Exercises Week 12

DSP

1. A signal $x[n]$ has a Z transform with one pole $p_1 = -0.5$ and one zero $z_1 = 0.9$. It is known that at $\omega = \pi$, the modulus of the Fourier transform is $|X(\omega = \pi)| = 1$.

   a. Find the signals's Z transform $X(z)$
   b. Compute the expression of $|X(\omega)|$ and $\angle X(\omega)$
   c. Find the values $|X(\frac{\pi}{2})|$, $|X(\frac{-\pi}{2})|$ and $|X(0)|$
   d. Sketch $|X(\omega)|$

2. Design the pole-zero plot of a signal with:

   - low frequency content
   - frequency content around the frequency $\omega = \frac{\pi}{2}$

3. A digital filter has the following properties:

   - it is a high-pass filter of order 1
   - the pole is situated at a distance 0.9 from the origin
   - continuous signals are completely blocked by the filter

   Requirements:

   a. Draw the pole-zero diagram and find the system function $H(z)$
   b. Compute the amplitude response and the phase response of the filter
   c. Normalize the filter such that $|H(\pi)| = 1$
   d. Find the output signal $y[n]$ if the input signal is $x[n] = 2\cos(\frac{\pi}{6}n + \frac{\pi}{4}), n \in \mathbb{Z}$

4. Design two filters of order 2 of the following types, and write their difference equation:

   - a low-pass filter
   - a band-pass filter with central frequency around the frequency $\omega = \frac{3\pi}{4}$