SDP LAB 10
(PSS)

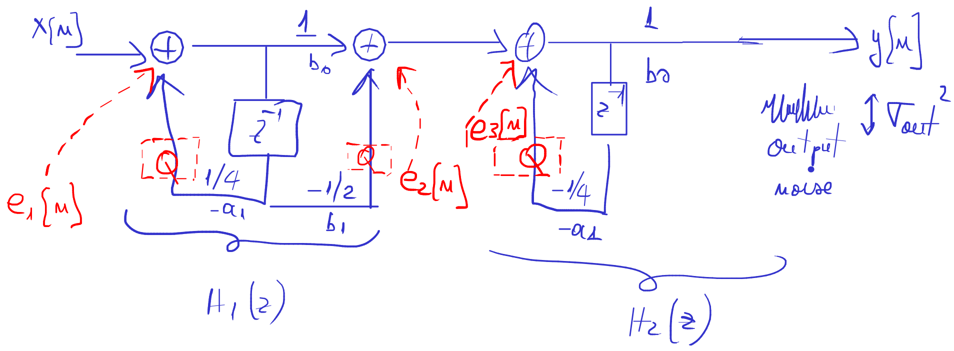
1

$H_1(z)$

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$H_2(z)$

a) series implementation : $H(z) = H_1(z) \cdot H_2(z)$



$e_1[n]$: ↕ σ_1^2

$e_2[n]$: ↕ σ_2^2

$e_3[n]$: — " — ↕ σ_3^2

noise signals

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_e^2 = \frac{\Delta^2}{12} = \frac{2^{-2b}}{12}$$

$$\Delta = 2^{-b}$$

$$\sigma_{out}^2 = \sigma_1^2 \cdot \underbrace{\frac{1}{2\pi j}\oint_C H(z)\cdot H(z^{-1})\, z^{-1} dz}_{=1.26} + \sigma_2^2 \cdot \underbrace{\frac{1}{2\pi j}\oint_C H_2(z)\cdot H_2(z^{-1})\, dz}_{=16/15}$$

$$+ \sigma_3^2 \cdot \underbrace{\frac{1}{2\pi j}\oint_C H_2(z)\cdot H_2(z^{-1})\, dz}_{=16/15}$$

will be computed in the following

Only the part of the system through which that noise travels !

$$\frac{1}{2\pi j}\oint H(z)\cdot H(z^{-1})\cdot z^{-1} = \text{sum of residuals inside unit circle of the poles}$$

=
- write $H(z)\cdot H(z^{-1})\cdot z^{-1}$
- decompose in simple fractions
- sum their coefficients, for the poles inside unit circle

$$\bullet \quad \frac{1}{2\pi j} \oint_C H_2(z) \cdot H_2(z^{-1}) z^{-1} dz = \frac{16}{15}$$

$$H_2(z) = \frac{1}{1+\frac{1}{4}z^{-1}}$$

$$H_2(z) \cdot H_2(z^{-1}) \cdot z^{-1} = \frac{z \cdot 1}{z\left(1+\frac{1}{4}z^{-1}\right)} \cdot \frac{1}{1+\frac{1}{4}z} \cdot \frac{1}{z} = \frac{\cancel{z}}{z+\frac{1}{4}} \cdot \frac{4 \cdot 1}{4 \cdot \left(1+\frac{1}{4}z\right)} \cdot \frac{1}{\cancel{z}} =$$

$$= \frac{1}{z+\frac{1}{4}} \cdot \frac{4}{4+z} = \frac{4}{\left(z+\frac{1}{4}\right)(z+4)}$$

Simple fractions:

$$= \frac{A}{z+\frac{1}{4}} + \frac{B}{z+4}$$

$$A = \left.\frac{4}{\cancel{\left(z+\frac{1}{4}\right)}(z+4)}\right|_{z=-\frac{1}{4}} = \frac{4}{-\frac{1}{4}+4} = \frac{4}{\frac{15}{4}} = \boxed{\frac{16}{15}}$$

$$\bullet \quad \frac{1}{2\pi j} \oint H(z) \cdot H(z^{-1}) \cdot z^{-1} = A + B = 1.26$$

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z)\, H(z^{-1})\, z^{-1} = \frac{z \cdot z \cdot \left(1-\frac{1}{2}z^{-1}\right)}{z \cdot z \cdot \left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \cdot \frac{\left(1-\frac{1}{2}z\right) \cdot 4 \cdot 4}{\left(1-\frac{1}{4}z\right)\left(1+\frac{1}{4}z\right) \cdot 4 \cdot 4} \cdot \frac{1}{z} =$$

$$= \frac{\cancel{z}\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)} \cdot \frac{(16-8z)}{(4-z)(4+z)} \cdot \frac{1}{\cancel{z}} =$$

$$= \frac{\left(z-\frac{1}{2}\right) \cdot 8 \cdot (z-2)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)(z-4)(z+4)} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z+\frac{1}{4}} + \frac{C}{z-4} + \frac{D}{z+4}$$

$$A = \text{replace } z = \frac{1}{4} \Rightarrow \frac{\left(\frac{1}{4}-\frac{1}{2}\right) \cdot 8 \cdot \left(\frac{1}{4}-2\right)}{\cancel{\left(\frac{1}{4}-\frac{1}{4}\right)}\left(\frac{1}{4}+\frac{1}{4}\right)\left(\frac{1}{4}-4\right)\left(\frac{1}{4}+4\right)} = -0.43$$

ignore

$$B = \text{replace } z = -\frac{1}{4} \quad \frac{\left(-\frac{1}{4}-\frac{1}{2}\right) \cdot 8 \cdot \left(-\frac{1}{4}-2\right)}{\left(-\frac{1}{4}-\frac{1}{4}\right) \cdot \left(-\frac{1}{4}-4\right)\left(-\frac{1}{4}+4\right)} = \ldots 1.26$$

Final answer :

$$\sigma_{out}^2 = \frac{2^{-2b}}{12}\left(1.26 + \frac{16}{15} + \frac{16}{15}\right)$$