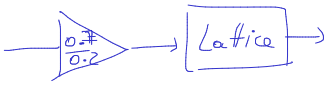
Curs 5

Lattice pt. sisteme IIR doar cu poli

$$H(z) = \frac{1}{1 + a_1 \cdot z^{-1} + \dots + a_N z^{-N}}$$

$$H(z) = \frac{0.7}{0.2 + 5z^{-1} + 1 \cdot z^{-2}} = \frac{0.7}{0.2} \cdot \frac{1}{1 + 25 \cdot z^{-1} + 5 \cdot z^{-2}}$$



$$y[n] = -a_1 y[n-1] - \dots - a_N y[n-N] + x[n]$$

Inversăm y cu x:

$$x[n] = -a_1 x[n-1] - \dots - a_N x[n-N] + y[n]$$

$$\Leftrightarrow$$

$$y[n] = x[n] + a_1 \cdot x[n-1] + \dots + a_N x[n-N]$$

$$Y(z) = X(z) \cdot \left(1 + a_1 \cdot z^{-1} + \dots + a_N \cdot z^{-N}\right)$$

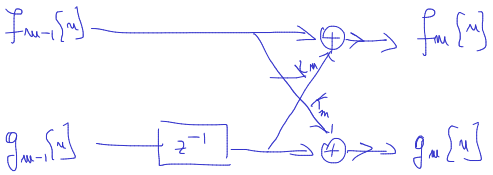
$$H(z) = \frac{Y(z)}{X(z)} = 1 + a_1 z^{-1} + \dots + a_N z^{-N}$$

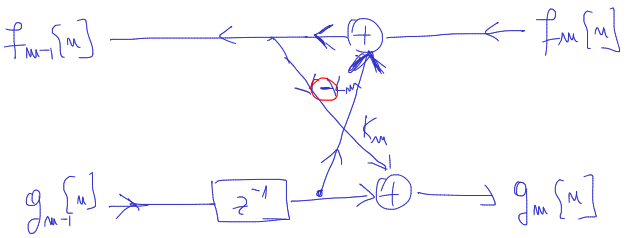
FIR, implementabil cu Lattice

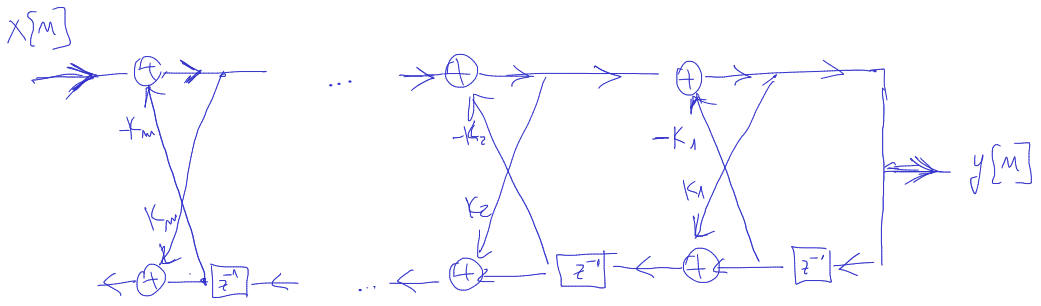
Lattice FIR:

$$f_m[n] = f_{m-1}[n] + K_m g_{m-1}[n-1]$$
$$g_m[n] = K_m f_{m-1}[n] + g_{m-1}[n-1]$$

$$\Rightarrow \quad f_{m-1}[n] = f_m[n] - K_m \cdot g_{m-1}[n-1]$$
$$g_m[n] = K_m \cdot f_{m-1}[n] + g_{m-1}[n-1]$$







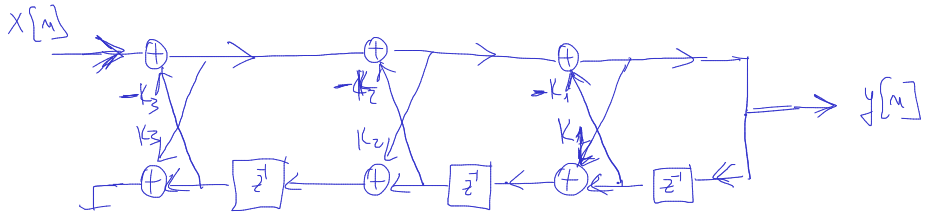
$$H(z) = \frac{1}{A_m(z)}$$

$K_1, K_2, \dots K_m$ se obțin din numitorul lui $H(z)$, ca la FIR

Exercitiul 2, Lab 6

$$H(z) = \frac{1}{1 + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \frac{1}{2}z^{-3}} = A_3(z)$$

implem. în forma lattice



K_1, K_2, K_3 : din numitor :

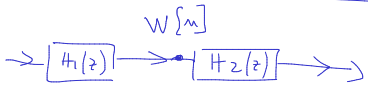
$$A_3(z) = 1 + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \frac{1}{2}z^{-3}$$

De la exercitiul precedent $\Rightarrow$ $K_3 = \frac{1}{2}$

$K_2 = 1/5$

$K_1 = 1/4$

Lattice IIR general



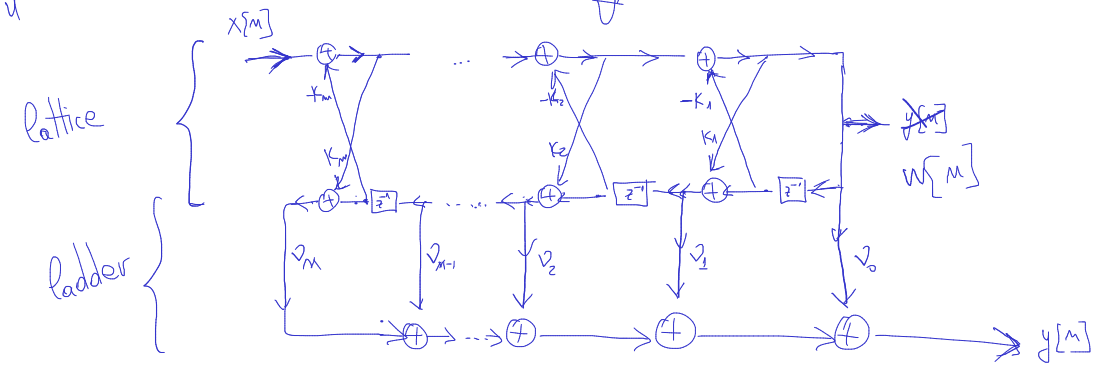
$$H(z) = \frac{C_M(z)}{A_N(z)} = \frac{1}{A_N(z)} \cdot C_M(z)$$

$C_M(z) \Rightarrow v_0, v_1, \ldots v_M$

$A_N(z) \Rightarrow K_1, K_2, \ldots K_M$

FIR $\rightarrow$ $y[n] = c_0 \cdot w[n] + c_1 \cdot w[n-1] + \ldots + c_M w[n-M]$

Lattice - ladder



v_k se calc. din numaratorul $C_M(z)$

$$C_m(z) = C_{m-1}(z) + v_m B_m(z)$$

$$C_{m-1}(z) = C_m(z) - v_m B_m(z)$$

v_m = ultimul coef. al polinomului $C_m(z)$

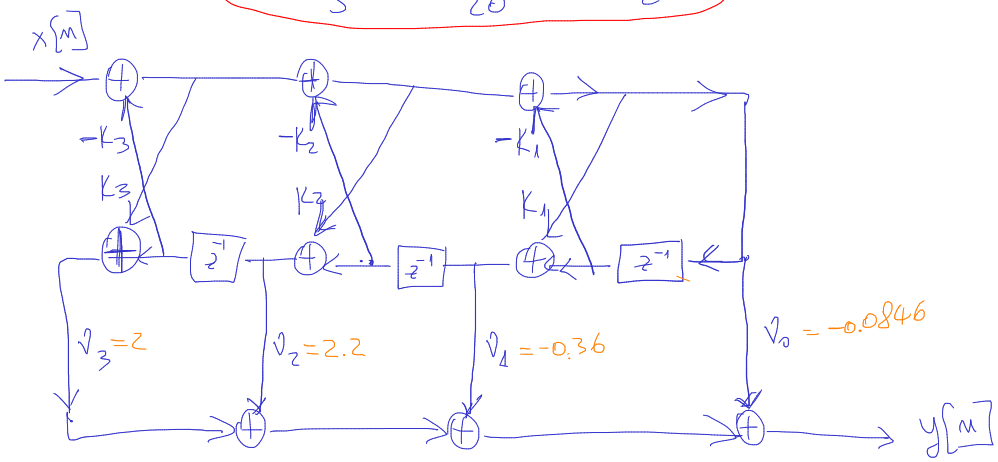
Exercițiul 1, Lab 0.6

$$H(z) = \frac{0.5 + 2z^{-1} + 3z^{-2} + 2z^{-3}}{1 + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \frac{1}{2}z^{-3}}$$

$= C_3(z) \Rightarrow V$ (numărător)

$= A_3(z) \Rightarrow K$ (numitor)

- Implem. în formă lattice.



- Dacă toți $|K_m| < 1$, sist. e stabil
- Dacă unul dintre K_m este $|K_m| \geq 1$, sist. e instabil !!

1) Calcul K_m

$$A_3(z) = \underset{\text{numerar } 1}{1} + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \underset{K_3}{\frac{1}{2}}z^{-3}$$

$A_2(z) = \ldots\ldots K_2 = \frac{1}{5}$

$A_1(z) = \ldots\ldots K_1 = \frac{1}{4}$

ca la exercițiul de săptămâna trecută

2). Calcul V_m

$$C_3(z) = \underset{\text{original}}{0.5} + 2z^{-1} + 3z^{-2} + \underset{V_3}{2}z^{-3}$$

$$C_2(z) = C_3(z) - V_3 \cdot B_3(z) = 0.5 + 2z^{-1} + 3z^{-2} + 2z^{-3} - 2\cdot\left(\frac{1}{2} + \frac{7}{20}z^{-1} + \frac{2}{5}z^{-2} + 1\cdot z^{-3}\right)$$

$B_3(z)$ de la calculul K_m = $A_3(z)$ cu coef. inversați

$$= -0.5 + z^{-1}\left(2 - \frac{14}{20}\right) + z^{-2}\left(3 - \frac{4}{5}\right)$$

$$= -0.5 + \frac{3}{10}z^{-1} + \underset{V_2}{2.2}z^{-2}$$

$$C_1(z) = C_2(z) - V_2 \cdot B_2(z)$$

$$= -0.5 + \frac{3}{10}z^{-1} + 2.2z^{-2} - 2.2\left(\frac{1}{5} + \frac{3}{10}z^{-1} + 1\cdot z^{-2}\right)$$

$$= \left(-0.5 - \frac{2.2}{5}\right) + z^{-1}\left(\frac{3}{10} - 2.2\cdot\frac{3}{10}\right)$$

$$= -0.94 \underset{V_1}{-0.36}z^{-1}$$

$$C_0(z) = C_1(z) - V_1 \cdot B_1(z) = -0.94 - 0.36z^{-1} + 0.36\cdot\left(\frac{1}{4} + 1\cdot z^{-1}\right) =$$

$$= -0.94 + \frac{0.36}{4} = \underset{V_0}{-0.0846}$$