Exercises Week 7 : Source Coding Algorithms

(1) $H(S) = 20\,b$

a) b) $\eta = \dfrac{H(S)}{\bar{\ell}}$ $\quad \underbrace{H(S)}_{20} \le \bar{\ell} < \underbrace{H(S)+1}_{21} \Rightarrow \bar{\ell} \in [20, 21)$

$$\eta = \left(\frac{20}{21}, \frac{20}{20}\right] \Rightarrow \eta \in (0.95, 1]$$

$$\rho = 1 - \eta \Rightarrow \rho \in [0, 0.05)$$

c). H_{max} = when all mess. have equal prob $= \frac{1}{n}$, $H_{max} = \log_2 n$

$$H(S) = 20\,b \le H_{max} = \log_2 n \Rightarrow 20 \le \log_2 n$$

$$\underbrace{2^{20}}_{\simeq 1000000} \le n$$

$$2^{10} = 1024 \simeq 1000$$

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(2) $S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ 0.05 & 0.4 & 0.1 & 0.25 & 0.2 \end{pmatrix}$

a) Shannon:

		$\ell_i = \lceil -\log_2(p(s_i)) \rceil$	Cum. sum	Codeword
s_2	0.4	$\lceil 1.3 \rceil = 2$	0	00
s_4	0.25	2	0.4	01
s_5	0.2	$\lceil 2.3 \rceil = 3$	0.65	101
s_3	0.1	$\lceil 3.3 \rceil = 4$	0.85	1101
s_1	0.05	$\lceil 4.3 \rceil = 5$	0.95	11110

Shannon-Fano

s_2	0.4	0			
s_4	0.25	1	0		
s_5	0.2	1	1	0	
s_3	0.1	1	1	1	0
s_1	0.05	1	1	1	1

CumSum $* 2^{\ell_i}$ => in binary with ℓ_i bits

$0 \cdot 2^2 = 0 \Rightarrow 00$

$0.4 * 2^2 = 1.\cancel{6} \Rightarrow 01$

$0.65 * 2^3 = 5.\cancel{2} \Rightarrow 101$

$0.85 * 2^4 = 13.\cancel{6} \Rightarrow 1101$ (8 4 2 1)

$0.95 \times 2^5 = 30.\cancel{4} \Rightarrow 11110$ (16 8 4 2)

Huffman

Code							
0	s_2	0.4	0.4	0.4		0.6	1
11	s_4	0.25	0.25	0.35 (10)		0.4	0
100	s_5	0.2	0.2 (100)	0.25 (11)			
1011	s_3	0.1 (1011)	0.15 (101)				
1010	s_1	0.05 (1010)					

$\bar{\ell}_S = 0.4 \cdot 2 + 0.25 \cdot 2 + 0.2 \cdot 3 + 0.1 \cdot 4 + 0.05 \cdot 5 = 2.55\,b$

$\bar{\ell}_{SF} = 0.4 \cdot 1 + 0.25 \cdot 2 + 0.2 \cdot 3 + 0.1 \cdot 4 + 0.05 \cdot 4 = 2.1\,b$

$\bar{\ell}_H = 0.4 \cdot 1 + 0.25 \cdot 2 + 0.2 \cdot 3 + 0.1 \cdot 4 + 0.05 \cdot 4 = 2.1\,b$

b). $\eta_H = \dfrac{H(S)}{\bar{\ell}} = \dfrac{2.04}{2.1} = \dots$

$\rho_H = 1 - \eta = 1 - \dots = \dots$

$$H(S) = -0.4 \log 0.4 - 0.25 \log 0.25 - 0.2 \log 0.2 - 0.1 \log 0.1 - 0.05 \cdot \log 0.05 = 2.04$$

c). $p(0) = ? = \dfrac{\bar{\ell}_0}{\bar{\ell}} = \dfrac{1}{2.1}$ $\qquad \bar{\ell}_0 = 0.4 \cdot 1 + 0.25 \cdot 0 + 0.2 \cdot 2 + 0.1 \cdot 1 + 0.05 \cdot 2 = 1\,b$

$p(1) = ? = \dfrac{\bar{\ell}_1}{\bar{\ell}} = \dfrac{1.1}{2.1}$ $\qquad \bar{\ell}_1 = \bar{\ell} - \bar{\ell}_0 = 2.1 - 1 = 1.1\,b$

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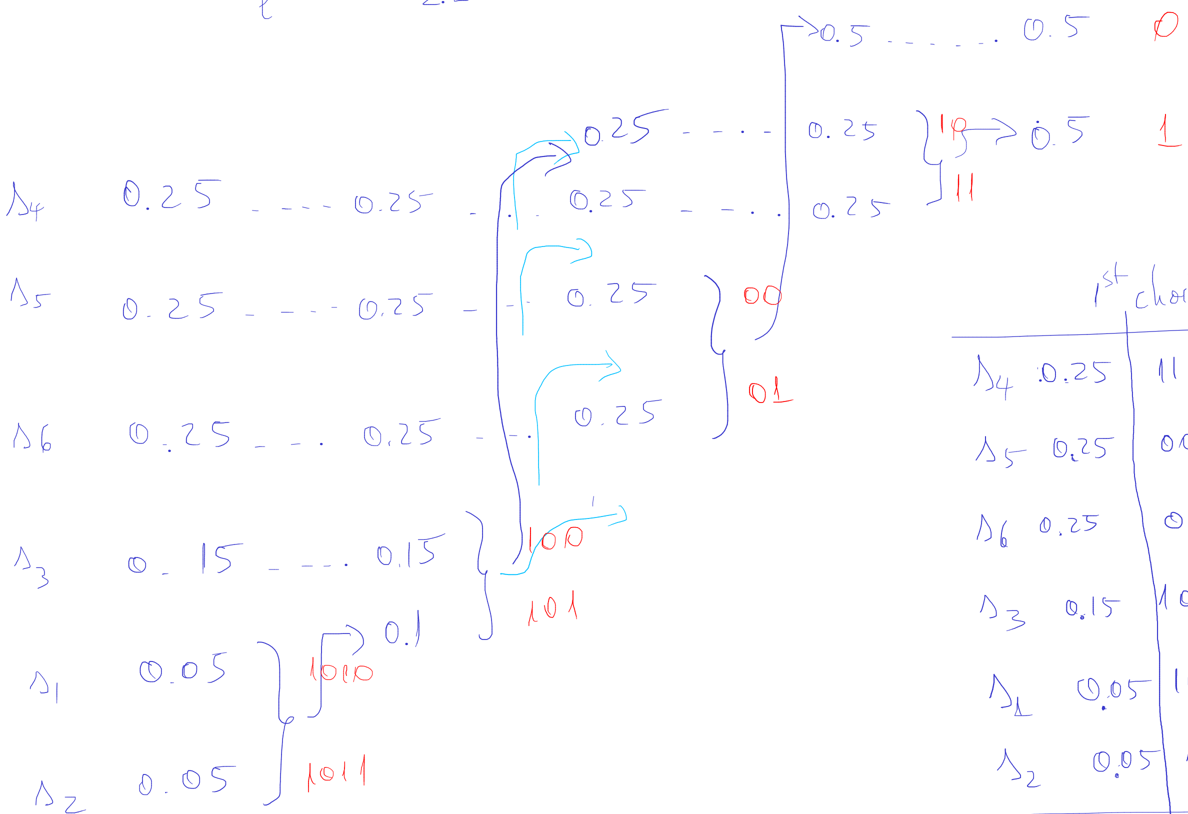

		1st choice	2nd choice
s_4	0.25	11	10
s_5	0.25	00	11
s_6	0.25	01	00
s_3	0.15	100	010
s_1	0.05	1010	0110
s_2	0.05	1011	0111