



$$A_0(z) = 1 \quad A_m(z) = A_{m-1}(z) + K_m \cdot z^{-1} \cdot B_{m-1}(z)$$

$$B_0(z) = 1 \quad B_m(z) = z^{-m} \cdot A_m(z^{-1})$$

①  $K_1 = \boxed{\frac{1}{2}} \quad \boxed{K_2 = 0.6} \quad \boxed{K_3 = -0.7} \quad K_m = \text{last coeff. of } A_m(z)$

Find filter coefficients in direct form (i.e. find  $H(z)$ )

$$A_0(z) = 1$$

$$B_0(z) = 1$$

$$A_1(z) = A_0(z) + K_1 \cdot z^{-1} \cdot B_0(z) = \boxed{1} + \boxed{\frac{1}{2}} \cdot z^{-1}$$

$$B_1(z) = z^{-1} \cdot A_1(z^{-1}) = z^{-1} \cdot \left(1 + \frac{1}{2}z\right) = \boxed{\frac{1}{2}} + \boxed{1} \cdot z^{-1}$$

$$A_2(z) = 1 + \frac{1}{2}z^{-1} + 0.6 \cdot z^{-1} \cdot \left(\frac{1}{2} + z^{-1}\right) = 1 + z^{-1} \left(\frac{1}{2} + 0.6 \cdot \frac{1}{2}\right) +$$

$$= 1 + 0.8z^{-1} + \boxed{0.6}z^{-2}$$

$$B_2(z) = A_2(z) \text{ inverse!}$$

$$A_3(z) = \underbrace{1 + 0.8z^{-1} + 0.6z^{-2}}_{A_2(z)} - 0.7 \cdot z^{-1} \cdot \left(0.6 + 0.8z^{-1} + 1 \cdot z^{-2}\right)$$

$$= 1 + z^{-1}(0.8 - 0.42) + z^{-2}(0.6 - 0.56) + z^{-3} \cdot 0.7$$

$$= \boxed{1} + \boxed{0.38}z^{-1} + \boxed{0.04}z^{-2} + \boxed{-0.7}z^{-3} = H(z)$$

$b_0 \quad b_1 \quad b_2 \quad b_3 \quad K_3$

$$(2) \quad H(z) = 1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \boxed{\frac{1}{2}} z^{-3} = A_3(z)$$

$$K_1 = ? \quad K_2 = ? \quad K_3 = ?$$

$$A_{m-1}(z) = \frac{A_m(z) - K_m \cdot B_m(z)}{1 - K_m^2}$$

$$A_2(z) = \frac{\overbrace{1 + \frac{2}{5} z^{-1} + \frac{7}{20} z^{-2} + \frac{1}{2} z^{-3}}^{A_3(z)} - \frac{1}{2} \overbrace{\left( \frac{1}{2} + \frac{7}{20} z^{-1} + \frac{2}{5} z^{-2} + 1 \cdot z^{-3} \right)}^{B_3(z)}}{1 - \frac{1}{2^2}}$$

$$= \frac{1 - \frac{1}{4} + z^{-1} \left( \frac{2}{5} - \frac{7}{40} \right) + z^{-2} \left( \frac{7}{20} - \frac{1}{5} \right)}{1 - \frac{1}{4}}$$

$$= \frac{\frac{3}{4} + \frac{9}{40} z^{-1} + \frac{3}{20} z^{-2}}{3/4}$$

$$= 1 + \frac{3}{10} z^{-1} + \boxed{\frac{1}{5}} z^{-2}$$

$$A_1(z) = \frac{\overbrace{1 + \frac{3}{10} z^{-1} + \frac{1}{5} z^{-2}}^{A_2(z)} - \frac{1}{5} \overbrace{\left( \frac{1}{5} + \frac{3}{10} z^{-1} + 1 \cdot z^{-2} \right)}^{K_2 B_2(z)}}{1 - \left( \frac{1}{5} \right)^2}$$

$= A_2$  with coeff. in opposite order

$$= \frac{1 - 1/25 + z^{-1} \left( \frac{3}{10} - \frac{3}{50} \right)}{1 - 1/25}$$

$$= \frac{\frac{24}{25} + \frac{12}{50} z^{-1}}{24/25} = 1 + \boxed{\frac{1}{4}} z^{-1}$$