

①  $\Theta = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$$1 \cdot 1 \oplus 0 \cdot 0 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 1$$

$$\frac{1 \cdot 0 \oplus 0 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0}{0} = 0$$

a)  $i = [1011]$

$$i \cdot \Theta = c$$

$\underbrace{[1 \ 0 \ 1 \ 1]}_i \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = \underbrace{[1 \ 0 \ 1 \ 1]}_i \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = c$

b).

$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$   
 $\underbrace{\hspace{10em}}_{I_{3 \times 3}}$

c)  $R = 1010111$

$$1 \cdot 1 + 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 +$$

$$1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = 1$$

$\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_z = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}}_H \cdot \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}}_{R^T} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{H \cdot R^T}$

$z \neq 0$  error

e	$z^T$
$[10000000]$	$[110]^T$
$01000000$	$111$
$\vdots$	$\vdots$
$0000100$	$[100]$

$\Rightarrow$  error at pos 5  
 $C = 101011$   
 $i = 1010$

d).  $e_d = 2$

$e_c = 1$

$$0 = c_4 \oplus i_5 \oplus i_6 \oplus i_7$$

$$0 = 0 \oplus c_2 + 1 \cdot (c_2 \oplus i_3 \oplus i_6 \oplus i_7)$$

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$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ i_3 \\ c_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}$$

$$0 = c_1 \oplus i_3 \oplus i_5 \oplus i_7$$

$$i_A = [1001] \rightarrow c_H = [0011001]$$

Hamming SECDED (8,4)

$$i_B = [1110] \rightarrow c_B = [0010110] \Rightarrow \left[ \begin{array}{c} 10010110 \\ \hline P \end{array} \right]$$

Hamming (7,4) parity

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$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$z \neq 0 \Rightarrow$  Error

$$(110)_2 = 6 \Rightarrow \text{error pe poz. 6 } (i_6)$$

correct = 10110101  
 $c_1 c_2 i_3 c_4 i_5 i_6 i_7$

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$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$z^T$

$z = 0 \Rightarrow$  no errors

Alternative:

$$z = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 1) \text{ Sunt erori } \checkmark \\ 2) z_0 = 1 \Rightarrow \underline{1 \text{ eroare}} \\ 3) \cdot \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} = \text{pozitia erorii} \end{matrix}$$

poz = 0 (C0)

$$z = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} 1) \text{ Sunt erori} \\ 2) z_0 = 1 \Rightarrow 1 \text{ eroare} \\ 3) \text{ Poz erorii } = 3 \end{matrix}$$

(13)

$$z = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 1) \text{ Sunt erori} \\ 2) z_0 = 0 \Rightarrow \underline{2 \text{ erori}} \\ \text{Nu putem sa le corectam} \end{matrix}$$

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Hamming (15, 11)

	H											13	14	15
												x	x	
0 0 0 0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
	0	0	0	1	1	1	1	0	0	0	0	1	1	1
	0	1	1	0	0	1	1	0	0	1	1	0	0	1
	1	0	1	0	1	0	1	0	1	0	1	0	1	0
	x x x													

a) b)

$$e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \quad b) \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

grupul de  
a) erori pe poz 1, 2, 3  
nu pot fi det

b) erori pe poz 1 si 2

$$\Rightarrow z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

erori pe poz 13, si 14

$$z = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$z \neq 0 \Rightarrow$  detectate  
acelasi  $z \Rightarrow$  nu pot fi det