$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$H(\omega)$

$$h[n] = b_0 + \sum_{k} (a_k)^n u[n]$$

$b_1 = \frac{1}{3}$

$= 0.333333333\ldots$

$b_2 = 0.251379\ldots$

≈ 0.251379

FIR systems:

$$H(z) = b_0 + b_1 z^{-1} + \ldots + b_M z^{-M} \quad (M = \text{order}) = \underbrace{\left(1 - z_1 z^{-1}\right)\cdot\left(1 - z_2 z^{-1}\right)}_{H_1(z)} \underbrace{\cdots \left(1 - z_M z^{-1}\right)}_{H_2(z)}$$

(M zeros)

$$h[n] = \{ b_0, b_1, b_2, \ldots b_M \}\ 0, 0, 0, \ldots 0$$

($n=0$; $M+1$ values)

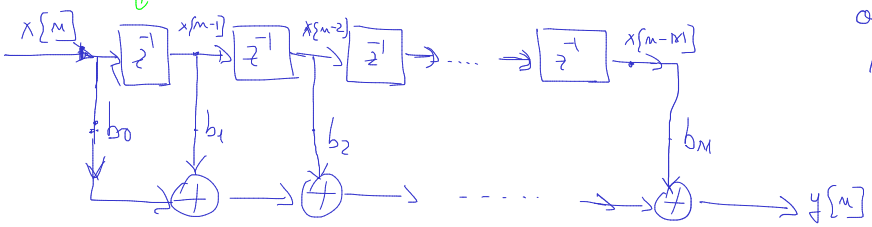
$$y[n] = b_0 \cdot x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

Example

$$H(z) = 1 - 1.5 z^{-1} + 0.56 z^{-2} = (1 - 0.7 z^{-1})\cdot(1 - 0.8 z^{-1})$$

$$= \frac{z^2 - 1.5z + 0.56}{z^2} = \frac{(z - 0.7)(z - 0.8)}{z^2} = (1 - 0.7 z^{-1})(1 - 0.8 z^{-1})$$

Direct form — delay block (z^{-1})



order M

$M+1$ multiplications

$b_0 = h[0]$

$b_1 = h[1]$

$\vdots$

Optim. for linear-phase

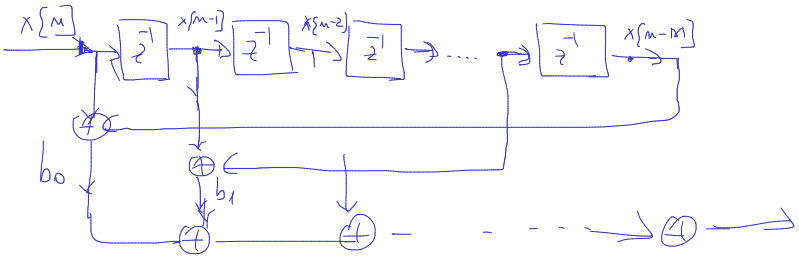
b_0 $h[0]$; b_M $h[M]$

Example: $h[n] = \{ 1, 2, 3, 7, 0, -5, 0, 7, 3, 2, 1 \}$

$h[n] = \{ 1, 2, 3, 7, 0, 0, 0, -7, -3, -2, -1 \}$

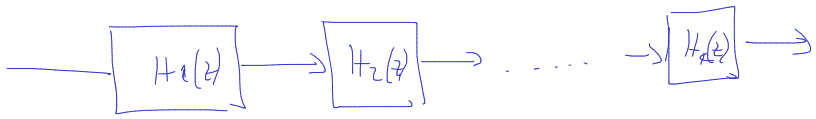
$h[n] = \pm h[M-n]$

$b_0 = b_M$



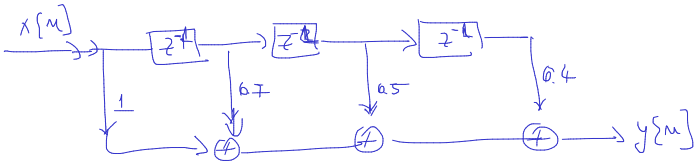
Cascade (series) form

$$H(z) = \prod H_k(z) = \underbrace{H_1(z)}_{\text{order 2}} \cdot \underbrace{H_2(z)}_{\text{order 2}} \cdot \ldots \cdot H_K(z)$$



Example:

$$H(z) = 1 + 0.7z^{-1} + 0.5z^{-2} + 0.4z^{-3}$$



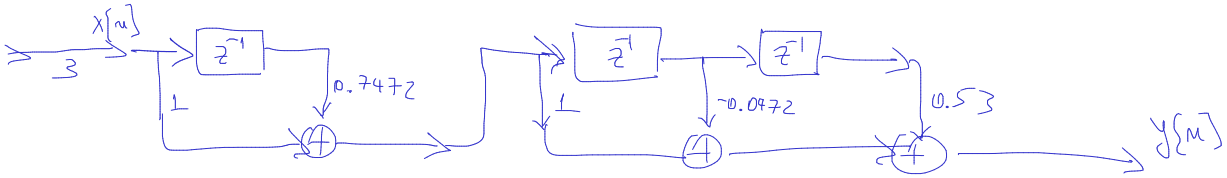
Direct form

Cascade form

$$H(z) = \underbrace{3}_{b_0}\left(1 + \underbrace{0.7472}_{-z_1} z^{-1}\right) \cdot \left(1 - \underbrace{(0.0236 + 0.7313j)}_{z_2} z^{-1}\right)\left(1 - \underbrace{(0.0236 - 0.7313j)}_{z_3} z^{-1}\right)$$

$H_1(z)$ $\qquad$ $H_2(z)$

$$H_1(z) = 1 + 0.7472z^{-1} \qquad H_2(z) = 1 - 0.0472z^{-1} + 0.53z^{-2}$$



Linear phase:

$z_1 = 0.7$ $\qquad$ reciprocal pair

$z_2 = \frac{1}{0.7}$

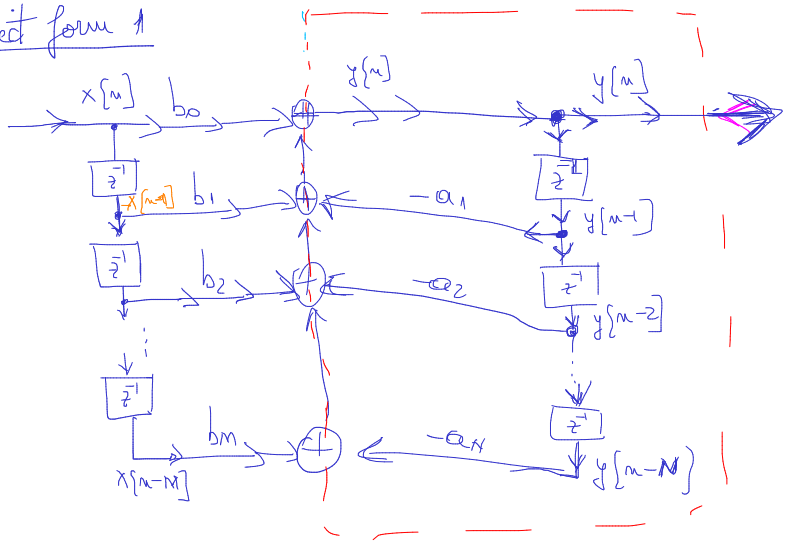
IIR systems

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \dots b_M z^{-M}}{1 + a_1 z^{-1} + \dots a_N z^{-N}}$$

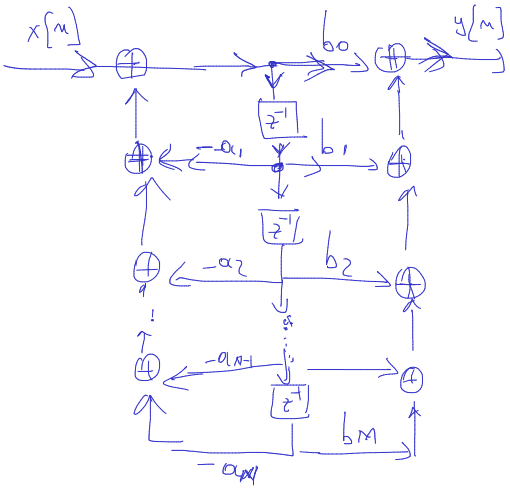
$$h[n] = \dots\dots$$

$$y[n] = -a_1 \cdot y[n-1] - \dots - a_N \cdot y[n-N] + \underbrace{b_0 \cdot x[n] + b_1 \cdot x[n-1]}_{\delta} + \dots + b_M \cdot x[n-M]$$

Direct form 1

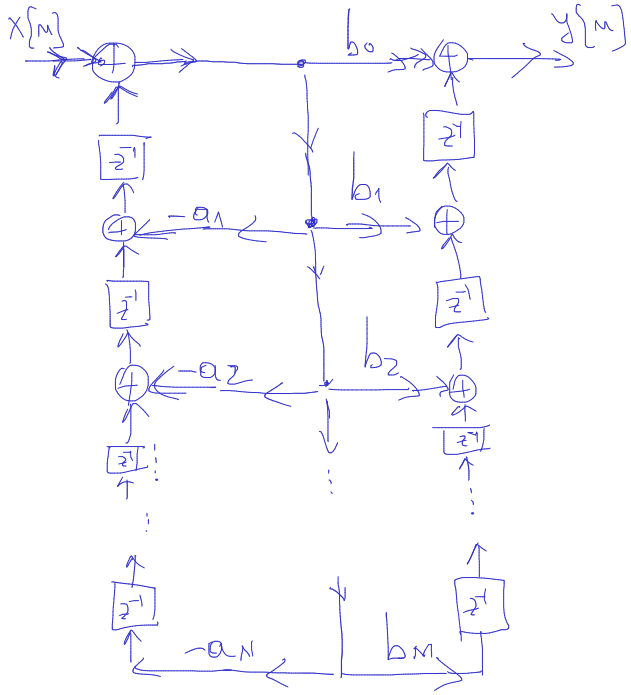


Direct form 2

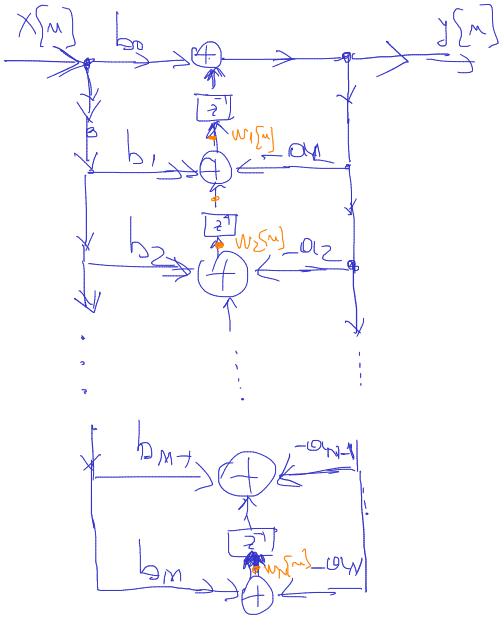


Transposed forms

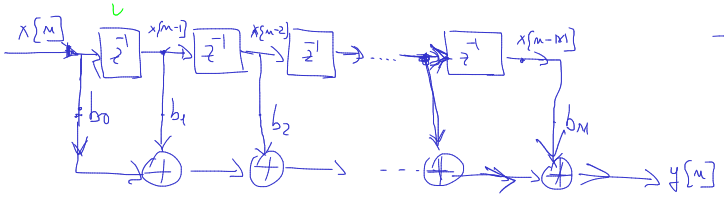
Direct form 1 transposed



Direct form 2 transposed



Transposed FIR direct form



Transposed ⟹