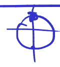
Exercises Week 11

(1)

$$x[n] = \begin{array}{cccc} x[0] & x[1] & & x[3] \\ \left[\left[1\right.\right. & 1 & 0 & \left.0\right] \end{array} \; \vdots \; 1 \quad 1 \quad 0 \quad 0 \; \vdots \; 1 \quad 1 \quad 0 \quad 0 \; \vdots \; \ldots$$

$N = 4$

$$\boxed{e^{jx} = \cos x + j \sin x}$$



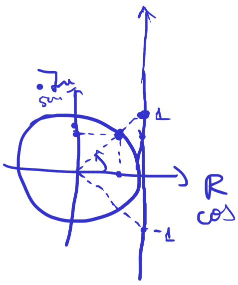
DFT: $\boxed{X_k = \sum_n x[n] \cdot e^{-j 2\pi \frac{k}{N} n}}$ $\quad N = 4$

$$X_0 = \sum_{n=0}^{3} x[n] \cdot \underbrace{e^{-j 2\pi \frac{0}{N} n}}_{1} = \sum_{n=0}^{3} x[n] \cdot 1 = 2$$

$$X_1 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{1}{4} n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{0}}_{1} + \underbrace{x[1]}_{1} \cdot e^{-j 2\pi \cdot \frac{1}{4} \cdot 1} + \underbrace{x[2]}_{0} \cdot e^{-j 2\pi \frac{1}{4} 2} + \underbrace{x[3]}_{0} \cdot e^{-j 2\pi \frac{1}{4} 3}$$

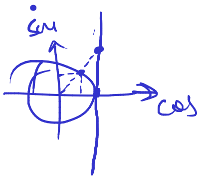
$$= 1 + 1 \cdot e^{-j 2\pi \frac{1}{4}} = 1 + e^{-j \pi/2} = 1 + \underbrace{\cos\left(-\pi/2\right)}_{0} + j \underbrace{\sin\left(-\pi/2\right)}_{-1} =$$

$$= 1 - j$$



$$X_2 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{2}{4} \cdot n} = \underbrace{x[0] \cdot e^{0}}_{1} + \underbrace{x[1]}_{1} \cdot e^{-j 2\pi \frac{2}{4} \cdot 1} + \underbrace{x[2] \cdot e^{-j 2\pi \frac{2}{4} \cdot 2}}_{0} + \underbrace{x[3] \cdot e^{-j 2\pi \frac{2}{4} 3}}_{0}$$

$$= 1 + e^{-j\pi} = 1 + \underbrace{\cos(-\pi)}_{-1} + j \underbrace{\sin(-\pi)}_{0} = 0$$



$$X_3 = \sum_{n=0}^{3} x[n] \, e^{-j 2\pi \frac{3}{4} \cdot n} = \underbrace{x[0] \cdot e^{0}}_{1} + \underbrace{x[1]}_{1} \cdot e^{-j 2\pi \frac{3}{4} 1} + \underbrace{x[2] \cdot e^{-j 2\pi \frac{3}{4} 2}}_{0} + \underbrace{x[3] \cdot e^{-j 2\pi \frac{3}{4} \cdot 3}}_{0}$$

$$= 1 + e^{-j \frac{3\pi}{2}} = 1 + \underbrace{\cos\left(-\frac{3\pi}{2}\right)}_{0} + j \underbrace{\sin\left(-\frac{3\pi}{2}\right)}_{1} = 1 + j$$

$$X_k = \begin{array}{cccc} X_0 & X_1 & X_2 & X_3 \\ \left[\, 2, \right. & 1 - j, & 0, & \left. 1 + j \,\right] \end{array}$$

$$|1 - j| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\angle 1 - j = \operatorname{atan} \frac{-1}{1} = -\pi/4$$

$$x[n] = \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 2 \cdot \sqrt{2} \cdot \cos\left(2\pi \cdot \frac{1}{4} \cdot n + \left(\frac{-\pi}{4}\right)\right) + \frac{1}{4} \cdot 0 \cdot \cos\left(2\pi \frac{2}{4} n + 0\right)$$

DC. $\frac{1}{N} \cdot X(0)$ $\qquad 0 = |X_2| \qquad 0 = \angle X(2)$

② DTFT

$$x[n] = \dots 0\ 0\ [1\ \ 1\ \ 0\ \ 0]\ \ 0\ \ 0\ \ 0\ \ 0 \dots$$

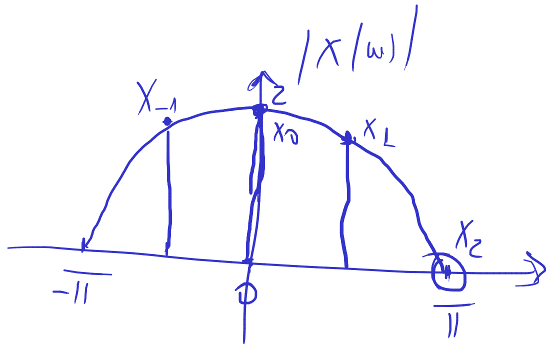
$$X(\omega) = \sum_{n=0}^{3} x[n]\, e^{-j\omega n} \quad \text{- continuous function}$$

$$= \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j\cdot\omega\cdot 0}}_{1} + \underbrace{x[1]}_{1} \cdot e^{-j\cdot\omega\cdot 1}$$

$$= 1 + e^{-j\omega} = 1 + \cos(-\omega) + j\sin(-\omega)$$

$$= \underbrace{1 + \cos(\omega)}_{R} - j\underbrace{\sin(\omega)}_{I}$$

$$|X(\omega)| = \sqrt{(1+\cos(\omega))^2 + (-\sin(\omega))^2}$$



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③ $p_1 = -0.5$

$z_1 = 0.9$

$\rightarrow \left|X(\omega = \pi)\right| = 1$

a) $X(z) = ?$

$$X(z) = C \cdot \frac{(z - 0.9)}{(z + 0.5)}$$

$$\boxed{X(z) = C \cdot \frac{(z - z_1)}{(z - p_1)}} \qquad \boxed{z = e^{j\omega}}$$

$$e^{j\pi} = -1$$

$$X(\omega) = C \cdot \frac{e^{j\omega} - 0.9}{e^{j\omega} + 0.5}$$

$$X(\pi) = C \cdot \frac{e^{j\pi} - 0.9}{e^{j\pi} + 0.5} = C \cdot \frac{-1.9}{-0.5} = \frac{19}{5} \cdot C$$

$$|X(\pi)| = 1 \Rightarrow |C| = \frac{5}{19} \Rightarrow C = \pm \frac{5}{19}$$

$$X(z) = \pm \frac{5}{19} \cdot \frac{z-0.9}{z+0.5} \quad \cancel{\text{of}} \text{ (pick +)}$$

b). $$X(\omega) = \pm \frac{5}{19} \cdot \frac{e^{j\omega} - 0.9}{e^{j\omega} + 0.5}$$

$$|X(\omega)| = \left| \pm \frac{5}{19} \right| \cdot \frac{|e^{j\omega} - 0.9|}{|e^{j\omega} + 0.5|}$$

$$= \frac{5}{19} \cdot \frac{\sqrt{(\cos\omega - 0.9)^2 + (\sin(\omega))^2}}{\sqrt{(\cos\omega + 0.5)^2 + (\sin(\omega))^2}}$$

$$\angle X(\omega) = \angle \pm \frac{5}{19} + \angle e^{j\omega} - 0.9 - \angle e^{j\omega} + 0.5$$

$$= 0 \text{ or } \pi + \operatorname{atan} \frac{\sin(\omega)}{\cos\omega - 0.9} - \operatorname{atan} \frac{\sin(\omega)}{\cos\omega + 0.5}$$