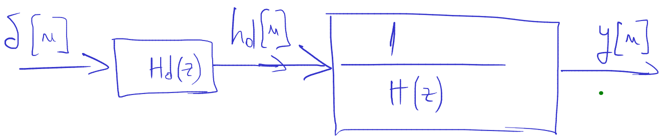
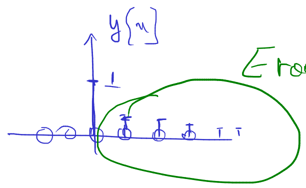
Curs 02 04.03.2021



$$H(z) = \frac{b_0}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}}$$

$H(z) = H_d(z) \Rightarrow y[n] = \delta[n]$

$H(z) \simeq H_d(z) \Rightarrow y[n] \approx \delta[n]$



$$\text{Eroare} = E = \sum_{n=1}^{\infty} (y[n])^2$$

M. celor mai mici pătrate

$$\boxed{b_0 = h_d[0]}$$

$K = 1$: $\displaystyle\sum_{l=1}^{N} a_l \cdot r_{dd}[1, l] = -r_{dd}[1]$

$\Leftrightarrow a_1 \cdot r_{dd}[1,1] + a_2 r_{dd}[1,2] + \ldots + a_N r_{dd}[1,N] = -r_{dd}[1]$

$K = 2$: $a_1 \cdot r_{dd}[2,1] + a_2 r_{dd}[2,2] + \ldots + a_N r_{dd}[2,N] = -r_{dd}[2]$

$\vdots$

$K = N$: $a_1 \cdot r_{dd}[N,1] + a_2 r_{dd}[N,2] + \ldots + a_N r_{dd}[N,N] = -r_{dd}[N]$

$$\begin{bmatrix} r_{dd}[1,1] & r_{dd}[1,2] & \ldots & r_{dd}[1,N] \\ r_{dd}[2,1] & r_{dd}[2,2] & \ldots & r_{dd}[2,N] \\ & \vdots & & \\ r_{dd}[N,1] & r_{dd}[N,2] & \ldots & r_{dd}[N,N] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} -r_{dd}[1] \\ -r_{dd}[2] \\ \vdots \\ -r_{dd}[N] \end{bmatrix}$$

$$r_{dd}[k,l] = \sum_{n=1}^{\infty} h_d[n-k] \cdot h_d[n-l]$$

$$r_{dd}[k] = \sum_{n=1}^{\infty} h_d[n] \cdot h_d[n-k]$$

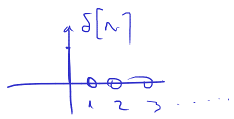
→ Aceleași rezultat, cu metoda pred. liniare:

$$\boxed{H(z) = \frac{b_0}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}}}$$

$y[n] = -a_1 y[n-1] - \ldots - a_N y[n-N] + b_0 \cdot \delta[n]$

$x[n] = \delta[n]$

$y[n] = h[n]$



$h[n] = -a_1 h[n-1] - \ldots - a_N h[n-N] + \underbrace{b_0 \cdot \delta[n]}_{\text{dispare pt } n \geq 1}$

$$\boxed{b_0 = h_d[0]}$$

pt $n \geq 1$:

$$\boxed{h[n] = -a_1 h[n-1] - \ldots - a_N h[n-N]}$$

Predicție liniară pt. $h_d[n]$:→ $\hat{h}_d[n] = -a_1 \cdot h_d[n-1] - \ldots - a_N h_d[n-N]$

$h_d[n] \simeq \hat{h}_d[n]$ $\quad E = \displaystyle\sum_{n=1}^{\infty} \left(h_d[n] - \hat{h}_d[n]\right)^2$ vrem cât mai mic

$$E = \sum_{n=1}^{\infty} \Big(h_d[n] + \underbrace{a_1 h_d[n-1] + \dots + a_N h_d[n-N]}_{\sum a_k h_d[n-k]} \Big)^2$$

Metoda Prony

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

$$y[n] = -a_1 y[n-1] - \dots - a_N y[n-N] + b_0 \cdot x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

$x[n] = \delta[n]$
$y[n] = h[n]$

$$h[n] = -a_1 h[n-1] - \dots - a_N h[n-N] + \underbrace{b_0 \cdot \delta[n] + b_1 \delta[n-1] + \dots + b_M \delta[n-M]}_{\text{dispare pt } n \geq M+1}$$

$$\boxed{b_0 = h_d[0]}$$

pt $n \geq M+1$

$$\boxed{h[n] = -a_1 h[n-1] - \dots - a_N h[n-N]}$$

Predicție liniară pt. $h_d[n]$: $\rightarrow$ $\hat{h}_d[n] = -a_1 \cdot h_d[n-1] - \dots - a_N h_d[n-N]$

$$h_d[n] \simeq \hat{h}_d[n] \qquad E = \sum_{n=M+1}^{\infty} \left(h_d[n] - \hat{h}_d[n] \right)^2$$ vrem cât mai mic

$$E = \sum_{n=M+1}^{\infty} \Big(h_d[n] + \underbrace{\sum_{\ell=1}^{N} a_\ell h_d[n-\ell]}_{(a_1 h_d[n-1] + \dots + a_k h_d[n-k] + \dots a_N h_d[n-N])} \Big)^2$$ = vrem minim

$$\frac{\partial E}{\partial a_k} = 0, \ \forall k = 1, 2, 3, \dots N$$

$$\frac{\partial E}{\partial a_k} = \sum_{n=M+1}^{\infty} \left(\cancel{2} \cdot \left(h_d[n] + \sum_{\ell=1}^{N} a_\ell \cdot h_d[n-\ell] \right) \cdot h_d[n-k] \right) = 0$$

$$= \underbrace{\sum_{n=M+1!}^{\infty} h_d[n] h_d[n-k]}_{r_{dd}[k]} + \underbrace{\sum_{n=M+1}^{\infty} \sum_{\ell=1}^{N} a_\ell h_d[n-\ell] \cdot h_d[n-k]}_{\sum_{\ell=1}^{N} a_\ell \underbrace{\sum_{n=M+1!}^{\infty} h_d[n-\ell] h_d[n-k]}_{r_{dd}[k,\ell]}} = 0$$

$$\boxed{\sum_{\ell=1}^{N} a_\ell \, r_{dd}[k,\ell] = -r_{dd}[k], \quad k = 1, 2, \dots N}$$

$$\Leftrightarrow \begin{bmatrix} r_{dd}[1,1] & r_{dd}[1,2] & \dots & r_{dd}[1,N] \\ r_{dd}[2,1] & r_{dd}[2,2] & \dots & r_{dd}[2,N] \\ \vdots & & & \\ r_{dd}[N,1] & r_{dd}[N,2] & \dots & r_{dd}[N,N] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} -r_{dd}[1] \\ -r_{dd}[2] \\ \vdots \\ -r_{dd}[N] \end{bmatrix}$$

Coef. b_k se obțin ca la metoda Padé

Exemplu: (lab. 3) Prony:

$$H_d(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \qquad M$$

$$h_d[n] = 0, \{\overset{h_d[0]}{1}, \overset{h_d[1]}{2}, \underset{h_d[2]}{3}, 2, 1, 2, \overset{h_d[7]}{3}\}, 0 \ldots$$

$$R_{dd}[k,l] = \sum_{n=M+1}^{\infty} h_d[n-k]\cdot h_d[n-l] \qquad n = 3$$

$$R_{dd}[k] = \sum_{n=3}^{\infty} h_d[n]\cdot h_d[n-k]$$

$$\begin{bmatrix} R_{dd}[1,1] & R_{dd}[1,2] \\ R_{dd}[2,1] & R_{dd}[2,2] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -R_{dd}[1] \\ -R_{dd}[2] \end{bmatrix}$$

$$\underset{k\ l}{R_{dd}[1,1]} = \sum_{n=3}^{\infty} h_d[n-1]\cdot h_d[n-1] = h_d[2]^2 + h_d[3]^2 + \ldots = 3^2 + 2^2 + 1^2 + 2^2 + 3^2 = 27$$

hd[0]						
1	2	3	2	1	2	3
1	2	3	2	1	2	3

$1 + 4 + 9 + 4 + 1 + 4 + 9 = 27$

$$R_{dd}[1,2] = \sum_{n=3}^{\infty} h_d[n-1]\cdot h_d[n-2] = h_d[2]\cdot h_d[1] + h_d[3]\cdot h_d[2] + \ldots$$

$$= \qquad =$$

$$R_{dd}[2,1] = \sum_{n=3}^{\infty} h_d[n-2]\cdot h_d[n-1] = 22$$

hd[0]							
1	2	3	2	1	2	3	
	1	2	3	2	1	2	3
	2	6	6	2	2	6	

$= 22$

$$\underset{k\ l}{R_{dd}[2,2]} = \sum_{n=3}^{\infty} h_d[n-2]\cdot h_d[n-2] = h_d[1]^2 + h_d[2]^2 + \ldots$$

$$= 2^2 + 3^2 + 2^2 + 1^2 + 2^2 + 3^2 = 4 + 9 + 4 + 1 + 4 + 9 = 31$$

$$R_{dd}[1] = \sum_{n=3}^{\infty} h_d[n]\cdot h_d[n-1] =$$

1	2	3	2	1	2	3	
	1	2	3	2	1	2	3
	2	6	6 + 2 + 2 + 6				

$= 16$

$$R_{dd}[2] = \sum_{n=3}^{\infty} h_d[n]\cdot h_d[n-2]$$

1	2	3	2	1	2	3		
		1	2	3	2	1	2	3
			4 + 3 + 4 + 3					

$= 14$

$$\begin{bmatrix} 27 & 22 \\ 22 & 31 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -16 \\ -14 \end{bmatrix} \Rightarrow \begin{cases} 27 a_1 + 22 a_2 = -16 \\ 22 a_1 + 31 a_2 = -14 \end{cases}$$

$a_1 = -0.53$

$a_2 = -0.07$

b_k: Padé

$$\begin{cases} h_0 = b_0 = h_d[0] \Rightarrow b_0 = 1 \\ h[1] = -\underbrace{a_1}_{-0.53} \underbrace{h_d[0]}_{1} + b_1 = \underbrace{h_d[1]}_{2} \Rightarrow b_1 = 2 - 0.53 = 1.47 \\ h[2] = -\underbrace{a_1}_{-0.53} \underbrace{h[1]}_{2} - \underbrace{a_2}_{-0.07} \underbrace{h[0]}_{1} + b_2 = \underbrace{h_d[2]}_{3} \Rightarrow b_2 = 3 - 0.53\cdot 2 - 0.07 = \ldots \end{cases}$$

Filtrul FIR invers prin metoda celor mai mici □