Exercises Week 12

(1) = (3) from Week 11, continued
points c) and d)

c) $|X(\omega)| = \ldots$ from Week 11, b)

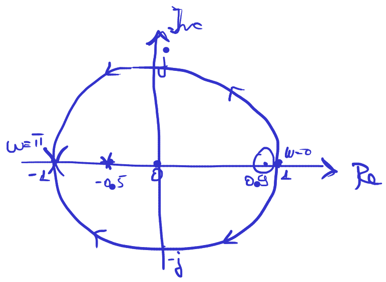
$\omega = \pi/2$: $\left| X(\pi/2) \right| =$

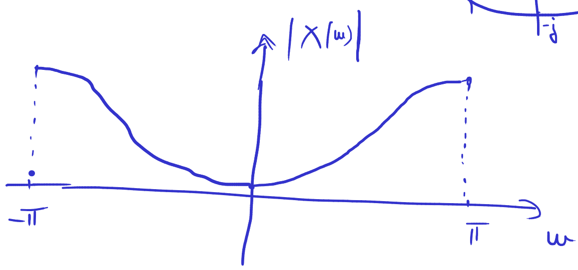
$\omega = -\pi/2$:

$\omega = 0$:

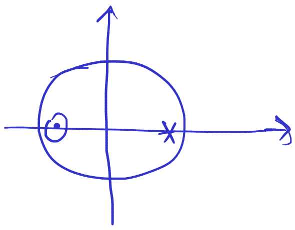
d). Sketch $|X(\omega)|$

$$X(z) = \pm \frac{5}{19} \cdot \frac{z - 0.9}{z + 0.5}$$

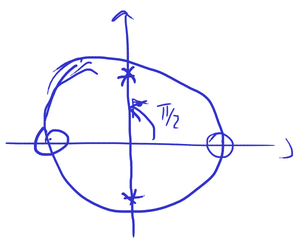


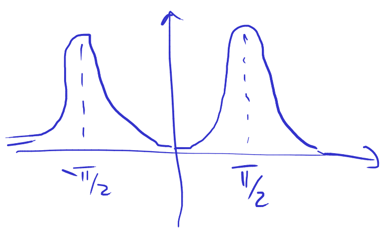


(2) a) "a signal with low frequencies and very little high frequencies"
" a low-pass filter "



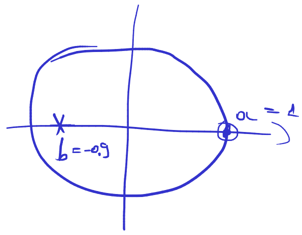
b).





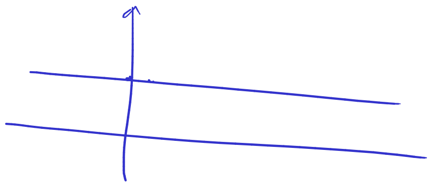
3 · HPF, order 1

a) $H(z) = C \cdot \dfrac{z - a}{z - b}$

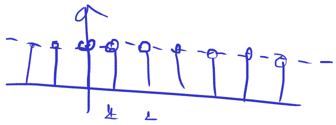


~~zeros~~ : a

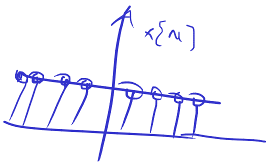
poles : b



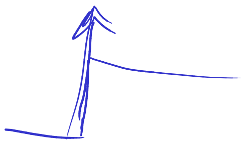
$\boxed{b = -0.9}$ because it is at dist. 0.9 from origin and on left side because high-pass



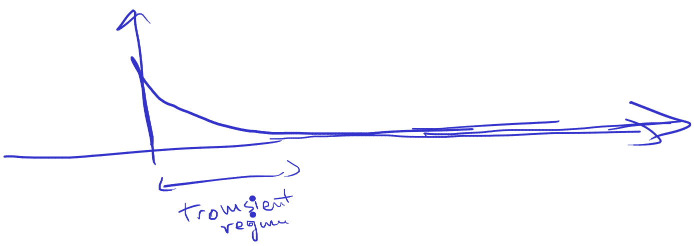
$\boxed{\text{constant signal} = \text{frequency } \underline{0}}$



$x[n] = A \cdot \cos(2\pi \cdot 0 \cdot n + \varphi) = A \cdot \cos(\varphi) = \text{constant}$



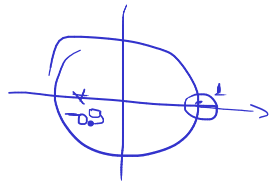
$x[n] = A \cdot \cos(2\pi 0 n) \cdot u[n]$



$\boxed{|H(0)|_{w=0} = 0}$ Amplif. at $f = 0$ is $\underline{0}$

$\Rightarrow$ the zero $a = 1$ (is precisely on unit circle at $w = 0$)

$$H(z) = C \cdot \frac{z-1}{z+0.9}$$



b). $|H(\omega)|$, $\angle H(\omega)$

$$e^{j\omega} = \cos\omega + j\sin\omega$$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = C \cdot \frac{e^{j\omega} - 1}{e^{j\omega} + 0.9}$$

$$|H(\omega)| = |C| \cdot \frac{\sqrt{(\cos\omega - 1)^2 + \sin^2(\omega)}}{\sqrt{(\cos\omega + 0.9)^2 + \sin^2(\omega)}}$$

$$\angle H(\omega) = \angle C + \operatorname{atan}\frac{\sin\omega}{(\cos\omega - 1)} - \operatorname{atan}\frac{\sin\omega}{(\cos\omega + 0.9)}$$

c). "Normalize the filter" = find the value of C

$$|H(\pi)| = 1 \iff |C| \cdot \frac{\sqrt{4+0}}{\sqrt{(-0.1)^2 + 0}} = 1$$

$$\iff |C| \cdot \frac{2}{0.1} = 1 \Rightarrow \boxed{C = \pm \frac{0.1}{2}}$$

d). $x[n] = 2 \cdot \cos\left(\overset{\omega}{\frac{\pi}{6}} n + \pi/4\right)$, $n \in \mathbb{Z}$

$$y[n] = 2 \cdot \underbrace{|H(\pi/6)|}_{\text{from } |H(\omega)|,\ \omega = \pi/6} \cdot \cos\left(\frac{\pi}{6} n + \pi/4 + \underbrace{\angle H(\pi/6)}_{\text{from } \angle H(\omega),\ \omega = \pi/6}\right)$$