Exercises Week 13 (Lecture 12)

Ex. Set 6:

$c = [\,c_0\ c_1\ c_2\ i_3\ c_4\ i_5\ i_6\ i_7\,]$ $= [\,c_0\ c_1\ c_2\ 1\ c_4\ 1\ 1\ 0\,]$

(2) SECDED: $i = [\,1\ 1\ 1\ 0\,]$ ($i_3\ i_5\ i_6\ i_7$) — Hamming (8,4) SECDED

$$\tilde{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & & & H & & & & \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}}_{\tilde{H}} \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ 1 \\ c_4 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 0 \\ 0 \\ \\ 0 \\ \\ \\ \end{matrix}$$

$0 = c_0 \oplus c_1 \oplus c_2 \oplus i_3 \oplus c_4 \oplus i_5 \oplus i_6 \oplus i_7 \Rightarrow$

$\Rightarrow c_0 = c_1 \oplus c_2 \oplus i_3 \oplus c_4 \oplus i_5 \oplus i_6 \oplus i_7 = 1$

$0 = \overbrace{c_4 \oplus 1 \oplus 1 \oplus 0}^{i_5\ i_6\ i_7} \Rightarrow c_4 = 0$

$0 = \overbrace{c_2 \oplus 1 \oplus 1 \oplus 0}^{i_3\ i_6\ i_7} \Rightarrow c_2 = 0$

$0 = \overbrace{c_1 \oplus 1 \oplus 1 \oplus 0}^{i_3\ i_5\ i_7} \Rightarrow c_1 = 0$

$c_4 = i_5 \oplus i_6 \oplus i_7$

$c_2 = i_3 \oplus i_6 \oplus i_7$

$c_1 = i_3 \oplus i_5 \oplus i_7$

$\Rightarrow c = [\,1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\,]$ (1 0 0: control bits; 1 0 1 0: inf. bits)

(4) $r = [\,1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\,]$ — Hamming (8,4) SECDED

$$\tilde{z} = \tilde{H} \cdot r^T \Rightarrow \begin{matrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}}_{\tilde{H}} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \leftarrow \text{inf bits}$$

z $\tilde{H}$ r^T

$z = [\,0\ 0\ 0\,]^T \Rightarrow$ No errors!

$\Downarrow$

$i = [\,0\ 0\ 1\ 0\,]$

What if: $r = [\,0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\,]$

$$\begin{matrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{matrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}}_{\tilde{H}} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$\tilde{z}$ $\tilde{H}$

$\tilde{z} = [\,1\ 0\ 0\ 0\,]^T \Rightarrow$ we have errors $\Rightarrow z_0 = 1$ (1 error) $\Rightarrow$ position of error is $(0\,0\,0)_{(2)} = (0)_{(10)} \Rightarrow$

$\Rightarrow$ true codeword $= [\,1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\,] \Rightarrow$ inf. bits $= 0\,0\,1\,0$

What if $r = [10110010]$?

$$\begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1 \\ 0&1&0&1&0&1&0&1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$\tilde{z}$ $\qquad$ $\tilde{H}$

$\tilde{z} \neq 0$ We have errors! $\Rightarrow$ $z_0 = 0$ 2 errors! $\Rightarrow$ Can't fix them.

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(5) Hamming $(15, 11)$ (n, k)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1 \\ 1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{bmatrix} \cdot \begin{bmatrix} e \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{15}$$

H

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1 \\ 1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{bmatrix} \cdot \begin{bmatrix} e \\ 1 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}_{15}$$

H