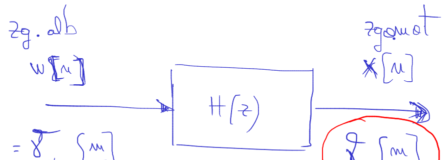
Curs 14 PSS

Ex 1 / Lab 12

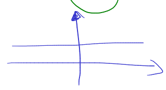


$$\sigma_w^2\cdot\delta[n] = \gamma_{ww}[n] \qquad \gamma_{xx}[n] \quad \xleftarrow{\text{transf. } Z \text{ invers}}$$

$$x[n] = \underbrace{\frac{1}{2}}_{-a_1}\cdot x[n-1] + \underbrace{1}_{b_0}\cdot w[n] + \underbrace{1}_{b_1}\cdot w[n-1]$$

$$\boxed{\Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1}) = \Gamma_{xx}(z)}$$

$\Gamma_{ww}(z)$ = constant = σ_w^2



$$(y[n] = (-a_1)y[n-1]\ldots + (b_0)x[n] + (b_1)x[n-1]\ldots)$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} = \frac{1 + 1\cdot z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

$$H(z^{-1}) = \frac{1 + 1\cdot z}{1 - \frac{1}{2}\cdot z}$$

$$\Gamma_{xx}(z) = \sigma_w^2\cdot\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}\cdot\frac{1+z}{1-\frac{1}{2}z}$$

$$= \sigma_w^2\cdot\frac{z+1}{z-\frac{1}{2}}\cdot\frac{z+1}{-\frac{1}{2}(z-2)} = -2\sigma_w^2\cdot\boxed{\frac{(z+1)^2}{(z-\frac{1}{2})(z-2)}}\ A(z)$$

$$\frac{A(z)}{z} = \frac{(z+1)^2}{z\cdot(z-\frac{1}{2})\cdot(z-2)} = \frac{A}{z} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-2}$$

$$A = \frac{1^2}{(-\frac{1}{2})\cdot(-2)} = \frac{1}{1} = 1$$

$$B = \frac{(3/2)^2}{\frac{1}{2}\cdot(-\frac{3}{2})} = \frac{9/4}{-3/4} = -3$$

$$C = \frac{9}{2\cdot\frac{3}{2}} = 3$$

$$1 \longleftrightarrow \delta[n]$$

$$\frac{z}{z-a} \longleftrightarrow a^n u[n],\ |z|>|a| \qquad \longleftrightarrow -a^n u[-n-1],\ |z|<|a|$$

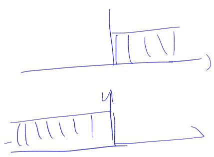
$$A(z) = A + B\cdot\frac{z}{z-\frac{1}{2}} + C\cdot\frac{z}{z-2}$$

$$\Gamma_{xx}(z) = -2\sigma_w^2\left(A + B\cdot\frac{z}{z-\frac{1}{2}} + C\cdot\frac{z}{z-2}\right)$$

$$\updownarrow$$

$$\gamma_{xx}[n] = -2\sigma_w^2\left(A\cdot\delta[n] + B\cdot\left(\frac{1}{2}\right)^n\cdot u[n] - C\cdot 2^n\cdot u[-n-1]\right)$$

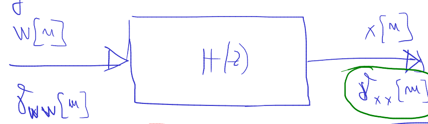
$$= -2\sigma_w^2\left(\delta[n] - 3\cdot\left(\frac{1}{2}\right)^n\cdot u[n] - 3\cdot 2^n\cdot u[-n-1]\right)$$



$$\frac{1}{2}^{|n|}$$

$$= -2\sigma_w^2\left(\delta[n] - 3\cdot\frac{1}{2}^{|n|}\right)$$

Ex.2 / Lab 12



$$\gamma_{ww}[n] \qquad \gamma_{xx}[n] \quad \text{transf. } Z$$

$$\Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1}) = \Gamma_{xx}(z)$$

$$\Gamma_{ww}(z) = \sigma_w^2$$

$$\gamma_{xx}[n] = \frac{1}{4}^{|n|} = \begin{cases}\left(\frac{1}{4}\right)^n, & n \geq 0\\ \left(\frac{1}{4}\right)^{-n}, & n < 0\end{cases}$$

$$\Gamma_{xx}(z) = \sum_{n=-\infty}^{\infty}\gamma_{xx}[n]\cdot z^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{-1}\left(\frac{1}{4}\right)^{-n} z^{-n} \quad -n \to n$$

$$= \sum_{n=0}^{\infty}\Big(\underbrace{\frac{1}{4z}}_{q}\Big)^n + \sum_{n=1}^{\infty}\Big(\underbrace{\frac{z}{4}}_{q}\Big)^n$$

Progr. geometrice

$$q^0 + q^1 + q^2 + q^3 + \ldots = \frac{1}{1-q}$$

$$\frac{1}{1-\frac{1}{4z}} = \frac{4z}{4z-1} = \frac{z}{z-1/4} \qquad \frac{1}{1-\frac{z}{4}} - 1 = \frac{4}{4-z} - 1 = \frac{4-4+z}{4-z} = \frac{z}{4-z} = -\frac{z}{z-4}$$

$$\frac{1}{4}z - 4z = \frac{-15z}{16}$$

$$\Gamma_{xx}(z) = \frac{z}{z-\frac{1}{4}} - \frac{z}{z-4} = \frac{\cancel{z^2} - 4z - \cancel{z^2} + \frac{1}{4}z}{\left(z-\frac{1}{4}\right)\left(z-4\right)} = \frac{-15/16 \; z \cdot \frac{1}{z}}{\left(z-\frac{1}{4}\right)\left(z-4\right)\cdot\frac{1}{z}} = \frac{-15/16}{\left(z-\frac{1}{4}\right)\left(1-4z^{-1}\right)} = \frac{15/16}{\left(\frac{1}{4}-z\right)\left(\frac{1}{4}-z^{-1}\right)\cdot 4} =$$

$$= \frac{15}{16\cdot 4}\cdot\frac{1}{\left(\frac{1}{4}-z\right)\left(\frac{1}{4}-z^{-1}\right)} \qquad p = \frac{1}{4} \qquad p = 4$$

$$\left(= \overbrace{\frac{15}{4}}^{\sigma_w^2} \cdot \overbrace{\frac{1}{4\cdot\left(\frac{1}{4}-z\right)}}^{H(z)} \cdot \overbrace{\frac{1}{4\cdot\left(\frac{1}{4}-z^{-1}\right)}}^{H(z^{-1})} \right)$$

$$\boxed{\overbrace{\Gamma_{ww}(z)}^{\sigma_w^2} \cdot H(z) \cdot H(z^{-1}) = \Gamma_{xx}(z)}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots}{1 + a_1 z^{-1} + \dots}$$

$$H(z) = \frac{1}{\frac{1}{4} - z}\cdot\frac{z^{-1}}{z^{-1}} = \frac{z^{-1}}{\frac{1}{4}z^{-1} - 1} = \frac{\overbrace{-1}^{b_1}\cdot z^{-1}}{1\underbrace{-\frac{1}{4}}_{a_1}z^{-1}}$$

$$y[n] = -a_1 y[n-1] + b_0 x[n] + b_1 x[n-1] + \dots$$

$$x[n] = \frac{1}{4}\cdot x[n-1] - w[n-1]$$