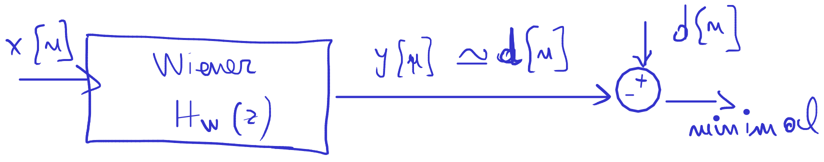


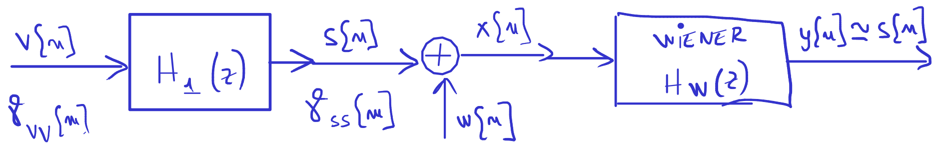
$$\gamma_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]\cdot x[n+m]$$

$$\gamma_{dx}[m] = \sum_{n=-\infty}^{\infty} d[n]\cdot x[n+m]$$

Usual scenarios:

x	d
$s[n]$ + noise	$s[n]$
$s[n]$	$s[n+1]$

1



$$\Gamma_{vv}(z)\cdot H_1(z)\cdot H_1(z^{-1}) = \Gamma_{ss}(z)$$

$$\gamma_{xx}[m]$$
$$\Gamma_{xx}(z)$$

a) $v[n]$ = white noise $\Rightarrow$ $\gamma_{vv}[m] = \sigma_v^2 \cdot \delta[m]$

$$\Gamma_{vv}(z) = \sigma_v^2$$

$w[n]$ = —— " —— $\Rightarrow$ $\gamma_{ww}[m] = \sigma_w^2 \cdot \delta[m]$

$$\Gamma_{ww}(z) = \sigma_w^2$$

$$H_1(z) = \frac{S(z)}{V(z)} = \frac{1}{1-0.6z^{-1}}$$

$$\Gamma_{ss}(z) = \sigma_v^2 \cdot \frac{1}{1-0.6z^{-1}} \cdot \frac{1}{1-0.6z} = \frac{0.64\, z}{(z-0.6)(1-0.6z)} = \frac{0.64}{-0.6}\cdot\frac{z}{(z-0.6)(z-\frac{1}{0.6})}$$

$$\frac{\Gamma_{ss}(z)}{z} = \frac{0.64}{-0.6}\cdot\frac{1}{(z-0.6)(z-\frac{1}{0.6})} = \frac{0.64}{-0.6}\left(\frac{A}{z-0.6} + \frac{B}{z-\frac{1}{0.6}}\right)$$

A, B = ...

$$\gamma_{ss}[m] = \frac{0.64}{-0.6}\cdot\left(\underbrace{A\cdot 0.6^{m}\, u[m] + B\cdot\left(\frac{1}{0.6}\right)^{m} u[-m-1]}_{0.6^{|m|}}\right)$$

$$x = s + w$$

$$\gamma_{xx}[m] = \gamma_{(s+w)(s+w)}[m] = \gamma_{ss}[m] + \underbrace{\gamma_{sw}[m]}_{=0} + \underbrace{\gamma_{ws}[m]}_{=0} + \underbrace{\gamma_{ww}[m]}_{= \sigma_w^2 \cdot \delta[m]}$$

(white noise)

$$= \gamma_{ss}[m] + 1 \cdot \delta[m]$$

$$\gamma_{ss}[m] = 0.6^{|m|}$$

$$\gamma_{xx}[m] = 0.6^{|m|} + \delta[m]$$

b). $x[n]$

Wiener, FIR, $M = 2$ $\quad H_w = b_0 + b_1 z^{-1}$

$d[n] = s[n]$

$$\begin{bmatrix} \gamma_{dx}[0] \\ \gamma_{dx}[1] \\ \vdots \end{bmatrix} = \begin{bmatrix} \gamma_{xx}[0] & \gamma_{xx}[1] & \cdots \\ \gamma_{xx}[-1] & \gamma_{xx}[0] & \cdots \\ \vdots & & \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \end{bmatrix}$$

$$\gamma_{xx}[m] = 0.6^{|m|} + \delta[m]$$

$$\gamma_{xx}[0] = \underline{1} + \underline{1} = 2$$

$$\gamma_{xx}[-1] = \gamma_{xx}[1] = 0.6$$

$$\gamma_{dx}[m] = \gamma_{sx}[m] = \gamma_{s(s+w)}[m] = \gamma_{ss}[m] + \underbrace{\gamma_{sw}[m]}_{=0} = \gamma_{ss}[m]$$

$$\gamma_{dx}[0] = \gamma_{ss}[0] = \underline{1}$$

$$\gamma_{dx}[1] = \gamma_{ss}[1] = 0.6$$

$$\begin{bmatrix} 1 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 2 & 0.6 \\ 0.6 & 2 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \Rightarrow \ldots \Rightarrow \begin{matrix} b_0 = 0.45 \\ b_1 = 0.16 \end{matrix}$$

$$H_w(z) = 0.45 + 0.16\, z^{-1}$$

c).

$$\underset{\substack{EPMM\\(MMSE)}}{E} = \gamma_{dd}[0] - \sum_{k=0}^{\infty} b_k \cdot \gamma_{dx}[k]$$

$$b_k = h[k]$$

$$\gamma_{dd}[0] = \gamma_{ss}[0] = \underline{1}$$

$$E = \underline{1} - b_0 \cdot \gamma_{dx}[0] - b_1 \cdot \gamma_{dx}[1] = \underline{1} - b_0 \cdot \gamma_{ss}[0] - b_1 \cdot \gamma_{ss}[1] = \ldots$$

$$= 0.45$$