}^1 = \underbrace{F(1)}_{} - \underbrace{F(-\infty)}_{P(A\le -\infty)=0}$$

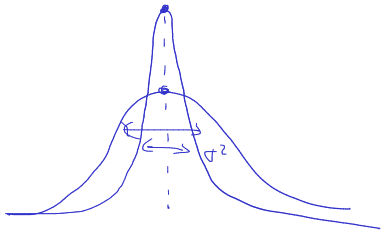
Discutie

$P(A < 1) = P(A \le 1)$ (v. a. continuă)

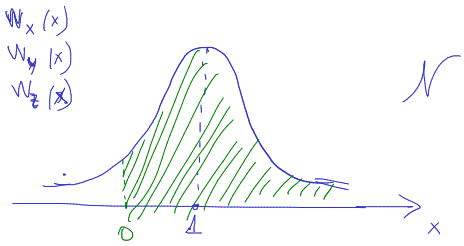
$$-\frac{(1-1)^2}{2\cdot 20}$$

c) $w(x)$ își atinge maximul la $x = 1\,(\mu)$, și val. max. $= \frac{1}{\sqrt{20\cdot 2\pi}} \cdot e = \frac{1}{\sqrt{40\pi}}$

$= \frac{1}{\sigma\cdot\sqrt{2\pi}}$



(4)



$\mathcal{N}(\mu = 1,\ \sigma^2 = 1)$

(3 zaruri:
$P(5,5,5) = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$
$P(\text{zar1}=5 \text{ și } \text{zar2}=5 \text{ și } \text{zar3}=5) = P(\text{zar1}=5)\cdot P(\text{zar2}=5)\cdot P(\text{zar3}=5)$)

$P(X>0 \text{ și } Y>0 \text{ și } Z>0) = \underbrace{P(X>0)}_{0.85}\cdot\underbrace{P(Y>0)}_{0.85}\cdot\underbrace{P(Z>0)}_{0.85} = (0.85)^3$

$$P(X>0) = \int_0^\infty w(x)\,dx = \underbrace{F(\infty)}_{1} - F(0) = 1 - 0.15 = 0.85$$

$$F(0) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{0-1}{\sqrt{2}}\right)\right) = 0.15$$

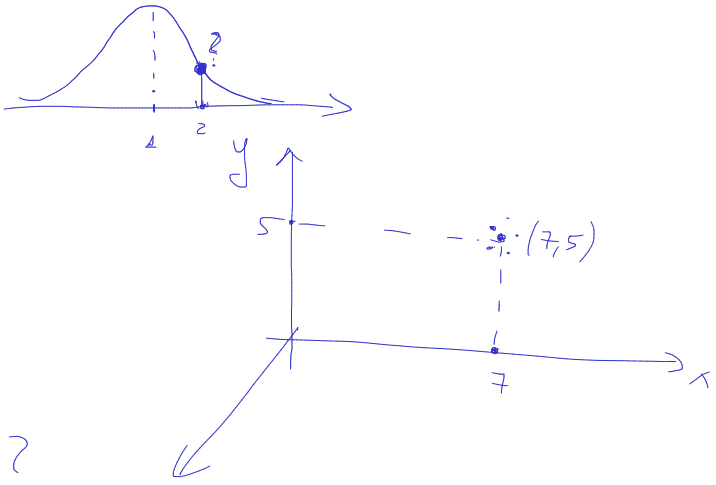
(5)

A: $\mathcal{N}(\mu = 1, \sigma^2 = 3)$

B: $\mathcal{N}(\mu = -4, \sigma^2 = 3)$

C: $\mathcal{N}(\mu = 5, \sigma^2 = 3)$

$\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$



$P((A,B,C) \text{ în jurul } (2,-6,3)) = ?$

$= P(A \text{ în jurul } 2 \text{ și } B \text{ în jurul } -6 \text{ și } C \text{ în jurul } 3)$

$= \underbrace{P(A \text{ în jurul } 2)}_{w_A(2)}\cdot\underbrace{P(B \text{ în jurul } -6)}_{w_B(-6)}\cdot\underbrace{P(C \text{ în jurul } 3)}_{w_C(3)}$

$w_A(2)$: $\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{1}{6}}$

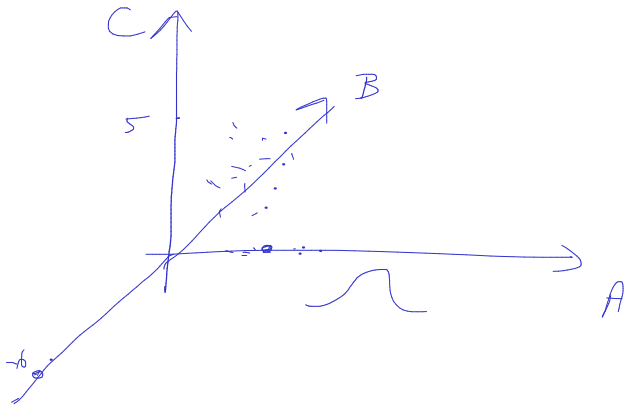
$w_B(-6)$: $\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{4}{6}}$

$w_C(3)$: $\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{4}{6}}$

$= \left(\frac{1}{\sqrt{6\pi}}\right)^3\cdot\underbrace{e^{-9/6}}_{\frac{1}{e^{9/6}}}$

$P((A,B,C) \text{ în jurul } (-2,-3,2)) = ?$

$\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{9}{6}}\cdot\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{1}{6}}\cdot\frac{1}{\sqrt{6\pi}}\cdot e^{-\frac{9}{6}} = \left(\frac{1}{\sqrt{6\pi}}\right)^3\cdot\underbrace{e^{-19/6}}_{\frac{1}{e^{19/6}}}$


C
B
5
A
-6