Discrete Memoryless Sources

Information Theory Exercises

1. Consider the following game: I think of a number between 1 and 8, and you have to guess it by asking yes/no questions.

   - How much uncertainty does the problem have?
   - How is the best way to ask questions? Why?
   - What if the questions are not asked in the best way?
   - On average, what is the number of questions required to find the number?

2. Guess the number: what is the optimal decision tree for guessing a number chosen according to the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

3. Guess the number: what is the optimal decision tree for guessing a number chosen according to the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ 0.14 & 0.29 & 0.4 & 0.17 \end{pmatrix}$$

4. A DMS has the following distribution

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ \frac{1}{2} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \end{pmatrix}$$

   a. Compute the information of message s_1, s_2 and s_3
   b. Compute the average information of a message
   c. Compute the efficiency, absolute redundancy and relative redundancy of the source

5. Compute the KL distance between the following two probability distributions:

$$P = [0 \quad 0 \quad 0 \quad 1], \qquad Q = [0.1 \quad 0.05 \quad 0.05 \quad 0.8]$$