Curs 04

Lattice pt. sisteme FIR

$$H_m(z) = A_m(z) = \overset{\alpha_m[0]}{1} + \alpha_m[1]\, z^{-1} + \alpha_m[2]\, z^{-2} + \dots + \alpha_m[m]\, z^{-m}$$

$$h[n] = \{ \underset{\uparrow}{1}, \alpha_m[1], \alpha_m[2], \dots\dots \alpha_m[m] \} = \alpha_m[n]$$

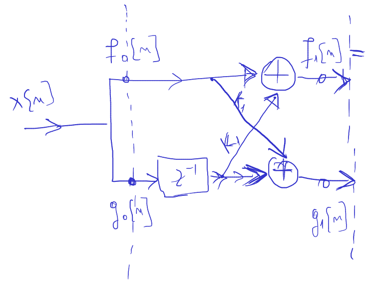
$x[n]$

$$\rightarrow y[n] = x[n] * h[n] = \sum_{k=\cancel{-\infty}\ 0}^{\cancel{\infty}\ m} \underbrace{h[k]}_{\alpha_m[k]} \cdot x[n-k] = \sum_{k=0}^{\cancel{\infty}\ m} \alpha_m[k] \cdot x[n-k]$$

$$= x[n] + \sum_{k=1}^{\cancel{\infty}\ m} \alpha_m[k] \cdot x[n-k]$$

$$\rightarrow A_1(z) = 1 + \alpha_1[1]\, z^{-1}$$

$$y[n] = x[n] + \alpha_1[1] \cdot x[n-1]$$



$$f_0[n] = g_0[n] = x[n]$$

$$f_1[n] = 1 \cdot f_0[n] + K_1 \cdot g_0[n-1] = y[n] =$$

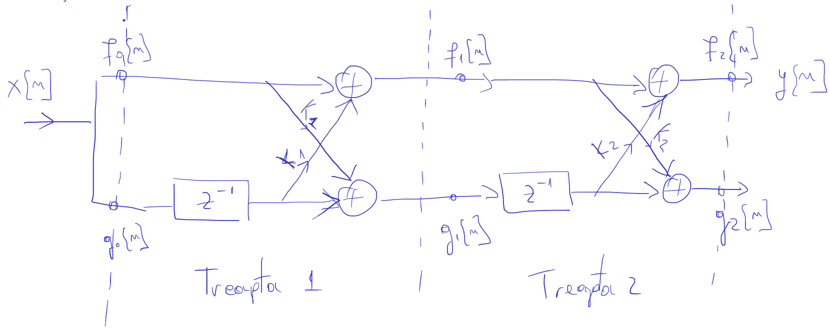
$$g_1[n] = K_1 \cdot f_0[n] + 1 \cdot g_0[n-1]$$

Un sistem de ord 1 cu $H(z) = A_1(z) = 1 + \overset{\alpha_1[1]}{\text{ceva}} \cdot z^{-1}$

se implem. cu o treaptă lattice cu K_1 = ceva $\qquad K_1 = \alpha_1[1]$

Ordin $m = 2$

$$H(z) = A_2(z) = 1 + \alpha_2[1] \cdot z^{-1} + \alpha_2[2] \cdot z^{-2} \Rightarrow y[n] = x[n] + \alpha_2[1] \cdot x[n-1] + \alpha_2[2] \cdot x[n-2]$$



$$f_2[n] = \underbrace{f_1[n]}_{} + K_2 \cdot g_1[n-1]$$

$$= \underbrace{f_0[n]}_{x[n]} + K_1 \cdot \underbrace{g_0[n-1]}_{x[n-1]} + K_2 \cdot \Big(K_1 \cdot \underbrace{f_0[n-1]}_{x[n-1]} + 1 \cdot \underbrace{g_0[n-2]}_{x[n-2]}\Big)$$

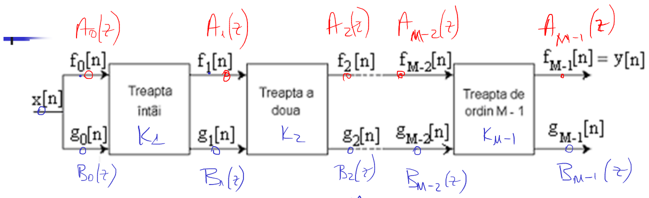
$$= x[n] + K_1(1+K_2) \cdot x[n-1] + K_2 \cdot x[n-2]$$

Un sistem de ord 2 cu $H(z) = A_2(z) = 1 + \alpha_2[1]\, z^{-1} + \alpha_2[2]\, z^{-2}$

se implem. cu 2 trepte lattice, cu $K_2 = \alpha_2[2]$

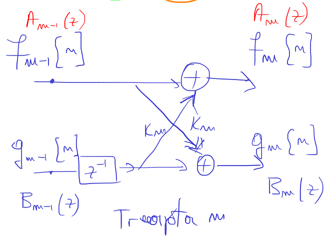
$$K_1(1+K_2) = \alpha_2[1] \Rightarrow K_1 = \frac{\alpha_2[1]}{1+\alpha_2[2]}$$

Ordinul m:



$$A_m(z) = 1 + \alpha_m[1]z^{-1} + \dots + \alpha_m[m]z^{-m}$$

$$B_m(z) = \alpha_m[m] + \beta_m[1]z^{-1} + \dots + 1\, z^{-m}$$



$$H(z) = A_M(z) = 1 + \alpha_M[1]z^{-1} + \dots + \alpha_M[M]z^{-M}$$

$$f_m[n] = f_{m-1}[n] + K_m \cdot g_{m-1}[n-1]$$

$$g_m[n] = K_m f_{m-1}[n] + g_{m-1}[n-1]$$

Ex: $A_2(z) = 1 + 7z^{-1} + 0.3z^{-2}$

$B_2(z) = 0.3 + 7z^{-1} + 1 \cdot z^{-2}$

$$\boxed{B_m(z) = z^{-m} \cdot A_m(z^{-1})}$$

$$\boxed{A_m(z) = A_{m-1}(z) + K_m \cdot z^{-1} \cdot B_{m-1}(z)}$$

$$B_m(z) = K_m \cdot A_{m-1}(z) + z^{-1} \cdot B_{m-1}(z)$$

$$B_m(z) = z^{-m} A_m(z^{-1})$$

$$A_0(z) = B_0(z) = 1$$

$$A_{m-1}(z) = A_m(z) - K_m \cdot z^{-1} \cdot B_{m-1}(z)$$

$$B_{m-1}(z) = \frac{B_m(z) - K_m \cdot A_{m-1}(z)}{z^{-1}}$$

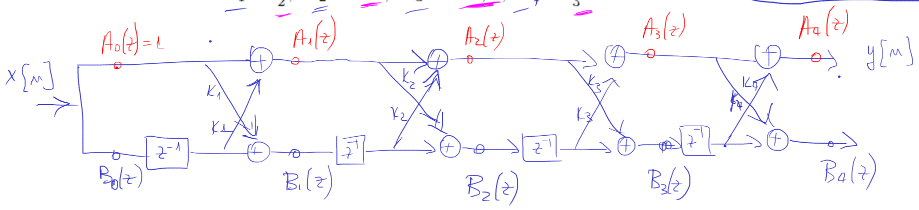
$$A_{m-1}(z) = A_m(z) - K_m \cdot \cancel{z^{-1}} \cdot \frac{B_m(z) - K_m \cdot A_{m-1}(z)}{\cancel{z^{-1}}}$$

$$A_{m-1}(z) = A_m(z) - K_m \cdot B_m(z) + K_m^2 \cdot A_{m-1}(z)$$

$$\boxed{A_{m-1}(z) = \frac{A_m(z) - K_m \cdot B_m(z)}{1 - K_m^2}}$$

Exercițiul
Exemplul 1

1. Determinați coeficienții filtrului FIR în forma directă dacă se cunosc coeficienții de reflexie ai structurii *lattice*: $K_1 = \frac{1}{2}$, $K_2 = 0.6$, $K_3 = -0,7$, $K_4 = \frac{1}{3}$.



$H(z) = ?$

$$H(z) = A_4(z) = ?$$

$$A_0(z) = B_0(z) = 1$$

$$A_1(z) = A_0(z) + K_1 \cdot z^{-1} \cdot B_0(z) = 1 + \frac{1}{2} \cdot z^{-1} \cdot 1 = 1 + \frac{1}{2} z^{-1}$$

K_1

$z \to z^{-1}$

$z^{-1} \to (z^{-1})^{-1} = z$

$$B_1(z) = z^{-1} \cdot A_1(z^{-1}) = z^{-1} \cdot \left(1 + \frac{1}{2} z\right) = z^{-1} \cdot 1 + \cancel{z^{-1}} \cdot \frac{1}{2} \cancel{z} = \frac{1}{2} + 1 z^{-1}$$

$$A_2(z) = A_1(z) + K_2 \cdot z^{-1} \cdot B_1(z) = 1 + \frac{1}{2} z^{-1} + 0.6 \cdot z^{-1} \cdot \left(\frac{1}{2} + 1 \cdot z^{-1}\right) = 1 + \frac{1}{2} z^{-1} + 0.3 z^{-1} + 0.6 z^{-2}$$

$$= 1 + 0.8 z^{-1} + 0.6 z^{-2}$$

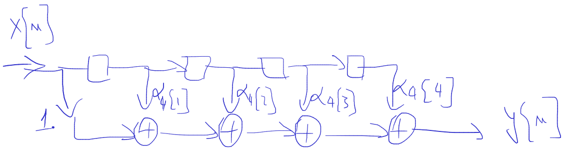
$$B_2(z) = 0.6 + 0.8 z^{-1} + 1 \cdot z^{-2}$$

$$A_3(z) = \underline{1 + 0.8z^{-1} + 0.6z^{-2}} + (-0.7) \cdot z^{-1} (0.6 + 0.8z^{-1} + z^{-2}) = 1 + z^{-1}\underbrace{(0.8 - 0.7 \cdot 0.6)}_{0.38} + z^{-2}\underbrace{(0.6 - 0.7 \cdot 0.8)}_{0.04} - 0.7 \cdot z^{-3}$$

$$B_3(z) = \dots \cancel{\text{invers}} \; -0.7 + 0.04 z^{-1} + 0.38 z^{-2} + 1 \cdot z^{-3}$$

$$A_4(z) = H(z) = \underbrace{1 + 0.38z^{-1} + 0.04z^{-2} - 0.7z^{-3}}_{A_3(z)} + \frac{1}{3} \cdot z^{-1} (-0.7 + 0.04z^{-1} + 0.38z^{-2} + z^{-3})$$

$$= 1 + z^{-1}\underbrace{\left(0.38 - 0.7 \cdot \frac{1}{3}\right)}_{\alpha_4[1]} + z^{-2}\underbrace{\left(0.04 + \frac{1}{3} \cdot 0.04\right)}_{\alpha_4[2]} + z^{-3}\underbrace{\left(-0.7 + \frac{1}{3} \cdot 0.38\right)}_{\alpha_4[3]} + \underbrace{\frac{1}{3}}_{\alpha_4[4]} \cdot z^{-4}$$

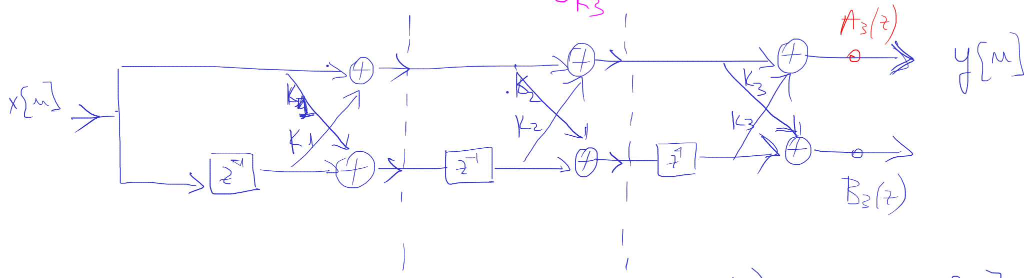


Exercitiul 2

2. Determinați coeficienții structurii *lattice* pentru un filtru FIR cu funcția de sistem:

$$H(z) = 1 + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \frac{1}{2}z^{-3} = A_3(z) \qquad K_1, K_2, K_3 = ?$$

K_3



! K_m = ultimul coef. al lui $A_m(z) = \alpha_m[m]$!

$K_3 = \frac{1}{2}$ = ult. coef. al lui $A_3(z) = H(z)$

$$A_{m-1}(z) = \frac{A_m(z) - K_m \cdot B_m(z)}{1-K_m^2}$$

$A_3(z) = H(z)$

$$B_3(z) = \frac{1}{2} + \frac{7}{20}\cdot z^{-1} + \frac{2}{5}\cdot z^{-2} + 1\cdot z^{-3}$$

$$A_2(z) = \frac{A_3(z) - K_3\cdot B_3(z)}{1-K_3^2} = \frac{1+\frac{2}{5}z^{-1}+\frac{7}{20}z^{-2}+\cancel{\frac{1}{2}z^{-3}} - \frac{1}{2}\left(\frac{1}{2}+\frac{7}{20}z^{-1}+\frac{2}{5}z^{-2}+\cancel{1\cdot z^{-3}}\right)}{1-\left(\frac{1}{2}\right)^2}$$

$$= \frac{3/4 + z^{-1}\left(\frac{2}{5}-\frac{7}{40}\right) + z^{-2}\left(\frac{7}{20}-\frac{1}{5}\right)}{3/4} = \frac{3/4 + \frac{9}{40}z^{-1} + \frac{3}{20}z^{-2}}{3/4} = 1 + \frac{3}{10}z^{-1} + \frac{1}{5}z^{-2}$$

K_2

$\frac{9}{40}\cdot\frac{4}{3} =$

$\frac{3}{20}\cdot\frac{4}{3} = \frac{1}{5}$

$$B_2(z) = \frac{1}{5} + \frac{3}{10}z^{-1} + 1\cdot z^{-2}$$

$$A_1(z) = \frac{1+\frac{3}{10}z^{-1}+\cancel{\frac{1}{5}z^{-2}} - \frac{1}{5}\cdot\left(\frac{1}{5}+\frac{3}{10}z^{-1}+\cancel{1\cdot z^{-2}}\right)}{1-\left(\frac{1}{5}\right)^2} = \frac{24/25 + z^{-1}\overbrace{\left(\frac{3}{10}-\frac{3}{50}\right)}^{12/50}}{24/25} = 1 + \frac{1}{4}z^{-1}$$

K_1

$\frac{12}{50}\cdot\frac{25}{24} = \frac{1}{4}$