Exercises Week 11

Information Theory

1. Consider a code with generator matrix

$$[G] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   a. Compute the codewords $\mathbf{c_1}$ and $\mathbf{c_2}$ for transmitting the information words $\mathbf{i_1} = [1001]$ and $\mathbf{i_2} = [1110]$
   b. Compute $\mathbf{c_3} = \mathbf{c_1} \oplus \mathbf{c_2}$. Is this a codeword? If yes, what is its corresponding information word $\mathbf{i_3}$?

2. Alice builds codewords $\mathbf{c}'$ in the following way: it computes the codeword $\mathbf{c}$ with $[G]$ from Exercise 1, and then it writes all the bits $\mathbf{c}$ in the opposite order.

   a. Is this also a linear block code?
   b. If yes, what is its generator matrix? If no, why?
   c. Similar question: Bob doesn't reverse $\mathbf{c}$, but instead it always removes the third bit from $\mathbf{c}$. Repeat the two questions.

3. Consider a systematic code with generator matrix

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   a. Compute the codewords $\mathbf{c_1}$ and $\mathbf{c_2}$ for transmitting the information words $\mathbf{i_1} = [0100]$ and $\mathbf{i_2} = [1011]$
   b. Compute the parity-check matrix $[H]$ and check that $\mathbf{0} = [H] \cdot \mathbf{c_1}^T$ and $\mathbf{0} = [H] \cdot \mathbf{c_2}^T$.

4. Consider a systematic code with parity-check matrix

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

We receive two sequences $\mathbf{r_1} = [0101100]$ and $\mathbf{r_2} = [1101100]$. Find if there are errors in the received data, and, if yes, perform correction and retrieve the transmitted information bits.