IT Sample Exam

Exercises (14p)

1. A discrete, complete and memoryless source has the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & \frac{3}{16} \end{pmatrix}$$

   a. (1p) Compute the entropy of the source;
   b. (3p) Encode it with Huffman coding and write the resulting codewords.

2. (2p) Suppose we encode the source from Exercise 1 with the code below. Compute the efficiency and redundancy of this code.

Message	Code
s_1	00
s_2	011
s_3	010
s_4	101
s_5	100
s_6	11

3. At the input of a binary symmetric channel with the following channel matrix

$$P(y_j|x_i) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

   we apply two inputs x_1 and x_2 with probabilities $p(x_1) = \frac{3}{4}$ and $p(x_2) = \frac{1}{4}$.

   a. (1p) Draw the graph of the channel;
   b. (1p) Compute the joint probability matrix $P(X, Y)$;
   c. (2p) Compute the mutual information $I(X, Y)$ on this channel.

4. (2p) Design a block code consisting of 4 codewords which is able to detect 2 errors in a codeword. Justify that it is able to detect 2 errors.

5. (2p) Consider a systematic code with parity-check matrix

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   We receive the sequence $\mathbf{r} = [0100010]$. Find if there are errors in the received data, and, if yes, perform correction and find the transmitted information bits.

Theory questions (13p)

1. (4p) Prove that the entropy of any second-order extension, $H(S^2)$, is 2 times larger than $H(S)$:

$$H(S^2) = 2 \cdot H(S)$$

2. (1p) Consider two different joint probability matrices $P_A(X,Y)$ and $P_B(X,Y)$. Which one is better for a communication process? Justify (in words).

$$P_A(X,Y) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.2 \end{bmatrix} \quad P_B(X,Y) = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}$$

3. (1p) Give an example of a channel matrix $P(Y|X)$ for a channel with zero mean error $H(Y|X) = 0$.
4. (4p) State and prove the Kraft inequality theorem
5. (1p) The family of Hamming codes contains many possible Hamming codes. What are the next two Hamming codes in the following sequence? Justify the answers.

   `Hamming(7,4), Hamming(15,11), Hamming(?,?), Hamming(?,?), ...`

6. (2p) Consider the following block code:

   a. Is it a linear code?
   b. Is it a systematic code? Justify your answers.

Information word i	Codeword c
00	00110
01	11111
10	00011
11	11001

Notes

- 3p are awarded from start. 30p in total.
- Time available: 2h