

Exercises Week 10

①

$$X_1 = \begin{bmatrix} 1 & 3 & 1 & 3 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 2 & 2 & 5 & 5 \end{bmatrix}$$

$$X_1 \otimes X_2 = ? \quad [28 \ 28 \ 28 \ 28]$$

$$\begin{array}{|cccc|} \hline 2 & 2 & 5 & 5 \\ 15 & 6 & 6 & 15 \\ 5 & 5 & 2 & 2 \\ 6 & 15 & 15 & 6 \\ \hline 28 & 28 & 28 & 28 \\ \hline \end{array}$$

②

Circular convolution in $N=7$ points
"add zeros until length = 7"

$$X_1' = \begin{bmatrix} 1 & 3 & 1 & 3 & 0 & 0 & 0 \end{bmatrix}$$

$$X_2' = \begin{bmatrix} 2 & 2 & 5 & 5 & 0 & 0 & 0 \end{bmatrix}$$

$$X_1' \otimes X_2' = ?$$

$$\begin{array}{|ccccccc|} \hline 2 & 2 & 5 & 5 & 0 & 0 & 0 \\ 0 & 6 & 6 & 15 & 15 & 0 & 0 \\ 0 & 0 & 2 & 2 & 5 & 5 & 0 \\ 0 & 0 & 0 & 6 & 6 & 15 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 8 & 13 & 28 & 26 & 20 & 15 \\ \hline \end{array}$$

$$X_1' \otimes X_2' = X_1 \otimes X_2 = X_1 * X_2$$

$$e^{jx} + e^{-jx} = 2 \cos(x)$$

③

$$x[n], N=6$$

$$X_k = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & X_4 & X_5 \\ 21 & -3 + 5.19j & -3 + 1.73j & -3 & -3 - 1.73j & -3 - 5.19j \end{bmatrix}$$

$$x[n] = \text{write as sum of sinusoids (cos)} \quad \begin{matrix} X_{-2} & X_{-1} \end{matrix}$$

$$X_{N-k} = X_{-k}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi k}{N} n} = \frac{1}{N} \sum_{k=-2}^3 X_k e^{j \frac{2\pi k}{N} n}$$

$$x[n] = \frac{1}{6} \left(\underbrace{X_{-2} \cdot e^{j2\pi(-2)n/6}}_{3.46 \cdot e^{j0.52}} + \underbrace{X_{-1} \cdot e^{j2\pi(-1)n/6}}_{5.99 \cdot e^{j1.04}} + \underbrace{X_0 \cdot e^{j2\pi(0)n/6}}_{21 \cdot e^0 = 21} + \underbrace{X_1 \cdot e^{j2\pi(1)n/6}}_{5.99 \cdot e^{-j1.04}} + \underbrace{X_2 \cdot e^{j2\pi(2)n/6}}_{3.46 \cdot e^{-j0.52}} + \underbrace{X_3 \cdot e^{j2\pi(3)n/6}}_{-3} \right)$$

$$X_{-2} = -3 - 1.73j \quad \begin{matrix} | \cdot | = 3.46 \\ \angle = 0.52 \end{matrix}$$

$$= |X_2| \cdot e^{j\angle X_{-2}}$$

$$= 3.46 \cdot e^{j0.52}$$

$$X_2 = -3 + 1.73j \quad \begin{matrix} | \cdot | = 3.46 \\ \angle = -0.52 \end{matrix}$$

$$! X_{+1} = -3 + 5.19j \quad \begin{matrix} | \cdot | = 5.99 \\ \angle = -1.04 \end{matrix}$$

$$= 5.99 \cdot e^{-j1.04}$$

$$X_{-1} = 5.99 \cdot e^{j1.04}$$

$$e^{jx} = \cos(x) + j\sin(x)$$

$$e^{jx} + e^{-jx} = 2 \cdot \cos(x)$$

$$\cos(2\pi f_m + \varphi)$$

$$x[n] = \frac{1}{6} \cdot 21 + \frac{1}{6} \cdot 5.99 \left(\underbrace{e^{j(-2\pi n/6 + 1.04)} + e^{-j(-2\pi n/6 + 1.04)}}_{2 \cdot \cos(-2\pi n/6 + 1.04)} \right) +$$

$$+ \frac{1}{6} \cdot 3.46 \left(\underbrace{e^{j(-2\pi \frac{2}{6}n + 0.52)} + e^{-j(-2\pi \frac{2}{6}n + 0.52)}}_{2 \cdot \cos(-2\pi \frac{2}{6}n + 0.52)} \right) - \frac{1}{6} \cdot 3 \cdot e^{j2\pi \frac{3}{6}n}$$

$$\cos(2\pi \frac{3}{6}n) + j\sin(2\pi \frac{3}{6}n)$$

$$\underbrace{\cos(2\pi \frac{3}{6}n)}_{\cos(n\pi)} + \underbrace{j\sin(n\pi)}_{=0}$$

$$= \underbrace{\frac{21}{6}}_{\text{D.C.}} + \frac{2 \cdot 5.99}{|X_1|} \cdot \cos\left(2\pi \underbrace{\frac{1}{6}}_{f=\frac{1}{6}}n - \underbrace{1.04}_{\angle X_1}\right) + \frac{1}{6} \cdot 2 \cdot \frac{3.46}{|X_2|} \cdot \cos\left(2\pi \underbrace{\frac{2}{6}}_{f=\frac{1}{3}}n - \underbrace{0.52}_{\angle X_2}\right) +$$

$$+ \frac{3 \cdot \frac{1}{6}}{|X_3|} \cdot \cos\left(2\pi \underbrace{\frac{3}{6}}_{f=0.5}n\right)$$

$$x[n] = \frac{1}{N} \cdot X_0 + \frac{1}{N} \cdot 2 \cdot |X_1| \cdot \cos\left(2\pi \underbrace{\frac{1}{6}}_{f=\frac{1}{6}}n + \angle X_1\right) + \frac{1}{N} \cdot 2 \cdot |X_2| \cdot \cos\left(2\pi \underbrace{\frac{2}{6}}_{f=\frac{1}{3}}n + \angle X_2\right) +$$

$$\text{D.C.} \quad + \frac{1}{N} \cdot X_3 \cdot \cos\left(2\pi \underbrace{\frac{3}{6}}_{f=0.5}n\right)$$