Exercises Week 3

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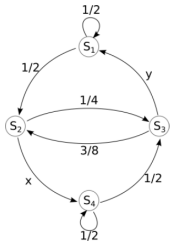

a) $1/4 + x = 1 \Rightarrow x = 3/4$

$3/8 + y = 1 \Rightarrow y = 5/8$

b) $T =$

	S_1	S_2	S_3	S_4
S_1	1/2	1/2	0	0
S_2	0	0	1/4	3/4
S_3	5/8	3/8	0	0
S_4	0	0	1/2	1/2

4×4

c). $H(S_4) = -0 \log 0 - 0 \log 0 - \frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2}$

$= -2 \cdot \frac{1}{2}\log\frac{1}{2} = 1$ bit

d). $H(S) =$

$H(S_1) = 1$ bit

$H(S_2) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} = \frac{2}{4} - \frac{3}{4}(\log_2 3 - \log_2 4) = \frac{2}{4} - \frac{3}{4}\log 3 + \frac{6}{4} = 2 - \frac{3}{4}\log 3$

$= 2 - \frac{3}{4} \cdot 1.58 = 0.81$ b

$H(S_3) = -\frac{5}{8}\log\frac{5}{8} - \frac{3}{8}\log\frac{3}{8} = \frac{5}{8}(\log 8 - \log 5) + \frac{3}{8}(\log 8 - \log 3) = \frac{15}{8} - \frac{5}{8}\log 5 + \frac{9}{8} - \frac{3}{8}\log 3$

$= 0.95$ b

$H(S_4) = 1$ b

Find P_k:

$[P_1\ P_2\ P_3\ P_4] \cdot \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 \\ 5/8 & 3/8 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix} = [P_1\ P_2\ P_3\ P_4]$

$\begin{cases} \frac{1}{2}P_1 + \frac{5}{8}P_2 = P_1 \\ \frac{1}{2}P_1 + \frac{3}{8}P_3 = P_2 \\ \frac{1}{4}P_2 + \frac{1}{2}P_4 = P_3 \\ \frac{3}{4}P_2 + \frac{1}{2}P_4 = P_4 \\ P_1 + P_2 + P_3 + P_4 = 1 \end{cases}$

$\frac{5}{8}P_3 = \frac{1}{2}P_1 \Rightarrow P_3 = \frac{4}{5}P_1$

$\frac{1}{2}P_1 + \frac{3}{8}\cdot\frac{4}{5}P_1 = P_2 \Leftrightarrow \frac{1}{2}P_1 + \frac{3}{10}P_1 = P_2 \Rightarrow P_2 = \frac{4}{5}P_1$

$\frac{3}{4}P_2 = \frac{1}{2}P_4 \Rightarrow P_4 = \frac{3}{2}P_2 = \frac{3}{2}\cdot\frac{4}{5}P_1 = \frac{6}{5}P_1$

$P_1 + \frac{4}{5}P_1 + \frac{4}{5}P_1 + \frac{6}{5}P_1 = 1 \Rightarrow P_1 = \frac{5}{19}$

$P_2 = \frac{4}{19}$ $P_3 = \frac{4}{19}$ $P_4 = \frac{6}{19}$

$H(S) = P_1 \cdot H(S_1) + P_2 \cdot H(S_2) + P_3 \cdot H(S_3) + P_4 \cdot H(S_4)$

$= \frac{5}{19}\cdot 1 + \frac{4}{19}\cdot 0.81 + \frac{4}{19}\cdot 0.95 + \frac{6}{19}\cdot 1$

e). $m = 2$

$n = 2$

f) $\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}_{1\times 4} \cdot \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 \\ 5/8 & 3/8 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}_{4\times 4} \cdot \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 \\ 5/8 & 3/8 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} \frac{5}{32} & \frac{3}{32} & \frac{2}{8} & \frac{3}{8} \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1/4 & 3/4 \end{bmatrix}$