24.03.2020

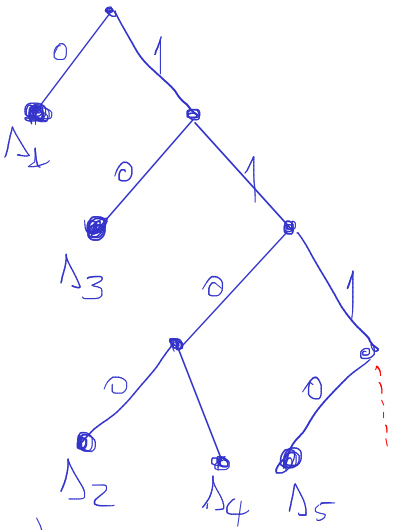
Exercises set 3

Information theory

(1)

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ \frac{1}{2} & \frac{1}{10} & \frac{3}{10} & \frac{1}{10} & \frac{1}{10} \end{pmatrix}$$

Message	$P(s_i)$	$l_i = \lceil -\log_2(p) \rceil$	Codeword
s_1	1/2	1	0
s_3	3/10	2 (1.73)	10
s_2	1/10	4 (3.32)	1100
s_4	1/10	4	1101
s_5	1/10	4	1110



$$H(S) = \frac{1}{2} \cdot 1 + \frac{3}{10} \cdot 1.73 + 3 \cdot \frac{1}{10} \cdot 3.32 = 2.15 \text{ b}$$

$$\bar{l} = \frac{1}{2} \cdot 1 + \frac{3}{10} \cdot 2 + \frac{1}{10} \cdot 4 \times 3 = 2.3 \text{ b}$$

$$\eta = \frac{2.15}{2.3} = \dots \qquad \rho = 1 - \eta = \dots$$

(2) $S: \begin{pmatrix} \Delta_1 & \Delta_2 \\ 0.95 & 0.05 \end{pmatrix}$

		ℓ_i	Codeword	b) Optimized
Δ_1	0.95	1 (0.07)	0	0
Δ_2	0.05	5 (4.32)	10000	1

a) $\overline{\ell} = 0.95 \cdot 1 + 0.05 \cdot 5$

$= 1.2$ b

$H(S) = 0.95 \cdot 0.07 + 0.05 \cdot 4.32$

$= 0.28$

$\eta = \frac{0.28}{1.2} = 23.3\%$

$\rho = 1 - \eta = 76.7\%$

c). $\overline{\ell}_o = 1$ $\quad \eta = \frac{0.28}{1} = 28\%$

$\rho = 72\%$

(3)

Λ_1	0
Λ_2	10
Λ_3	100
Λ_4	11

$p(\Lambda_1) = ? = 2^{-1} = 0.5$

$p(\Lambda_2) = ? = 2^{-3} = 0.125$

$p(\Lambda_3) = ? = 2^{-3} = 0.125$

$p(\Lambda_4) = ? = 2^{-2} = 0.25$

Optimal code $\Leftrightarrow$

$\eta = 1 \Rightarrow l_i = -\log_2(p(\Lambda_i))$ fara rotunjiri

$\Downarrow$

$p(\Lambda_i) = 2^{-\text{cuvo}}$