DSP Exam

No.1

Exercises (16p)

1. (3p) Consider the following signal:

$$x_a(t) = 2 + 0.5\ cos(400\pi t) + 0.5\ cos(800\pi t)$$

   a. (1p) Compute the minimum sampling frequency necessary for avoiding alias.
   b. (1p) The signal is sampled with 500Hz. Write the discrete signal obtained via sampling.
   c. (1p) What is the analog signal reconstructed from the samples via ideal D/A conversion?

2. (2p) Compute the convolution y of the two sequences $x_1 = \{...0, 0, \underset{\uparrow}{1}, 2, 1, 0, 0, ...\}$ and $x_2 = \{...0, 0, 1, 2, \underset{\uparrow}{3}, 4, 5, 0, 0, ...\}$.

3. A causal LTI system has the property that if the input signal is

$$x[n] = 2\left(\frac{1}{4}\right)^n u[n],$$

   then the output signal is

$$y[n] = u[n]$$

   a. (1p) Find the system function H(z), specify the Region of Convergence, draw the pole-zero diagram, justify if the system is stable or not.
   b. (2p) Compute the impulse response $h[n]$ of the system

4. (8p) Consider a causal LTI system with system function

$$H(z) = 3\frac{1 + z^{-2}}{1 - 0.81z^{-2}}$$

   a. (1p) Draw the pole-zero diagram
   b. (2p) Sketch the amplitude response of the system. What type of filter is it?.
   c. (2p) Write the expression of the amplitude response $|H(\omega)|$.
   d. (2p) Write a non-zero sinusoidal signal $x[n]$ for which the output signal $y[n] = 0$.

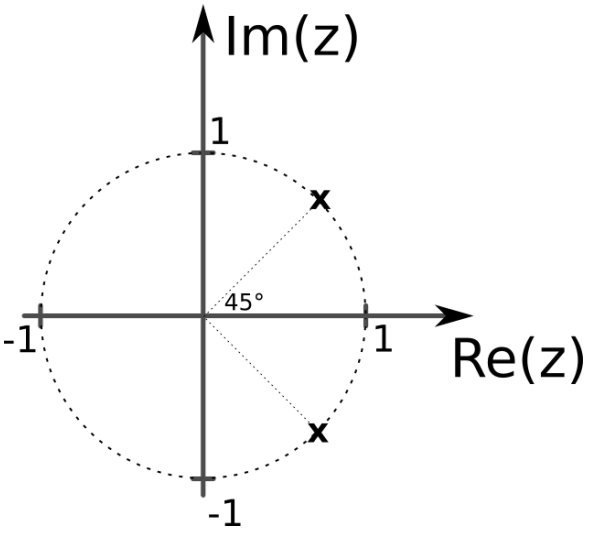
Known formulas

$$a^n \cdot u[n] \quad \overset{\text{Z}}{\longleftrightarrow} \quad \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z - a}, ROC : |z| > |a|$$

$$-a^n \cdot u[-n-1] \quad \overset{\text{Z}}{\longleftrightarrow} \quad \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z - a}, ROC : |z| < |a|$$

Theory (15p)

1. (1p) Prove that $cos(2\pi(\frac{1}{2}+\frac{1}{5})n)$ is identical to $cos(2\pi(\frac{1}{2}-\frac{1}{5})n)$.
2. (2p) We apply the unit step signal $u[n]$ to a system and we observe that the output $y[n]$ slowly increases up to infinity. What property can we state about the system? Explain.
3. (4p) Derive the convolution equation. If a linear and time-invariant system has an input $x[n]$ which can be written as $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, derive the expression of the output signal (based on the impulse response $h[n]$).
4. (1p) A signal $x[n]$ has the Region of Convergence $0.2 < |z| < 1.4$. Is the signal causal / anti-causal / bilateral?
5. (2p) Specify all the possible frequencies that can appear in the spectrum of a a discrete periodic signal with period N.
6. (3p) Prove that an input signal $x[n] = Ae^{j\omega_0 n}$, when applied to the input of an LTI system, produces an output that is proportional to the input $y[n] = x[n] \cdot H(\omega_0)$.
7. (2p) What type of digital system has the following pole-zero diagram? What is its output y[n]?.



Notes

- 3p are awarded from start. 34p max total, obtain 30p for grade 10.
- Time available: 2h