Information Theory Exam

Nr.1

Exercises (16p)

1. A discrete, complete and memoryless source has the following distribution:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

   a. (1p) Compute the entropy of the source;
   b. (1p) Compute the efficiency and redundancy of this source;
   c. (1p) Write the messages and the probabilities of its second order extension, S^2.

3. For the source in Exercise 1:

   a. (1p) Find a Shannon code and draw the graph of the code
   b. (1p) Encode the following sequence:

$$s_3 s_1 s_2 s_4 s_2 s_2 s_2$$

3. Consider a systematic code with parity-check matrix

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   a. (1p) Write the corresponding generator matrix
   b. (2p) We receive the sequence $\mathbf{r} = [0111100]$. Find if there are errors in the received data, and, if yes, perform correction.
   c. (1p) Which bits are information bits, and which are control bits?

3. (3p) Find the systematic cyclic codeword for the sequence $\mathbf{i} = [1011]$, considering a cyclic code with generator polynomial $g(x) = 1 \oplus x \oplus x^3$.

4. Consider a communication with the following **joint probability matrix**:

$$P(x_i \cap y_j) = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} \end{bmatrix}$$

   a. (1p) Compute the marginal probabilities and the marginal entropies $H(X)$ and $H(Y)$;
   b. (1p) Find the matrix $P(Y|X)$, the matrix $P(X|Y)$, and draw the graph of the channel;
   c. (2p) Compute the mutual information $I(X, Y)$, and draw the geometrical representation.

Theory questions (17p)

1. (2p) What is the difference between an information source *with memory* and a *memoryless* one?

2. (3p) Prove that the entropy of a 2-nd order extension, $H(S^2)$, is 2 times larger than $H(S)$:

$$H(S^2) = 2 \cdot H(S)$$

3. (2p) Show that the following code is not uniquely-decodable

Message	Codeword
s_1	10
s_2	101
s_3	01
s_4	00

4. (2p) A source is encoded with the following code. The code is optimal (efficiency $\eta = 1$). What are the probabilities of the messages, $p(s_i)$?

Message	Codeword
s_1	100
s_2	101
s_3	11
s_4	0

5. (2p) Consider the Hamming(31,26) code. How many bits does the codeword have? How many of them are information bits, how many of them are parity bits?

6. (4p) Cyclic codes: Prove that any cyclic shift of a codeword is also a codeword

7. (2p) Consider two different joint probability matrices $P_A(X, Y)$ and $P_B(X, Y)$. Which one is better for a communication process? Justify your answer.

$$P_A(X,Y) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.2 \end{bmatrix} \qquad P_B(X,Y) = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}$$

Notes

- You start with 3p ("punctul din oficiu").
- Grade = total points / 2 (e.g. 30p = grade 10, 15p = grade 5 etc).
- Time available: 2h