

Exercises Week 12

DSP

1. A signal $x[n]$ has a Z transform with one pole $p_1 = -0.5$ and one zero $z_1 = 0.9$. It is known that at $\omega = \pi$, the modulus of the Fourier transform is $|X(\omega = \pi)| = 1$.
 - a. Find the signals's Z transform $X(z)$
 - b. Compute the expression of $|X(\omega)|$ and $\angle X(\omega)$
 - c. Find the values $|X(\frac{\pi}{2})|$, $|X(\frac{-\pi}{2})|$ and $|X(0)|$
 - d. Sketch $|X(\omega)|$
2. Design the pole-zero plot of a signal with:
 - content mainly at low frequencies
 - frequency content around the frequency $\omega = \frac{\pi}{2}$
3. A digital filter has the following properties:
 - it is a high-pass filter of order 1
 - the pole is situated at a distance 0.9 from the origin
 - constant signals are completely blocked by the filter

Requirements:

- a. Draw the pole-zero diagram and find the system function $H(z)$
- b. Compute the amplitude response and the phase response of the filter
- c. Normalize the filter such that $|H(\pi)| = 1$
- d. Find the output signal $y[n]$ if the input signal is $x[n] = 2 \cos(\frac{\pi}{6}n + \frac{\pi}{4})$, $n \in \mathbb{Z}$