Exercises Week 10

Information Theory

1. For the following code, do the following:
   a. compute the minimum Hamming distance and indicate how many errors it can detect and how many errors it can correct, with the nearest neighbor decoding algorithm;
   b. considering two received codewords, $\mathbf{r_1} = 11100$ and $\mathbf{r_2} = 00011$, perform decoding and say if there are errors, and if so then correct the errors (find the correct codeword and indicate where the errors are located);
   c. Give an example of errors the code cannot detect, and another example of errors it can detect but it cannot correct, using the nearest neighbor decoding algorithm.

Message	Codeword
s_1	$c_1 = 00000$
s_2	$c_2 = 10011$
s_3	$c_3 = 11100$
s_4	$c_4 = 00111$

2. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to detect 3 errors in a codeword.

3. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to correct 2 errors in a codeword.

4. If we send one codeword of the previous code from exercise 2 over a Binary Symmetric Channel with error probability $p = 0.1\%$, what is the probability that nearest neighbor error detection algorithms fails?

   Hint: compute the probability that we get a number of errors more than the code can handle (4, 5 or 6 errors).