① $H(S) = 20\,b$ Shannon coding $\Rightarrow \bar{\ell}$

a) $\eta = ?$ $\eta = \frac{H(S)}{\bar{\ell}}$ $\eta \in \left(\frac{20}{21} , \frac{20}{20} \right] \Leftrightarrow \eta > \frac{20}{21}$

b) $\rho = ?$ $\rho = 1 - \eta$ $\rho = \left[0 , 1 - \frac{20}{21} \right)$

c). 2 messages $\Rightarrow H_{max} = 1\,b$

10 messages $\Rightarrow H_{max} = \log_2(10)$ $\left(\frac{1}{10} \; \frac{1}{10} \; \dots \; \frac{1}{10} \right) \Rightarrow$

n messages $\Rightarrow H_{max} = \log_2(n)$ $\left(\frac{1}{n} \; \frac{1}{n} \; \dots \; \frac{1}{n} \right)$

1000 messages $\Rightarrow H_{max} = \log_2(1000) \simeq 10$

X messages $\Rightarrow H_{max} = \log_2(X) \geq 20\,b$

$\Rightarrow X \geq 2^{20} = 1.000.000$ messages

----- .... --- (20 bits)

② $S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ 0.05 & 0.4 & 0.1 & 0.25 & 0.2 \end{pmatrix}$

Shannon

$l_i = \lceil \log_2(p_i) \rceil$

s_2	0.4	2	00
s_4	0.25	2	01
s_5	0.2	3	100
s_3	0.1	4	1010
s_1	0.05	5	1011

Shannon – Fano

s_2	0.4	0			
s_4	0.25	1	0		
s_5	0.2	1	1	0	
s_3	0.1	1	1	1	0
s_1	0.05	1	1	1	1

Huffman:

s_2 0.4 ..... 0.4 ..... 0.4 ..... 0.4 → 0.6 → 0 / 1

s_4 0.25 ..... 0.25 ..... 0.25 → 0.35 → 00 / 01

s_5 0.2 ..... 0.2 → 000 / 001

s_3 0.1 → 0.15 → 0010 / 0011

s_1 0.05 → 0010 / 0011

s_1	0011
s_2	1
s_3	0010
s_4	01
s_5	000

b) $\overline{\ell} = \sum p(A_i) \cdot \ell_i = 0.05 \cdot 4 + 0.4 \cdot 1 + 0.1 \cdot 4 +$

$+ 0.25 \cdot 2 + 0.2 \cdot 3 = 2.1\,b$

$H(S) = 2.04\,b \qquad \eta = \frac{2.04}{2.1} \qquad S = 1 - \eta$

c) $\overline{\ell_0} = 0.05 \cdot 2 + 0.4 \cdot 0 + 0.1 \cdot 3 + 0.25 \cdot 1 + 0.2 \cdot 3 = 1.25\,b$

$\overline{\ell_1} = 0.05 \cdot 2 + 0.4 \cdot 1 + 0.1 \cdot 1 + 0.25 \cdot 1 + 0.2 \cdot 0 = 0.85$

$$p_0 = \frac{1.25}{2.1} = 0.59$$

$$p_1 = \frac{0.85}{2.1} = 0.41$$