DSP Exam - Subject 2

Exercises (16 points)

1. (1p) Consider the following signal:

$$x_a(t) = \cos(400\pi t + \frac{\pi}{2}) + \cos(2000\pi t)$$

   The signal is sampled with 5000Hz. Write the discrete signal obtained via sampling.

2. (2p) Consider the following discrete signal $x[n]$:

$$x[n] = \begin{cases} -4, & -2 \leq n < 0 \\ n-4, & 0 \leq n < 4 \\ 0, & elsewhere \end{cases}$$

   a. (1p) Find the values of $x[n]$ and represent the signal graphically
   b. (1p) Represent graphically the signal $x[-n+2]$

3. (4p) A causal LTI system has the system function

$$H(z) = \frac{2 - 0.5z^{-1} + 1z^{-2}}{1 + 0.5z^{-1} - 0.8z^{-2}}$$

   a. (2p) Find the difference equation of the system.
   b. (2p) Find the amplitude response and the phase response of the filter.

4. (4p) 1. Consider the system with the following difference equation:

$$y[n] = 0.6y[n-1] - 2x[n] - 0.4x[n-1]$$

   a. (2p) Compute the impulse response $h[n]$ of the system.
   b. (2p) Compute the response of the system to the signal $x[n] = 2\left(\frac{1}{3}\right)^n u[n]$

5. (2p) A causal signal $x[n]$ has a Z transform with one pole $p_1 = -0.7$ and one zero $z_1 = 0.7$. It is known that at $\omega = \pi$, the Fourier transform is $X(\omega = \pi) = 1$.

   Find the signals's Z transform $X(z)$, and specify it's Region Of Convergence.

6. (3p) Design an IIR low-pass filter.
   - (1p) Draw the pole-zero plot
   - (1p) Specify the system function $H(z)$
   - (1p) Sketch the magnitude response and argue that it is a low pass filter

Known formulas

$$a^n \cdot u[n] \quad \overset{Z}{\longleftrightarrow} \quad \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z-a}, ROC: |z| > |a|$$

$$-a^n \cdot u[-n-1] \quad \overset{Z}{\longleftrightarrow} \quad \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z-a}, ROC: |z| < |a|$$

Theory (17 points)

1. (2p) Fill in the blanks: "Sampling with frequency $F_s = 30000$Hz an analog cosine signal of frequency $F_1 = 7500$Hz is the same as sampling with frequency $F_s = 45000$Hz an analog cosine signal with frequency $F_2 =$ ___________ Hz". Justify your answer!

2. (2p) A general signal $x[n]$ is subsampled by a factor of 2, then interpolated by a factor of 2. Do we get back the original signal? Justify.

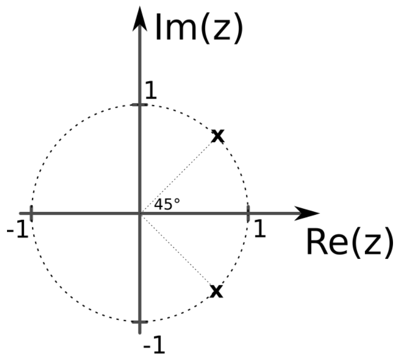
3. (4p) Derive the convolution equation. If a linear and time-invariant system has an input $x[n]$ which can be written as $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, derive the expression of the output signal (based on the impulse response $h[n]$).

4. (1p) Is a system with impulse response $h[n] = \left(\frac{-1}{2}\right)^n$ a FIR or a IIR system? Explain.

5. (2p) State the relationship between the type of a signal (causal / anti-causal / bilateral) and the shape of the Region of Convergence of its Z transform.

6. (1p) Fill in the blanks:
   - A signal which is **periodic in time** is ______________ in frequency
   - A signal which is **discrete in time** is ______________ in frequency
   - A signal which is **periodic and discrete in time** is ______________ in frequency

7. (2p) What type of digital system has the following pole-zero diagram? What is the expression of its output signal? Justify.



8. (3p) Show that the effect of **linear phase** in a filter means delaying the signal.

Notes: Obtain 30p for grade 10. 3p are awarded from start. Time available: 2h