Exercises Week 8

Information Theory

1. A discrete memoryless source has the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ 0.05 & 0.4 & 0.1 & 0.25 & 0.2 \end{pmatrix}$$

   a. Encode the source with Shannon, Shannon-Fano coding and Huffman coding and compute the average length in every case.
   b. Find the efficiency and redundancy of the Huffman code
   c. Compute the probabilities of the symbols 0 and 1, for the Huffman code

2. For the following source, perform Huffman coding and obtain three different codes with same average length, but different individual codeword length.

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ 0.05 & 0.05 & 0.15 & 0.25 & 0.25 & 0.25 \end{pmatrix}$$

   a. Compute the average length in all three cases and show it is the same
   b. Which code is better for sending over a binary channel which reaches its capacity when $p(0) = 0.9$ and $p(1) = 0.1$?
   c. Which code is better for sending over a binary symmetric channel?

3. A discrete memoryless source has the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$$

   a. Find the average code length obtained with Huffman coding on the original source and on its second order extension.
   b. Encode the sequence $s_7 s_7 s_3 s_7 s_7 s_7 s_1 s_3 s_7 s_7$ with both codes.

4. A discrete memoryless source has the following distribution

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 0.4 & 0.3 & 0.2 & 0.04 & 0.03 & 0.02 & 0.009 & 0.001 \end{pmatrix}$$

Find the Huffman code for a code with 4 symbols, x_1, x_2, x_3 and x_4, and encode the sequence

$$s_1 s_7 s_8 s_3 s_3 s_1$$