Chapter V. Frequency Analysis of Discrete Systems

Response of LTI systems to harmonic signals

- We consider an LTI system with $h[n]$
- Input signal = complex harmonic (exponential) signal

$$x[n] = Ae^{j\omega_0 n}$$

- Output signal = convolution

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[n]x[k-n] \\ &= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega_0 k} Ae^{j\omega_0 n} \\ &= H(\omega_0) \cdot x[n] \end{aligned}$$

- $H(\omega_0)$ = Fourier transform of $h[[n]$ evaluated for $\omega = \omega_0$

Eigen-function

- Complex exponential signals are **eigen-functions** (functii proprii) of LTI systems:
  - output signal $=$ input signal $\times$ a (complex) constant
- $H(\omega_0)$ is a constant that multiplies the input signal
  - Amplitude of input gets multiplies by $|H(\omega_0)|$
  - Phase of input signal is added with $\angle H(\omega_0)$
- Why are sin/cos/exp functions important?
  - If input signal $=$ sum of complex exponential ($=$ coses $+$ sinuses),
  - since the system is linear,
  - then output $=$ same sum of complex exponentials, each scaled with some coefficients

Response to cosine and sine

- Cosine / sine = sum of two exponentials, via Euler
- System is linear and real =>
  - amplitude is multiplied by $|H(\omega_0)|$
  - phase increases by $\angle H(\omega_0)$
- See proof at blackboard

Frequency response

- Names
  - $H(\omega)$ = frequency response of the system
  - $|H(\omega)|$ = amplitude response
  - $\angle H(\omega)$ = phase response
- Phase response might have jumps of 2π
- Stitching the pieces in a continuous function = phase *unwrapping*
  - Example: at blackboard
- Wrapped phase: $\in [-\pi, \pi]$, may have jumps of 2π
- Unwrapped phase: continuous function, may go outside interval

Permanent and transient response

- The above harmonic signals start at $n = -\infty$, not at 0.
- What if the signal starts at some time $n = 0$?
- Total response $=$ transient response $+$ permanent response
  - transient response goes towards 0 as $n \to \infty$
  - permanent response $=$ the above
- So the above relations are valid only in **permanent regime**
  - i.e. after the transient regime has passed
  - i.e. after the transient response has practically vanished
  - i.e. when the signal started very long ago (from $n = -\infty$)
- Example at blackboard