Exercises Week 10

Information Theory

1. For the following code, do the following:
   a. compute the minimum Hamming distance and indicate how many errors it can detect and how many errors it can correct, with the nearest neighbor decoding algorithm;
   b. Give an example of errors the code cannot detect, and another example of errors it can detect but it cannot correct, using the nearest neighbor decoding algorithm.

Message	Codeword
s_1	$c_1 = 00000$
s_2	$c_2 = 10011$
s_3	$c_3 = 11100$
s_3	$c_4 = 00111$

2. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to detect 3 errors in a codeword.

3. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to correct 2 errors in a codeword.

4. If we send one codeword of the previous code from exercise 3 over a Binary Symmetric Channel with error probability $p = 0.1\%$, what is the probability that nearest neighbor error correction algorithms fails?

   Hint: compute the probability that we get 3, 4 or more errors in codeword.

5. Prove the following property of Hamming distance:

$$d_H(\mathbf{a}, \mathbf{b}) + d_H(\mathbf{b}, \mathbf{c}) \geq d_H(\mathbf{a}, \mathbf{c}),$$

   where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are sequences of N bits.