IIR filter design with Prony method

Lab 3, SDP

Objective

Using the Prony method for designing IIR filters of various types

Theoretical notions

The Prony method has the same purpose as the Pade method: to design a system function $H(z)$ of a specified order n:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_N z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}}$$

such that the impulse response $h[n]$ is approximately equal to the desired impulse response $h_d[n]$:

$$h[n] \approx h_d[n]$$

The difference is from the Pade method is in **how** this is done.

The Prony method operates as follows:

1. The denominator coefficients a_k are found by minimizing the **energy of the difference** signal between $h[n]$ and $h_d[n]$:

   $$E = \sum_{n=-\infty}^{\infty} (h[n] - h_d[n])^2$$

   Replacing $h[n]$ with the same formula used in the Pade method leads to an equation system using the autocorrelation function:

   $$to \quad draw \quad at \quad whiteboard$$

The **autocorrelation function** of $h_d[n]$, $\Gamma_{hh}[k]$, is defined as:

$$\Gamma_{hh}[k] = \sum_{n=-\infty}^{\infty} h_d[n] \cdot h_d[n+k]$$

2. Once a_k are known, the numerator coefficients b_k are found just like in the Pade method, from the same equations, in the same way.

Note: Because the b_k coefficients, found like in the Pade method, will make the first M coefficients of $h[n]$ equal to those of $h_d[n]$ (just like Pade), when computing the autocorrelation function $\Gamma_{hh}[k]$ we can consider only the part of $h_d[n]$ which starts after the first M samples. That's because we don't need to worry about the first M samples, they will be equal anyway.

Shank's method

An improved method would be to find the coefficients b_k not from the Pade equations (suboptimal), but from another energy optimization problem similar to the one used for finding a_k.

This method, known as **Shank's method**, is implemented in Matlab as `prony()`

Exercises

1. Design with the Prony method an IIR filter of order 2 which approximates the following desired impulse response:

$$h_d[n] = \{...0, \underset{\uparrow}{1}, 2, 3, 2, 1, 2, 3, 0, ...\}$$

   (the origin of time $n = 0$ is at the first value of 1 in the sequence).

2. Implement in Matlab a function for creating and then solving the equation system resulting from the **Prony method**:

   ```
   [b,a] = pronymet(order, hd)
   ```

   The function shall have the following arguments:

   - `order`: the order of the designed filter
   - `hd`: a vector holding the first samples of the desired impulse response

   The function shall return the coefficients of the system function for the resulting filter:

   - `b`: the numerator coefficients
   - `a`: the denominator coefficients

3. Use the function above to design a second order filter with the Pade method, for approximating the desired impulse response given below:

$$h_d[n] = \left(\frac{1}{3}\right)^n \cdot \cos(\frac{\pi}{4}n) \cdot u[n]$$

4. Use the function above to design with the Prony method a filter of order 2 which approximates the following higher-order filter (3):

$$H(z) = \frac{0.0736 + 0.0762z^{-1} + 0.0762z^{-1} + 0.0736z^{-3}}{1 - 1.3969z^{-1} + 0.8778z^{-1} - 0.1812z^{-3}}$$

   a. Use the function `impz()` to generate a sufficiently long impulse response of the given filter;
   b. Use your function `pronymet()` to actually design the filter;
   c. Plot on the same figure the impulse response of the given filter and the impulse response of the designed filter, for the first 50 samples.

5. Load a sample audio file in Matlab and filter it with the filter found above. Play the filtered signal. How does it sound like? Compare it with the original signal.

Final questions

1. TBD