Digital Signal Processing

Chapter VI. Implementation of Digital Systems

VI.1. Direct-Form structures

Structures for implementation

- We will see different methods of implementing systems
  - mostly LTI systems
- Differences
  - computational complexity (number of operations)
  - memory requirements
  - finite-precision effects
  - flexibility
- Block diagrams (structures)
  - can be implemented either in HW or SW

Direct-Form I

- A LTI system is described by the difference equation

$$
\begin{aligned}
y[n] &= -\sum_{k=1}^{N} a_k y[n-k] + -\sum_{k=1}^{M} b_k x[n-k] \\
&= -a_1 y[n-1] - a_2 y[n-2] - ... - a_N y[n-N] + b_0 x[n] + b_1 x[n -
\end{aligned}
$$

- **Direct-Form I** structure = directly implementing this equation
- Main disadvantage: too many delay blocks (approx. 2x filter order)

Direct-Form I

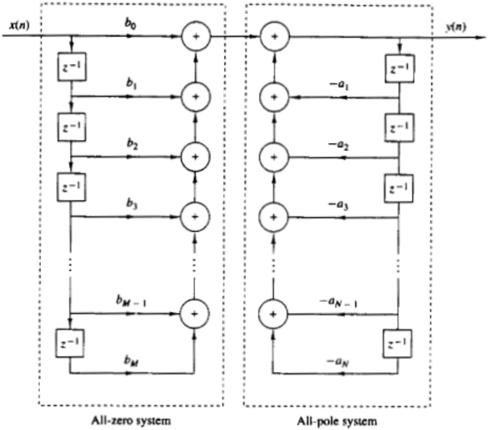

Figure 1: Direct-Form I structure

[image from "Digital Signal Processing", Proakis & Manolakis, 3rd ed.]

Direct-Form II

- Swap the two halves of a Direct-Form I structure
  - (convolution is commutative)
- Advantage: number of delay blocks $=$ filter order
- Is not straightforwardly related to the difference equation
- Known as Direct-Form II or **canonical form**

Direct-Form II

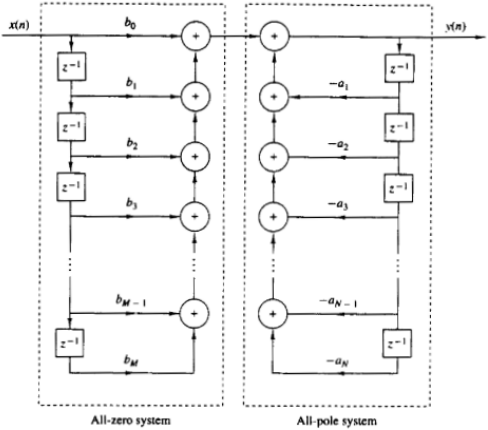

Figure 2: Direct-Form II structure

[image from "Digital Signal Processing", Proakis & Manolakis, 3rd ed.]

Transposed forms

- **Transposition of a graph** = reverse the direction of all branches, swap input and output
- Theorem: If a structure is transposed, the transfer function stays the same
  - summing nodes become branching nodes
  - branching nodes become sum nodes
- Direct-Form I and II **Transposed**
  - transpose the form
  - different structures than the originals

FIR systems

- For FIR systems, $a_i = 0$ so the graphs become simpler
- There is a single Direct-Form, and a single Direct-Form Transposed

Cascade and parallel implementations

- If a system function $H(z)$ can be written as a **sum** of smaller parts, the system can be implemented in a **parallel structure**
  - implement each smaller part
  - same input, sum the outputs
- If a system function $H(z)$ can be written as a **product** of smaller parts, the system can be implemented in a **cascade structure** (or **series**)
  - implement each smaller part, connect in series
  - order does not matter

Cascade and parallel implementations

- A system function $H(z)$ can always be written as a sum of **partial fractions**
  - a parallel implementation is always possible
- A system function $H(z)$ can always be written as a product of $\frac{(z-z_k)}{(p-p_k)}$ terms
  - a series implementation is always possible
- To avoid complex-number coefficients, must group conjugate zeros and conjugate poles together
  - resulting in polynomials of degree 2

Second-order sections

- In practice, due to finite-precision calculations, small rounding errors may appear in coefficients or signal values
- The **most robust** implementation to these errors is the **series implementation**
  - using as many terms as possible
  - but always keeping conjugate zeros and conjugate poles together
- **Second-order sections** structure = implementation as a series of small systems of degree at most 2
  - very robust to finite-precision errors