Information Theory - Homework 3

1. Consider a systematic code with codeword length $n = 8$ and information word length $k = 4$. The 4 control bits c_0, c_1, c_2, c_3 are defined by the following equations (the information bits are i_0, i_1, i_2, i_3):

$$\begin{cases} c_0 = & i_1 + i_2 + i_3 \\ c_1 = & i_0 + i_1 + i_2 \\ c_2 = & i_0 + i_1 + i_3 \\ c_3 = & i_0 + i_2 + i_3 \end{cases}$$

   a. Find the generator matrix [G] and the parity-check matrix [H] for this code
   b. Find how many errors this code can detect and how many it can correct (based on the columns of [H])
   c. From the result b), deduce the minimum Hamming distance of the code

2. Consider the following source code:

A	C	D	E	G	I	N	O
000	100	010	001	110	101	011	111

The letters are encoded with an error correcting code with the following generator matrix:

$$[G] = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Use [G] to encode the word "ENCODING" (letter by letter).

3. Some data is transmitted over a Binary Symmetric Channel with probability of error $p = 10^{-6}$. The data is encoded with the Hamming(7,4) code used for correction.

   a. How many errors can this code correct?
   b. What is the probability that the correction fails? (i.e. a codeword has errors that go undetected or are wrongly corrected). *Hint*: Find the smallest number of errors the code can *not* correct, and compute the probability to have this many errors in a codeword (we neglect the probability of having more errors than that). You might need the value $C(7, 2) = 21$ (combinations of 7 taken by 2).