Exercises Week 7

(1) $y[n] = -0.1\, y[n-1] + 0.2\, y[n-2] + 1\, x[n] + 1\, x[n-1]$

a) $$H(z) = \frac{1 + 1\cdot z^{-1}}{1 + 0.1 z^{-1} - 0.2 z^{-2}} = \frac{1 + z^{-1}}{1 + 0.1 z^{-1} - 0.2 z^{-2}} \cdot \frac{z^2}{z^2} = \frac{z^2 + z}{z^2 + 0.1 z - 0.2}$$

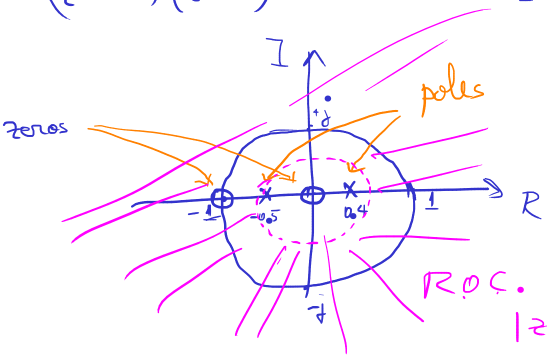
ROC = ?

$|z| > 0.5$

$$z^2 + z = z(z+1) \Rightarrow \begin{aligned} z_1 &= 0 \\ z_2 &= -1 \end{aligned}$$

$$z^2 + 0.1 z - 0.2 = 0 \Rightarrow p_{1,2} = \frac{-0.1 \pm \sqrt{0.1^2 + 0.8}}{2} = \frac{-0.1 \pm 0.9}{2} = -0.5\ ;\ 0.4$$

$$= (z + 0.5)(z - 0.4)$$



System is causal
+
Poles inside unit circle

$\Rightarrow$ stable

b) $h[n] = ?$

$$H(z) = \frac{z(z+1)}{(z+0.5)(z-0.4)}$$

$$\frac{H(z)}{z} = \frac{(z+1)}{(z+0.5)(z-0.4)\cancel{z}} = \frac{A}{z+0.5} + \frac{B}{z-0.4} + \cancel{\frac{}{z-0}} \quad \Big|\cdot z$$

$$A = \frac{-0.5+1}{-0.5-0.4} = \frac{0.5}{-0.9} = -5/9$$

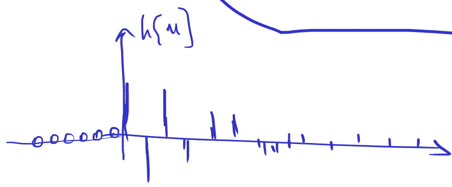
$$B = \frac{1.4}{0.9} = \frac{14}{9}$$

$$\frac{z}{z-a} \leftrightarrow \begin{cases} a^n u[n], & |z| > |a| \\ -a^n u[-n-1], & |z| < |a| \end{cases}$$

$$H(z) = A \cdot \frac{z}{z + 0.5} + B \cdot \frac{z}{z - 0.4}$$

$a = -0.5$; $a = 0.4$

$$h[n] = A\cdot(-0.5)^n u[n] + B \cdot (0.4)^n u[n]$$



c). $x[n] = u[n]$

$y[n] = ?$

$$\boxed{u[n] \longleftrightarrow \frac{z}{z-1}, \quad |z| > 1}$$

$$Y(z) = \underbrace{X(z)}_{\frac{z}{z-1}} \cdot \underbrace{H(z)}_{\frac{z(z+1)}{(z+0.5)(z-0.4)}}$$

ROC: $|z| > 1$

$$Y(z) = \frac{z}{z-1} \cdot \frac{z(z+1)}{(z+0.5)(z-0.4)}$$

$$= \frac{z^2(z+1)}{(z-1)(z+0.5)(z-0.4)}$$

$$\frac{Y(z)}{z} = \frac{z(z+1)}{(\ )(\ )(\ )} = \frac{A}{z-1} + \frac{B}{z+0.5} + \frac{C}{z-0.4}$$

$A, B, C = \ldots$

$$Y(z) = A \cdot \frac{z}{z-1} + B \cdot \frac{z}{z+0.5} + C \cdot \frac{z}{z-0.4}$$

R.O.C. $|z| > 1$

$$y[n] = A \cdot 1^n u[n] + B \cdot (-0.5)^n u[n] + C \cdot (0.4)^n u[n]$$

$$a^n u[n] \longleftrightarrow \frac{z}{z-a}, \quad |z| > |a|$$

d). $x[n] = \left(\frac{1}{3}\right)^n u[n]$

$$X(z) = \frac{z}{z-\frac{1}{3}}, \quad \text{R.O.C. } |z| > \frac{1}{3}$$

$$Y(z) = \underbrace{\frac{z}{z-\frac{1}{3}}}_{X(z)} \cdot \underbrace{\frac{z(z+1)}{(z+0.5)(z-0.4)}}_{H(z)}$$

$$\frac{Y(z)}{z} = \frac{z(z+1)}{\left(z-\frac{1}{3}\right)(z+0.5)(z-0.4)} = \frac{A}{z-\frac{1}{3}} + \frac{B}{z+0.5} + \frac{C}{z-0.4}$$

$$Y(z) = A \cdot \frac{z}{z-\frac{1}{3}} + B \cdot \frac{z}{z+0.5} + C \cdot \frac{z}{z-0.4}$$

$$y[n] = A \cdot \left(\frac{1}{3}\right)^n u[n] + B \cdot (-0.5)^n u[n] + C \cdot (0.4)^n u[n]$$