Information Theory - Homework 1

1. A fair coin is flipped until the first *head* occurs. Let X denote the number of flips required until the first *head* occurs.
   a. Find the distribution of X (Note: it has an infinite number of terms).
   b. Compute the entropy H(X). You can use the following relation:

$$\sum_{i=0}^{\infty} nq^n = \frac{q}{(1-q)^2}$$

   c. What is the best way to ask a series of yes-no questions in order to find the precise value of X? Explain.

2. A source with memory has the graph depicted below.
   a. Find the probabilities on all the transitions. Justify.
   b. If the source is initially in state S_2, in what state will the source be after two messages? What is the most likely and the least likely state to be in?
   c. Compute the entropy of the source.

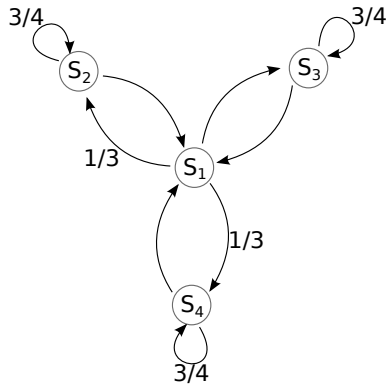

Figure 1: Graph of the source