Exercises Week 5

Information Theory

1. For the following channels, compute the channel capacity and the input probabilities required to reach it:

   a. Noiseless binary channel
   b. Noisy binary non-overlapping channel
   c. "Noisy typewriter"
   d. Binary symmetric channel
   e. Binary erasure channel
   f. Symmetric channel of order n

2. Consider a discrete transmission channel with the following channel matrix:

$$P(Y|X) = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \end{bmatrix}$$

   a. Draw the graph of the channel, and compute the equivocation $H(X|Y)$;
   b. If $p(x_1) = \frac{1}{2}$, $p(x_2) = \frac{1}{4}$ and $p(x_3) = \frac{1}{4}$, compute the average information transmitted on the channel $I(X, Y)$;
   c. Compute the redundancy and the efficiency of the channel.

3. Two binary channels are connected in series, as in the picture below:

   [see on whiteboard]

   a. Find q such that the equivalent channel is a binary symmetric channel;
   b. Find the channel capacity of the equivalent channel from a).

4. A logical AND gate with two inputs can be considered a transmission channel, with four input symbols $x_1 = 00$, $x_2 = 01$, $x_3 = 10$, $x_4 = 11$, and two output symbols $y_1 = 0$, $y_2 = 1$.

   a. If $p(x_1) = p(x_2) = p(x_3) = p(x_4) = \frac{1}{4}$, find the average information transmitted on this channel;
   b. Find the channel capacity and the probabilities of the inputs required to reach it.