$\boxed{\text{Exercises Week 8}}$

$$a^n u[n] \longleftrightarrow \frac{z}{z-a}, \quad |z|>|a|$$

$$-a^n u[-n-1] \longleftrightarrow \frac{z}{z-a}, \quad |z|<|a|$$

$$a^{n-1} u[n-1] \longleftrightarrow z^{-1}\cdot\frac{z}{z-a}, \quad |z|>|a|$$

Ex. Week 7, Ex 2

$$\rightarrow x[n] = \left(\frac{1}{3}\right)^n u[n] - \underbrace{\frac{1}{4}\cdot\left(\frac{1}{3}\right)^n u[n-1]}_{\frac{1}{4}\cdot\frac{1}{3}\cdot\left(\frac{1}{3}\right)^{n-1}\cdot u[n-1]}$$

$$\rightarrow y[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$$

a) $$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = \frac{z}{z-\frac{1}{4}}, \quad |z| > \left|\frac{1}{4}\right|$$

$$X(z) = \frac{z}{z-1/3} - \frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{z-\frac{1}{3}}, \quad |z| > \frac{1}{3}$$

$$= \frac{z}{z-1/3} - \frac{1}{12}\cdot\frac{1}{z-1/3} = \frac{z-1/12}{z-1/3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-\frac{1}{4}}\cdot\frac{z-1/3}{z-1/12} = \frac{z(z-1/3)}{(z-\frac{1}{4})(z-1/12)}, \quad |z| > \frac{1}{3}$$

$p_1 = 1/4$, $p_2 = 1/12$ => Stable

=> Same as Ex. 1

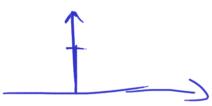
b). Same as Ex. 1

c). $y[n] =$

$$H(z) = \frac{z^2 - 1/3\, z \;:z^2}{z^2 - \frac{4}{12}z + \frac{1}{48} \;:z^2} = \frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{3}z^{-1} + \frac{1}{48}z^{-2}} \quad \frac{Y(z)}{X(z)}$$

$$(1)\quad y[n] = \frac{1}{3}\cdot y[n-1] - \frac{1}{48} y[n-2] + 1\cdot x[n] - \frac{1}{3} x[n-1]$$

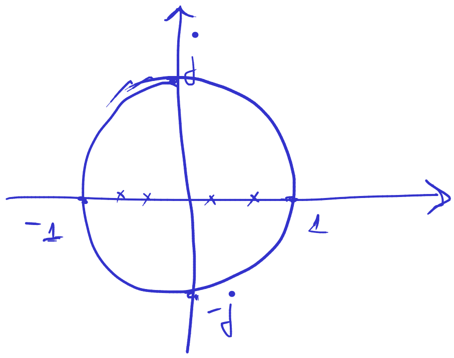
d).



a). Poles exist => IIR

b). Recursive equation

c). Stable (poles inside unit circle)



j
-1
1
-j