

# Exercises Week 10: Linear Block Codes

$$G = \begin{bmatrix} I & Q \end{bmatrix}$$

$$H = \begin{bmatrix} Q^T & I \end{bmatrix}$$

1

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

a)  $i = [1011]$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$4 \times 7$   
 $7 \times 7$

(+)

~~$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$~~   
 $5+5=10$

$$1 \cdot 1 \oplus 0 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 0 = 1 \oplus 0 \oplus 1 \oplus 0 = 1 \oplus 1 = 0$$

$$\underbrace{1 \oplus 1}_{0} \oplus 1 \oplus 0 \oplus 1 = 0 \oplus 1 = 1$$

b)  $H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$   
 $3 \times 7$

c)  $r = [1010000]$

Error detection:

$$z = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \neq 0 \Rightarrow \text{We have errors!}$$

$7 \times 7$   
 $C^T \oplus R^T$

Error correction

$$z = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = H \cdot e^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$e = ?$

Lookup table

e	z
1000000	110
0100000	111
0010000	001
0001000	011
1111111	

We found  $e = [0001000]$   $\Rightarrow$  there is an error on 4<sup>th</sup> position in your received word!

$$r = [1010000] \Rightarrow c = [1011000] \Rightarrow i = 1011$$

$r \oplus e = c$

c).  $\mathbf{r} = [1010111]$

$\mathbf{z} = \mathbf{H} \cdot \mathbf{r}^T$

$$\mathbf{z} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \neq 0 \Rightarrow \text{Errors detected!}$$

Correction:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e \\ e \\ e \\ e \\ 1 \\ e \\ e \end{bmatrix}$$

$\mathbf{z}$   $\mathbf{e}^T$

$\mathbf{e}$	$\mathbf{z}$
1000000	
0100000	
0010000	
0001000	
0000100	100

$\Rightarrow$  error on 5<sup>th</sup> position!

$\Rightarrow \mathbf{c} = [1010011]$

$\Rightarrow \hat{\mathbf{x}} = 1010$