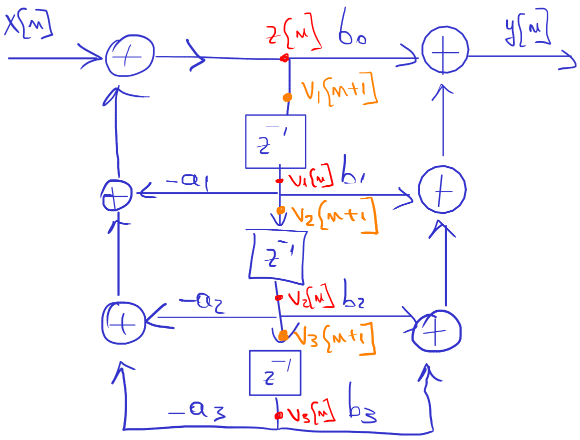


Direct form 2

$$y[n] = b_0 \cdot z[n] + b_1 \cdot v_1[n] + b_2 \cdot v_2[n] + b_3 \cdot v_3[n]$$

$$z[n] = x[n] - a_1 \cdot v_1[n] - a_2 \cdot v_2[n] - a_3 \cdot v_3[n]$$

$$\Rightarrow y[n] = (b_1 - b_0 a_1) \cdot v_1[n] + (b_2 - b_0 a_2) \cdot v_2[n] + (b_3 - b_0 a_3) \cdot v_3[n] + b_0 \cdot x[n]$$

$$\begin{cases} v_1[n+1] = z[n] = -a_1 \cdot v_1[n] - a_2 v_2[n] - a_3 v_3[n] + x[n] \\ v_2[n+1] = v_1[n] \\ v_3[n+1] = v_2[n] \end{cases}$$

$$y[n] = \begin{bmatrix} (b_1 - b_0 a_1) & (b_2 - b_0 a_2) & (b_3 - b_0 a_3) \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underbrace{b_0}_{} \cdot x[n]$$

Output equation

State -space type 1

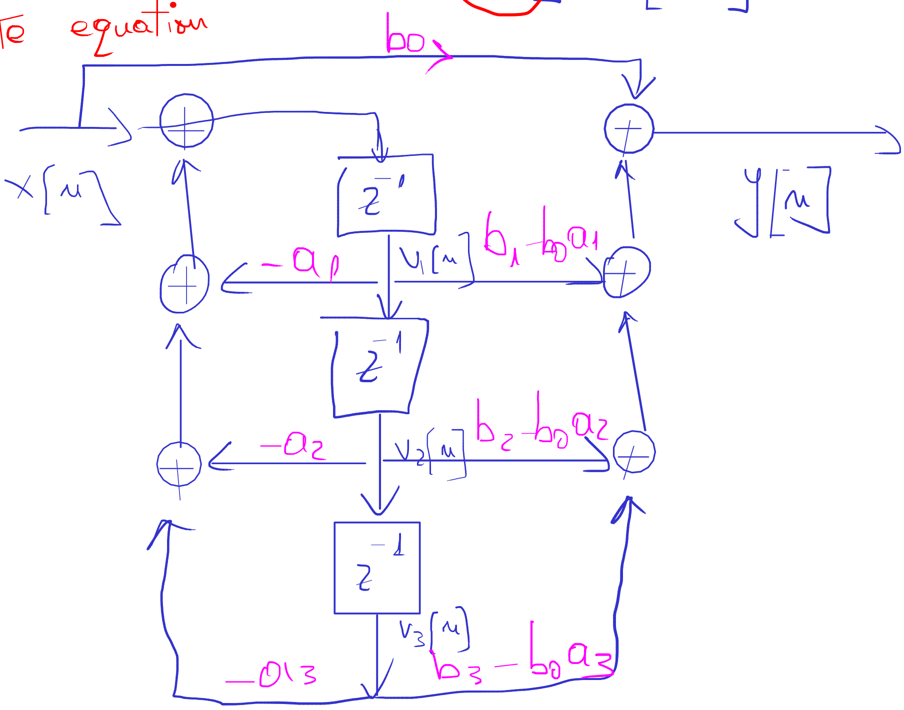
T

state

next state

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot x[n]$$

State equation

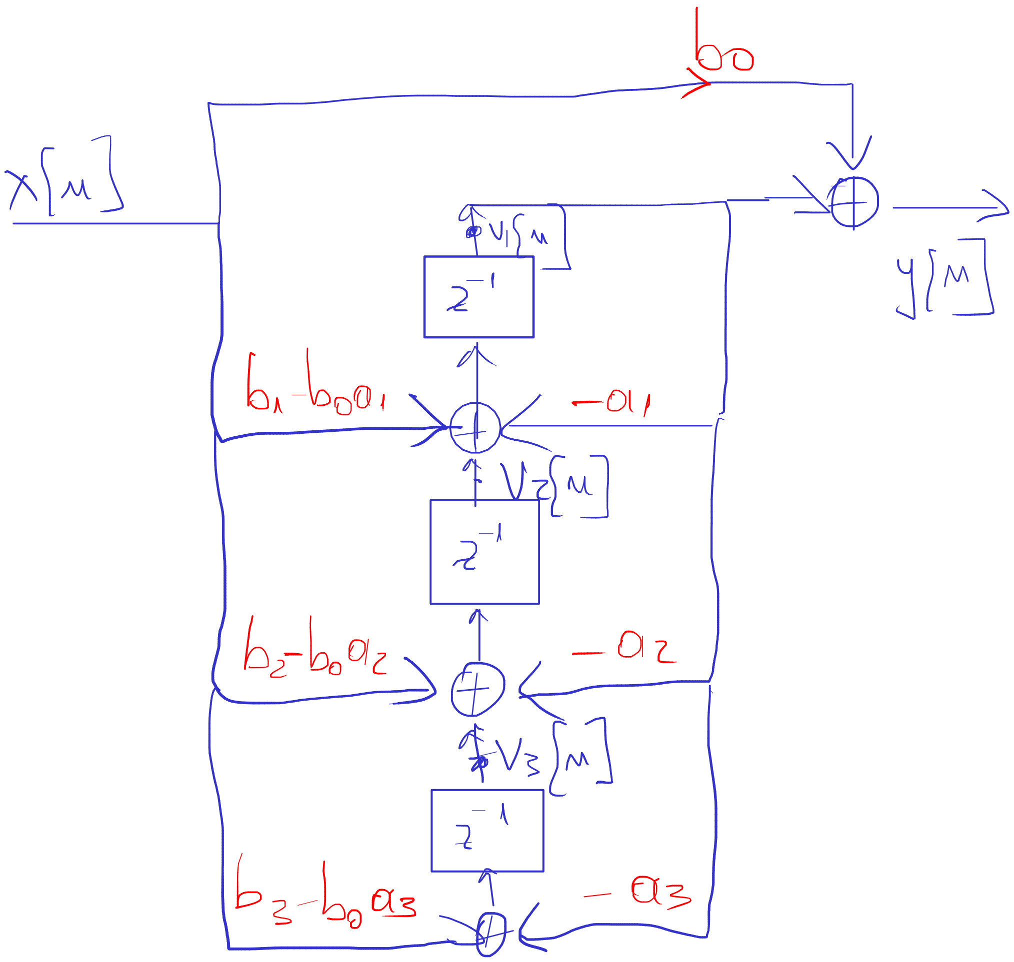


State space type 2

= transpose matrix, exchange vectors

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} b_1 - b_0 a_1 \\ b_2 - b_0 a_2 \\ b_3 - b_0 a_3 \end{bmatrix} \cdot x[n]$$

$$y[n] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underbrace{b_0} \cdot x[n]$$



1

$$H(z) = \frac{\overset{b_0}{1} + \overset{b_1}{2} z^{-1} + \overset{b_2}{3} z^{-2} + \overset{b_3}{2} z^{-3}}{\underset{a_0}{1} + \underset{a_1}{0.9} z^{-1} + \underset{a_2}{0.8} z^{-2} + \underset{a_3}{0.5} z^{-3}}$$

a) state space type 1 and 2

type 1:

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} -0.9 & -0.8 & -0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot x[n]$$

$$y[n] = \begin{bmatrix} 1.1 & 2.2 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underbrace{1} \cdot x[n]$$

$$b_1 - b_0 a_1 = 1.1$$
$$b_2 - b_0 a_2 = 2.2$$

type 2:

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \\ v_3[n+1] \end{bmatrix} = \begin{bmatrix} -0.9 & 1 & 0 \\ -0.8 & 0 & 1 \\ -0.5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \begin{bmatrix} 1.1 \\ 2.2 \\ 1.5 \end{bmatrix} \cdot x[n]$$

$$y[n] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1[n] \\ v_2[n] \\ v_3[n] \end{bmatrix} + \underbrace{1} \cdot x[n]$$

b). $v_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad x[n] = u[n] \quad y[0] = ? \quad y[1] = ? \quad y[2] = ?$

$$y[0] = \begin{bmatrix} 1.1 & 2.2 & 1.5 \end{bmatrix} \cdot \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}^{v[0]} + 1 \cdot \overbrace{1}^{x[0]} = 2.5$$

$$v[1] = \begin{bmatrix} -0.9 & -0.8 & -0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}^{v[0]} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \overbrace{1}^{x[0]} = \overbrace{\begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}}^{v[1]}$$

$$y[1] = \begin{bmatrix} 1.1 & 2.2 & 1.5 \end{bmatrix} \cdot \overbrace{\begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}}^{v[1]} + 1 \cdot \overbrace{1}^{x[1]} = 1.55$$

$$v[2] = \begin{bmatrix} -0.9 & -0.8 & -0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \overbrace{\begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}}^{v[1]} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \overbrace{1}^{x[1]} = \overbrace{\begin{bmatrix} 0.55 \\ 0.5 \\ 0 \end{bmatrix}}^{v[2]}$$

$y[2] =$

$v[3] =$

$y[3] =$

$v[4] =$

$\vdots$

② $v[n+1] = \begin{array}{cc} -a_1 & -a_2 \end{array}$
$$v[n+1] = \begin{bmatrix} 1 & -0.81 \\ 1 & 0 \end{bmatrix} \cdot v[n] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot x[n]$$

$$y[n] = \begin{bmatrix} 1 & -1.81 \end{bmatrix} \cdot v[n] + \overset{b_0}{1}\, x[n]$$

$b_1 - b_0 a_1 \quad b_2 - b_0 a_2$

$\Rightarrow$

$a_1 = -1$

$a_2 = 0.81$

$b_0 = 1$

$b_1 = 1 + b_0 \cdot a_1 = 1 + (-1) = 0$

$b_2 = -1.81 + 1 \cdot 0.81 = -1$

$$H(z) = \frac{\overset{b_0}{1} + \overset{b_1}{0} \cdot z^{-1} - \overset{b_2}{z^{-2}}}{\underset{a_0}{1} - \underset{a_1}{z^{-1}} + \underset{a_2}{0.81 z^{-2}}} \Leftarrow$$