Memoryless Sources

1, 2, 3) Done during lecture

$$\text{exponential} > \text{poly} > \text{log}$$
$$a^p \qquad p^{\alpha} \qquad \log_m P$$

$$\lim_{p \to 0} \underbrace{p \cdot \log_2 p}_{\text{poly} \quad \text{log}} = \lim_{p \to \infty} \frac{p}{\left(\frac{1}{\log_2 p}\right)} =$$

$$= \lim_{p \to 0} \frac{\overline{P}}{(-1) \frac{1}{\log_2 p} \cdot \frac{1}{p}}$$

4). $S: \begin{pmatrix} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 & \Delta_5 \\ 1/2 & 0 & 1/8 & 1/4 & 1/8 \end{pmatrix}$

a) $i(\Delta_1) = -\log_2 \frac{1}{2} = \log_2 2 = 1\,b$

$i(\Delta_2) = -\log_2(0) = \infty$
$\underbrace{\qquad}_{-\infty}$

$i(\Delta_3) = -\log_2 \frac{1}{8} = 3\,b$

b. $H(S) = -\sum_i p_i \log_2 p_i = -\frac{1}{2}\underbrace{\log \frac{1}{2}}_{-1} - 0 \cdot \underbrace{\log 0}_{0} - \frac{1}{8}\underbrace{\log \frac{1}{8}}_{-3} - \frac{1}{4}\underbrace{\log \frac{1}{4}}_{-2} - \frac{1}{8}\underbrace{\log \frac{1}{8}}_{-3}$

$$\underbrace{}_{1/2} \quad \underbrace{}_{3/8} \quad \underbrace{}_{1/2} \quad \underbrace{}_{3/8}$$

$$= \frac{1}{2} + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = \frac{7}{4} = 1.75\,b$$

c). $\eta = \frac{H(S)}{H_{max}} = \frac{1.75}{\log_2 5} = 0.75 = 75\%$

$\rho = 1 - \eta = 0.25$

$R = H_{max} - H(S) = 2.32 - 1.75 = 0.57\,b$

5) $\underset{\text{D(P,Q)}}{D} = \sum_i p_i \log_2 \frac{p_i}{q_i} = 0 \cdot \underbrace{\log \frac{0}{0.1}}_{0} + 0 \cdot \underbrace{\log \frac{0}{0.05}}_{0} + 0 \cdot \underbrace{\log \frac{0}{0.05}}_{0} + 1 \cdot \underbrace{\log \frac{1}{0.8}}_{} = 0.32$

$P = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$Q = \begin{bmatrix} 0.1 & 0.05 & 0.05 & 0.8 \\ q_1 & q_2 & q_3 & q_4 \end{bmatrix}$

$D(P,Q) \neq D(Q,P)$