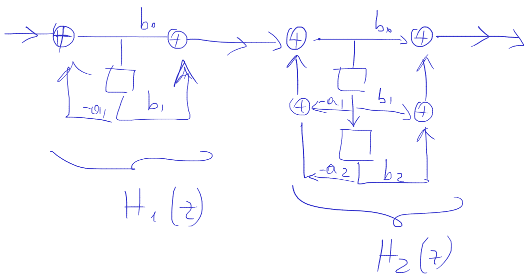
SDP Lecture 4

Series implem.:

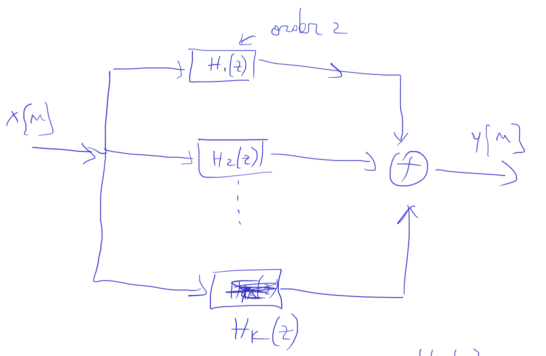
$$H(z) = \underbrace{\frac{2(1-z^{-1})}{(1+0.5z^{-1})}}_{H_1(z)} \underbrace{\frac{(1+\sqrt{(2)}z^{-1}+z^{-2})}{(1-0.9z^{-1}+0.81z^{-2})}}_{H_2(z)},$$



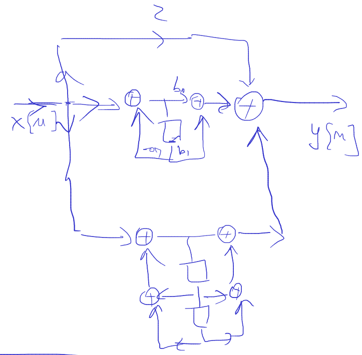
Parallel implem.:

$$H(z) = H_1(z) + H_2(z) + \dots + H_K(z)$$

$$\boxed{\frac{A_k}{z-a_k}} + \frac{A_k}{z-a_k^*} = \frac{Bz+C}{z^2+\cdot z+\cdot}$$



$$H(z) = \frac{5-6z^{-1}+3.72z^{-2}-0.74z^{-3}}{1-1.5z^{-1}+1.24z^{-2}-0.37z^{-3}} = \underbrace{2}_{H_1(z)} + \overset{H_2(z)}{\boxed{\frac{1}{1-0.5z^{-1}}}} + \overset{H_3(z)}{\boxed{\frac{2-z^{-1}}{1-z^{-1}+0.74z^{-2}}}}$$



Lattice

$$A_m(z) = \underbrace{1}_{\alpha_m[0]} + \alpha_m[1]\cdot z^{-1} + \alpha_m[2]\cdot z^{-2} + \dots + \alpha_m[m]\cdot z^{-m}$$

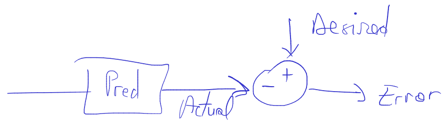
m = order of the system

$$h[n] = \{\underbrace{\alpha_m[0]}_{1}, \alpha_m[1], \dots \alpha_m[m]\}$$

$x[n]$ input

$$y[n] = x[n] * h[n] = \sum_{k=0}^{m} \alpha_m[k]\cdot x[n-k] = x[n] + \sum_{k=1}^{m} \alpha_m[k]\cdot x[n-k]$$

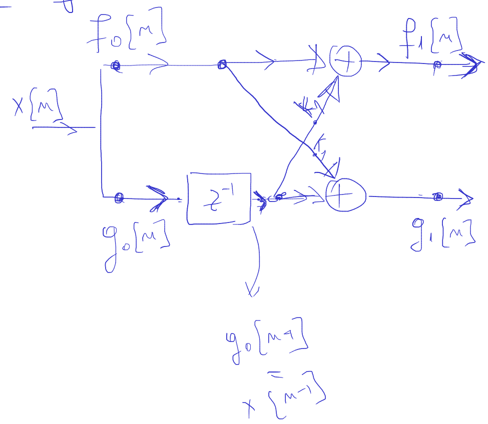
Pred:



For degree $m = 1$:

$$y[n] = x[n] + \alpha_1[1]\cdot x[n-1]$$

Lattice form:



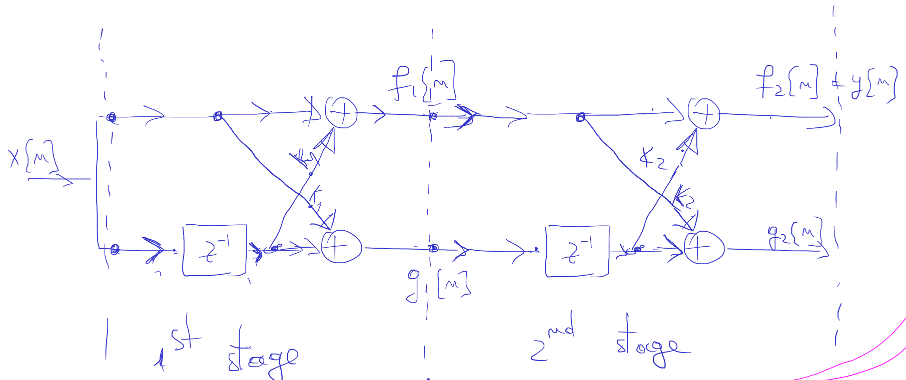
$$y[n] = f_1[n] = 1\cdot x[n] + K_1\cdot \underbrace{g_0[n-1]}_{x[n-1]}$$

$$g_1[n] = K_1\cdot x[n] + x[n-1]$$

We can implement a system of degree 1, $A_1(z) = 1 + \alpha_1[1]z^{-1}$, with a lattice structure with 1 stage, using $K_1 = \alpha_1[1]$

Degree 2:

$$A_2(z) = \boxed{1 + \alpha_2[1]\cdot z^{-1} + \alpha_2[2]\cdot z^{-2}}$$



replace

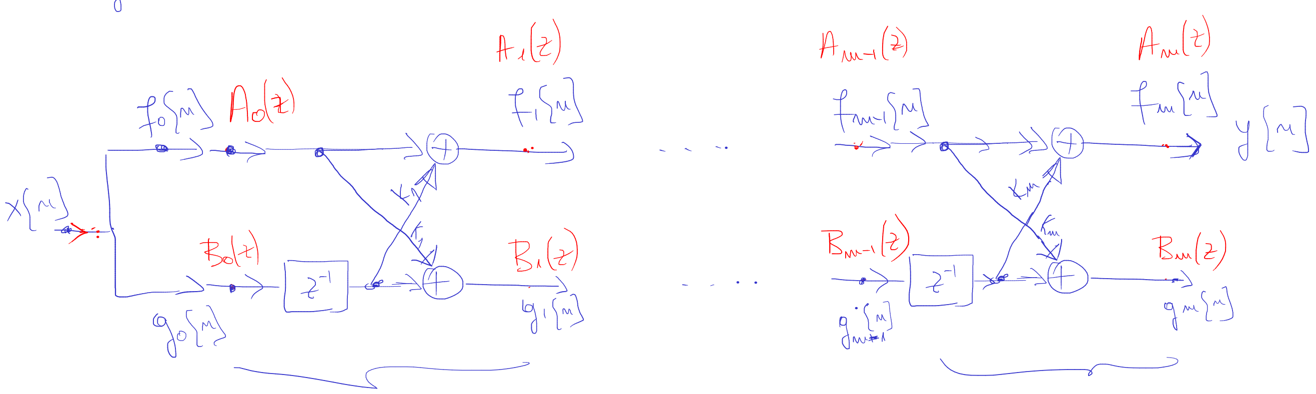
$$f_2[n] = f_1[n] + K_2\cdot g_1[n-1] = \boxed{x[n] + K_1\cdot x[n-1] + K_2\left(K_1 x[n-1] + x[n-2]\right)}$$

$$= 1\cdot x[n] + K_1(1+K_2)\cdot x[n-1] + K_2\, x[n-2]$$

$$g_2[n] = K_2\cdot f_1[n] + g_1[n-1]$$

$$g_2[n] = K_2\, x[n] + K_1(1+K_2)\, x[n-1] + 1\, x[n-2]$$

In general, order m:



$$A_m(z) = \frac{\mathcal{Z}\{f_m[n]\}}{\mathcal{Z}\{x[n]\}} = \frac{Y(z)}{X(z)}$$

$A_m(z)$ = system function from the input to output of stage m

$B_m(z)$ = " " " " 2nd output of stage m

$$\begin{cases} f_m[n] = f_{m-1}[n] + K_m \cdot g_{m-1}[n-1] \\ g_m[n] = K_m f_{m-1}[n] + g_{m-1}[n-1] \end{cases}$$

$$A_m(z) = \mathcal{Z}\{\alpha_m[0], \alpha_m[1], \dots \alpha_m[m]\} = \mathcal{Z}\{\alpha_m[n]\}$$

$$A_1(z) = 1 + K_1 z^{-1}$$
$$B_1(z) = K_1 + 1 \cdot z^{-1}$$

$B_m(z) =$ $A_m(z)$ with coef. in reversed order

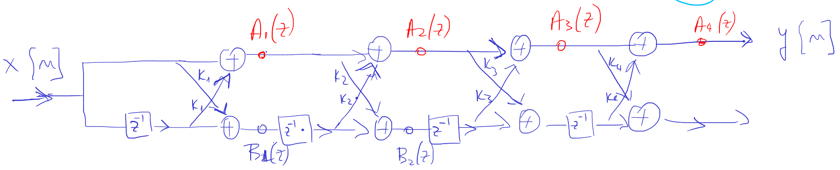
$$\boxed{B_m(z) = z^{-m} \cdot A_m(z^{-1})}$$

$$\rightarrow \begin{cases} A_m(z) = A_{m-1}(z) + K_m \cdot z^{-1} \cdot B_{m-1}(z) \\ B_m(z) = K_m \cdot A_{m-1}(z) + z^{-1} \cdot B_{m-1}(z) \\ A_0(z) = B_0(z) = 1 \end{cases}$$

Exercise 1, Lab 5:

$K_1 = \frac{1}{2}$, $K_2 = 0.6$, $K_3 = -0.7$, $K_4 = \frac{1}{3}$

$A_4(z) = H(z) = ? = 1 + \alpha_4[1] z^{-1} + \alpha_4[2] z^{-2} + \alpha_4[3] z^{-3} + \alpha_4[4] z^{-4}$



$A_0(z) = B_0(z) = 1$

$$A_1(z) = A_0(z) + K_1 \cdot z^{-1} \cdot B_0(z) = 1 + \frac{1}{2} \cdot z^{-1} \cdot 1 = 1 + \frac{1}{2} z^{-1}$$

$$B_1(z) = z^{-1} \cdot A_1(z^{-1}) = z^{-1} \cdot \left(1 + \frac{1}{2} \cdot z\right) = \frac{1}{2} + 1 \cdot z^{-1}$$

$z \to z^{-1}$

$z^{-1} \to z$

$$A_2(z) = A_1(z) + K_2 \cdot z^{-1} \cdot B_1(z) = 1 + \frac{1}{2} z^{-1} + 0.6 \cdot z^{-1} \cdot \left(\frac{1}{2} + z^{-1}\right) = 1 + \frac{1}{2} z^{-1} + 0.3 z^{-1} + 0.6 \cdot z^{-2}$$
$$= 1 + 0.8 z^{-1} + 0.6 z^{-2}.$$

$$B_2(z) = 0.6 + 0.8 z^{-1} + 1 \cdot z^{-2}$$

$$A_3(z) = A_2(z) + K_3 z^{-1} B_2(z) = 1 + 0.8 z^{-1} + 0.6 z^{-2} + (-0.7) \cdot z^{-1} \cdot (0.6 + 0.8 z^{-1} + 1 \cdot z^{-2})$$
$$= 1 + z^{-1} \cdot (0.8 - 0.7 \cdot 0.6) + z^{-2} \cdot (0.6 - \underbrace{0.7 \cdot 0.8}_{0.56}) - 0.7 z^{-3}$$
$$= 1 + 0.38 z^{-1} + 0.04 z^{-2} - 0.7 z^{-3}$$

$$B_3(z) = -0.7 + 0.04 z^{-1} + 0.38 z^{-2} + 1 \cdot z^{-3}$$

$$A_4(z) = A_3(z) + K_4 z^{-1} B_3(z) = \underbrace{1 + 0.38 z^{-1} + 0.04 z^{-2} - 0.7 z^{-3}}_{A_3(z)} + \frac{1}{3} \cdot z^{-1} \left(-0.7 + 0.04 z^{-1} + 0.38 z^{-2} + 1 \cdot z^{-3}\right)$$
$$= 1 + z^{-1}\left(0.38 - \frac{0.7}{3}\right) + z^{-2} \cdot \left(0.04 + \frac{1}{3} \cdot 0.04\right) + z^{-3}\left(-0.7 + \frac{1}{3} \cdot 0.38\right) + \frac{1}{3} z^{-4}$$

$$H(z) = A_4(z) = \underbrace{1}_{\alpha_4[0]} + \underbrace{(\ldots)}_{\alpha_4[1]} \cdot z^{-1} + \underbrace{\frac{0.16}{3}}_{\alpha_4[2]} z^{-2} + \underbrace{(\ldots)}_{\alpha_4[3]} \cdot z^{-3} + \underbrace{\frac{1}{3}}_{\alpha_4[4]} z^{-4}$$