Exercises Week 1

(1) a) $x[n] = \cos\left(\pi \cdot \frac{3}{10} \cdot n\right) = \cos(2\pi f n)$

$x[n] = x[n+N]$, $N \in \mathbb{N}$

$x[n+N] = \cos\left(\pi \frac{3}{10}(n+N)\right) = \cos\left(\pi \frac{3}{10} \cdot n + \underbrace{\pi \cdot \frac{3}{10} \cdot N}_{\text{multiple of } 2\pi}\right)$

~~$N = 0$~~ $\quad N = 20 \Rightarrow \pi \cdot \frac{3}{10} \cdot N = 6\pi$

$\frac{1}{f} = \frac{20}{3}$ $\quad f = \frac{3}{20}$

$N = 20 = 3 \cdot \frac{1}{f}$

$f = \frac{1}{N} \iff N = \frac{1}{f}$

$\left(N = \frac{1}{f}\right)$ not always

N = multiple of $\frac{1}{f}$

b). $\cos(7 \cdot 2\pi n)$ $\quad f = 3.6$

$n \to n+N$

$\cos\left(7 \cdot 2\pi n + \underbrace{7 \cdot 2\pi N}_{2K\pi}\right)$ $\quad N = 5$

c) $\sin(3n)$ $\quad f = \frac{3}{2\pi}$ $\quad f \notin \mathbb{Q}$

$2\pi f n = 2\pi \frac{3}{2\pi} n = 3n$

$\sin(3n + \underbrace{3N}_{2K\pi})$ $\quad N$ nu există!

d). $x[n] = \sin \frac{\pi n}{2} + \cos \frac{3\pi n}{4}$

N_1 N_2

$\sin \frac{\pi n}{2} = \sin\left(\frac{\pi n}{2} + \frac{\pi N_1}{2}\right)$ with $N_1 = 4$

$\cos \frac{3\pi n}{4} = \cos\left(\frac{3\pi n}{4} + \underbrace{\frac{3\pi N_2}{4}}_{6\pi}\right)$ with $N_2 = 8$

N total $= 8 =$ c.m.m.m.c $(4, 8)$

$=$ least common multiple

(2) $x_a(t) = (1 + 0.5 \cos 400\pi t) \cdot \cos(8000\pi t)$

a) $F_{s_{min}} = 2 \cdot F_{max} = 8400\ Hz$

$x_a(t) = \cos 8000\pi t + 0.5 \cdot \frac{1}{2}\left(\cos 8400\pi t + \cos 7600\pi t\right)$

$F = 3600\ Hz$ $\quad F = 4000\ Hz$

$= 0.25 \cdot \cos(7600\pi t) + \cos(8000\pi t) + 0.25 \cdot \cos(8400\pi t)$

$2\pi F t$ $\quad F = 4200\ Hz$

b). $t \rightarrow n \cdot T_s = \frac{n}{F_s} = \frac{n}{8000}$

f_1 $\quad f_2$

$x[n] = 0.25 \cos\left(2\pi \frac{3.6}{8} \cdot n\right) + \cos\left(2\pi \frac{1}{2} \cdot n\right)$

$+ 0.25 \cos\left(2\pi \frac{4.2}{8} n\right)$ $\quad f_3$

$> \frac{1}{2}$

$f_3 = \frac{4.2}{8} = \frac{4.2}{8} - \frac{1}{} = \frac{-3.8}{8}$

$x[n] = 0.25 \ldots + \cos(\ldots) + 0.25 \cos\left(2\pi \cdot \frac{-3.8}{8} \cdot n\right)$

c) $n \rightarrow t \cdot F_s \qquad (f \rightarrow f \cdot F_s)$

$$x_r(t) = 0.25 \cos(2\pi \cdot 3600 t) +$$

$$+ \cos(2\pi\, 4000\, t)$$

alias

$$+ 0.25 \cos\left(2\pi \cdot \frac{-3.8}{8} \cdot 8000 \cdot t\right)$$

$$= 0.25 \cos(2\pi\, 3800\, t)$$