Exercises, Week 11

1).d)- $H = \left[\begin{array}{ccc|cccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$ $Q^T \mid I_3$ — 3×7

Check the conditions in the lecture:

⇒ Errors detected: 1 ✓ 2 ✓ ~~3 ✗~~ : $\begin{bmatrix}1\\1\\0\end{bmatrix} \oplus \begin{bmatrix}1\\1\\1\end{bmatrix} \oplus \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ (1 + 2 + 7)

Errors corrected: 1 ✓ ~~2~~

(2) $i_1 = [1\ 0\ 0\ 1]$ — Hamming (7, 4)

$H = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$

$c = [c_1\ c_2\ i_3\ c_4\ i_5\ i_6\ i_7]$

$0 = H \cdot c^T$

$\begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix}0\\0\\1\\1\\0\\0\\1\end{bmatrix} \begin{matrix} c_1 \\ c_2 \\ i_3 \\ c_4 \\ i_5 \\ i_6 \\ i_7 \end{matrix}$

$\begin{cases} 0 = 0 \cdot c_1 \oplus c_4 \oplus i_5 \oplus i_6 \oplus i_7 \Rightarrow c_4 = i_5 \oplus i_6 \oplus i_7 = 1 \\ 0 = c_2 \oplus i_3 \oplus i_6 \oplus i_7 \Rightarrow c_2 = i_3 \oplus i_6 \oplus i_7 = 0 \\ 0 = c_1 \oplus i_3 \oplus i_5 \oplus i_7 \Rightarrow c_1 = i_3 \oplus i_5 \oplus i_7 = 0 \end{cases}$ (1 0 1)

$i = [1\ 0\ 0\ 1] \longrightarrow c = [0\ 0\ 1\ 1\ 0\ 0\ 1]$

(3) $r = [1\ 0\ 1\ 0\ 1\ 1\ 1]$, Hamming (7, 4)

$z = \begin{bmatrix}1\\1\\0\end{bmatrix} = H \cdot r^T$ (1) (2) (3) (4) (5) (6) (7)

$\begin{bmatrix}0\\1\\0\end{bmatrix} = H \cdot e^T$

$z \neq 0 \Rightarrow$ We have errors

Correction:

e	z
~~[1 0 0 0 0 0 0]~~	~~[0 0 1]~~
~~[0 1 0 0 0 0 0]~~	~~[0 1 0]~~
⋮	
[0 0 0 0 0 1 0]	[1 1 0]

not necessary

$110_{\text{binary}} = 6_{(\text{base } 10)}$ ⇒ error on position 6 ⇒ $c = [1\ 0\ 1\ 0\ 1\ 0\ 1]$ (0 correct)

$c_1\ c_2\ i_3\ c_4\ i_5\ i_6\ i_7$

$\hat{i} = [1\ 1\ 0\ 1]$