

1

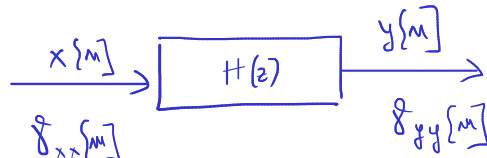
$$y[m] = 0.8 y[m-1] + x[m] + x[m-1]$$

$$x[m] = \text{Random process with } \delta_{xx}[m] = \left(\frac{1}{2}\right)^{|m|}$$

a) $\Gamma_{yy}(z) = ?$

b) $\delta_{yy}[m] = ?$

c) $\sigma_y^2 = ?$



$$\underbrace{\Gamma_{xx}(z)}_{X(z) \cdot X(z^{-1})} \cdot H(z) \cdot H(z^{-1}) = \underbrace{\Gamma_{yy}(z)}_{Y(z) \cdot Y(z^{-1})}$$

$$H(z) = \frac{1 + z^{-1}}{1 - 0.8 z^{-1}}$$

$$\Gamma_{xx}(z) = \sum_{m=-\infty}^{\infty} \delta_{xx}[m] \cdot z^{-m} = \sum_{m=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|m|} \cdot z^{-m} =$$

$$\text{Same as in L11 } \left(\frac{1}{4}\right)^{|m|} \Rightarrow = \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 - \frac{1}{2} z} - \frac{1}{4}$$

$$= \frac{\left(1 - \frac{1}{2} z\right) \left(1 - \frac{1}{2} z^{-1}\right) - \left(1 - \frac{1}{2} z\right) \left(1 - \frac{1}{2} z^{-1}\right)}{\left(1 - \frac{1}{2} z^{-1}\right) \left(1 - \frac{1}{2} z\right)} = \frac{2 - \frac{1}{2} z - \frac{1}{2} z^{-1} - 1 + \frac{1}{2} z + \frac{1}{2} z^{-1} - \frac{1}{4}}{\left(1 - \frac{1}{2} z^{-1}\right) \left(1 - \frac{1}{2} z\right)}$$

$$= \frac{3/4}{\left(1 - \frac{1}{2} z^{-1}\right) \left(1 - \frac{1}{2} z\right)} = \frac{3/4}{\left(1 - \frac{1}{2} z^{-1}\right) \cdot \frac{1}{2} (2 - z)} = \frac{-3/2}{\left(1 - \frac{1}{2} z^{-1}\right) (z - 2)} = \frac{-3/2 \cdot z}{\left(z - \frac{1}{2}\right) (z - 2)}$$

$$\Gamma_{yy}(z) = \Gamma_{xx}(z) \cdot H(z) \cdot H(z^{-1}) = \frac{-3/2 \cdot z}{\left(z - \frac{1}{2}\right) (z - 2)} \cdot \frac{1 + z^{-1}}{1 - 0.8 z^{-1}} \cdot \frac{1 + z}{1 - 0.8 z}$$

$$= \frac{-3/2 \cdot z (z+1) (z+1)}{\left(z - \frac{1}{2}\right) (z - 2) (z - 0.8) (1 - 0.8 z)} = \frac{-3/2}{-0.8} \cdot \frac{z (z+1)^2}{\left(z - \frac{1}{2}\right) (z - 2) (z - 0.8) \left(z - \frac{1}{0.8}\right)}$$

$$\Gamma_{yy}(z) \stackrel{z=e^{j\omega}}{=} \dots$$

b) $\delta_{yy}[m] = \mathcal{Z}^{-1} \left\{ \frac{\Gamma_{yy}(z)}{z} \right\}$

$$\frac{\Gamma_{yy}(z)}{z} = \frac{-3/2}{-0.8} \cdot \frac{(z+1)^2}{\left(z - \frac{1}{2}\right) (z - 2) (z - 0.8) \left(z - \frac{1}{0.8}\right)} = \frac{1.5}{0.8} \left(\frac{A}{z - \frac{1}{2}} + \frac{B}{z - 2} + \frac{C}{z - 0.8} + \frac{D}{z - \frac{1}{0.8}} \right)$$

$$A = \left(z - \frac{1}{2}\right) = \frac{\left(\frac{1}{2} + 1\right)^2}{\left(\frac{1}{2} - 2\right) \left(\frac{1}{2} - 0.8\right) \left(\frac{1}{2} - \frac{1}{0.8}\right)} = \dots \quad B, C, D = \dots$$

$$\delta_{yy}[m] = \frac{1.5}{0.8} \left(A \cdot \left(\frac{1}{2}\right)^m u[m] + B \cdot 2^{-m} u[-m-1] + C \cdot 0.8^m u[m] + D \cdot \left(\frac{1}{0.8}\right)^{-m} u[-m-1] \right)$$

$(\frac{1}{2} < 1)$
 $(2 > 1)$
 $(0.8 < 1)$
 $(\frac{1}{0.8} > 1)$

$$c). \quad \sigma_{yy}[0] = \overline{(y[m])^2} = \sigma_y^2 + \mu^2$$

$$\Rightarrow \sigma_y^2 = \sigma_{yy}[0] - \mu^2 = \sigma_{yy}[0] - 0^2 = \sigma_{yy}[0] = \dots$$