SDP Lecture 07

Exercises with state-space equations:

Ex. 1 / Lab 7:

$$H(z) = \frac{\overset{b_0}{1} + \overset{b_1}{2z^{-1}} + \overset{b_2}{3z^{-2}} + \overset{b_3}{2z^{-3}}}{\underset{a_0}{1} + \underset{a_1}{0.9z^{-1}} + \underset{a_2}{0.8z^{-2}} + \underset{a_3}{0.5z^{-3}}}$$

$b_3 - b_0 a_3 = 2 - 1 \cdot 0.5 = 1.5$

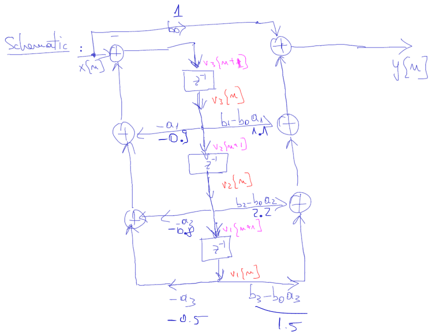
$b_2 - b_0 a_2 = 3 - 1 \cdot 0.8 = 2.2$

$b_1 - b_0 a_1 = 2 - 1 \cdot 0.9 = 1.1$

a) S.S. type 1:

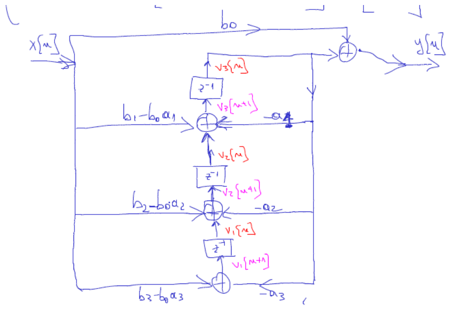
$$\begin{cases} y[n] = \begin{bmatrix} 1.5 & 2.2 & 1.1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + 1 \cdot x[n] \\ \begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot x[n] \end{cases}$$

Schematic:



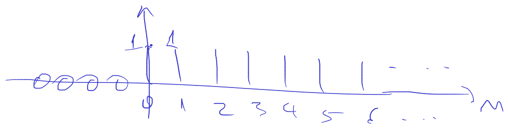
S.S. type 2

$$\begin{cases} y[n] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + 1 \cdot x[n] \\ \begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.5 \\ 1 & 0 & -0.8 \\ 0 & 1 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 1.5 \\ 2.2 \\ 1.1 \end{bmatrix} \cdot x[n] \end{cases}$$



$-0.9 + 1 = 0.1$

b) $x[n] = u[n] =$



$y[n] = ?$

At $n=0$: $y[0] = \begin{bmatrix} 1.5 & 2.2 & 1.1 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{V[0]} + 1 \cdot \underbrace{x[0]}_{1} = 2.1$

$$\begin{bmatrix} v_1[1] \\ v_2[1] \\ v_3[1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \underbrace{x[0]}_{1} = \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix}$$

At time $n=1$:

$y[1] = \begin{bmatrix} 1.5 & 2.2 & 1.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix} + 1 \cdot \underbrace{x[1]}_{1} = 2.2 + 0.11 + 1 = 3.32$

$-0.8 - 0.09 + 1 = 0.2 - 0.09 = 0.11$

$$\begin{bmatrix} v_1[2] \\ v_2[2] \\ v_3[2] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.8 & -0.9 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0.1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \underbrace{x[1]}_{1} = \begin{bmatrix} 1 \\ 0.1 \\ 0.11 \end{bmatrix}$$

At time $n=2$:

$y[2] = \ldots$

Ex 2 / Lab 7

$$v[n+1] = \begin{bmatrix} 0 & 1 \\ \underbrace{-0.81}_{-a_2} & \underbrace{1}_{-a_1} \end{bmatrix} v[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x[n]$$

$\Rightarrow$ type 1

$\Rightarrow$ order = 2

$$y[n] = \begin{bmatrix} \underbrace{-1.81}_{b_2 - b_0 a_2} & \underbrace{1}_{b_1 - b_0 a_1} \end{bmatrix} + \underbrace{1}_{b_0} x[n]$$

a) $H(z) = ? = \dfrac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$

$a_2 = 0.81$

$a_1 = -1$

$b_0 = 1$

$b_2 - \underbrace{b_0}_{1} \underbrace{a_2}_{0.81} = -1.81 \Rightarrow b_2 = 0.81 - 1.81 = -1$

$b_1 - \underbrace{b_0}_{1} \underbrace{a_1}_{-1} = 1 \Rightarrow b_1 + 1 = 1 \Rightarrow b_1 = 0$

$\Rightarrow H(z) = \dfrac{1 - 1 \cdot z^{-2}}{1 - z^{-1} + 0.81 z^{-2}}$

b). At time $n = 0$: $y[0] = [-1.81 \quad 1] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \cdot \underbrace{x[0]}_{1} = 2$

$$v[1] = \begin{bmatrix} 0 & 1 \\ -0.81 & 1 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{v[0]} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \underbrace{x[0]}_{1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\underline{n = 1}$: $y[1] = [-1.81 \quad 1] \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1 \cdot \underbrace{x[0]}_{1} = -1.81 + 2 + 1 = 1.19$

$$v[2] = \begin{bmatrix} 0 & 1 \\ -0.81 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \underbrace{x[1]}_{1} = \begin{bmatrix} 2 \\ -0.81 + 2 + 1 = 2.19 \end{bmatrix} = \begin{bmatrix} 2 \\ 2.19 \end{bmatrix}$$

$\underline{n = 2}$: $y[2] = \ldots$

$v[2] = \ldots$

$y[3] = \ldots$

$v[3] = \ldots$

Chapter 3:

$\boxed{\frac{1}{3}} = \boxed{0.33333}333\ldots$

73.4 in binary?

$73 = 64 + 8 + 1$

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$1\ 0\ 0\ 1\ 0\ 0\ 1$

73	2
36	2
18	2
9	2
4	2
2	2
1	2
$\boxed{0}$	

Remainders: 1, 0, 0, 1, 0, 0, 1 (read upward)

0.4:

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$

$0.6 \times 2 = 1.2$

$0.2 \times 2 = 0.4$

$0.4 \times 2 = 0.8$ — repeat

0 ...

$73.4 : \underbrace{1001001}_{73}.0110011001100\ldots$

$2^{-2}\ 2^{-3}\ \ 2^{-6}\ 2^{-7}\ \ 2^{-10}\ 2^{-11}$

MSB LSB

$(0\ 0\ 0)\ 1\ 0\ 0\ 1\ 0 = 18$, value = ?

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

MSB LSB

$\underbrace{1\ 0\ 0\ 1\ 0}_{18}.0\ 1\ 1\ 0$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0\ \ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ \ldots$

resolution is 2^{-4}

$= 18 + \cancel{0 \cdot 2^{-1}} + 1 \cdot \underbrace{2^{-2}}_{\frac{1}{4} = 0.25} + 1 \cdot \underbrace{2^{-3}}_{\frac{1}{8} = 0.125} + \cancel{0 \cdot 2^{-4}}$

$0.375 = \frac{3}{8}$

$= 18 + 0.25 + 0.125 = 18.375$

$~~73~~.\ 88.24 = 8 \cdot 10^1 + 8 \cdot 10^0 + 2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

$0.1111 = 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16} = 1 - 2^{-4}$

Negative numbers in binary:

Format: 1S 5i 4F

$\underbrace{\_}_{S}\ \underbrace{\_\ \_\ \_\ \_\ \_}_{i}\ .\ \underbrace{\_\ \_\ \_\ \_}_{F}$

$X = -18$

1) Sign-magnitude

$\underbrace{\_}_{0:+,\ 1:-}\ \underbrace{\_\_\_\_\_}_{18}$

$18: \underbrace{0}_{+}\ \underbrace{10010}_{18}$

$-18: \underbrace{1}_{-}\ \underbrace{10010}_{18}$

$\underbrace{\qquad\qquad}_{b+1 \text{ bits}}$

Format: Fixed point 1S 0i 7F

$\underbrace{\_}_{S}\ .\ \underbrace{\_\_\_\_\_\_\_}_{F}$

$0: \quad \underbrace{0}_{+}.\ 0000\ldots0 \quad +0$

$\quad\ \ \underbrace{1}_{-}\ 00000.\ 0 \quad -0$

carry bit

```
 10100 +     20 +
  1101       13
------      ---
100001       33
```

In binary

$5 + (-5) = 0$

$\underbrace{0}_{+}\ \underbrace{101}_{5}\ +$

$\underbrace{1}_{-}\ \underbrace{101}_{5}$

11010

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2). One's complement: 1C C1

1S5i0F:

$18: \underbrace{0}_{+}10010$

$-18: \underbrace{1}_{-}01101$

01101 → -18 / 13 ?

$0 \rightarrow \underbrace{0}_{+}\ 00000 \quad +0$

$\quad\ \ \underbrace{1}_{-}\ 11111 \quad -0$

$18 + (-18) = 0$

```
 010010 +
 101101
-------
 111111  = -0
```

3) Two's complement (2C) (C2)

ISSIOF: 18 : 010010

-18 : 101101 +

1

101110

$1\cancel{4} = 32 - 18$

0 : 000000

-0 : 111111 +

1

000000

0 is only: 0000..0

$$\begin{array}{r} 18 \ + \\ -18 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 010010 \ + \\ 101110 \\ \hline 000000 \end{array}$$

4). Binary offset