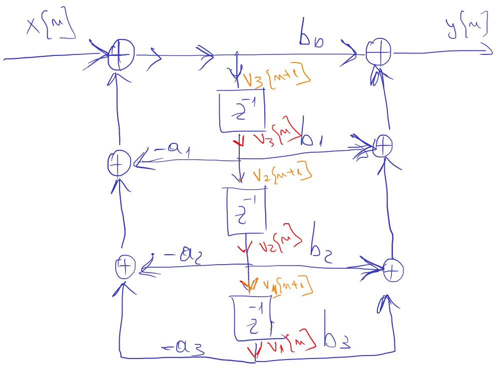
Curs 06 — Spațiul stărilor

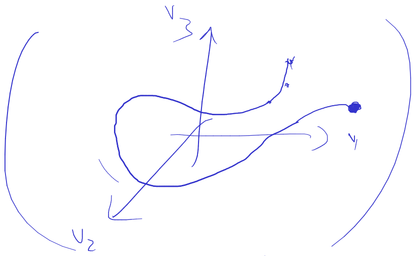


Forma directă 2

$v_{1,2,3}$ = variabile de stare

$$y[n] = b_3 \cdot v_1[n] + b_2 \cdot v_2[n] + b_1 \cdot v_3[n] + b_0 \cdot \big(x[n] - a_1 \cdot v_3[n] - a_2 \cdot v_2[n] - a_3 \cdot v_1[n]\big)$$

$$= (b_3 - b_0 a_3) \cdot v_1[n] + (b_2 - b_0 a_2) \cdot v_2[n] + (b_1 - b_0 a_1) \cdot v_3[n] + b_0 \cdot x[n]$$



Ecuația de ieșire

$$y[n] = \underbrace{\begin{bmatrix} b_3 - b_0 a_3 & b_2 - b_0 a_2 & b_1 - b_0 \cdot a_1 \end{bmatrix}}_{g^T} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + b_0 \cdot x[n]$$

ieșirea de la momentul n

starea la momentul n

intrarea la momentul n

Ecuațiile de stare

$$\begin{cases} v_1[n+1] = v_2[n] \\ v_2[n+1] = v_3[n] \\ v_3[n+1] = -a_3 \cdot v_1[n] - a_2 v_2[n] - a_1 v_3[n] + x[n] \end{cases}$$

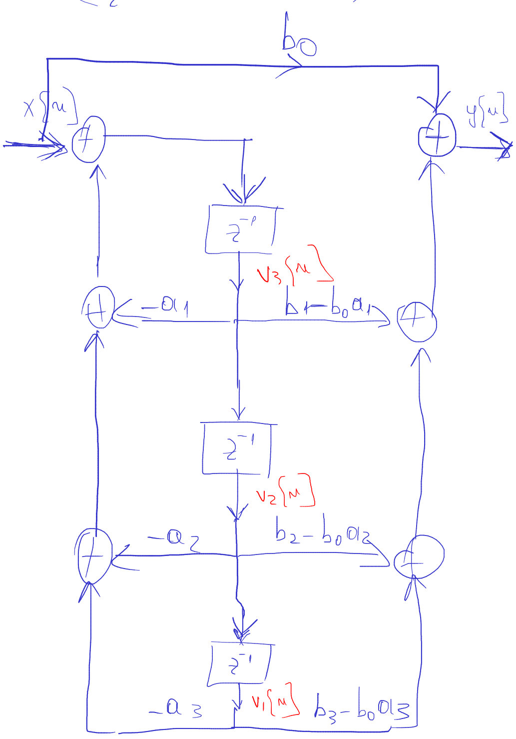
$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}}_{F} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{q} \cdot x[n]$$

starea de la mom. viitor (n+1)

starea de la momentul curent

intrarea de la mom. curent

$\boxed{\text{Sp. stărilor tip I}}$



Sp. stărilor de tip II

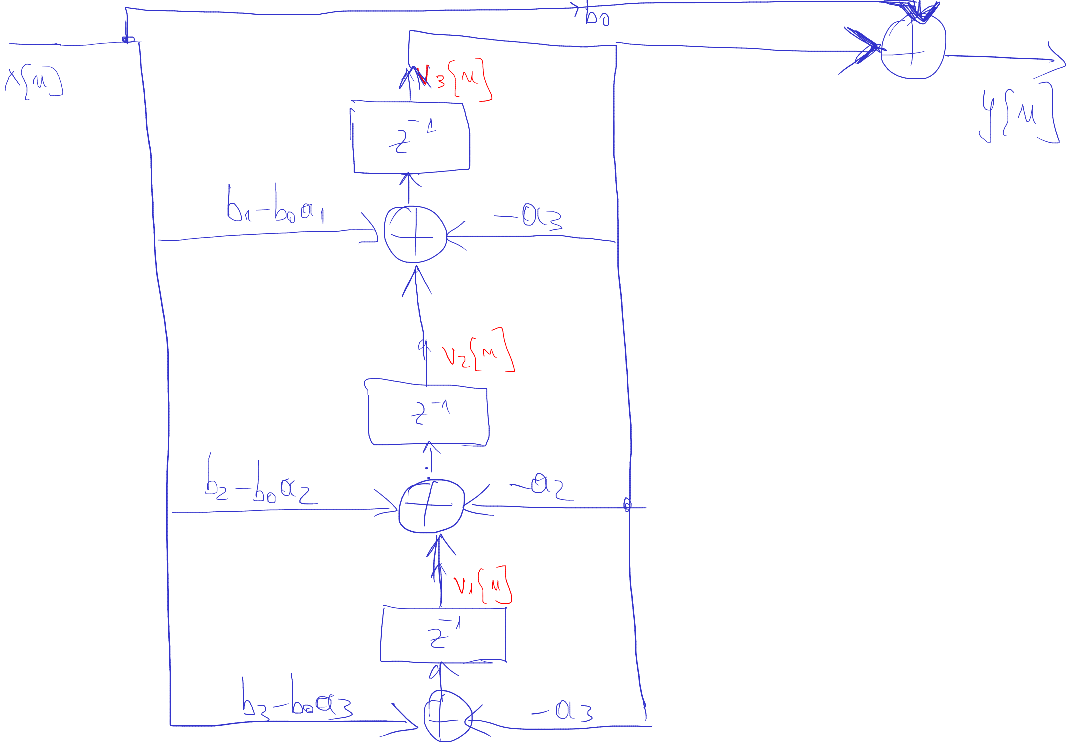
- Se pleacă de la f. d. II Transpusă
- Ecuații: − F se transpune
  - g, q se schimbă între ei

$$y[n] = v_3[n] + b_0 x[n]$$

$$y[n] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + b_0 \cdot x[n]$$

$$\begin{cases} v_1[n+1] = -a_3 v_3[n] + (b_3 - b_0 a_3)\, x[n] \\ v_2[n+1] = v_1[n] - a_2 v_3[n] + (b_2 - b_0 a_2) \cdot x[n] \\ v_3[n+1] = v_2[n] - a_3 \cdot v_3[n] + (b_1 - b_0 a_1)\, x[n] \end{cases}$$

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} b_3 - b_0 a_3 \\ b_2 - b_0 a_2 \\ b_1 - b_0 a_1 \end{bmatrix} \cdot x[n]$$



Exerciții:

Ex. 1, Lab 7:

$$H(z) = \frac{\overset{b_0}{1} + \overset{b_1}{2}z^{-1} + \overset{b_2}{3}z^{-2} + \overset{b_3}{2}z^{-3}}{1 + \underset{a_1}{0.9}z^{-1} + \underset{a_2}{0.8}z^{-2} + \underset{a_3}{0.5}z^{-3}}$$

$b_3 - b_0 a_3 = 2 - 1 \cdot 0.5 = 1.5$

$b_2 - b_0 a_2 = 3 - 0.8 = 2.2$

$b_1 - b_0 a_1 = 2 - 0.9 = 1.1$

$b_0 = 1$

a). tip I:

$$y[n] = \begin{bmatrix} 1.5 & 2.2 & 1.1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underline{1} \cdot x[n]$$

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot x[n]$$

+ desen

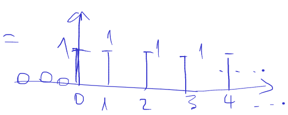
tip II:

$$y[n] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underline{1} \cdot x[n]$$

+ desen

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.5 \\ 1 & 0 & -0.8 \\ 0 & 1 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 1.5 \\ 2.2 \\ 1.1 \end{bmatrix} \cdot x[n]$$

b). $x[n] = u[n] =$



$y[n] = ?$

$$y[0] = \begin{bmatrix} 1.5 & 2.2 & 1.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 1 = 2.1$$

Cond. inițiale: (starea sist. la momentul 0)

$$v[0] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v[1] = \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \underline{1}$$

$$\Rightarrow \quad v[1] = \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix}$$ starea sist. la mom. $n=1$

$$y[1] = [1.5 \quad 2.2 \quad 1.1] \cdot \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix} + 1 = 2.2 + 0.11 + 1 = 3.31$$

$$v[2] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot 1 = \begin{bmatrix} 1 \\ 0.1 \\ -0.8-0.09+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.1 \\ 0.11 \end{bmatrix}$$

$$y[2] = [1.5 \quad 2.2 \quad 1.1] \cdot \begin{bmatrix} 1 \\ 0.1 \\ 0.11 \end{bmatrix} + 1 = 1.5 + 0.22 + 0.121 + 1 = 2.841$$

$v[3] = \dots$

$y[3] = \dots$

$v[4] = \dots$

$y[3] = \dots$

Exercițiul 2, Lab 7

$$v[n+1] = \begin{bmatrix} 0 & 1 \\ \underset{-a_2}{-0.81} & \underset{-a_1}{1} \end{bmatrix} v[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} \underset{b_2 - b_0 a_2}{-1.81} & \underset{b_1 - b_0 a_1}{1} \end{bmatrix} + \underset{b_0 = 1}{1}\, x[n]$$

$\Rightarrow$ din forma ecuațiilor, e vorba de sp. stărilor tip I

a) $H(z) = ?$

Prin identificare $\Rightarrow$

$-a_2 = -0.81$

$-a_1 = 1$

$\Rightarrow a_2 = 0.81$

$a_1 = -1$

$b_2 - b_0 a_2 = -1.81 \Rightarrow b_2 = b_0 a_2 - 1.81 = 1 \cdot 0.81 - 1.81 = -1$

$b_1 - b_0 a_1 = 1 \qquad b_1 = b_0 a_1 + 1 = 1 \cdot (-1) + 1 = 0$

$b_0 = 1$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{1 - z^{-2}}{1 - z^{-1} + 0.81 z^{-2}}$$

b). La fel: $\underline{n=0}$: $y[0] = \begin{bmatrix} -1.81 & 1 \end{bmatrix} \cdot \overset{v[0]}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} + \underline{1} = 2$

$n=1$: $v[1] = \begin{bmatrix} 0 & 1 \\ -0.81 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \underline{1} = \begin{bmatrix} \underline{1} \\ 2 \end{bmatrix}$

$y[1] = \begin{bmatrix} -1.81 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \underline{1} = -1.81 + 2 + \underline{1} = \underline{1}.19$

$v[2] = \ldots$

$y[2] = \ldots$

$v[3] = \ldots$

$y[3] = \ldots$

$\vdots$

c). Desene