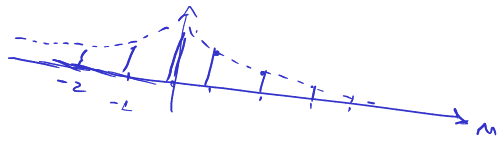
Exercises Week 5

(1) b). $x[n] = \begin{cases} \left(\frac{1}{3}\right)^n & , n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & , n < 0 \end{cases}$



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = \underbrace{\sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} \cdot z^{-n}}_{\left(\frac{1}{2}z\right)^{-n}} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n} \cdot z^{-n}}_{\left(\frac{1}{3}\cdot z^{-1}\right)^{n}}$$

$$= \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{2}z\right)^{n}}_{q} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{3}z^{-1}\right)^{n}}_{q}$$

$q^0 + q + q^2 + q^3 + \ldots =$ (missing q^0)

$$= \frac{1}{1-q} - 1$$

$$= \frac{1}{1-\frac{1}{2}z} - 1$$

$q^0 + q^1 + q^2 + q^3 + \ldots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{3}z^{-1}}$, $|q| < 1$

$\left|\frac{1}{3}z^{-1}\right| < 1$

$|z| > \frac{1}{3}$

RoC

$|q| < 1 \Leftrightarrow \left|\frac{1}{2}z\right| < 1$

$\Leftrightarrow \left|\frac{1}{2}\right| \cdot |z| < 1 \Leftrightarrow \boxed{|z| < 2}$ RoC

$$X(z) = \frac{1}{1-\frac{1}{2}z} - 1 + \frac{1}{1-\frac{1}{3}z^{-1}}\,z \quad , \quad \text{RoC: } \frac{1}{3} < |z| < 2$$

$$= \frac{1}{1-\frac{1}{2}z} - 1 + \frac{z}{z-\frac{1}{3}} = \frac{z-\frac{1}{3} - \left(1-\frac{1}{2}z\right)\left(z-\frac{1}{3}\right) + 1 - \frac{1}{2}z}{\left(1-\frac{1}{2}z\right)\left(z-\frac{1}{3}\right)}$$

$$= \frac{\cancel{z} - \frac{1}{3} - \cancel{z} + \frac{1}{3} + \frac{1}{2}z^2 - \frac{1}{6}z + 1 - \frac{1}{2}z}{\left(1-\frac{1}{2}z\right)\left(z-\frac{1}{3}\right)} = \frac{\frac{1}{2}z^2 - \frac{2}{3}z + 1}{\left(1-\frac{1}{2}z\right)\left(z-\frac{1}{3}\right)} \cdot 2$$

$(-1)\left(\frac{1}{2}z - 1\right)$

$-1\,(z-2)$

$$= -\frac{z^2 - \frac{4}{3}z + 1}{(z-2)\left(z-\frac{1}{3}\right)}$$

$$\Rightarrow z_{1,2} = \frac{\frac{4}{3} \pm \sqrt{\frac{16}{9} - 4}}{2} = \frac{2}{3} \pm j\,\frac{\sqrt{5}}{3}$$

$$\Rightarrow p_1 = 2 \in \mathbb{R}, \quad p_2 = \frac{1}{3}$$

(2)

$$X(z) = \frac{7}{(1-2z^{-1})(1+0.5z^{-1})} \cdot z^2 \cdot z^{-2} = \frac{7 \cdot z^2}{(z-2)(z+0.5)}$$

$x[n] = ?$

$$\frac{X(z)}{z} = \frac{7 \cdot z}{(z-2)(z+0.5)} = \frac{A}{z-2} + \frac{B}{z+0.5} \cdot z$$

$z = 2$ $\qquad$ $z = -0.5$

$$A = \frac{7 \cdot 2}{2+0.5} = \frac{14}{2.5}$$

$$B = \frac{7 \cdot (-0.5)}{(-0.5-2)} = \frac{-3.5}{-2.5} = \frac{7}{5}$$

$$\frac{z}{z-\alpha} \rightarrow \alpha^n u[n], \ |z| > |\alpha| \qquad -\alpha^n u[-n-1], \ |z| < |\alpha|$$

$$X(z) = \frac{14}{2.5} \cdot \frac{z}{z-2} + \frac{7}{5} \cdot \frac{z}{z+0.5}$$

$\alpha = 2$ $\qquad$ $\alpha = -0.5$

$2^n u[n]$	$-2^n u[-n-1]$	$(-0.5)^n u[n]$	$-(-0.5)^n u[-n-1]$
$\lvert z \rvert > 2$	$\lvert z \rvert < 2$	$\lvert z \rvert > 0.5$	$\lvert z \rvert < 0.5$

ROC.

I $\quad |z| < 0.5 < 2$

$$x[n] = \frac{14}{2.5} \cdot \left(-2^n u[-n-1]\right) + \frac{7}{5} \cdot \left(-(-0.5)^n u[-n-1]\right)$$

II $\quad 0.5 < |z| < 2$

$$x[n] = \frac{14}{2.5} \cdot \left(-2^n u[-n-1]\right) + \frac{7}{5} \left((-0.5)^n u[n]\right)$$

III $\quad |z| > 2 > 0.5$

$$x[n] = \frac{14}{2.5} \cdot 2^n \cdot u[n] + \frac{7}{5} \cdot (-0.5)^n \cdot u[n]$$