Wiener Filter for signal denoising

Laboratory 13, SDP

Objective

Students should design and use a Wiener filter in a denoising application.

Theoretical notions

Exercises

1. Consider the signal $x[n] = s[n] + w[n]$, where $s[n]$ is an autoregressive (AR) random process of order 1, with the difference equation:

$$s[n] = 0.6 \cdot s[n-1] + v[n].$$

   The signals $w[n]$ and $v[n]$ are white noises, uncorrelated, with variances $\sigma_w^2 = 1$ and $\sigma_v^2 = 0.64$.

   a. Find the autocorrelation function of the signals s and x, $\gamma_{ss}[m]$ and $\gamma_{xx}[m]$;
   b. Find the Wiener filter of length $M = 2$ for estimating $s[n]$ from $x[n]$;
   c. Find the minimum mean squared error for $M = 2$.

2. In Matlab, consider the following signal:

$$s[n] = \sin(2\pi f_1 n) + \sin(2\pi f_2 n),$$

   where $f_1 = 0.013$, $f_2 = 0.051$ and $n = 0 : 999$.

   To the signal $s[n]$ we add a white noise with variance $\sigma_w^2 = 0.25$, the resulting signal being $x[n] = s[n] + w[n]$.

   a. Using the function `wienerfir()`, find the coefficients of the Wiener FIR filter with M=20 and filter the signal $x[n]$ with this filter;

b. Plot on the same figure the three signals $s[n]$, $x[n]$ and the result of the filtering;

c. Compute the resulting mean squared error;

c. Repeat all the above for different values of M=40, 100. What do you notice?

Final questions

1. TBD