# VARIABLE ENTROPY GENERATOR

## 1. Theoretical introduction

For a set of disjoint events

$$[X] = [x_1, x_2, \ldots, x_n]$$

with probabilities:

$$[P(X)] = [p(x_1), p(x_2), \ldots, p(x_n)]$$

the **entropy** is the average value of the information or, equivalently, the average uncertainty of the events. The entropy is defined as follows:

$$H(X) = \sum_{i=1}^{n} p(x_i) \cdot i(x_i) = -\sum_{i=1}^{n} p(x_i) \cdot \log p(x_i)$$

The entropy reaches its maximum value whenever the uncertainty is maximum, which happens when the events $x_i$ have equal probability: $\max H(X) = \log n$.

Fig.1 shows the value of the entropy in the case of two events (n = 2).

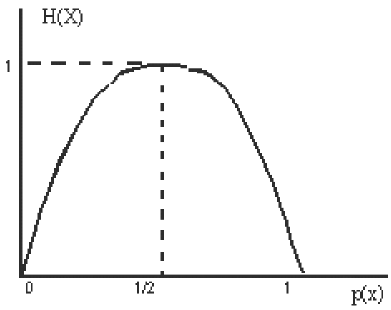


**Figure 1**

## 2. Practical device

The practical device allows taking of a simple psychological test: finding a human subject's decision speed upon the lighting of a light bulb.

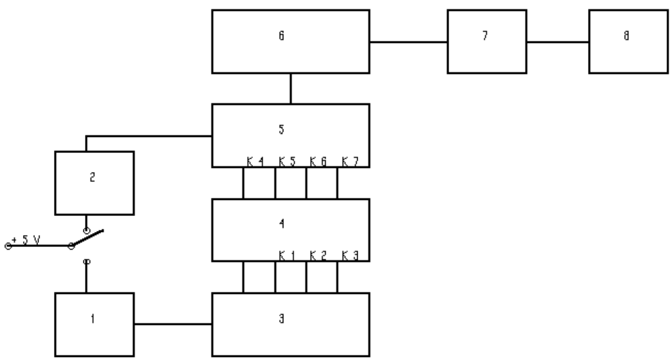


**Figure 2**

The block scheme of the device is presented in Fig.2 and is composed of the following units:

1. CBA 1
2. CBA 2
3. Feedback shift register
4. Combinatorial circuit
5. Counter clock circuit
6. Binary counter
7. Decoder
8. Display

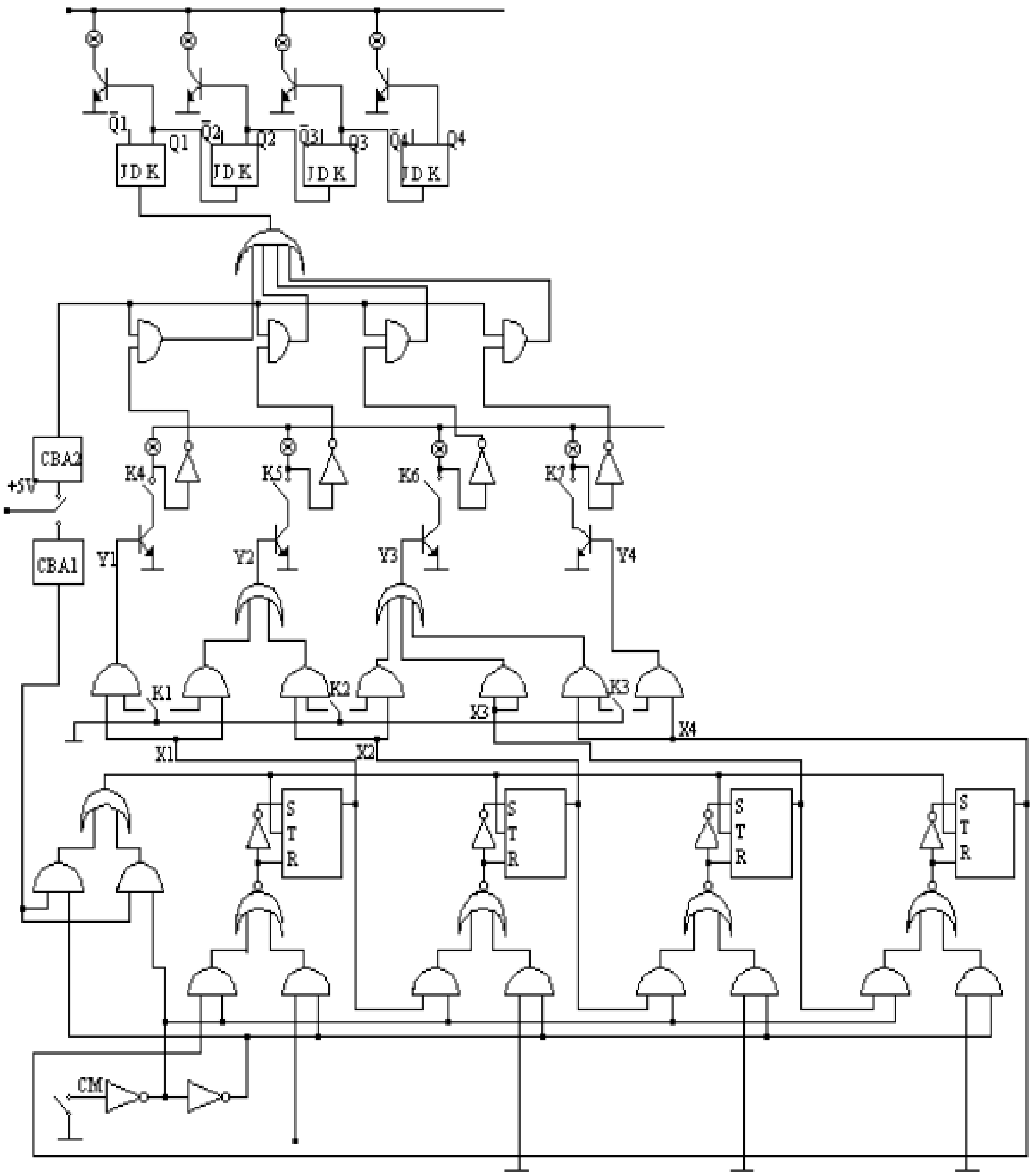


**Figure 3**

The electronic schematic of the device is presented in Fig. 3. The main component of the device is a feedback shift register (CDB 495). Setting the Mode Control switch (CM) on position 1 will set logical 1 in the first cell and logical 0 in all the others. Setting CM on position 0 creates the feedback shift register, and the logical 1 value will successively travel through all the four cells with the clock signal. When the clock signal is stopped, only one cell has state 1, the others being 0. We define the event $x_i$ as the fact that cell "i" has state 1 when the clock signal is stopped. In this case, the set of events and their probabilities is as follows:

$$[X]=[x_1,x_2,x_3,x_4],$$

$$[P(X)]=[1/4,1/4,1/4,1/4],$$

and therefore the entropy has maximum value:

$$H(X)=\log_2 4=2 \text{ bits/message.}$$

For the modification of the entropy we use a combinatorial circuit with four outputs. At its input we apply the primary set of events [*X*]. The outcome is the secondary set of events [*Y*]:

$$[Y]=[y_1,y_2,y_3,y_4]$$

$$[P(Y)]=[p(y_1),p(y_2),p(y_3),p(y_4)]$$

where the probabilities $p(y_i)$ depend on the switches K1, K2, K3.

Table 1 presents the possible probabilities $[P(Y)]$ of the secondary set as well as their entropies, for the various combinations of the three switches (for the switches, 0 designates a closed switch and 1 an open switch).

**Table 1**

| Switches | | | | | | P(Y) | | | | H(Y) |
|---|---|---|---|---|---|---|---|---|---|---|
| K1 | | K2 | | K3 | | p(y1) | p(y2) | p(y3) | p(y4) | bit/ msg |
| A | B | C | D | E | F | | | | | |
| 1 | 0 | 1 | 0 | 0 | 1 | 1/4 | 1/4 | 1/4 | 1/4 | 2 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1/2 | 1/4 | 1/4 | 1,5 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1/4 | 1/4 | 1/2 | 0 | 1,5 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1/4 | 1/2 | 1/4 | 1,5 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1/4 | 0 | 1/2 | 1/4 | 1,5 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1/2 | 1/2 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1/4 | 0 | 3/4 | 0 | 0,81 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1/4 | 3/4 | 0 | 0,81 |

## 3. Using the practical device

a) Set the clock switch on position CBA1 and make sure that the switches K4-K7 (of the lights) are closed. Using the switches K1-K3 one possible set of events [Y] is chosen , according to Table 1.

b) Power on the devices using the main supply switch.

c) Initialize the shift register by setting CM on position 1, in which case the four cells of the register are loaded with a 1 and three 0's.

d) When setting the switch CM in position 0, the values contained in the register's cells are shifting towards the right, as long as the clock signal is running.

e) Set the clock switch in position CBA2, in which case the clock circuit CBA1 is powered off and the second circuit CBA2 is powered on.

When CBA1 is turned off, the clock signal of the shift register is stopped, and one of the cells will contain the logical 1 value. Therefore, one of the outputs $y_i$ has a logical value of 1, which will determine the corresponding light emitter to be turned on and the corresponding AND gate to be valid.

The circuit CBA2 is turned on simultaneously, and the provided clock signal is applied via the AND gate to the counter, which shows the time elapsed from the moment the light $y_i$ is turned on.

f) Open the switch corresponding to the light, and therefore the clock signal applied to the counter is stopped. Read the time elapsed from lighting the bulb.

g) For repeating the experiment, it is necessary to perform the following:

-Close the switch corresponding to the light emitter;

-Initialize again the shift register by setting CM to 1 and then back to 0;

-Set the clock switch on position CBA1;

-Initialize the counter by setting the corresponding switch to 0 and then back to 1;

-Repeat the experiment.

## 4. Practical exercises

a) Study the functioning of the device based on the block schematic and the electronic schematic.

b) Write the expressions of the probabilities $p(y_i)$ as a function of the probabilities $p(x_i)$ and the position of the switches K1-K3, and check them against the ones in Table 1.

c) **Using MATLAB -> Simulink:**

Launch the *Matlab* application, and open the current model with 'File->Open'. Check the results obtained in Table 1 in the following way:

- Identify the blocks in the schematic;

- With double-click on the blocks A-B, C-D, E-F, select the row from the table which you would like to verify;

- With the buttons 'Start/Pause Simulation' make at least 50 measurements; the "Display" blocks indicate the position of the cell which contains the logical 1. Compute the probabilities of the secondary set of events in Matlab’s Command Window, as follows (use the division symbol “/” for dividing values):

```
>>p1=
>>p2=
>>p3=
>>p4=
```

- Compute the entropy of the set Y via the following command (followed by Enter):

```
>> H= -(p1*log2(p1)+ p2*log2(p2)+ p3*log2(p3)+ p4*log2(p4))
```

where p1, p2, p3, p4 are the probability values found before (exclude from this expression the terms for which the probability is). Compare the obtained value of the entropy with the corresponding one from the table.