

Exercises Week 4

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

① $x_1[n] = \{ \dots, 0, \overset{x[0]}{1}, \overset{x[1]}{2}, 3, 4, 0, \dots \} \longleftrightarrow X_1(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

$x_2[n] = \{ \dots, 0, \overset{x[0]}{2}, \overset{x[1]}{2}, \overset{x[2]}{3}, \overset{x[3]}{3}, 0, \dots \} \longleftrightarrow X_2(z) = 2z + 2 + 3z^{-1} + 3z^{-2}$

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ z^1 & z^0 & z^{-1} & z^{-2} \end{matrix}$

$$y[n] = x_1[n] * x_2[n] =$$

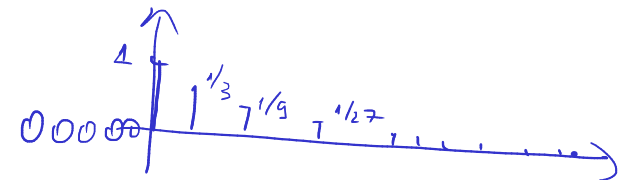
$$\longleftrightarrow Y(z) = X_1(z) \cdot X_2(z) =$$

$$\begin{aligned} &= (1 + 2z^{-1} + 3z^{-2} + 4z^{-3}) (2z + 2 + 3z^{-1} + 3z^{-2}) = \\ &= (2) \cdot z + (2+4) \cdot z^0 + (3+4+6) \cdot z^{-1} + \\ &\quad + (6+9+8) \cdot z^{-2} + (6+9+8) \cdot z^{-3} \\ &\quad + (9+12) \cdot z^{-4} + (12) \cdot z^{-5} \end{aligned}$$

$$\begin{aligned} Y(z) &= 2z + 6 + 13z^{-1} + 23z^{-2} + 23z^{-3} + 21z^{-4} + 12z^{-5} \\ y[n] &= ? \\ &= \{ 0, \dots, 2, 6, 13, 23, 23, 21, 12, 0, \dots \} \end{aligned}$$

\uparrow

② a) $x[n] = \left(\frac{1}{3}\right)^n \cdot \mu[n]$



$$= \{ \dots, 0, \overset{x[0]}{1}, \overset{x[1]}{\frac{1}{3}}, \overset{x[2]}{\left(\frac{1}{3}\right)^2}, \overset{x[3]}{\left(\frac{1}{3}\right)^3}, \dots \}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = 1 + \frac{1}{3} \cdot z^{-1} + \left(\frac{1}{3}\right)^2 \cdot z^{-2} + \left(\frac{1}{3}\right)^3 \cdot z^{-3} + \dots$$

\neq Notation: $\left(\frac{1}{3} z^{-1}\right) = z$

$$X(z) = \underset{z^0}{\frac{1}{3}} + z + z^2 + z^3 + \dots$$

geometric progression —

$$= \sum_{n=0}^{\infty} z^n = \sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n$$

$$\boxed{\frac{1}{1-z}, \text{ if } |z| < 1} \quad \text{Sum of geometric progression}$$

$$\boxed{X(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}}$$

· z

$$= \frac{z}{z - \frac{1}{3}}$$

· z

↙

~~z < 0~~

$$\text{Roc: } |z| < 1$$

$$\left|\frac{1}{3} z^{-1}\right| < 1$$

$$\left|\frac{1}{3}\right| \cdot |z^{-1}| < 1$$

$$|z^{-1}| < 3$$

$$\boxed{\text{Roc:}}$$

$$\boxed{|z| > \frac{1}{3}}$$

b), c): next time