Exercises Week 9

Information Theory

1. For the following code, compute the minimum Hamming distance and indicate how many errors it can detect and how many errors it can correct, with the nearest neighbor decoding algorithm.
   a. Give an example of errors the code cannot detect, and another example of errors it can detect but it cannot correct, using the nearest neighbor decoding algorithm.

Message	Codeword
s_1	$c_1 = 00000$
s_2	$c_2 = 10011$
s_3	$c_3 = 11100$
s_3	$c_4 = 00111$

2. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to detect 3 errors in a codeword.

3. Design a block code consisting of 4 codewords, with minimum codeword length, which is able to correct 2 errors in a codeword.

4. Prove the following property of Hamming distance:

$$d_H(\mathbf{a}, \mathbf{b}) + d_H(\mathbf{b}, \mathbf{c}) \geq d_H(\mathbf{a}, \mathbf{c}),$$

   where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are sequences of N bits.