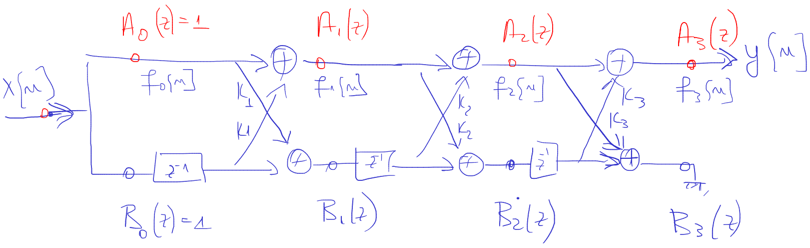
Lecture 05 : Exercise 2, Lab05 :

$$H(z) = 1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \underset{K_3}{\frac{1}{2}} z^{-3} = A_3(z)$$

$K_{1,2,3} = ?$ Implement in lattice form (tf2latc())



$$A_m(z) = A_{m-1}(z) + K_m \boxed{z^{-1} B_{m-1}(z)}$$

$$B_m(z) = K_m A_{m-1}(z) + \boxed{z^{-1} B_{m-1}(z)}$$

$$\boxed{z^{-1} B_{m-1}(z)} = B_m(z) - K_m A_{m-1}(z)$$

$$\Downarrow$$

$$A_m(z) = A_{m-1}(z) + K_m \cdot \left(B_m(z) - K_m A_{m-1}(z)\right)$$

$$\Leftrightarrow$$

$$A_m(z) - K_m B_m(z) = A_{m-1}(z) \cdot (1 - K_m^2)$$

$$\Downarrow$$

$$\boxed{A_{m-1}(z) = \frac{A_m(z) - K_m B_m(z)}{1 - K_m^2}}$$

$B_{m-1}(z) = A_{m-1}(z)$ reversed coefficients

!! $\boxed{K_m = \text{the last coefficient in } A_m(z)}$!!

$$A_3(z) = 1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \frac{1}{2} z^{-3}$$

$$B_3(z) = \frac{1}{2} + \frac{7}{20} z^{-1} + \frac{2}{5} z^{-2} + 1 \cdot z^{-3}$$

$$\rightarrow A_2(z) = \frac{A_3(z) - K_3 \cdot B_3(z)}{1 - K_3^2} = \underline{\qquad\qquad}$$

$$= \frac{1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \cancel{\frac{1}{2} z^{-3}} - \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{7}{20} z^{-1} + \frac{2}{5} z^{-2} + \cancel{1 \cdot z^{-3}} \right)}{1 - \left(\frac{1}{2}\right)^2} = \frac{3/4 + z^{-1} \cdot \overbrace{\left(\frac{2}{5} - \frac{7}{40}\right)}^{9/40} + z^{-2} \cdot \overbrace{\left(\frac{7}{20} - \frac{1}{5}\right)}^{3/20}}{3/4}$$

$\frac{\cancel{9}^{3}}{\cancel{40}_{10}} \cdot \frac{\cancel{4}}{\cancel{3}} = \frac{3}{10}$

$\frac{\cancel{12}}{\cancel{50}_{2}} \cdot \frac{\cancel{25}}{\cancel{24}_{2}} = \frac{1}{4}$

$$= 1 + \frac{3}{10} z^{-1} + \underset{K_2}{\frac{1}{5}} z^{-2}$$

$$A_1(z) = \frac{A_2(z) - K_2 B_2(z)}{1 - K_2^2} = \frac{1 + \frac{3}{10} z^{-1} + \cancel{\frac{1}{5} z^{-2}} - \frac{1}{5} \cdot \left(\frac{1}{5} + \frac{3}{10} z^{-1} + \cancel{1 \cdot z^{-2}} \right)}{1 - \frac{1}{25}} = \frac{24/25 + z^{-1} \overbrace{\left(\frac{3}{10} - \frac{3}{50}\right)}^{12/50}}{24/25}$$

$$= 1 + \underset{K_1}{\frac{1}{4}} z^{-1}$$

Lattice for IIR systems (all-pole)

$$\boxed{H(z) = \frac{1}{1 + \sum_{k=1}^{m} \alpha_m[k] z^{-k}}}$$

$$H(z) = \frac{0.7}{0.5 + z^{-1} + \frac{1}{3} z^{-2}} = \frac{0.7}{0.5} \cdot \boxed{\frac{1}{1 + 2z^{-1} + \frac{2}{3} z^{-2}}}$$

$$= \frac{1}{1 + 0.5 z^{-1} + 0.22 z^{-2}}$$

$$y[n] = -a_1 y[n-1] - \dots - a_N y[n-N] + x[n]$$

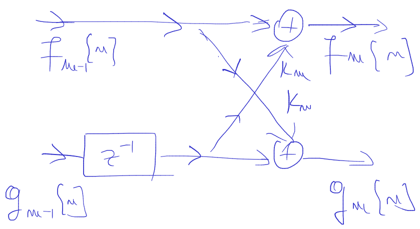
What if $y[n]$ = input and $x[n]$ is output?
$= f_N[n]$ $\quad$ $f_0[n]$

$$x[n] = y[n] + a_1 y[n-1] + \dots + a_N y[n-N]$$

$$H(z) = \frac{X(z)}{Y(z)} = 1 + a_1 z^{-1} + \dots + a_N z^{-N} \longrightarrow \text{implement with Lattice FIR}$$

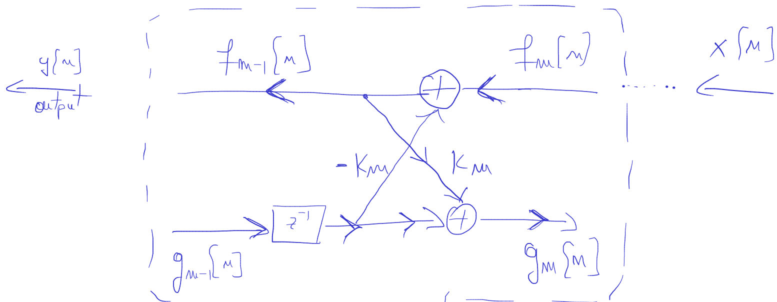
Lattice FIR:

$$\begin{cases} f_m[n] = f_{m-1}[n] + K_m \cdot g_{m-1}[n-1] \\ g_m[n] = K_m f_{m-1}[n] + g_{m-1}[n-1] \end{cases}$$



Lattice IIR all-pole:

$$\begin{cases} f_{m-1}[n] = f_m[n] - K_m g_{m-1}[n-1] \\ g_m[n] = K_m f_{m-1}[n] + g_{m-1}[n-1] \end{cases}$$

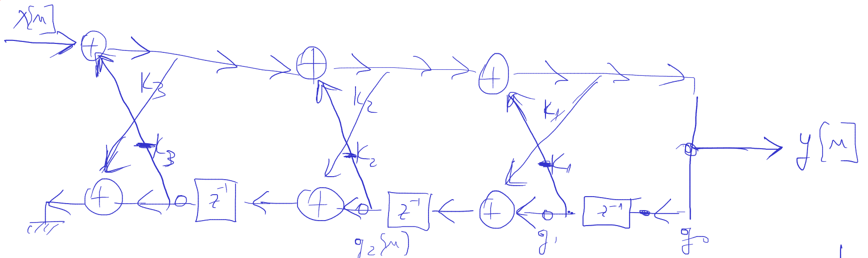


One stage

Exercise 2, Lab 06

Example:

$$H(z) = \frac{1}{\underbrace{1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \frac{1}{2} z^{-3}}_{A_3(z)}}$$



Find Lattice implem.

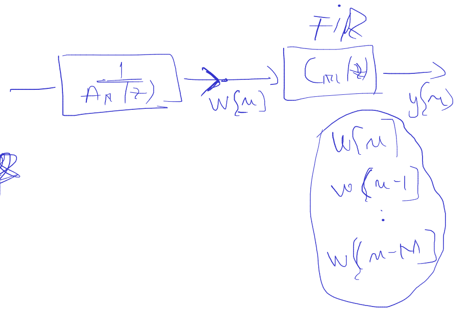
K_1, K_2, K_3?

!

Find with the same procedure, for the denominator $\Rightarrow K_1 = \frac{1}{4}, K_2 = \frac{1}{5}, K_3 = \frac{1}{2}$, just like in the prev. exercise!

General case: $H(z) = \dfrac{C_M(z)}{A_N(z)} = \dfrac{0.5+0.7z^{-1}+0.2z^{-2}}{1+0.4z^{-1}+0.1z^{-2}+0.9z^{-3}}$

$= \dfrac{1}{A_N(z)} \cdot \underbrace{C_M(z)}_{\text{FIR system}} \Rightarrow$



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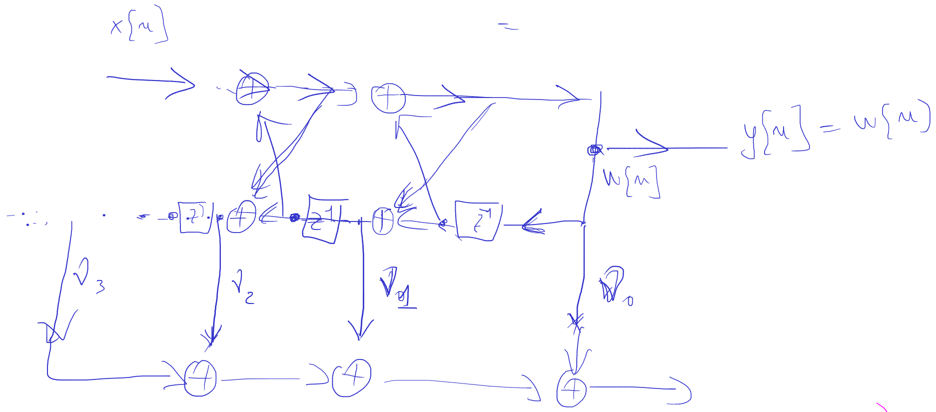


Exercise 1 / Lab 06

not required to be 1

$H(z) = \dfrac{0.5 + 2z^{-1} + 3z^{-2} + 2z^{-3}}{1 + \frac{2}{5}z^{-1} + \frac{7}{20}z^{-2} + \frac{1}{2}z^{-3}}$

fixed

$= C_3(z) \Rightarrow \nu_0, \nu_1, \nu_2, \nu_3$ implement in lattice-ladder form

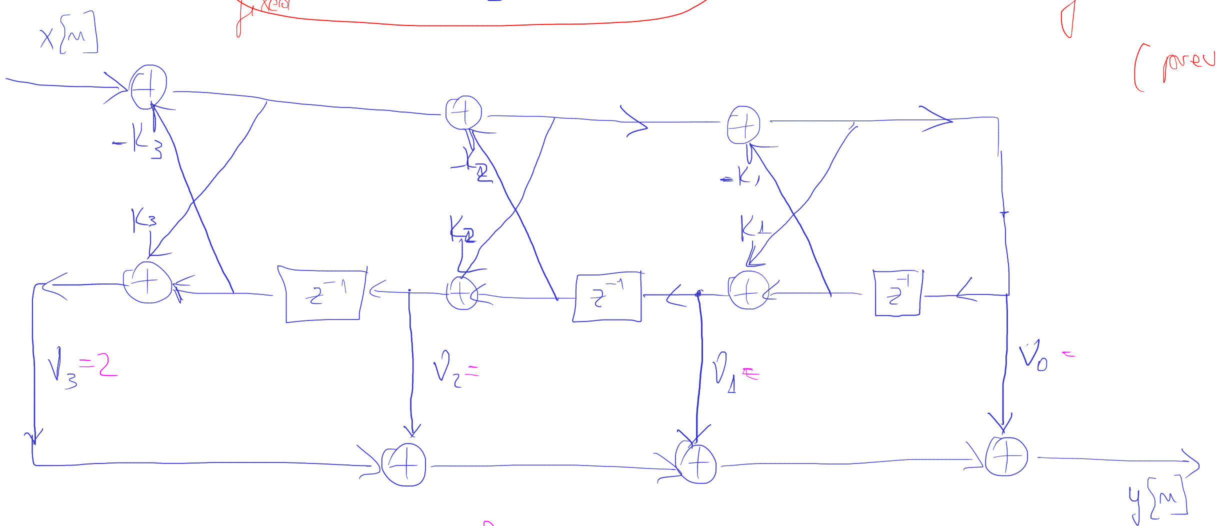
$= A_3(z) \Rightarrow K_1, K_2, K_3$ just like for FIR systems (prev. exercises)

$\Downarrow$

$K_3 = \frac{1}{2}$

$K_2 = \frac{1}{5}$

$K_1 = \frac{1}{4}$



$C_3(z) = 0.5 + 2z^{-1} + 3z^{-2} + \overset{\nu_3}{2}z^{-3}$

$C_2(z) = C_3(z) - \nu_3 \cdot B_3(z)$

$= 0.5 + 2z^{-1} + 3z^{-2} + \cancel{2z^{-3}} - 2\left(\frac{1}{2} + \frac{7}{20}z^{-1} + \frac{2}{5}z^{-2} + \cancel{z^{-3}}\right)$

$= -0.5 + z^{-1}\left(2 - \frac{7}{10}\right) + z^{-2}\left(3 - \frac{4}{5}\right)$

$= -0.5 + 1.3z^{-1} + \underset{\nu_2}{2.2}\,z^{-2}$

$$\boxed{C_{m-1}(z) = C_m(z) - \nu_m B_m(z)}$$

$B_m(z)$ = the one found when computing K_m

= $A_m(z)$ reversed coefficients

$B_2(z) = A_2(z)$ with reversed coefficients

$$C_1(z) = C_2(z) - \nu_2 \cdot B_2(z)$$

$$= -0.5 + 1.3 \cdot z^{-1} + \cancel{2.2 \cdot z^{-2}} - 2.2\left(\frac{1}{5} + \frac{3}{10} \cdot z^{-1} + \cancel{1 \cdot z^{-2}}\right)$$

$$= \left(-0.5 - \frac{2.2}{5}\right) + z^{-1}\left(1.3 - \frac{6.6}{10}\right)$$

$$= -0.94 + 0.64\, z^{-1}$$

ν_1

$$C_0(z) = C_1(z) - \nu_1 \cdot B_1(z)$$

$$= -0.94 + \cancel{0.64\, z^{-1}} - 0.64 \cdot \left(\frac{1}{4} + \cancel{1 \cdot z^{-1}}\right)$$

$$= -0.94 - \underbrace{\frac{0.64}{4}}_{0.16}$$

$$= -0.78$$

ν_0

IIR stability: if any $|K_m| \geq 1 \Rightarrow$ system is unstable

if all $|K_m| < 1 \Rightarrow$ system is stable