PSEUDORANDOM BINARY SEQUENCES GENERATOR

1. Theoretical considerations

White noise is defined as a random process with power spectral density that is constant in an infinite frequency band.

Quasi-white noise is defined as a random process having power spectral density that is constant in a finite frequency band. Quasi-white noise can be generated using various methods; a simple method relies on using periodical pseudorandom sequences.

O periodical pseudorandom sequence is a sequence having the following properties:

a) The signal consists of a sequence of impulses of duration Δ and value +U or -U;

b) The total number of intervals Δ in one period is $N = 2^p - 1$, where *p* is an integer number;

c) In one period, the number of intervals with value +U is larger by one than the number of intervals of value -U;

d) Defining a *state* a succession of intervals Δ with the same value +U or -U, then the total number of states is $(N+1)/2 = 2^{p-1}$;

e) In one period, half of the states have a duration of Δ, a quarter have 2Δ, one eighth have 3Δ and so on, except one state of duration *p*Δ and value +U and one state of duration *(p-1)*Δ and value -U;

f) Performing a cyclical permutation of a pseudorandom sequence results in a new sequence for which, compared against the original one, the number of intervals with the same value of the original is smaller by one than the number of intervals where the value is different from the original.

An intuitive representation of a pseudorandom sequence is in Fig.1.

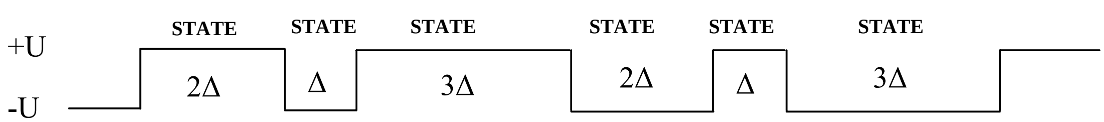

Figure 1

O periodical pseudorandom sequence corresponds to a binary pseudorandom sequence if we make the correspondence: $"1" \rightarrow +U$ şi $"0" \rightarrow -U$.

Periodical pseudorandom binary sequences can be obtained using feedback shift registers (FSR). The FSR, just like the multiplication and division circuits presented in the previous laboratory paper, can be implemented with D flip-flops (referred to as *cells*) and XOR gates (modulo-2 adders).

Example: Consider the register in Fig. 2, containing three cells. The outputs of cells 3 and 1 are connected to a XOR gate; the output of the gate is connected to the serial input of the register (therefore we implement the *"3:1 feedback"*). Assuming the initial state of the register to be 1 0 0 (cell C1 contains 1 and cells C2 and C3 0), and considering the functioning of a D flip-flop, it follows that as the clock signal is applied, the states of the register will be the ones in Table 1. Based on this example, we make the following observations:

- The binary sequences obtained at the output of the cells have the properties of a periodical pseudorandom sequence;
- The outputs of the cells are identical, only delayed;

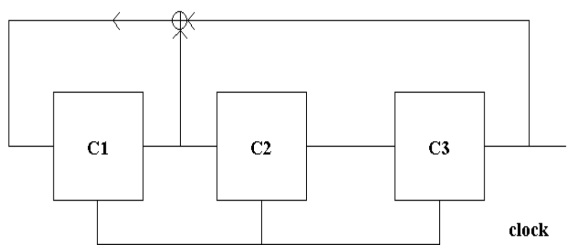

Figure 2

Table 1

1 0 0
1 1 0
1 1 1
0 1 1
1 0 1
0 1 0
0 0 1
1 0 0

In the general case of a register with *p* binary cells, not every feedback guarantees that the output will have the maximum period of length 2^p-1; in order to obtain a signal with maximum period, one of the following feedbacks must be used (Table 2):

Table 2

No. of cells *p*	2	3	4	5	6
Feedback	2:1	3:1 3:2	4:1 4:3	5:2 5:3	6:1 6:5

With one of these feedbacks, the output of a cell will be logical 1 a number of 2^{p-1} times, and logical 0 a number of 2^{p-1}-1 times, (one less, since the register state 000...0 is never reached). It follows that the probabilities of the output to be 1 or 0 are:

$$P(1)=2^{p-1}/2^p-1$$

$$P(0)=[2^{p-1}-1]/2^p-1$$

Note that $\lim_{p\to\infty} P(1) = \lim_{p\to\infty} P(0) = 1/2$.

The binary sequence obtained at the output of a cell, identical with every cell up to a certain delay, obeys all the properties of periodical pseudorandom sequences if the feedback is chosen from Table 2. The time interval Δ is identical to the period of the clock signal for the binary cells.

2. The autocorrelation function

Consider a binary sequence X of length N: $x_1\ x_2\ \ldots\ x_N$.

The autocorrelation function of a sequence is defined as:

$$R_{xx}(k) = \frac{1}{N}\sum_{h=1}^{N} x_h \cdot x_{h+k} \ , k = 0,1,2,\ldots,N-1$$

where x_h is the *h*-th symbol in the sequence, and x_{h+k} is the (*h*+*k*)-th. If $h+k > N$ we consider $x_{h+k} = x_{(h+k)-N}$ (periodical autocorrelation).

Example: Let X be $x_1\ x_2\ x_3\ x_4\ x_5$ (therefore N=5)

for $k = 0$, $R_{xx}(0) = 1/5 \cdot \left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2\right)$

for $k = 1$, $R_{xx}(1) = 1/5 \cdot \left(x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_5x_1\right)$

....

for $k = 4$, $R_{xx}(4) = 1/5 \cdot \left(x_1x_5 + x_2x_1 + x_3x_2 + x_4x_3 + x_5x_4\right)$

It can be shown that the autocorrelation function of a periodical pseudorandom sequence tends to the autocorrelation function of white noise when Δ becomes smaller and N (and therefore p) becomes larger. The function exhibits a maximum value U^2 for k = 0, N, 2N, 3N etc., and values equal to $-U^2/N$ elsewhere.

3. Practical device

The practical device, which allows visualization on the oscilloscope of a number of points equal to the periodical pseudorandom sequence length, consists of a shift register with 6 D flip-flops and one XOR gate, as in the schematic in Fig. 3. The shift register is constructed by connecting to outputs out of $Q_1, Q_2, \ldots, Q_6$ to the inputs R_1, R_2 of the XOR gate.

Points Y_1 and Y_2 are connected through three resistors having values of R, 2R, and 4R to the outputs $\overline{Q}$ of the first three, and the last three outputs of the flip-flops. By applying the corresponding voltages at Y_1 and Y_2 to the two pairs of deflection plates of the oscilloscope (horizontal and vertical), one point is obtained on the screen. The voltages change continually with the clock signal, showing on the screen a set of points equal to the number of the generated pseudorandom sequence.

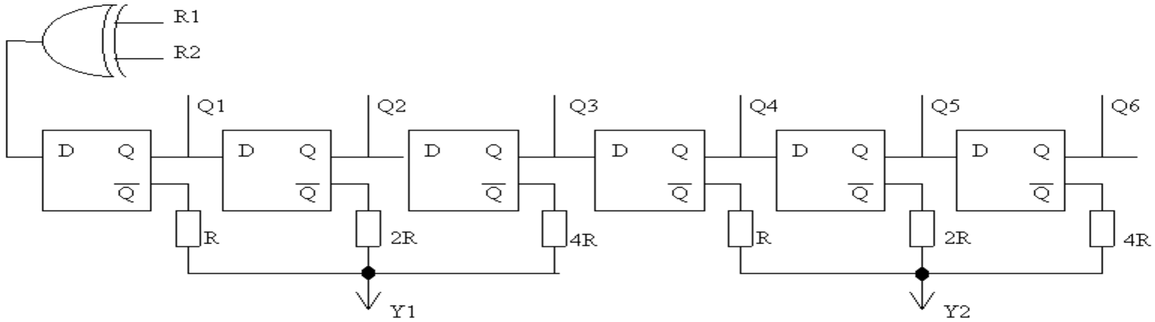

Figure 3

4. Practical exercises

4.1. Construct the tables containing the states of a feedback shift register with four cells, with 4:1 and 4:2 feedbacks, starting from the initial state 1000. What is the difference?

4.2. Draw the output sequences obtained with 3:1, 4:1 and 4:2 feedbacks, using the correspondence $+U \rightarrow 1$ şi $-U \rightarrow 0$. Check the properties of periodical pseudorandom sequences for the 3:1 and 4:1 feedbacks.

4.3. Compute the autocorrelation function for all the three feedbacks in exercise 4.2 considering k = 0, 1, 2,..... up to k = twice the period of the sequence. Draw the resulting graph of the function, putting the discrete values of k on the horizontal axis and the values of the function on the vertical axis. What are the differences?

4.4. Compute the horizontal and vertical coordinates of the points, according to the circuit in Fig. 3, for 3:1 feedback, considering: $1 \text{ logic} \rightarrow 5\text{ V}$ si $0 \text{ logic} \rightarrow 0\text{ V}$.

Example: if flip-flops 1, 2 and 3 have state 101, their $\overline{Q}$ outputs are 010, and the equivalent schematic for computing the voltage in Y_1 is in Fig. 4.

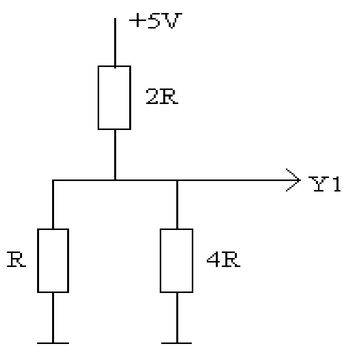

Figure 4

4.5. MATLAB -> Simulink Application:

Launch the *Matlab* application and open with 'File->Open' the model corresponding to the current laboratory. Check the results obtained in exercise 4.1. In this way:

- Identify the blocks in the schematic;
- Change the connections to obtain the desired feedbacks;
- Start the simulation by running 'Simulation->Start';

- Observe the waveforms plotted by the oscilloscopes ("Scope") and deduce the corresponding binary sequences. Indicate which are pseudorandom and which are not;

- Modify the feedbacks of the feedback register with six cells and observe all the states of the circuit;

- For the 4:3 feedback compute the horizontal and vertical coordinates of the points in the figure obtained after simulation in the block„XY Graph”. Fill the table with six columns corresponding to the circuit states, considering that the binary value of the first cell is multiplied with 1, the second with 0.5, and the third with 0.25; for the last three cells the same operations are done as for the first three.