Exercises Week 10

1

$x_1 = [1 \quad 3 \quad 1 \quad 3]$

$x_2 = [2 \quad 2 \quad 5 \quad 5]$

$x_1 \circledast x_2 = ?$ $[28 \quad 28 \quad 28 \quad 28]$

2	2	5	5
15	6	6	15
5	5	2	2
6	15	15	6
28	28	28	28

2

Circular convolution in $N = 7$ points

"add zeros until lengths = 7"

$x_1' = [1 \quad 3 \quad 1 \quad 3 \quad 0 \quad 0 \quad 0]$

$x_2' = [2 \quad 2 \quad 5 \quad 5 \quad 0 \quad 0 \quad 0]$

$x_1' \circledast x_2' = ?$

2	2	5	5	0	0	0
0	6	6	15	15	0	0
0	0	2	2	5	5	0
0	0	0	6	6	15	15
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
2	8	13	28	26	20	15

$x_1' \circledast x_2' = x_1 \circledast_7 x_2 = x_1 * x_2$

$e^{jx} + e^{-jx} = 2\cos(x)$

3

$x[n]$, $N = 6$

$X_k = [21, \; -3 + 5.19j, \; -3 + 1.73j, \; -3, \; -3 - 1.73j, \; -3 - 5.19j]$

$X_0 \quad X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5$

$X_{-2} \quad X_{-1}$

$x[n]$ = write as sum of sinusoids (cos(·))

$X_{N-k} = X_{-k}$

$$x[n] = \frac{1}{N} \sum X_k e^{j \omega n} = \frac{1}{N} \sum_{k=-2}^{3} X_k e^{j 2\pi kn/N}$$

$\omega = \frac{2\pi k}{N}$, $N = 6$

$$x[n] = \frac{1}{6}\left(X_{-2}\cdot e^{j2\pi(-2)n/6} + X_{-1}\cdot e^{j2\pi(-1)n/6} + X_0\cdot e^{j2\pi 0n/6} + X_1 e^{j2\pi n/6} + X_2\cdot e^{j2\pi 2n/6} + X_3\cdot e^{j2\pi 3n/6}\right)$$

$X_{-2} = 3.46\cdot e^{j\cdot 0.52}$ (since $X_{-2} = -3 - 1.73j$, $|\cdot| = 3.46$, $\angle\cdot = 0.52$)

$= |X_{-2}|\cdot e^{j\angle X_{-2}}$

$X_2 = -3 + 1.73j$, $|\cdot| = 3.46$, $\angle\cdot = -0.52$

! $X_{+1} = -3 + 5.19j$, $|\cdot| = 5.99$, $\angle\cdot = -1.04$

$= 5.99\cdot e^{-j\cdot 1.04}$

$X_{-1} = 5.99\cdot e^{j\cdot 1.04}$

$e^{jx} = \cos(x) + j\sin(x)$

$e^{jx} + e^{-jx} = 2\cdot\cos(x)$

$\cos(2\pi f n + \varphi)$

$$x[n] = \frac{1}{6}\cdot 21 + \frac{1}{6}\cdot 5.99\left(\underbrace{e^{j(-2\pi n/6 + 1.04)} + e^{-j(-2\pi n/6 + 1.04)}}_{2\cdot\cos(-2\pi n/6 + 1.04)}\right) +$$

$$+\frac{1}{6}\cdot 3.46\left(\underbrace{e^{j(-2\pi\frac{2}{6}n + 0.52)} + e^{-j(-2\pi\frac{2}{6}n + 0.52)}}_{2\cos(-2\pi\frac{2}{6}n + 0.52)}\right) - \frac{1}{6}\cdot e^{j2\pi\frac{3}{6}n}$$

$\cos(2\pi\frac{3}{6}n) + j\sin(2\pi\frac{3}{6}n) =$

$\cos 2\pi\frac{3}{6}n + j\sin(n\pi)$ ($\cos = (-1)^n$, $\sin = 0$)

$$= \underbrace{\frac{21}{6}}_{X_0} + \frac{2\cdot 5.99}{6}\cdot\cos\left(2\pi\tfrac{1}{6}n - 1.04\right) + \frac{2}{6}\cdot 3.46\cdot\cos\left(2\pi\tfrac{2}{6}n - 0.52\right) +$$

$|X_1|$, $f = \frac{1}{6}$, $\angle X_1$; $|X_2|$, $\angle X_2$

D.C

$$+\frac{3}{6}\cdot\cos\left(2\pi\tfrac{3}{6}n\right)$$

$|X_3|$, $\neq$

$$x[n] = \frac{1}{N}\cdot X_0 + \frac{2}{N}\cdot|X_1|\cdot\cos\left(2\pi\tfrac{1}{6}n + \angle X_1\right) + \frac{2}{N}\cdot|X_2|\cdot\cos\left(2\pi\tfrac{2}{6}n + \angle X_2\right) +$$

D.C.

$$+\frac{1}{N}X_3\cdot\cos\left(2\pi\tfrac{3}{6}n\right)$$