

Digital Signal Processing

VI.1. Direct-Form structures

Structures for implementation

- ▶ We will see different methods of implementing systems
 - ▶ mostly LTI systems
- ▶ Differences
 - ▶ computational complexity (number of operations)
 - ▶ memory requirements
 - ▶ finite-precision effects
 - ▶ flexibility
- ▶ Block diagrams (structures)
 - ▶ can be implemented either in HW or SW

- ▶ A LTI system is described by the difference equation

$$\begin{aligned}y[n] &= - \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k] \\&= -a_1 y[n-1] - a_2 y[n-2] - \dots - a_N y[n-N] + b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]\end{aligned}$$

- ▶ **Direct-Form I** structure = directly implementing this equation
- ▶ Main disadvantage: too many delay blocks (approx. $2 \times$ filter order)

Direct-Form I

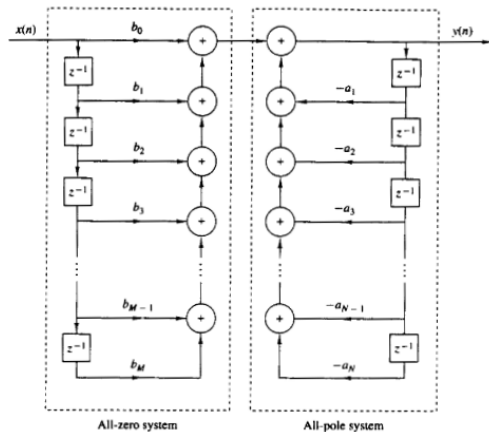


Figure 1: Direct-Form I structure

[image from "Digital Signal Processing", Proakis & Manolakis, 3rd ed.]

Direct-Form II

- ▶ Swap the two halves of a Direct-Form I structure
 - ▶ (convolution is commutative)
- ▶ Advantage: number of delay blocks = filter order
- ▶ Is not straightforwardly related to the difference equation
- ▶ Known as Direct-Form II or **canonical form**

Direct-Form II

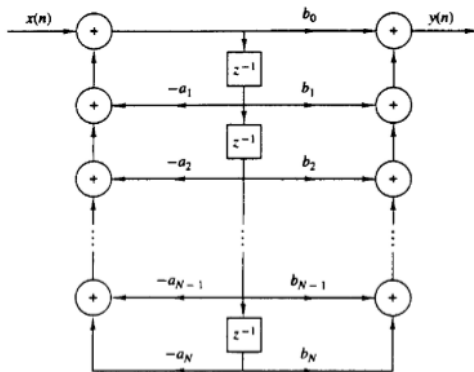


Figure 2: Direct-Form II structure

[image from "Digital Signal Processing", Proakis & Manolakis, 3rd ed.]

Transposed forms

- ▶ **Transposition of a graph** = reverse the direction of all branches, swap input and output
- ▶ Theorem: If a structure is transposed, the transfer function stays the same
 - ▶ summing nodes become branching nodes
 - ▶ branching nodes become sum nodes
- ▶ Direct-Form I and II **Transposed**
 - ▶ transpose the form
 - ▶ different structures than the originals

FIR systems

- ▶ For FIR systems, $a_i = 0$ so the graphs become simpler
- ▶ There is a single Direct-Form, and a single Direct-Form Transposed

Cascade and parallel implementations

- ▶ If a system function $H(z)$ can be written as a **sum** of smaller parts, the system can be implemented in a **parallel structure**
 - ▶ implement each smaller part
 - ▶ same input, sum the outputs
- ▶ If a system function $H(z)$ can be written as a **product** of smaller parts, the system can be implemented in a **cascade structure** (or **series**)
 - ▶ implement each smaller part, connect in series
 - ▶ order does not matter

Cascade and parallel implementations

- ▶ A system function $H(z)$ can always be written as a sum of **partial fractions**
 - ▶ a parallel implementation is always possible
- ▶ A system function $H(z)$ can always be written as a product of $\frac{(z-z_k)}{(p-p_k)}$ terms
 - ▶ a series implementation is always possible
- ▶ To avoid complex-number coefficients, must group conjugate zeros and conjugate poles together
 - ▶ resulting in polynomials of degree 2

Second-order sections

- ▶ In practice, due to finite-precision calculations, small rounding errors may appear in coefficients or signal values
- ▶ The **most robust** implementation to these errors is the **series implementation**
 - ▶ using as many terms as possible
 - ▶ but always keeping conjugate zeros and conjugate poles together
- ▶ **Second-order sections** structure = implementation as a series of small systems of degree at most 2
 - ▶ very robust to finite-precision errors