Slides

Introduction

Organization

Professors:

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- Laboratories: Daniel Matasaru (..@etti.tuiasi.ro)

Grades

$$\text{Final grade} = 0.75\ \text{Exam} + 0.25\ \text{Lab}$$

Time schedule

- 14 weeks of lectures (3h each)
- 14 weeks of laboratories (2h each)
  - 5 laboratories
  - 7 seminars
  - 1 recuperari
  - 1 test

Course structure

1. Introduction to probabilities

2. Pam

Bibliography

1. Pam Pam
2. HamHam
3. Yoyo

Introduction to probabilities

Basic notions of probability

Random variable = the outcome of an experiment

Distribution (probability mass function)

Discrete distribution

Alphabet

Basic properties

Two independent events:

$$P(A \cap B) = P(A) \cdot P(B)$$

Chapter I: Discrete information sources

Block diagram of a communication system

de pus poza

What is information?

Example:

I tell you the following sentence: "your favorite football team lost the last match".

Does this message carry information? How, why, how much?

Consider the following facts:

- the message carries information only because you didn't already know the result.
- if you already known the result, the message is useless (brings no information)
- since you didn't know the result, there were multiple results possible (win, equal or lose)
- the actual information in the message is that *lost* happened, and not *win* or *equal*
- if the result was to be expected, there is little information. If the result is highly unusual, there is more information in this message

Information source

We will always consider information in a context similar to the above example.
We will use terminology from probability theory to define information:

- there is a *probabilistic source* that can produce a number of different *events*.
- each event has a certain probability. We know all the probabilities beforehand.
- at one time, an event is randomly selected according to its probability.
- afterwards, a new message can be selected, and so on $==>$ a stream of messages is produced.

The source is called an *information source* and the selected event is a *message*.
A message carries the information that **it** happened, and not the other possible message events that could have been selected.
The quantity of information is dependent in its probability.

Discrete memoryless source

$=$ is an information source where the messages are **independent** , i.e. the choice of a message at one time does not depend on what were the previous message

Each message has a fixed probability. The set of probabilities is the *distribution* of the source.

$$S : \begin{pmatrix} s_1 & s_2 & s_3 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Properties:

- Discrete: it can take a value from a discrete set (alphabet)
- Complete: $\sum p(s_i) = 1$
- Memoryless: succesive values are independent of previous values (e.g. successive throws of a coin)

A message from a DMS is also called a *random variable* in probabilistics.

Examples

A coin is a discrete memoryless source (DMS) with two messages (head, tail):

$$S : \begin{pmatrix} s_1 & s_2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

A dice is a discrete memoryless source (DMS) with six messages:

$$S : \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Playing the lottery can be modeled as DMS:

$$S : \begin{pmatrix} s_1 & s_2 \\ 0.9999 & 0.0001 \end{pmatrix}$$

An extreme type of DMS containing the certain event:

$$S : \begin{pmatrix} s_1 & s_2 \\ 1 & 0 \end{pmatrix}$$

Information

When a DMS provides a new message, it gives out some new information, i.e. the information that a particular message took place.

The information attached to a particular event (message) is rigorously defined as:

$$i(s_i) = -log_2(p(s_i))$$

Properties:

- $i(s_i) \leq 0$
- lower probability means higher information
- higher probability means lower information
- a certain event brings no information: $-log(1) = 0$
- an event with probability 0 brings infinite information (but it never happens..)

Entropy of a DMS

We usually don't care about a single message. We are interested in a large number of them (think millions of bits of data).

We are interested in the *average* information of a message from a DMS.

Definition: the entropy of a DMS source S is **the average information of a message**:

$$H(S) = \sum_i p_i i(s_i) = -\sum_i p_i log(p_i)$$

where $p_i = p(s_i)$ is the probability of message i.

...

Example - Game

Game: I think of a number between 1 and 8. You have to guess it by asking yes/no questions.

- How much indetermination does the problem have?
- How is the best way to ask questions? Why?
- What if the questions are not asked in the best way?
- On average, what is the number of questions required to find the number?

Example - Game v2

Suppose I choose a number according to the following distribution:

...

- On average, what is the number of questions required to find the number?
- What distribution makes guessing the number the most difficult?
- What distribution makes guessing the number the easiest?

Entropy of a discrete memoryless source

Properties of entropy

1. It
2. is
3. cool

Sources with memory

A text can be considered as a sequence of symbols drawn from a memoryless source.

The distribution (frequencies) of letters in the Romanian language is close to: