IIR filter design with Pade method

Lab 2, SDP

Objective

Using the Pade method for designing IIR filters of various types

Theoretical notions

Exercises

1. Use the Pade method to find out the parameters of the system with the following system function
$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}},$$
considering the desired impulse response to be:
$$h_d[n] = \left(\frac{1}{3}\right)^n \cos\left(\frac{n\pi}{4}\right).$$

2. Use Matlab to solve numerically the Pade system for the previous exercise:
$$\begin{bmatrix} h_d[0] \\ h_d[1] \\ h_d[2] \\ h_d[3] \\ h_d[4] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ -h_d[0] & 0 & 0 & 1 & 0 \\ -h_d[1] & -h_d[0] & 0 & 0 & 1 \\ -h_d[2] & -h_d[1] & 0 & 0 & 0 \\ -h_d[3] & -h_d[2] & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

3. Implement in Matlab a function for creating and then solving the equation system resulting from the Pade method:
```
[b,a] = pade_method(order, hd)
```
The function shall have the following arguments:

- `order`: the order of the designed filter
- `hd`: a vector holding the first samples of the desired impulse response

The function shall return the coefficients of the system function for the resulting filter:

- `b`: the numerator coefficients
- `a`: the denominator coefficients

4. Use the `pade_method()` function above to design a second order filter with the Pade method, for approximating the desired impulse response given below:

$$h_d[n] = \left(\frac{1}{3}\right)^n \cdot \cos(\frac{\pi}{4}n) \cdot u[n]$$

5. Use `impz()` to find the impulse response of a filter with

$$H(z) = \frac{1 - 1.7z^{-1} + 0.7z^{-2}}{1 + 1.3z^{-1} + 0.4z^{-2}}.$$

   Then use our `pade_method()` to approximate a filter of order 2 from the impulse response. Do we obtain the same $H(z)$ back again?

6. Load a sample audio file in Matlab and filter it with the filter found above. Play the filtered signal. How does it sounde like? Compare it with the original signal.

Final questions

1. TBD