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$$w[n] \longrightarrow \boxed{H(z)} \longrightarrow x[n]$$

$$\gamma_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]\cdot x[n+m]$$

$\gamma_{ww}[m]$ $\qquad$ $\gamma_{xx}[m]$

z transform

$\Gamma_{ww}(z)$ $\qquad$ $\Gamma_{xx}(z)$

$\Gamma_{ww}(\omega)$ $\qquad$ $\Gamma_{xx}(\omega)$

$$\boxed{\Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1}) = \Gamma_{xx}(z)}$$

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w = white noise $\Rightarrow$ $\Gamma_{ww}[m] = \sigma_w^2\cdot\delta[m]$

$\Gamma_{ww}(z) = \sigma_w^2$ = constant

To Do: - find $\Gamma_{xx}(z)$

- inverse z transform $\Rightarrow$ $\gamma_{xx}[m]$

1

$$w[n] \longrightarrow \boxed{H(z)} \longrightarrow x[n]$$

$$\Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1}) = \Gamma_{xx}(z)$$

$$x[n] = \frac{1}{2}x[n-1] + w(n) + w[n-1] \Rightarrow H(z) = \frac{X(z)}{W(z)} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\Gamma_{ww}(z) = \sigma_w^2$$

$$\Gamma_{xx}(z) = \sigma_w^2\cdot\underbrace{\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}}_{H(z)}\cdot\underbrace{\frac{1+z}{1-\frac{1}{2}z}}_{H(z^{-1})} = \sigma_w^2\,\frac{(z+1)\cdot(1+z)\,(-z)}{\left(z-\frac{1}{2}\right)\cdot\left(1-\frac{1}{2}z\right)(-z)} = \sigma_w^2\cdot\frac{(-2)\cdot(z+1)^2}{\left(z-\frac{1}{2}\right)(z-2)}$$

$$\frac{\Gamma_{xx}(z)}{z} = -2\sigma_w^2\cdot\frac{(z+1)^2}{z\left(z-\frac{1}{2}\right)(z-2)} = \frac{A}{z} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-2}$$

$z=0$: $A = -2\sigma_w^2\cdot\frac{1}{\left(-\frac{1}{2}\right)\cdot(-2)} = -2\sigma_w^2$

$z=\frac{1}{2}$: $B = -2\sigma_w^2\cdot\frac{(1.5)^2}{1/2\cdot(-3/2)} = \ldots = 6\sigma_w^2$

$z=2$: $C = -2\sigma_w^2\cdot\frac{9}{2\cdot 3/2} = -6\sigma_w^2$

$$\Rightarrow \Gamma_{xx}(z) = A + B\cdot\frac{z}{z-\frac{1}{2}} + C\cdot\frac{z}{z-2}$$

$$\gamma_{xx}[m] = A\cdot\delta[m] + B\cdot\left(\frac{1}{2}\right)^m u[m] + C\cdot 2^m u[m]$$

where A, B, C = calculated

②

$$w[n] \longrightarrow \boxed{H(z)} \longrightarrow x[n]$$

$$\gamma_{xx}[m] = \frac{1}{4}^{|m|}$$

We need to find the system equation

$$\Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1}) = \Gamma_{xx}(z)$$

TO DO:
- compute $\Gamma_{xx}(z)$
- find $H(z)$
- write the equation

$$\Gamma_{xx}(z) = \mathcal{Z}\left(\gamma_{xx}[m]\right) = \sum_{m=-\infty}^{\infty} \gamma_{xx}[m]\, z^{-m} = \sum_{m=-\infty}^{\infty} \frac{1}{4}^{|m|} \cdot z^{-m}$$

$$1+q+q^2+\dots = \frac{1}{1-q}, \quad \text{if } |q|<1$$

$$= \underbrace{\sum_{m=0}^{\infty} \Big(\underbrace{\tfrac{1}{4}\cdot z^{-1}}_{q}\Big)^{m}}_{\frac{1}{1-\frac{1}{4}z^{-1}}} + \underbrace{\sum_{m=-\infty}^{-1} \left(\tfrac{1}{4}\right)^{-m}\cdot z^{-m}}_{=\ \sum_{m=1}^{\infty} \frac{1}{4}^{m}\cdot z^{m} \ =\ \frac{1}{1-\frac{z}{4}} - 1}$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{z}{4}} - 1 = \frac{\cancel{1}-\cancel{\frac{z}{4}} + 1 - \cancel{\frac{1}{4}z^{-1}} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{z}{4}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{4}z\right)}$$

$$-\left(\cancel{1} - \cancel{\tfrac{1}{4}z^{-1}} - \cancel{\tfrac{z}{4}} + \tfrac{1}{16}\right)$$

$$= \frac{\overset{\sigma_w^2}{15/16}\cdot 1 \cdot 1}{\underset{H(z)}{\left(1-\frac{1}{4}z^{-1}\right)}\underset{H(z^{-1})}{\left(1-\frac{1}{4}z\right)}} = \Gamma_{ww}(z)\cdot H(z)\cdot H(z^{-1})$$

$$\Rightarrow H(z) = \frac{1}{1-\frac{1}{4}z^{-1}} = \frac{X(\omega)}{W(\omega)} \Rightarrow x[n] = \frac{1}{4}x[n-1] + w[n]$$

Another choice: $\sigma_w^2 = 1$

$$H(z) = \frac{\sqrt{15/16}}{1-\frac{1}{4}z^{-1}} \Rightarrow x[n] = \frac{1}{4}x[n-1] + \sqrt{\frac{15}{16}}\cdot w[n]$$

$$H(z^{-1}) = \frac{\sqrt{15/16}}{1-\frac{1}{4}z}$$