Spectral estimation methods

Lab 11, SDP

Objective

Students should use some well-known spectral estimation methods and one of its applications.

Theoretical notions

Exercises

1. Find the average value and the autocorrelation function of the signal $x[n]$ obtained as the output of an ARMA(1,1) random process with the following difference equation:

$$x[n] = \frac{1}{2}x[n-1] + w[n] + w[n-1],$$

   where $w[n]$ is white noise with variance σ_w^2 and average value 0.

2. The autocorrelation function of an AR random process $x[n]$ is:

   $\gamma_{xx}[m] = \frac{1}{4}^{|m|}$.

   Find the difference equation of the random process $x[n]$. Is this unique? If not, find more than one possible solution.

3. In Matlab, create a signal of length $N = 1000$ defined as follows:

   $x[n] = \cos(2 \cdot \pi \cdot f_1 \cdot n) + 0.5 \cdot \cos(2 \cdot \pi \cdot f_2 \cdot n) + A \cdot noise$

   where $f_1 = 1/44100$ and $f_2 = 1.8/44100$, and the noise is random white gaussian noise (`randn()`).

   Try different values of A (e.g. 0.1, 0.3).

4. Estimate the power spectral density of the signal `x`:

   1. Compute the Fourier transform $X(f)$ (with `fft()`), and display $|X(f)|^2$
   2. Use the function `periodogram()`
   3. Split the signal in semnalul in 2, 4, or 10 segments of equal lengths, compute the periodogram of each one, and then average them. What are the differences?
   4. Use the Yule Walker method (`pyulear`).

   Pay attention to:

   - are the frequency peaks correctly located at f_1 and f_2?
   - are the frequency peaks wide or narrow?
   - is the noise spectrum flat or not?

5. In Matlab, create a script file which implements a live spectrum analyzer.

   a. Load the signal `music.wav` with the function `audioread()`.
   b. Use the function `buffer()` to split the signal into windows of length 30ms.
   c. Use `periodogram()` to estimate and plot, successively,the spectrum of each window signal.
   d. Optional: localize and plot the dominant frequency from the spectrum of each window. Convert the frequency to the corresponding musical note and output it.

Final questions

1. TBD