

# Exercises Week 1

① a)  $x[n] = \cos\left(\pi \cdot \frac{3}{10} \cdot n\right) = \boxed{\cos(2\pi f n)}$

$\boxed{x[n] = x[n+N]}$ ,  $\boxed{N \in \mathbb{N}}$

$$x[n+N] = \cos\left(\pi \cdot \frac{3}{10} (n+N)\right) = \cos\left(\pi \cdot \frac{3}{10} \cdot n + \underbrace{\pi \cdot \frac{3}{10} \cdot N}_{\text{multiple of } 2\pi}\right)$$

~~$N=0$~~   $\boxed{N=20} \Rightarrow \pi \cdot \frac{3}{10} \cdot N = 6\pi$

$$\frac{1}{f} = \frac{20}{3}$$

$$\boxed{f = \frac{3}{20}}$$

$$f = \frac{1}{N} \Leftrightarrow N = \frac{1}{f}$$

$$N = 20 = 3 \cdot \frac{1}{f}$$

$\left(N = \frac{1}{f}\right)$  not always

$N = \text{multiple of } \frac{1}{f}$

b).  $\cos(7.2 \pi n)$   $f = \underline{3.6}$

$n \rightarrow n+N$

$$\cos(7.2 \pi n + \underbrace{7.2 \pi N}_{2K\pi}) \quad \underline{N=5}$$

c)  $\sin\left(\underbrace{3n}_{2K\pi}\right)$

$f = \frac{3}{2\pi}$   $f \notin \mathbb{Q}$

$2\pi f n = 2\pi \cdot \frac{3}{2\pi} n = 3n$

$\sin(3n + 3N)$

$N$  nu există!

$$d). \quad x[n] = \sin \frac{\pi n}{N_1} + \cos \frac{3\pi n}{N_2}$$

$$\sin \frac{\pi n}{2} = \sin \left( \frac{\pi n}{2} + \frac{\pi N_1}{2} \right) \text{ with } N_1 = 4$$

$$\cos \frac{3\pi n}{4} = \cos \left( \frac{3\pi n}{4} + \underbrace{\frac{3\pi N_2}{4}}_{6\pi} \right) \text{ with } N_2 = 8$$

$$N_{\text{total}} = 8 = \text{c.m.m.m. c } (4, 8) \\ = \text{least common multiple}$$


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$$(2) \quad x_a(t) = (1 + 0.5 \cos 400\pi t) \cdot \cos(8000\pi t)$$

$$a) \quad f_{s_{\min}} = 2 \cdot f_{\max} = 8400 \text{ Hz}$$

$$\begin{aligned} x_a(t) &= \cos 8000\pi t + 0.5 \cdot \frac{1}{2} (\cos 8400\pi t + \cos 7600\pi t) \\ &= 0.25 \cdot \cos(7600\pi t) + \cos(8000\pi t) + 0.25 \cdot \cos(8400\pi t) \end{aligned}$$

$\overset{3800}{f} = \cancel{3600} \text{ Hz}$        $f = 4000 \text{ Hz}$        $\overset{\text{Hz}}{f} = 4200$

$2\pi f t$        $2\pi f t$        $2\pi f t$

$$b). \quad t \rightarrow n \cdot T_s = \frac{n}{f_s} = \frac{n}{8000}$$

$$\begin{aligned} x[n] &= 0.25 \cos\left(2\pi \frac{3.6}{8} \cdot n\right) + \cos\left(2\pi \frac{1}{2} \cdot n\right) \\ &\quad + 0.25 \cos\left(2\pi \frac{4.2}{8} \cdot n\right) \end{aligned}$$

$\overset{3.8}{f_1} = \frac{3.6}{8}$        $f_2 = \frac{1}{2}$        $f_3 = \frac{4.2}{8}$

$> \frac{1}{2}$

$$f_3 = \frac{4.2}{8} = \frac{4.2}{8} - 1 = -\frac{3.8}{8}$$

$$\checkmark \quad x[n] = 0.25 \dots + \cos(\dots) + 0.25 \cos\left(2\pi \cdot \frac{-3.8}{8} \cdot n\right)$$

$$c) \quad m \rightarrow t \cdot F_s \quad (f \rightarrow f \cdot F_s)$$

$$\begin{aligned}
 x_R(t) &= 0.25 \cos(2\pi \cdot \overset{3800}{\cancel{3600}} t) + \\
 &\quad + \cos(2\pi 4000 t) \\
 &\quad + 0.25 \cos(2\pi \cdot \overset{\text{alias}}{\left(\frac{-3.8}{2}\right)} \cdot 8000 \cdot t) \\
 &\quad \underbrace{\hspace{10em}} \\
 &= 0.25 \cos(2\pi \underline{3800} t)
 \end{aligned}$$