State-space implementations of digital IIR filters

Lab 9, SDP

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1 Objective

The students should become familiar with *state-space* type realization structure used for implementing IIR filters.

2 Theoretical notions

2.1 State-Space Type I implementation

The equations defining the state-space type I filter realization are given in Figure 1, for a IIR system of order 3.

$$\begin{bmatrix} \upsilon_1[n+1] \\ \upsilon_2[n+1] \\ \upsilon_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} \upsilon_1[n] \\ \upsilon_2[n] \\ \upsilon_3[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x[n]$$

$$y[n] = \left[(b_3 - b_0 a_3)(b_2 - b_0 a_2)(b_1 - b_0 a_1) \right] \begin{bmatrix} \upsilon_1[n] \\ \upsilon_2[n] \\ \upsilon_3[n] \end{bmatrix} + b_0 x[n]$$

Figure 1: State-Space Type I equations, for an IIR system of order 3

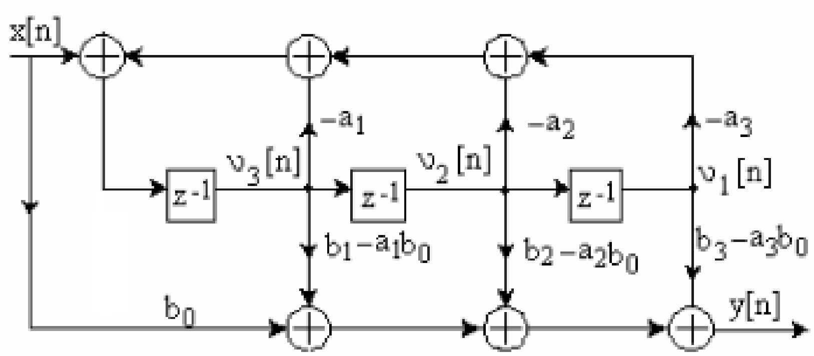

Figure 2: Schematic according to the equations (type I, order 3)

The schematic according to these equation is in Figure 2.

2.2 State-Space Type II implementation

$$
\begin{bmatrix} \upsilon_1[n+1] \\ \upsilon_2[n+1] \\ \upsilon_3[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \begin{bmatrix} \upsilon_1[n] \\ \upsilon_2[n] \\ \upsilon_3[n] \end{bmatrix} + \begin{bmatrix} (b_3 - b_0 a_3) \\ (b_2 - b_0 a_2) \\ (b_1 - b_0 a_1) \end{bmatrix} x[n]
$$

$$
y[n] = [0 \quad 0 \quad 1] \cdot \begin{bmatrix} \upsilon_1[n] \\ \upsilon_2[n] \\ \upsilon_3[n] \end{bmatrix} + b_0 x[n]
$$

Figure 3: State-Space Type II equations, for an IIR system of order 3

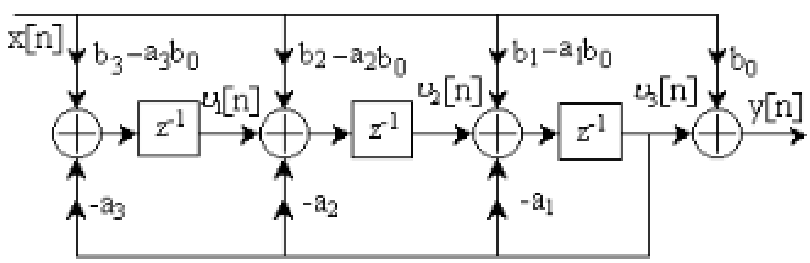

Figure 4: Schematic according to the equations (type II, order 3)

The equations defining the state-space type II filter realization are given in Figure 3, for a IIR system of order 3.

The schematic according to these equation is in Figure 4.

2.3 General equations

$$
\mathbf{v}[n+1] = \mathbf{F}\mathbf{v}[n] + \mathbf{q}x[n]
$$

$$
y[n] = \mathbf{g}^{\mathbf{t}} \mathbf{v}[n] + d x[n]
$$

Figure 5: General state-space equations

In the general case, there are always the two equations in Figure 5:

$$\mathbf{F}=\begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & 1 \\ -a_N & -a_{N-1} & \cdot & \cdot & \cdot & -a_2 & -a_1 \end{bmatrix} \quad \mathbf{q}=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{g}=\begin{bmatrix} b_N - b_0 a_N \\ b_{N-1} - b_0 a_{N-1} \\ \vdots \\ b_1 - b_0 a_1 \end{bmatrix} \quad d = b_0$$

Figure 6: Definition of the terms for type I

$$\mathbf{F}=\begin{bmatrix} 0 & 0 & \cdots & & & 0 & -a_N \\ 1 & 0 & \cdots & & & 0 & -a_{N-1} \\ \vdots & \vdots & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 & -a_2 \\ 0 & 0 & \cdots & \cdot & 0 & 1 & -a_1 \end{bmatrix} \quad \mathbf{q}=\begin{bmatrix} b_N - b_0 a_N \\ b_{N-1} - b_0 a_{N-1} \\ \vdots \\ b_2 - b_0 a_2 \\ b_1 - b_0 a_1 \end{bmatrix} \quad \mathbf{g}=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad d = b_0$$

Figure 7: Definition of the terms for type II

- the **state equation**: produces the next state $\mathbf{v}[n+1]$ depending on the current state $\mathbf{v}[n]$ and current input $x[n]$;
- the **output equation**: produces the current output $\mathbf{y}[n]$ depending on the current state $\mathbf{v}[n]$ and current input $x[n]$.

The definition of the general terms, according to type I or type II, is given in Figure 6 și Figure 7.

3 Theoretical exercises

1. Consider the IIR system with the system function

$$H(z) = \frac{1 + 2z^{-1} + 3z^{-2} + 2z^{-3}}{1 + 0.9z^{-1} + 0.8z^{-2} + 0.5z^{-3}}$$

   a. Write the equations and draw the type I and type II state-space implementations of this system
   b. Compute the first 5 values of the step response, considering the initial conditions $v[0] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2. Consider the system with the following state-space equations:

$$v[n+1] = \begin{bmatrix} 0 & 1 \\ -0.81 & 1 \end{bmatrix} v[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} -1.81 & 1 \end{bmatrix} + x[n]$$

a. Find the system function of this system
b. Compute the first 5 values of the step response, considering the initial conditions $v[0] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
c. Draw the the type I and type II state-space implementations of this system, as well as the direct form II implementation

4 Practical exercises

1. In the Matlab environment, use the `fdatool` tool to design one of the following filters:

   a. A low-pass filter, IIR, order 4, elliptic type, with cutoff frequency of 5kHz for a sampling frequency of 44.1 kHZ;
   b. A high-pass filter, IIR, order 4, elliptic type, with cutoff frequency of 1kHz for a sampling frequency of 44.1 kHZ;
   c. A band-pass filter, IIR, order 4, elliptic type, with pass-band between 700Hz and 4kHz for a sampling frequency of 44.1 kHZ.

   Export the coefficients to Matlab's Workspace.

2. Fill in the following code template to perform temporal filtering of a video sequence, using the filter designed above. Test the function on the video sequence `veh_small.mp4`.

   Use the state-space type I equations, but extend them to a system of order 4.

```
v = VideoReader('videofile.mp4');

% Read all the frames from the video, one frame at a time.

while hasFrame(v)
 frame = readFrame(v);

 % Do the processing heres

 % Output equation
 y = ...

 % State equation
```

```
    v1_next = ...
    v2_next = ...
    v3_next = ...
    v4_next = ...

    % Update for next iteration
    v1 = v1_next;
    v2 = v2_next;
    v3 = v3_next;
    v4 = v4_next;

end
```

3. Implement a function `filter_spst(b, a, x)` which filters a signal `x` with the filter defined by the coefficients `b` and `a`. Implementation shall follow the type I state-space equations.

4. Test the function written above with the coefficients designed at step 3, by filtering a sample audio signal.

5 Final questions

1. TBD