

① $H(S) = 2.06$ Shannon coding $\Rightarrow \bar{\ell}$

a) $\eta = ?$ $\eta = \frac{H(S)}{\bar{\ell}}$ $\eta \in \left(\frac{2.0}{2.1}, \frac{2.0}{2.0} \right] \Rightarrow \eta > \frac{2.0}{2.1}$

b) $\rho = ?$ $\rho = 1 - \eta$ $\rho = \left[0, 1 - \frac{2.0}{2.1} \right)$

c). 2 messages $\Rightarrow H_{\max} = 1 \text{ b}$

10 messages $\Rightarrow H_{\max} = \log_2(10)$ $\left(\frac{1}{10}, \frac{1}{10}, \dots, \frac{1}{10} \right) \Rightarrow$

n messages $\Rightarrow H_{\max} = \log_2(n)$ $\left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n} \right)$

1000 messages $\Rightarrow H_{\max} = \log_2(1000) \approx 10$

X messages $\Rightarrow H_{\max} = \log_2(X) \geq 2.06$

$\Rightarrow X \geq 2^{2.06} = 4.16 \text{ messages}$

$\frac{2.06}{2.0 \text{ bits}}$

② $S: \begin{pmatrix} \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_4 & \Lambda_5 \\ 0.05 & 0.4 & 0.1 & 0.25 & 0.2 \end{pmatrix}$

Shannon

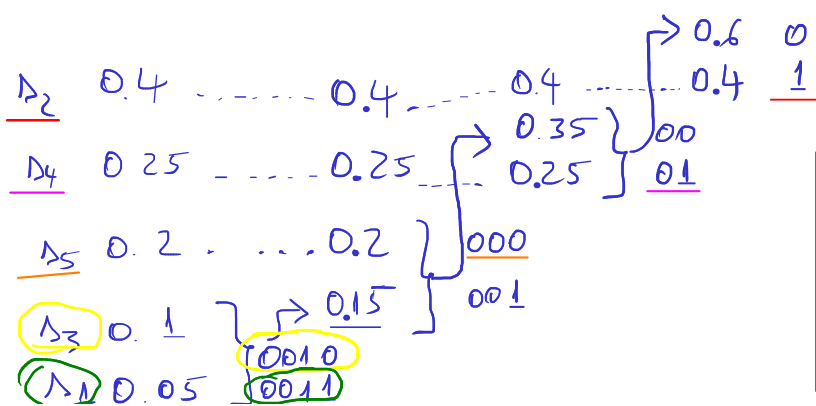
$\ell_i = \lceil \log_2(p_i) \rceil$

Λ_2	0.4	2	00
Λ_4	0.25	2	01
Λ_5	0.2	3	100
Λ_3	0.1	4	1010
Λ_1	0.05	5	10111

Shannon-Fano

Λ_2	0.4	0			
Λ_4	0.25	1	0		
Λ_5	0.2	1	1	0	
Λ_3	0.1	1	1	1	0
Λ_1	0.05	1	1	1	1

Huffman



Λ_1	00111
Λ_2	00
Λ_3	1010
Λ_4	01
Λ_5	100

$$b). \quad \bar{\ell} = \sum p(\Delta_i) \cdot \ell_i = 0.05 \cdot \underline{4} + 0.4 \cdot \underline{1} + 0.1 \cdot \underline{4} + \\ + 0.25 \cdot \underline{2} + 0.2 \cdot \underline{3} = \underline{2.1b}$$

$$H(s) = 2.04b$$

$$\eta = \frac{2.04}{2.1}$$

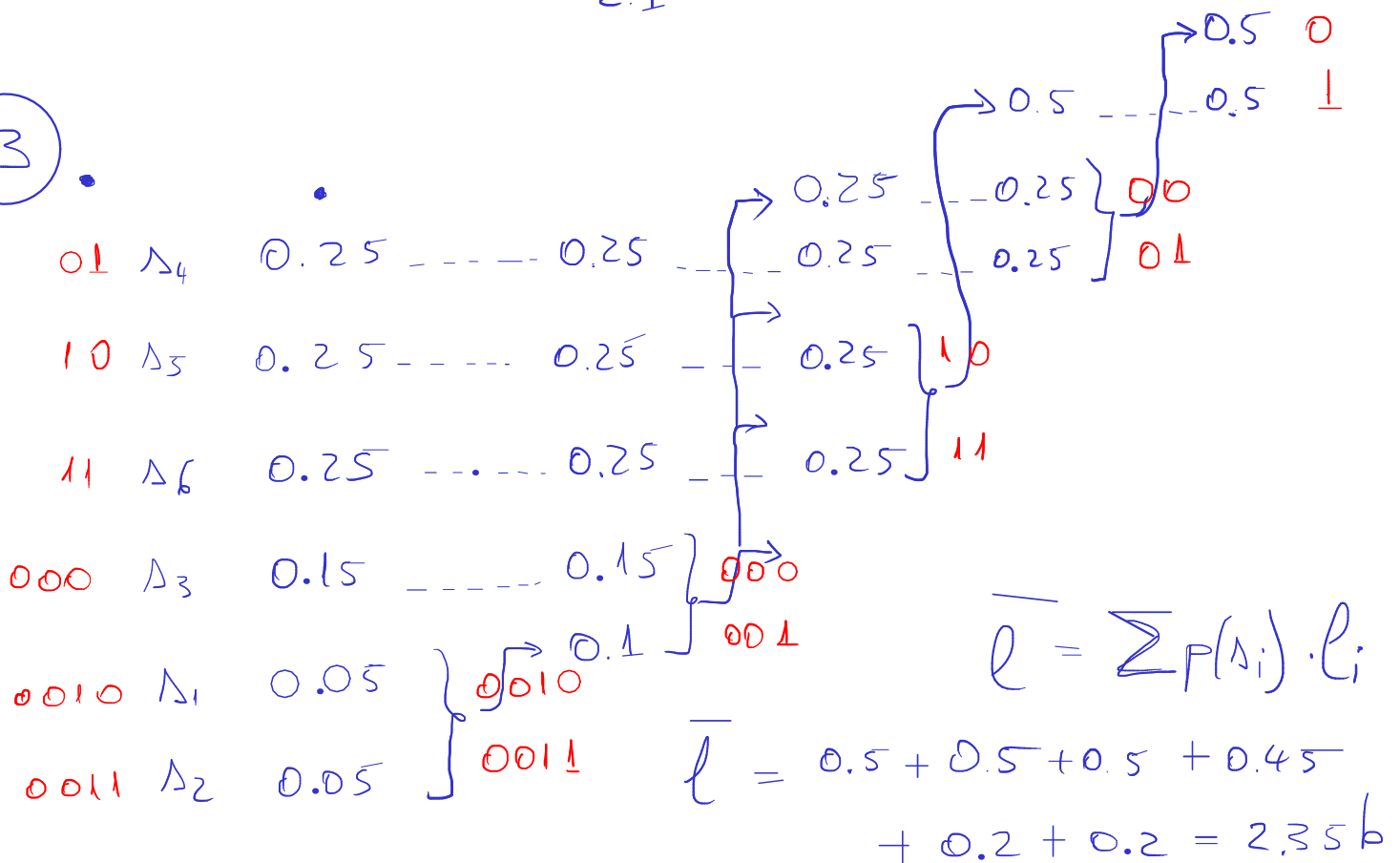
$$\xi = 1 - \eta$$

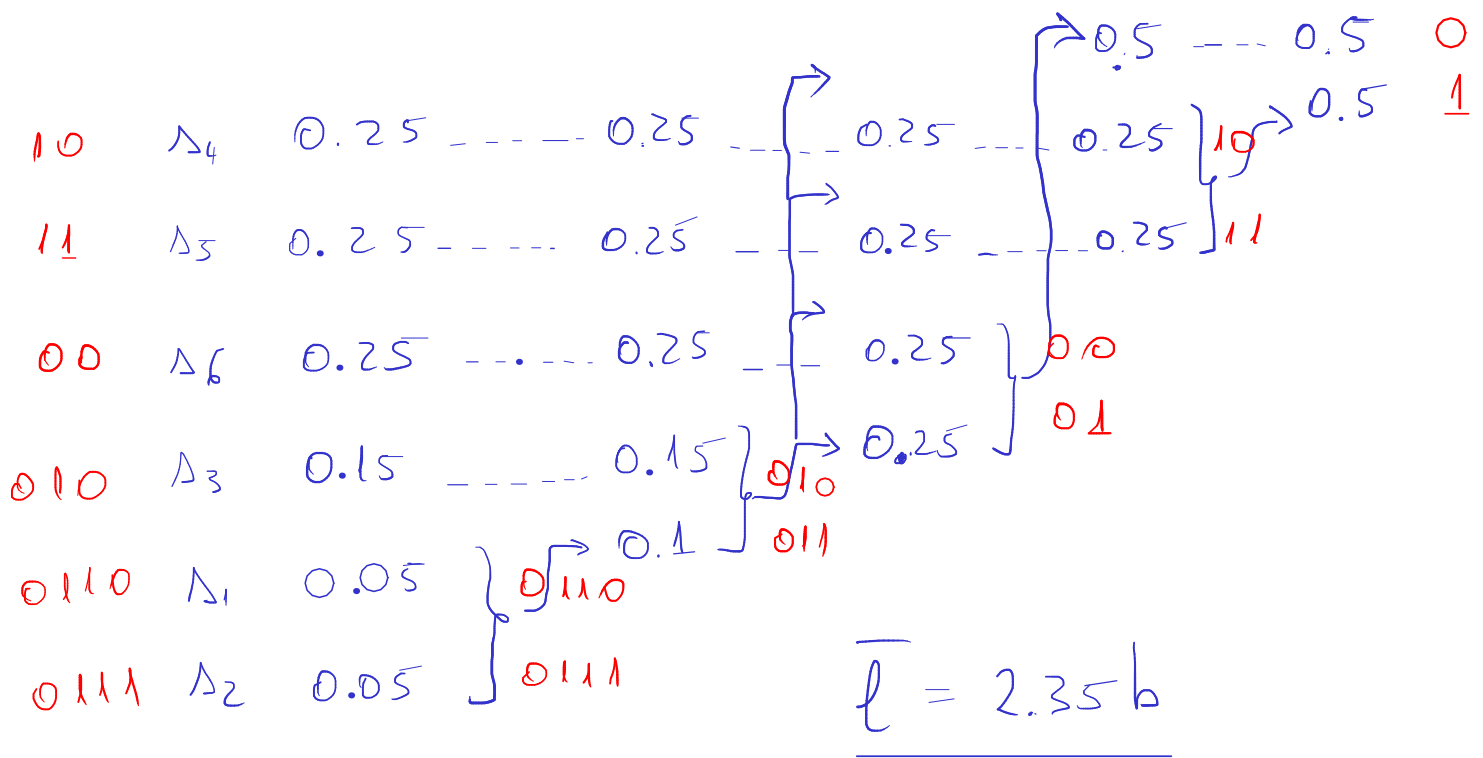
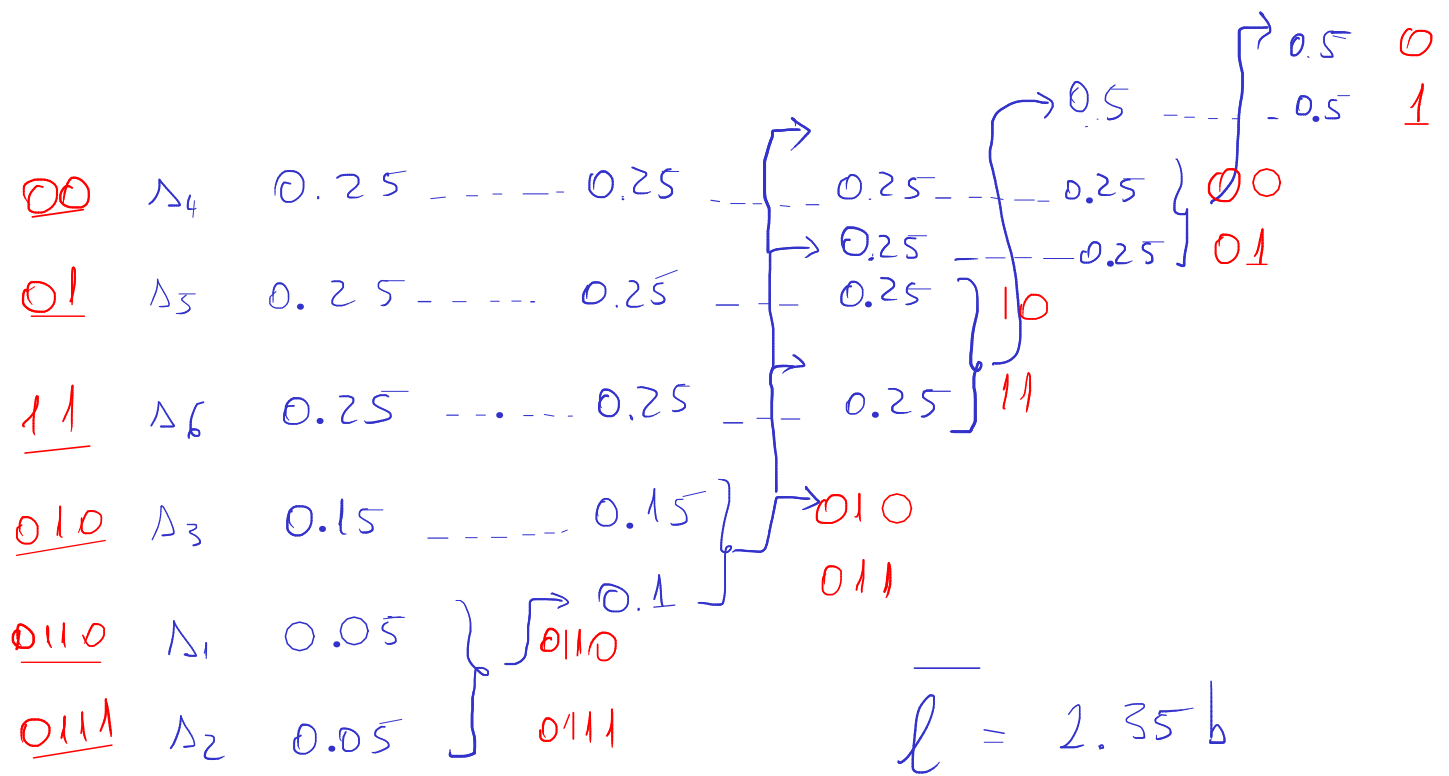
$$c). \quad \begin{aligned} \bar{\ell}_0 &= 0.05 \cdot 2 + 0.4 \cdot 0 + 0.1 \cdot 3 + 0.25 \cdot 1 + 0.2 \cdot 3 = \underline{1.25b} \\ \bar{\ell}_1 &= 0.05 \cdot 2 + 0.4 \cdot 1 + 0.1 \cdot 1 + 0.25 \cdot 1 + 0.2 \cdot 0 = \underline{0.85b} \end{aligned}$$

$$p_0 = \frac{1.25}{2.1} = 0.59$$

$$p_1 = \frac{0.85}{2.1} = 0.41$$

3.





$$S: \begin{pmatrix} \boxed{\Delta_1} & \textcircled{\Delta_2} & \boxed{\Delta_3} \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$$

$$\begin{array}{lcl} \Delta_2 & 0 & 0.7 \\ \Delta_3 & 10 & 0.2 \\ \Delta_4 & 11 & 0.1 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 0.7 \\ 0.3 \\ 11 \end{array}$$

$$\bar{l} = 0.1 \cdot 2 + 0.7 \cdot 1 + 0.2 \cdot 2 = \underline{\underline{1.36}}$$

$\Delta_2 \Delta_2 \Delta_3 \Delta_2 \Delta_2 \Delta_2 \Delta_1 \Delta_3 \Delta_2 \Delta_2$
 $(0010000111000)_{(13 \text{ bits})}$

$$S^2: \begin{pmatrix} \Lambda_1 \Lambda_1 & \Lambda_1 \Lambda_2 & \Lambda_1 \Lambda_3 & \Lambda_2 \Lambda_1 & \Lambda_2 \Lambda_2 & \Lambda_2 \Lambda_3 & \Lambda_3 \Lambda_1 & \Lambda_3 \Lambda_2 & \Lambda_3 \Lambda_3 \\ 0.01 & 0.07 & 0.02 & 0.07 & 0.49 & 0.14 & 0.02 & 0.14 & 0.04 \end{pmatrix}$$

[illegible]

$$\bar{l} = \left(\begin{array}{l} 0.49 + 0.84 + 0.72 \\ + 0.56 + 0.16 + \\ + 0.1 + 0.12 + 0.6 \\ 0.28 \end{array} \right) \cdot 1$$

$$\frac{2.33}{2} \approx 1.165$$

$l = 2.33b$

1

5

Λ_1 0.4 - - - - 0.4 - - - - 0.4 x_1

Λ_2 0.3 - - - - 0.3 - - - - 0.3 x_2

Λ_3 0.2 - - - - 0.2 - - - - 0.2 x_3

Λ_4 0.04 - - - 0.04 $x_4 x_1 \rightarrow 0.1 x_4$

Λ_5 0.03 - - - 0.03 $x_4 x_2$

Λ_6 0.02 - - - 0.02 $x_4 x_3$

Λ_7 0.009 $x_4 x_4 x_1 \rightarrow 0.01 x_4 x_4$

Λ_8 0.001 $x_4 x_4 x_2$

Λ_9 0 $x_4 x_4 x_3$ $8 \rightarrow 5 \rightarrow 2$

Λ_{10} 0 $x_4 x_4 x_4$ $10 \rightarrow 7 \rightarrow 4$ ✓

$\Lambda_1 \Lambda_7 \Lambda_8 \Lambda_3 \Lambda_3 \Lambda_1$

$x_1 x_4 x_4 x_1 x_4 x_4 x_2 x_3 x_3 x_1$