Exercises Week 2

Memoryless:
$y[10] = f(x[10])$
$y[5] = f(x[5])$

(3) a) $y[n] = n \cdot x[n^2]$

- M: with memory
- T.I.: Time variant $\quad H(x[n-k]) \overset{?}{=} y[n-k]$

$H(x[n-k]) = n \cdot x[(n-k)^2]$
$y[n-k] = (n-k) \cdot x[(n-k)^2]$
$\neq \Rightarrow$ T.V.

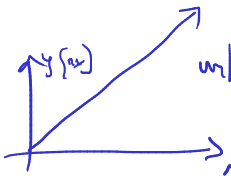
- L: $H(a \cdot x_1[n] + b \cdot x_2[n]) \overset{?}{=} a \cdot H(x_1[n]) + b \cdot H(x_2[n])$

$n(a \cdot x_1[n^2] + b \cdot x_2[n^2]) = a \cdot n \cdot x_1[n^2] + b \cdot n \cdot x_2[n^2]$

$= \Rightarrow$ yes, Linear

- C: $y[10] = 10 \cdot x[100]$ not causal

- S: $x[n] = 1$ for ever
$y[n] = n$



unbounded $\Rightarrow$ not stable

b). $y[n] = x[n] \cdot \cos(\omega_0 n)$

- M: memoryless
- L: $x \to x_1 + x_2$

$H(ax_1[n] + bx_2[n]) = \underbrace{(a x_1[n] + b x_2[n])}_{x[n]} \cdot \cos \omega_0 n = \overbrace{a \cdot x_1[n] \cdot \cos(\dots)}^{H(x_1)} + b \cdot \underbrace{x_2[n] \cdot \cos(\dots)}_{H(x_2)}$

- T: $H(x[n-k]) \neq y[n-k]$ because n in $\cos(\omega_0 n)$

Time - Variant

- C: causal because memoryless

- S: yes: $x[n]$ bounded, $\cos()$ bounded $\Rightarrow y[n]$ bounded $\Leftrightarrow$ stable

c). $y[n] = \sin\left(x[n]\right)$

- M : memoryless
- L : Not
- T : T. invar.
- C : Yes
- S : Yes, $\sin() \in [-1, 1]$

d). $y[n] = x[n] + n \cdot x[n+1]$

- M : with
- L : $H(a x_1[n] + b x_2[n]) = (a x_1 + b x_2) + n \cdot (a x_1[n+1] + b x_2[n+1])$

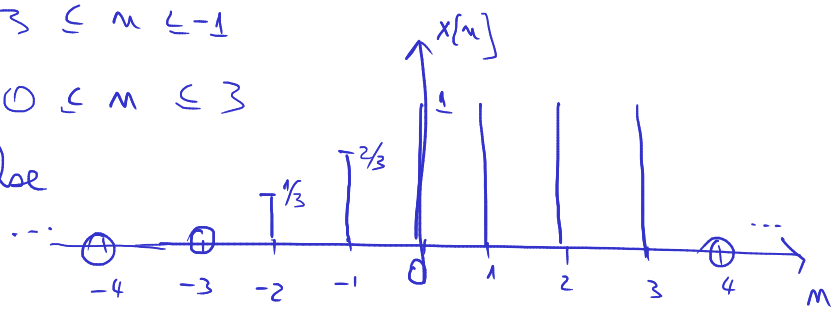
$= a \cdot (x_1[n] + n \cdot x_1[n+1]) + b \cdot (x_2[n] + n \cdot x_2[n+2])$

$= a \cdot H(x_1[n]) + b \cdot H(x_2[n])$

- T : Time variant (n)
- C : non-causal
- S : unstable ($x[n] = 1 \Rightarrow y[n] = n+1$ ↗)

(1) a)

$$x[n] = \begin{cases} 1 + n/3, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{else} \end{cases}$$



b). $x[-n+4] = y[n]$

n	-4	-3	-2	-1	0	1	2	3	4
$x[n]$	0	0	1/3	2/3	1	1	1	1	0
$y[n] = x[-n+4]$	0	0	0	0	0	1	1	1	1	2/3	1/3	0 ... 0