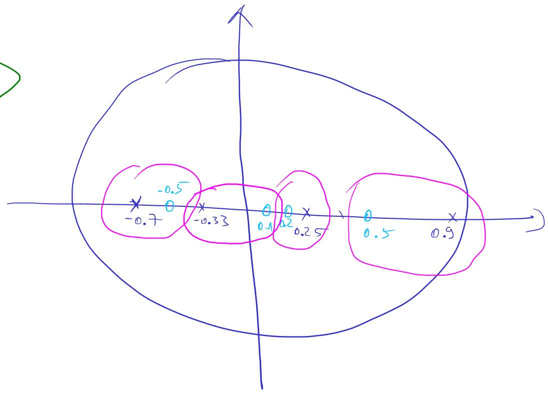
SDP Lecture 12

How to group poles and zeros for a series implementation:

$$H(z) = \frac{\overbrace{(z-0.1)}^{H_3(z)}\overbrace{(z-0.2)}^{H_4(z)}\overbrace{(z-0.5)}^{H_1(z)}\overbrace{(z+0.5)}^{H_2(z)}}{(z-0.25)(z-0.9)(z+0.7)(z+0.33)}$$



$$H_1(z) = \frac{(z-0.5)}{(z-0.9)}$$

$$H_2(z) = \frac{z+0.5}{z+0.7}$$

Exercise 1 / Lab 9:

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$

$$x[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$$

Initial cond. $= 0$

$$y[0] = \frac{1}{2}y[-1] + x[0] = 1$$

$$y[1] = \frac{1}{2}\cdot \underbrace{y[0]}_{1} + x[1] = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

a). Infinite precision:

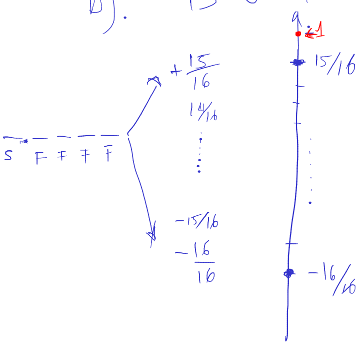
	$x[n]$	$y[n]$
$n=0$	1	1
$n=1$	1/4	3/4
$n=2$	1/16	7/16
$n=3$	1/64	15/64
$n=4$	1/256	31/256
$n=5$	1/1024	63/1024
	$\downarrow$ 0	$\downarrow$ 0

$$\frac{3}{8} + \frac{1}{16}$$

$$\frac{7}{32} + \frac{1}{64} = \frac{15}{64}$$

$$\frac{15}{128} + \frac{1}{256} =$$

b). Fixed-point 1S 0i 4F Truncation (Quant. with saturation)



	$x[n]$	$y[n]$	
$n=0$	15/16	15/16	(0.1111)
$n=1$	1/4 = 4/16	11/16	(0.1011)
$n=2$	1/16	6/16	(0.0110)
$n=3$	0	3/16	
$n=4$	0	1/16	
$n=5$	0	0	
	0 $\downarrow$ 0	0 $\downarrow$ 0	

$$[1]_Q = \frac{15}{16}$$

$$y[0] = \frac{1}{2}\cdot y[-1] + \underbrace{x[0]}_{15/16}$$

$$1/4 = 4/16$$

$$y[1] = \left[\frac{1}{2}\cdot\frac{15}{16}\right]_Q + \frac{4}{16} = \frac{11}{16}$$

$$= \left[\frac{7.5}{16}\right]_Q = \frac{7}{16}$$

8/16

7.5/16

7/16

$$n=3:\quad x[3] = \frac{1}{64} = \frac{0.25}{16} \xrightarrow{Q} \frac{0}{16}$$

c). Use <u>rounding</u>

	$x[n]$	$y[n]$
$n=0$	$15/16$	$15/16$
$n=1$	$1/4 = 4/16$	$12/16$
$n=2$	$1/16$	$7/16$
$n=3$	0	$4/16$
$n=4$	0	$2/16$
$n=5$	0	$1/16$
$n=6$	0	$1/16$
$n=7$	0	$1/16$
$\vdots$	$\vdots$	$1/16$ $\vdots$
	0	$1/16$

remaining constant

$\boxed{\text{Limit cycle}}$

$x[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$

$[1]_Q = \frac{15}{16}$

$y[0] = \frac{1}{2}\cdot 0 + \frac{15}{16}$

$x[3] = \frac{1}{64} = \frac{0.25}{16} \xrightarrow{Q} 0$

$1/16$

$0/16$

$y[1] = \underbrace{\frac{1}{2}\cdot\frac{15}{16}}_{} + \frac{4}{16}$

$\left[\frac{7.5}{16}\right]_Q = \frac{8}{16}$

$8/16$

$\frac{7.5}{16}$

$7/16$

$y[6] = \underbrace{\frac{1}{2}\cdot\frac{1}{16}}_{} + 0$

$\left[\frac{0.5}{16}\right]_Q = \frac{1}{16}$

$\boxed{\text{Note}}$:

Quantize every result!

(Rounding)

$$\underbrace{\underbrace{\frac{1}{2}\cdot\frac{11}{16}}_{Q} + \frac{13}{16}}_{Q} = \underbrace{\frac{6}{16} + \frac{13}{16}}_{Q} = \frac{15}{16} \quad (\text{Saturated})$$

(without saturation/ with overflow)

$$\frac{6}{16} + \frac{13}{16} = \frac{14}{16} \quad \frac{15}{16} \quad \cancel{\frac{16}{16}} = -\frac{13}{16}$$

$$\downarrow$$

$$-\frac{16}{16} \quad -\frac{15}{16} \quad -\frac{14}{16} \quad -\frac{13}{16}$$

Exercise 1 / Lab 10

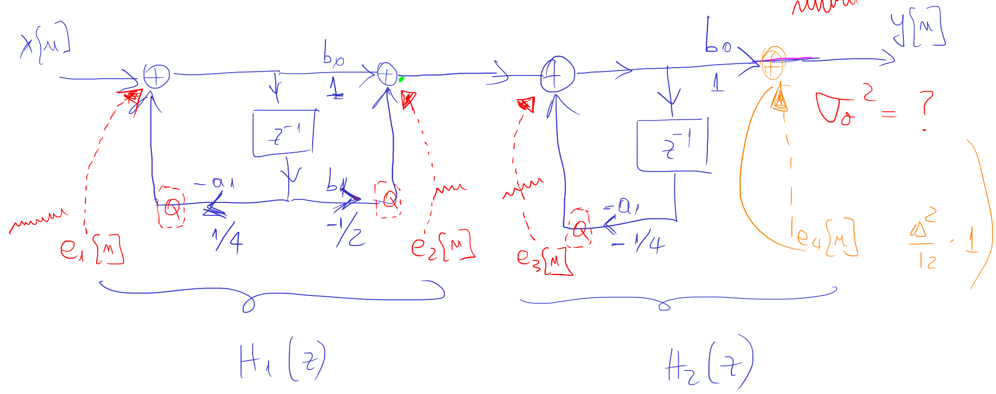
$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

$H_1(z)$ $\quad$ $H_2(z)$

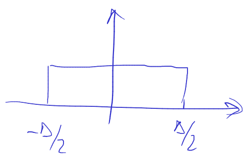
a) $H_1(z) = \dfrac{1 - \frac{1}{2} z^{-1}}{1 - \frac{1}{4} z^{-1}}$ $\quad$ b_0, b_1, a_1

$H_2(z) = \dfrac{1 \quad 0}{1 + \frac{1}{4} z^{-1}}$ $\quad$ $b_1 = 0$, a_1

$H(z) = H_1(z) \cdot H_2(z)$



b). $e_1[n]$, $e_2[n]$, $e_3[n]$ = quantization noise



Power of rounding noise $\sigma^2 = \dfrac{\Delta^2}{12}$ $\quad$ $\Delta = 2^{-b}$ = quant. step

$= \dfrac{2^{-2b}}{12}$ = variance of every rounding noise

Output variance σ_o^2 = sum of the effects of every internal noise !

. $e_1[n]$ travels through all the system $\Rightarrow$ output:

$\rightarrow$ $\underset{\text{output power}}{\sigma_o^2} = \underset{\text{input power}}{\sigma_i^2} \cdot \sum h[n]^2$

$\frac{\Delta^2}{12}$

. $e_2[n]$: $\sigma_o^2 = \underset{\frac{\Delta^2}{12}}{\sigma_i^2} \cdot \sum (h_2[n])^2$

$\rightarrow$ $h_2[n]$ because $e_2[n]$ travels only through $H_2(z)$

. $e_3[n]$: $\rightarrow$ ——— // ——— e_3 also travels through $H_2(z)$ only

Total Output power

$$\boxed{\sigma_o^2 = \frac{\Delta^2}{12} \sum_{n=0}^{\infty} (h[n])^2 + 2 \cdot \frac{\Delta^2}{12} \cdot \sum_{n=0}^{\infty} (h_2[n])^2}$$

? $\quad$? $\quad$ $= \frac{16}{15}$

$\Delta = 2^{-b}$

How to compute: $\boxed{\sum_n \left(h[n]\right)^2 = \sum_{\substack{\text{poles}\\ \text{inside}\\ \text{the}\\ \text{unit}\\ \text{circle}}} \text{residuals of } H(z)\cdot H(z^{-1})\cdot z^{-1}}$ General

$\sum \left(h_2[n]\right)^2$:

$$\underbrace{H_2(z)}_{\frac{1}{1+\frac{1}{4}z^{-1}}} \cdot \underbrace{H_2(z^{-1})}_{\frac{1}{1+\frac{1}{4}z}} \cdot z^{-1} = \frac{1}{1+\frac{1}{4}z^{-1}} \cdot \frac{1}{1+\frac{1}{4}z} \cdot z^{-1} = \frac{\cancel{z}}{z+\frac{1}{4}} \cdot \frac{1}{1+\frac{1}{4}z} \cdot \frac{1}{\cancel{z}}$$

$$= \frac{1}{\left(z+\frac{1}{4}\right)\left(1+\frac{1}{4}z\right)} = \frac{4}{\left(z+\frac{1}{4}\right)\left(z+4\right)}$$

poles: x $\frac{1}{x}$

residual for the pole in the unit circle

residuals

$$\frac{4}{\left(z+\frac{1}{4}\right)\left(z+4\right)} = \frac{A_1}{z+\frac{1}{4}} + \frac{A_2}{z+4}$$

$p = -\frac{1}{4}$ $\qquad p = -4$

$$A_1 \overset{z=-\frac{1}{4}}{=} \frac{4}{-\frac{1}{4}+4} = \frac{4}{\frac{15}{4}} = \frac{16}{15}$$

$$\Rightarrow \sum \left(h_2[n]\right)^2 = \frac{16}{15}$$

For $h[n]$:

$$H(z)\cdot H(z^{-1})\cdot z^{-1} = \underbrace{\frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}}_{H(z)} \cdot \underbrace{\frac{1-\frac{1}{2}z}{\left(1-\frac{1}{4}z\right)\left(1+\frac{1}{4}z\right)}}_{H(z^{-1})} \cdot z^{-1}$$

$$= \frac{\cancel{z^2 - \frac{1}{2}z}}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)} \cdot \frac{\left(1-\frac{1}{2}z\right)}{\left(1-\frac{1}{4}z\right)\left(1+\frac{1}{4}z\right)} \cdot \frac{1}{\cancel{z}}$$

$$= \frac{\left(z-\frac{1}{2}\right)\left(1-\frac{1}{2}z\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)\left(1-\frac{1}{4}z\right)\left(1+\frac{1}{4}z\right)} = -16 \cdot \frac{\left(z-\frac{1}{2}\right)\left(1-\frac{1}{2}z\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)\left(z-4\right)\left(z+4\right)}$$

$$= \frac{A_1}{z-\frac{1}{4}} + \frac{A_2}{z+\frac{1}{4}} + \frac{A_3}{z-4} + \frac{A_4}{z+4} \Rightarrow \boxed{A_1 + A_2} = \dots$$

— END —