

Cyclic codes

$$k = 10$$

$$m - k = 3$$

$\Rightarrow |m = 13|$ in the code word

1

$$i = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$g(x) = 1 \oplus x \oplus x^3$$

$$i(x) = 1 \oplus x^2 \oplus x^6 \oplus x^7$$

Non-syst: $c(x) = i(x) \cdot g(x) = 1 \oplus x \oplus x^2 \oplus x^3 \oplus x^5 \oplus x^6 \oplus x^7 \oplus x^8 \oplus x^9 \oplus x^{10}$
 $= 1 \oplus x \oplus x^2 \oplus x^3 \oplus x^5 \oplus x^6 \oplus x^8 \oplus x^9 \oplus x^{10} \oplus 0 \cdot x^{11} \oplus 0 \cdot x^{12} ?$

$$c = \underbrace{[1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0]}_{m=13}$$

Systematic: $c(x) = x^{m-k} \cdot i(x) \oplus b(x)$

$$x^3 \cdot i(x) = x^3 \oplus x^5 \oplus x^9 \oplus x^{10}$$

$$c(x) = 1 \oplus x \oplus x^3 \oplus x^5 \oplus x^9 \oplus x^{10}$$

$$c = \underbrace{[1 \ 1 \ 0]}_{b(x)} \underbrace{1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0}_i$$

000

$$\begin{array}{r} x^{10} \oplus x^9 \oplus x^5 \oplus x^3 \\ \underline{x^{10} \oplus x^8 \oplus x^7} \\ x^8 \oplus x^6 \oplus x^5 \oplus x^3 \\ \underline{x^8 \oplus x^6 \oplus x^5} \\ x^3 \\ \underline{x^3 \oplus x \oplus 1} \\ x \oplus 1 \\ \underline{x \oplus 1} \\ 0 \end{array}$$

Factor: $g = 1 \ 0 \ 1 \ 1$

$$\begin{array}{r|l} 0011000101 & 000 \\ \underline{1011} & \\ 01110101 & \\ \underline{1011} & \\ 101101 & \\ \underline{1011} & \\ 000001 & \\ \underline{1011} & \\ 011 & \end{array}$$

$$c = \underbrace{[1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0]}_{\text{CRC} \quad i}$$

2

3

$$k = 8$$

$$R = \frac{k}{n} \leq 0.6$$

$$\text{degree of } g(x) = m - k$$

$$\text{Answer } (m - k) \geq 6$$

$$\Rightarrow \frac{k}{n} \leq 0.6 \Leftrightarrow \frac{8}{n} \leq 0.6 \Leftrightarrow n \geq 13.3 \Rightarrow n \geq 14$$

$$8 \leq 0.6m \Rightarrow$$

$$\frac{8}{0.6} \leq m$$

$$m \geq 13.3 \Rightarrow m \geq 14$$

you need $g(x)$ with degree ≥ 6