Exercises Week 9 : Hamming Distance

(1)

Message	Codeword
s_1	$c_1 = 00000$
s_2	$c_2 = 10011$
s_3	$c_3 = 11100$
s_4	$c_4 = 00111$

a) $d(c_1, c_2) = 3$
$c_2 c_3 = 3$
$c_1, c_4 = 3$
$c_2, c_3 = 4$
$c_2, c_4 = 2$
$c_3, c_4 = 4$

$\Rightarrow \boxed{d_{H_{min}} = 2}$ $\Rightarrow$ Number of errors detected

$e_D = 1 \quad (\phi_{H_{min}} - 1)$

Number of errors corrected:

$e_c = \left\lfloor \frac{\phi_{H_{min}} - 1}{2} \right\rfloor =$

$= \lfloor 0.5 \rfloor = 0$

b). $r_1 = 11100$

Error detection: $r_1 = c_3 \Rightarrow$ no errors

$r_2 = 00011$

Error detection: r_2 is not a codeword $\Rightarrow$ there are errors!

$d(r_2, c_1) = 2$
$c_2 = 1$
$c_3 = 5$
$c_4 = 1$

$\searrow$ We don't know which to choose!

$r_3 = 00010$

Error det $\Rightarrow$ there are errors

Error corr.:

$d(r_3, c_1) = 1 \Rightarrow$ the correct codeword is $c_1 : 00000$
$c_2 = 2$ We got an error on 4th position $(r_2 = 00010)$
$c_3 = 4$
$c_4 = 2$

c). 1). If we send $c_1 = 00000$ and we have 3 errors on 1st, 4th and 5th positions,

$r_2 = 10011$ (or c_2 with 2 errors $\rightarrow e_4$)

2). $r_2 = 00011 \Rightarrow$ can detect, but can't correct

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(2) $e_c = 3 \Rightarrow d_{H_{min}} = 4$

$c_1 = 00000000$
$c_2 = 11110000$
$c_3 = 11001100$
$c_4 = 10101011$

(3) $e_c = 2 \Rightarrow d_{H_{min}} = 5 \Rightarrow$

$c_1 = 0000000000$
$c_2 = 1111100000$
$c_3 = 0000011111$
$c_4 = 1010101011$

5 5 5 10 5 5 ✓