IIR filter design with the Prony method

Lab 3, SDP

3 Theoretical exercises

1. Use the Prony method to find the parameters of the 2nd-order system with the following system function:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

which approximates the desired impulse response

$$h_d[n] = \{...0, \underset{\uparrow}{1}, 2, 3, 2, 1, 2, 3\}$$

$$\begin{bmatrix} r_{dd}[1,1] & r_{dd}[1,2] \\ r_{dd}[2,1] & r_{dd}[2,2] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -r_{dd}[1,0] \\ -r_{dd}[2,0] \end{bmatrix}$$

$$r_{dd}[k,l] = \sum_{n=M+1}^{\infty} h[n-k]\,h[n-l]$$

$M+1 = 3$

$r_{dd}[1,1] =$

n	0	1	2	3	...
$h[n-1]$	~~1~~	~~2~~	3	2	1 2 3
$h[n-1]$	~~1~~	~~2~~	3	2	1 2 3
	9	4	1	4	9 $\Rightarrow 27$

$r_{dd}[2,2] =$

0	1	2	3	...
~~1~~	~~2~~	3	2	1 2 3
~~1~~	~~2~~	3	2	1 2 3
4	9	4	1	4 9

$\Rightarrow 31$

$r_{dd}[1,2]$

n	0	1	2	3	...
$h[n-1]$	~~1~~	~~2~~	3	2	1 2 3
$h[n-2]$	~~1~~		2	3	2 1 2 3
	6	6	2	2	6 $\Rightarrow 22$

$r_{dd}[1,0] =$

0	1	2	3
~~1~~	~~2~~	3	2 1 2 3
~~1~~	~~2~~	~~3~~	2 1 2 3
6	2	2	6

$\Rightarrow 16$

$r_{dd}[2,1] =$

0	1	2	3	...
	~~1~~	2	3	2 1 2 3
~~1~~	~~2~~	3	2	1 2 3
6	6	2	2	6 $\Rightarrow 22$

$r_{dd}[2,0] =$

0	1	2	3
	~~1~~	2	3 2 1 2 3
~~1~~	~~2~~	~~3~~	2 1 2 3
4	3	4	3

$= 14$

$$\begin{bmatrix} 27 & 22 \\ 22 & 31 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -16 \\ -14 \end{bmatrix}$$

$$\begin{cases} 27a_1 + 22a_2 = -16 \\ 22a_1 + 31a_2 = -14 \end{cases} \Rightarrow a_1 = \frac{-16 - 22a_2}{27}$$

$$22 \cdot \frac{-16 - 22a_2}{27} + 31a_2 = -14$$

$$\Rightarrow \frac{-16 \cdot 22}{27} - \frac{22^2 \cdot a_2}{27} + 31a_2 = -14$$

$$\Rightarrow a_2 = \frac{-14 + \frac{16 \cdot 22}{27}}{31 - 22 \cdot 22/27} = -0.07$$

$$a_1 = \frac{-16 - 22a_2}{27} = -0.53$$

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} hd[0] \\ hd[1] \\ hd[2] \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ hd[0] & 0 \\ hd[1] & hd[0] \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}_{2\times 1}$$

$$\Rightarrow b_0 = 1$$

$$b_1 = 2 + 1 \cdot a_1 = \dots$$

$$b_2 = 3 + 2 \cdot a_1 + a_2 = \dots$$