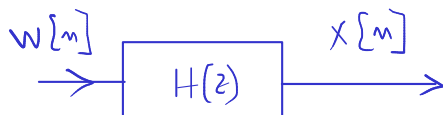


①



$$\delta_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n] \cdot x[n+m]$$

z transform \rightarrow

$$\begin{aligned} \delta_{ww}[m] &\rightarrow \Gamma_{ww}(z) \\ \delta_{xx}[m] &\rightarrow \Gamma_{xx}(z) \end{aligned}$$

$$\Gamma_{ww}(\omega) \quad \Gamma_{xx}(\omega)$$

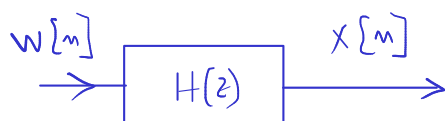
$$\Gamma_{ww}(z) \cdot H(z) \cdot H(z^{-1}) = \Gamma_{xx}(z)$$

$w = \text{white noise} \Rightarrow \Gamma_{ww}[m] = \sigma_w^2 \cdot \delta[m]$

$$\Gamma_{ww}(z) = \sigma_w^2 = \text{constant}$$

To Do : - find $\Gamma_{xx}(z)$
- inverse z transform $\Rightarrow \delta_{xx}[m]$

①



$$\Gamma_{ww}(z) \cdot H(z) \cdot H(z^{-1}) = \Gamma_{xx}(z)$$

$$x[n] = \frac{1}{2} x[n-1] + w[n] + w[n-1] \Rightarrow H(z) = \frac{X(z)}{W(z)} = \frac{1 + z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$\Gamma_{ww}(z) = \sigma_w^2$$

$$\Gamma_{xx}(z) = \sigma_w^2 \cdot \underbrace{\frac{1 + z^{-1}}{1 - \frac{1}{2} z^{-1}}}_{H(z)} \cdot \underbrace{\frac{1 + z}{1 - \frac{1}{2} z}}_{H(z^{-1})} = \sigma_w^2 \cdot \frac{(z+1) \cdot (1+z) \cdot (-z)}{(z - \frac{1}{2}) \cdot (1 - \frac{1}{2} z) \cdot (-z)} = \sigma_w^2 \cdot \frac{(-z) \cdot (z+1)}{(z - \frac{1}{2})(z-2)}$$

$$\frac{\Gamma_{xx}(z)}{z} = -2\sigma_w^2 \cdot \frac{(z+1)^2}{z(z - \frac{1}{2})(z-2)} = \frac{A}{z} + \frac{B}{z - \frac{1}{2}} + \frac{C}{z-2}$$

$$z=0 : A = -2\sigma_w^2 \cdot \frac{1}{(-\frac{1}{2}) \cdot (-2)} = -2\sigma_w^2$$

$$z=\frac{1}{2} : B = -2\sigma_w^2 \cdot \frac{(1.5)^2}{\frac{1}{2} \cdot (-3/2)} = -6\sigma_w^2$$

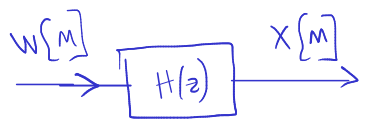
$$z=2 : C = -2\sigma_w^2 \cdot \frac{9}{2 \cdot 3/2} = -6\sigma_w^2$$

$$\Rightarrow \Gamma_{xx}(z) = A + B \cdot \frac{z}{z - \frac{1}{2}} + C \cdot \frac{z}{z-2}$$

$$\delta_{xx}[m] = A \cdot \delta[m] + B \cdot \left(\frac{1}{2}\right)^m u[m] + C \cdot 2^m u[m]$$

where A, B, C = calculated

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$$g_{xx}[m] = \frac{1}{4} |m|$$

We need to find the system equation

$$T_{ww}(z) \cdot H(z) \cdot H(z^{-1}) = T_{xx}(z)$$

- TO DO:
- compute $T_{xx}(z)$
 - find $H(z)$
 - write the equation

$$\begin{aligned}
 T_{xx}(z) &= \mathcal{Z}(g_{xx}[m]) = \sum_{m=-\infty}^{\infty} g_{xx}[m] z^{-m} = \sum_{m=-\infty}^{\infty} \frac{1}{4} |m| \cdot z^{-m} \\
 &= \underbrace{\sum_{m=0}^{\infty} \left(\frac{1}{4} \cdot z^{-1}\right)^m}_{= \frac{1}{1 - \frac{1}{4} z^{-1}}} + \underbrace{\sum_{m=-\infty}^{-1} \left(\frac{1}{4}\right)^{-m} \cdot z^{-m}}_{= \sum_{m=1}^{\infty} \frac{1}{4}^m \cdot z^m} \\
 &= \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{z}{4}} - 1 = \frac{\cancel{\frac{1}{4}} + 1 - \cancel{\frac{1}{4}} z}{\left(1 - \frac{1}{4} z^{-1}\right) \left(1 - \frac{z}{4}\right)} \\
 &= \frac{\sqrt{w}^2 \cdot \cancel{15/16} \cdot \cancel{1} \cdot \cancel{1}}{\cancel{H(z)} \left(1 - \frac{1}{4} z^{-1}\right) \left(1 - \frac{z}{4}\right) \cancel{H(z^{-1})}} = T_{ww}(z) \cdot H(z) \cdot H(z^{-1}) \\
 \Rightarrow H(z) &= \frac{1}{1 - \frac{1}{4} z^{-1}} = \frac{X(w)}{W(w)} \Rightarrow x[m] = \frac{1}{4} x[m-1] + w[m]
 \end{aligned}$$

Another choice:

$$\begin{aligned}
 \sqrt{w}^2 &= 1 \\
 H(z) &= \frac{\sqrt{15/16}}{1 - \frac{1}{4} z^{-1}} \Rightarrow x[m] = \frac{1}{4} x[m-1] + \sqrt{\frac{15}{16}} \cdot w[m] \\
 H(z^{-1}) &= \frac{\sqrt{15/16}}{1 - \frac{1}{4} z}
 \end{aligned}$$