Exercises Week 4

Information Theory

1. For the first two exercises of last week, also compute the equivocation $H(X|Y)$, the mutual information $I(X, Y)$, and draw the geometrical representation.
2. A communication process has the following joint probability matrix:

$$P(x_i \cap y_j) = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

   a. Find the noise matrix $P(Y|X)$ and the channel matrix P(X|Y)
   b. Compute the average information obtained on the input X when output symbol y_2 is received
   c. Compute the uncertainty remaining over the input X when output symbol y_2 is received

3. At the input of a binary symmetric channel with the following channel matrix

$$P(y_j|x_i) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

   we apply two inputs x_1 and x_2 with probabilities $p(x_1) = \frac{3}{4}$ and $p(x_2) = \frac{1}{4}$.

   a. Draw the graph of the channel
   b. Find $H(X)$, $H(Y)$ and $I(X, Y)$

4. Find the mutual information for a channel defined by

$$P(y_j|x_i) = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \end{bmatrix}$$

   if the input probabilities are $p(x_1) = \frac{1}{2}$, $p(x_2) = \frac{1}{4}$ and $p(x_3) = \frac{1}{4}$.