DSP Exam 2017-2018

No.2

Exercises

1. (3p) The signal $x_a(t) = cos(1000\pi t)) + cos(600\pi t)$ is sampled with frequency 3000Hz.

   a. Write the discrete signal obtained via sampling
   b. Can the signal be perfectly reconstructed from its samples? Justify.

2. (3p) Consider the causal system with the following difference equation:

   $$y[n] = -1.6 \cdot y[n-1] - 0.64 \cdot y[n-2] + x[n] - 2 \cdot x[n-1] + x[n-2]$$

   a. Find the system function H(z), draw the pole-zero diagram and indicate the Region Of Convergence
   b. Find the response of the system to a constant input signal $x[n] = A, \forall n \in \mathbb{Z}$

3. (3p) Compute the convolution of the two sequences $x_1 = \{...0, 0, 2, 2, \underset{\uparrow}{2}, 0, 0, ...\}$ and $x_2 = \{...0, 3, 2, \underset{\uparrow}{1}, 2, 3, 0, ...\}$.

4. (3p) Characterize the system with $H(z) = \frac{(z-2)(z-0.5)}{(z+2)(z+0.5)}$ in terms of the following:

   - stability
   - length of impulse response
   - implementation (recursive or not)

   Justify all the answers.

5. (3p) Find the output signal of the system with system function $H(z) = \frac{z-0.5}{z+0.8}$ if the input signal is the unit step $x[n] = u[n]$

6. (3p) Implement the following system in **Direct-Form I** structure:

   $$H(z) = \frac{2 - 0.5z^{-1} + 0.1z^{-2}}{1 - 0.4z^{-1} - 0.2z^{-2}}$$

Known formulas

$$
\begin{aligned}
a^n \cdot u[n] &\overset{Z}{\longleftrightarrow} \frac{1}{1-a\cdot z^{-1}} = \frac{z}{z-a}, ROC: |z| > |a| \\
-a^n \cdot u[-n-1] &\overset{Z}{\longleftrightarrow} \frac{1}{1-a\cdot z^{-1}} = \frac{z}{z-a}, ROC: |z| < |a|
\end{aligned}
$$

Theory

1. (2p) Fill in the blanks: “Sampling with frequency Fs=20000Hz an analog cosine signal of frequency F1=5000Hz is the same as samping with frequency Fs=30000Hz an analog cosine signal with frequency F2=__________ Hz”. Justify your answer!
2. (4p) Derive the convolution equation. If a linear and time-invariant system has an input $x[n]$ which can be written as $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, derive the expression of the output signal (based on the impulse response $h[n]$).
3. (2p) We apply the unit step signal $u[n]$ to a system and we observe that the output $y[n]$ slowly increases up to infinity. What property can we state about the system? Explain why.
4. (2p) What is the value of a Z transform $X(z)$ for a value z outside the Region of Convergence?
5. (3p) If a signal $x[n]$ is delayed, how does the magnitude of its Fourier transform $|X(\omega)|$ change? Justify the answer.
   a. $|X(\omega)|$ increases
   b. $|X(\omega)|$ remains the same
   c. $|X(\omega)|$ decreases
6. (2p) Considering the geometric interpretation, what is the effect of having **a pole in the origin of the plane (p=0)** for the modulus of a Fourier transform? Justify.
   a. The modulus of the Fourier transform is increased at low frequencies
   b. The modulus of the Fourier transform is increased at middle frequencies
   c. The modulus of the Fourier transform is increased at high frequencies
   d. No effect
7. (4p) Show that a FIR system of order $M-1$, M = odd, with negative symmetry $h[n] = -h[M-1-n]$, has linear phase.

Notes

- 40p total, solve 30p for grade 10. 3p are awarded from start.
- Time available: 2h