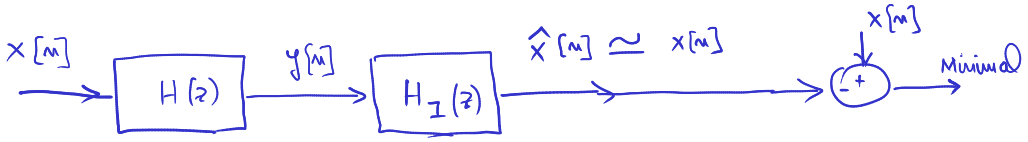
Designing the FIR inverse filter

Lab 4, SDP

3 Theoretical Exercise

1. Use the least squares method to find the inverse FIR filter of order 2 for the following filter:

$$H(z) = 0.2 + 0.8z^{-1} + 0.2z^{-2}$$



$$H_I(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} \quad \text{(order 2)}$$

$$\begin{bmatrix} h[0] \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_{hh}[0] & r_{hh}[1] & r_{hh}[-2] \\ r_{hh}[1] & r_{hh}[0] & r_{hh}[-1] \\ r_{hh}[2] & r_{hh}[1] & r_{hh}[0] \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

$$h[n] = \{ 0.2, 0.8, 0.2 \}$$
$$\uparrow$$

$r_{hh}[0]$:

$$\begin{array}{cccc} 0.2 & 0.8 & 0.2 & \\ 0.2 & 0.8 & 0.2 & \\ \hline 0.04 & 0.64 & 0.04 & \Rightarrow 0.72 \end{array}$$

$r_{hh}[1]$

$$\begin{array}{cccc} 0.2 & 0.8 & 0.2 & \\ & 0.2 & 0.8 & 0.2 \\ \hline & 0.16 & 0.16 & \Rightarrow 0.32 \end{array}$$

$= r_{hh}[-1]$

$$r_{hh}[2] = \begin{array}{ccc} 0.2 & 0.8 & 0.2 \\ & 0.2 & 0.8 & 0.2 \\ \hline & 0.04 & \end{array} = r_{hh}[-2] \qquad h[0] = 0.2$$

$$\begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.72 & 0.32 & 0.04 \\ 0.32 & 0.72 & 0.32 \\ 0.04 & 0.32 & 0.72 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

$0.2 \cdot 0.72^2 - 0.32^2 \cdot 0.2 = 0.08$

$$b_0 = \frac{\begin{vmatrix} 0.2 & 0.32 & 0.04 \\ 0 & 0.72 & 0.32 \\ 0 & 0.32 & 0.72 \end{vmatrix}}{\begin{vmatrix} 0.72 & 0.32 & 0.04 \\ 0.32 & 0.72 & 0.32 \\ 0.04 & 0.32 & 0.72 \end{vmatrix}}$$

$$b_1 = \frac{\begin{vmatrix} 0.72 & 0.2 & 0.04 \\ 0.32 & 0 & 0.32 \\ 0.04 & 0 & 0.72 \end{vmatrix}}{\begin{vmatrix} 0.72 & 0.32 & 0.04 \\ 0.32 & 0.72 & 0.32 \\ 0.04 & 0.32 & 0.72 \end{vmatrix}} = \text{same}$$

$0.2 \cdot 0.32 \cdot 0.04 - 0.2 \cdot 0.32 \cdot 0.72 = -0.04$

$0.72 \cdot 0.72 \cdot 0.72 + 0.32^2 \cdot 0.04 + 0.32^2 \cdot 0.04 - 0.72 \cdot 0.04^2 - 0.32^2 \cdot 0.72 - 0.32^2 \cdot 0.72 = 0.232$

$$b_2 = \frac{\begin{vmatrix} 0.72 & 0.32 & 0.2 \\ 0.32 & 0.72 & 0 \\ 0.04 & 0.32 & 0 \end{vmatrix}}{\begin{vmatrix} 0.72 & 0.32 & 0.04 \\ 0.32 & 0.72 & 0.32 \\ 0.04 & 0.32 & 0.72 \end{vmatrix}} = \text{same}$$

$= 0.32^2 \cdot 0.2 - 0.2 \cdot 0.72 \cdot 0.04 = 0.01$

$$\Rightarrow b_0 = \frac{0.08}{0.232} \qquad b_1 = \frac{-0.04}{0.232} \qquad b_2 = \frac{0.01}{0.232}$$