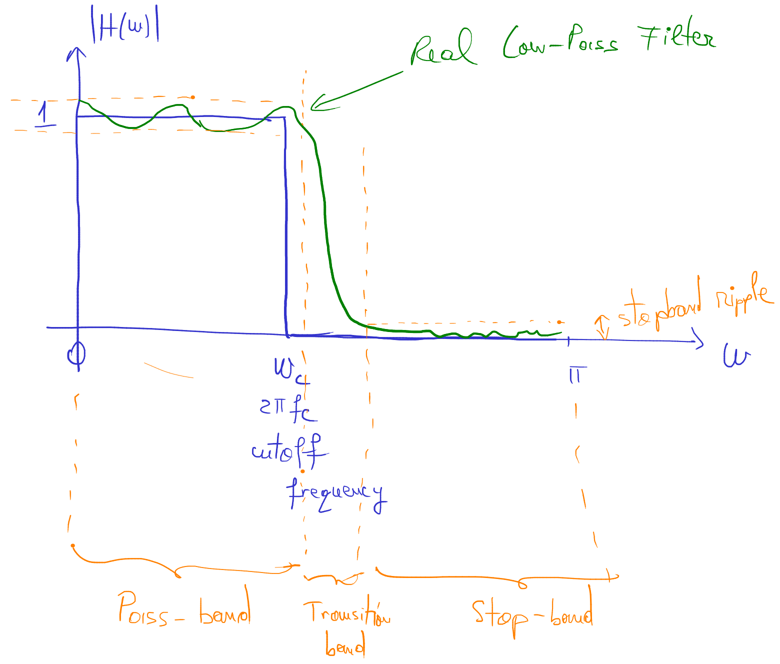
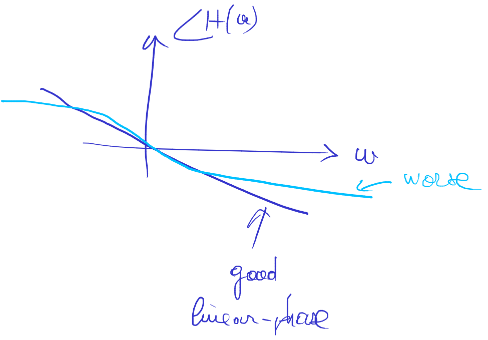
Lecture 1

Non-ideal filters:



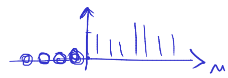
Symmetrical



Padé method:

We have $h_d[n]$

$h_d[n]$, $h[n]$ = causal

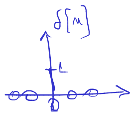


We have $H(z) = \dfrac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$

We want: b_k, a_k such that $h[n] = h_d[n]$ for the first $M+N+1$ samples

Method:

$$y[n] = -a_1 y[n-1] - \dots - a_N y[n-N] + b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$



$\delta[0] = 1$

$x[n] = \delta[n]$

$y[n] = h[n]$

$$\rightarrow h[n] = -a_1 h[n-1] - \dots - a_N h[n-N] + b_0 \delta[n] + b_1 \delta[n-1] + \dots + b_M \delta[n-M]$$

M+N+1 equations

$n = 0$: $h[0] = -a_1 \cdot 0 - a_2 \cdot 0 \dots - a_N\, 0 + b_0 \cdot 1 + b_1 \cdot 0 + \dots \; b_M \cdot 0$

$$\boxed{h[0] = b_0 = h_d[0]}$$

$n = 1$: $h[1] = -a_1 \cdot h_d[0] - a_2 \cancel{h} \cdot 0 - \dots + b_0 \cdot 0 + b_1 \cdot 1 + b + \dots \; b_M \cdot 0$

$$\boxed{h[1] = -a_1 \cdot h_d[0] + b_1 = h_d[1]}$$

$n = 2$:

⋮

$n = M+N$: $$\boxed{h[M+N] = -a_1 h_d[M+N-1] + a_2 \dots + 0 \cdot 0 \cdot 0 = h_d[M+N]}$$

$\Rightarrow$ system of $M+N+1$ equations with $M+N+1$ unknowns

$\xRightarrow{\text{solve}}$ find b_k, a_k

Padé Exercise

$$hd[0] = \left(\frac{1}{3}\right)^0 \cdot \cos(0) = 1$$
$$hd[1] = \frac{1}{3} \cdot \cos \frac{\pi}{4} = \frac{\sqrt{2}}{6}$$

$N=2$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \quad M=2$$

$$hd[n] = \left(\frac{1}{3}\right)^n \cdot \cos\left(\frac{n\pi}{4}\right)$$

$$\boxed{h[n] = -a_1 h[n-1] - a_2 h[n-2] + b_0 \delta[n] + b_1 \delta[n-1] + b_2 \delta[n-2]}$$

$n=0$: $h[0] = b_0 \cdot 1 = hd[0] = 1 \Rightarrow b_0 = 1$

$n=1$: $h[1] = -a_1 \cdot 1 + b_1 = hd[1] = \frac{\sqrt{2}}{6}$

$n=2$: $h[2] = -a_1 \cdot \frac{\sqrt{2}}{6} - a_2 \cdot 1 + b_2 = hd[2] = 0$

$n=3$: $h[3] = -a_1 \cdot 0 - a_2 \frac{\sqrt{2}}{6} = hd[3] = \frac{1}{27} \cdot \cos \frac{3\pi}{4} = \frac{-\sqrt{2}}{54}$

$n=4$: $h[4] = +a_1 \cdot \frac{\sqrt{2}}{54} - a_2 \cdot 0 = hd[4] = \frac{1}{81} \cos \pi = \frac{-1}{81}$

$\Rightarrow$ Solve this for a_1, a_2

$n=5$

$n=6$

$$\begin{bmatrix} 1 \\ \sqrt{2}/6 \\ 0 \\ -\sqrt{2}/54 \\ -1/81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{\sqrt{2}}{6} & -1 \\ 0 & 0 & 0 & -0 & -\frac{\sqrt{2}}{6} \\ 0 & 0 & 0 & +\frac{\sqrt{2}}{54} & -0 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\begin{cases} \frac{-\sqrt{2}}{54} = -a_2 \frac{\sqrt{2}}{6} \Rightarrow a_2 = \frac{\cancel{\sqrt{2}}}{\cancel{54}\,9} \cdot \frac{\cancel{6}}{\cancel{\sqrt{2}}} = \frac{1}{9} \\ -\frac{1}{81} = +a_1 \frac{\sqrt{2}}{54} \Rightarrow a_1 = \frac{-1}{\cancel{81}\,\cancel{9}\,3} \cdot \frac{\cancel{54}\,6\,2}{\sqrt{2}} = \frac{-2}{3\sqrt{2}} = \frac{-\sqrt{2}}{3} \end{cases}$$

$$-a_1 + b_1 = \frac{\sqrt{2}}{6} \Rightarrow b_1 = \frac{\sqrt{2}}{6} + a_1 = \frac{\sqrt{2}}{6} - \frac{\sqrt{2}}{3} = \frac{-\sqrt{2}}{6}$$

$$-a_1 \frac{\sqrt{2}}{6} - a_2 + b_2 = 0 \Rightarrow b_2 = a_1 \frac{\sqrt{2}}{6} + a_2 = \underbrace{\frac{-\sqrt{2}}{3} \cdot \frac{\sqrt{2}}{6}}_{-\frac{2}{18} = \frac{-1}{9}} + \frac{1}{9} = 0$$