

Linear Block Codes

Information Theory Exercises

1. Consider a systematic code with generator matrix

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

- a. Compute the codeword for sending the information bits $\mathbf{i}_2 = [1011]$
 - b. Compute the parity-check matrix $[H]$;
 - c. We receive a sequence $\mathbf{r} = 1010111$, which was encoded with this code. Find if there are errors in the received data, and, if yes, perform correction and retrieve the transmitted information bits.
 - d. Find out how many errors this code can detect, and how many it can correct.
2. Compute the codewords for transmitting the information words $\mathbf{i}_1 = [1001]$ with the Hamming (7,4) code, and $\mathbf{i}_2 = [1110]$ with the Hamming (8,4) SECDED code.
 3. We receive a sequence $\mathbf{r} = 1010111$, which was encoded with the Hamming (7,4) code. Find if there are errors in the received data, and, if yes, perform correction and retrieve the transmitted information bits.
 4. We receive a sequence $\mathbf{r} = 10101010$, which was encoded with the Hamming (8,4) SECDED code. Find if there are errors in the received data, and, if yes, perform correction and retrieve the transmitted information bits.
 5. Give an example of errors which the Hamming (15, 11) is not able to detect. Give another example of errors which the Hamming (15, 11) is able to detect, but is not able to correct.