Option-Based Estimation of the Price of Co-Skewness and Co-Kurtosis Risk

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Table of Content

1. Introduction

Paper stands for:

- Alternative - option-based - approach to measuring the price of co-skewness and co-kurtosis risk for the nonlinear exposure.
- Empirical results for co-skewness risk and comparison of the pricing performance of the new approach with regression-based estimates.
- Assess predictive performance of the new estimates.
- Option-based estimates of the prices of risk lead to reasonable values of the associated risk premia and improve the models' performance compared with regression-based estimates.

2. Main Findings

Table 1 Price of Co-Skewness; descriptive analysis

$$\widehat{\lambda}_{OI,t}^{COSK} = \widehat{E}_t^P(R_{m,t+1}^2) - \widehat{E}_t^Q(R_{m,t+1}^2)$$

TABLE 1
Option-Based Price of Coskewness Risk

	Physical Second Moment	Risk-Neutral Second Moment	Price of Coskewness Risk
Mean	0.3034	0.4499	−0.1464
Std. dev.	0.2168	0.4133	0.2289
Skewness	8.9863	3.4318	−2.2645
Kurtosis	109.6172	19.1077	12.8347
First-order autocorrelation	0.4996	0.7525	0.4352

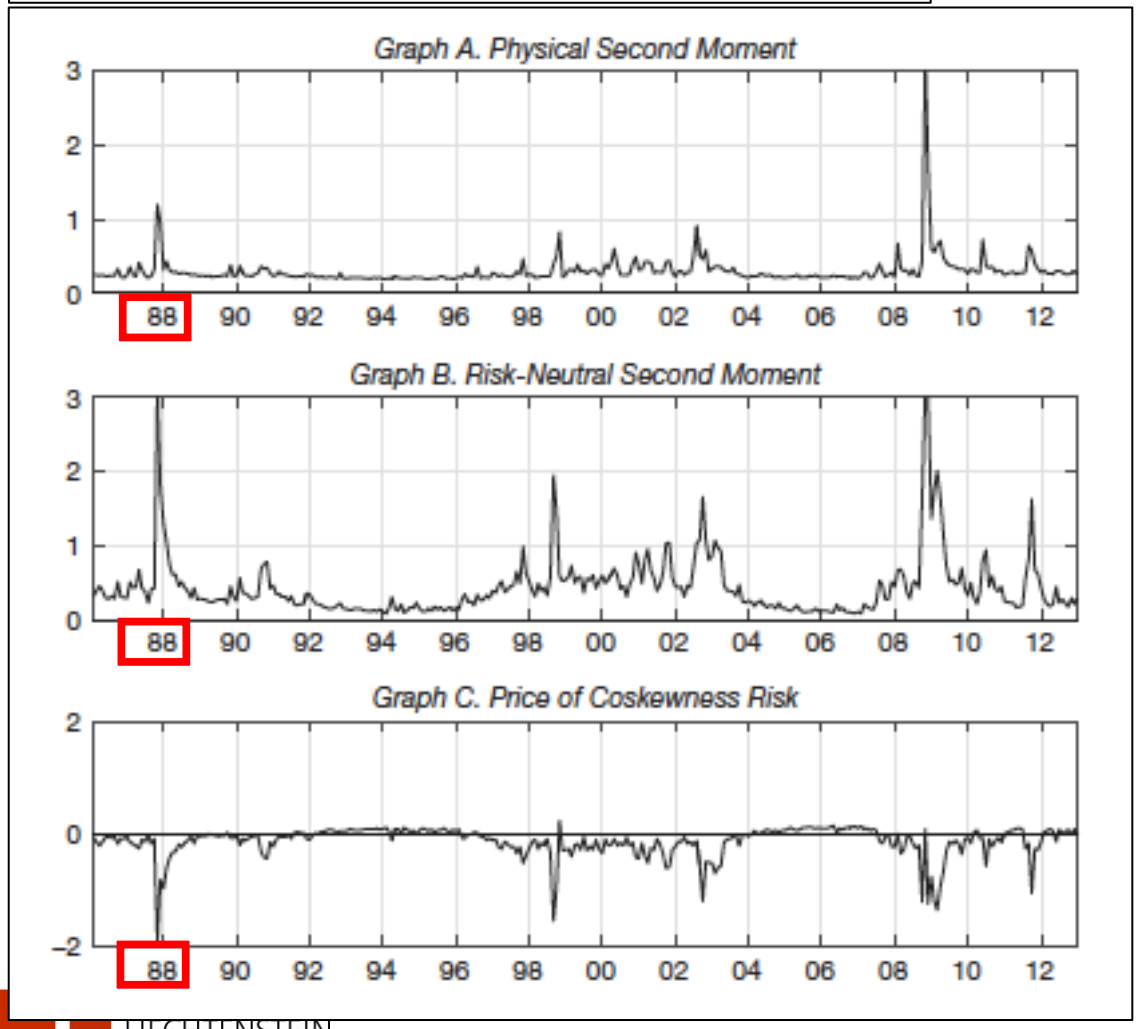
Consistent with theory: -0.1464

- **KL (1976);** Regression gives the value **-0.212**
- **HS(2000);** Regression gives the value **-0.305**

2. Main Findings

FIGURE 1

Option-Based Estimates of the Price of Coskewness Risk



Co-skewness price of risk is negative for almost all months

2. Main Findings

Table 2 Regression-Based Estimates of the Price of Coskewness Risk

TABLE 2

Regression-Based Estimates of the Price of Coskewness Risk

	25 Size/BM			25 Size/MOM			25 Size/STR			25 Size/LTR		
Variable	1	2	3	4	5	6	7	8	9	10	11	12
Panel A. 1986–2012												
λ^0	1.338	1.408	0.849	0.182	0.008	0.592	0.574	0.471	0.369	0.389	0.490	−0.208
	2.85	2.63	2.32	0.53	0.02	1.56	1.74	1.26	0.80	1.41	1.68	−0.54
λ^{MKT}	−0.637	−0.742	−0.231	0.472	0.585	0.089	0.073	0.206	0.218	0.370	0.236	0.829
	−1.27	−1.37	−0.56	1.11	1.33	0.22	0.17	0.44	0.46	1.12	0.68	1.92
λ^{HML}			0.017			0.069			−0.034			0.033
			0.10			0.35			−0.18			0.15
λ^{SMB}			0.333			0.200			0.497			0.542
			1.81			0.66			1.38			2.29
λ^{MOM}			0.436			0.570			−0.459			−0.055
			0.89			2.01			−0.96			−0.13
λ^{COSK}		−0.051	−0.073		−0.108	0.010		0.015	0.076		−0.080	−0.157
		−0.65	−1.13		−1.61	0.18		[illegible]	1.19		−1.12	[illegible]
Adj. R^2	16.01	31.14	44.99	13.78	27.26	56.22	14.37	32.73	51.11	7.84	21.86	43.75

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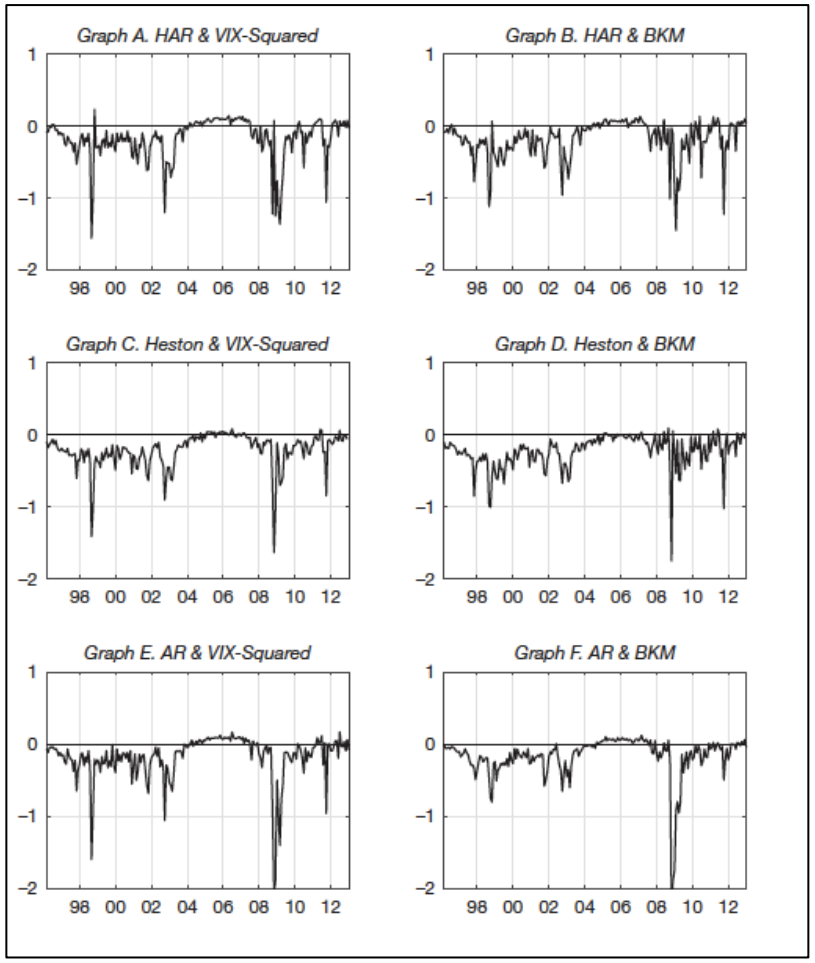
2. Main Findings

FIGURE 2
Various Option-Implied Estimates of the Price of Coskewness Risk Using Alternative Models for Physical and Risk Neutral Variance (1996–2012)



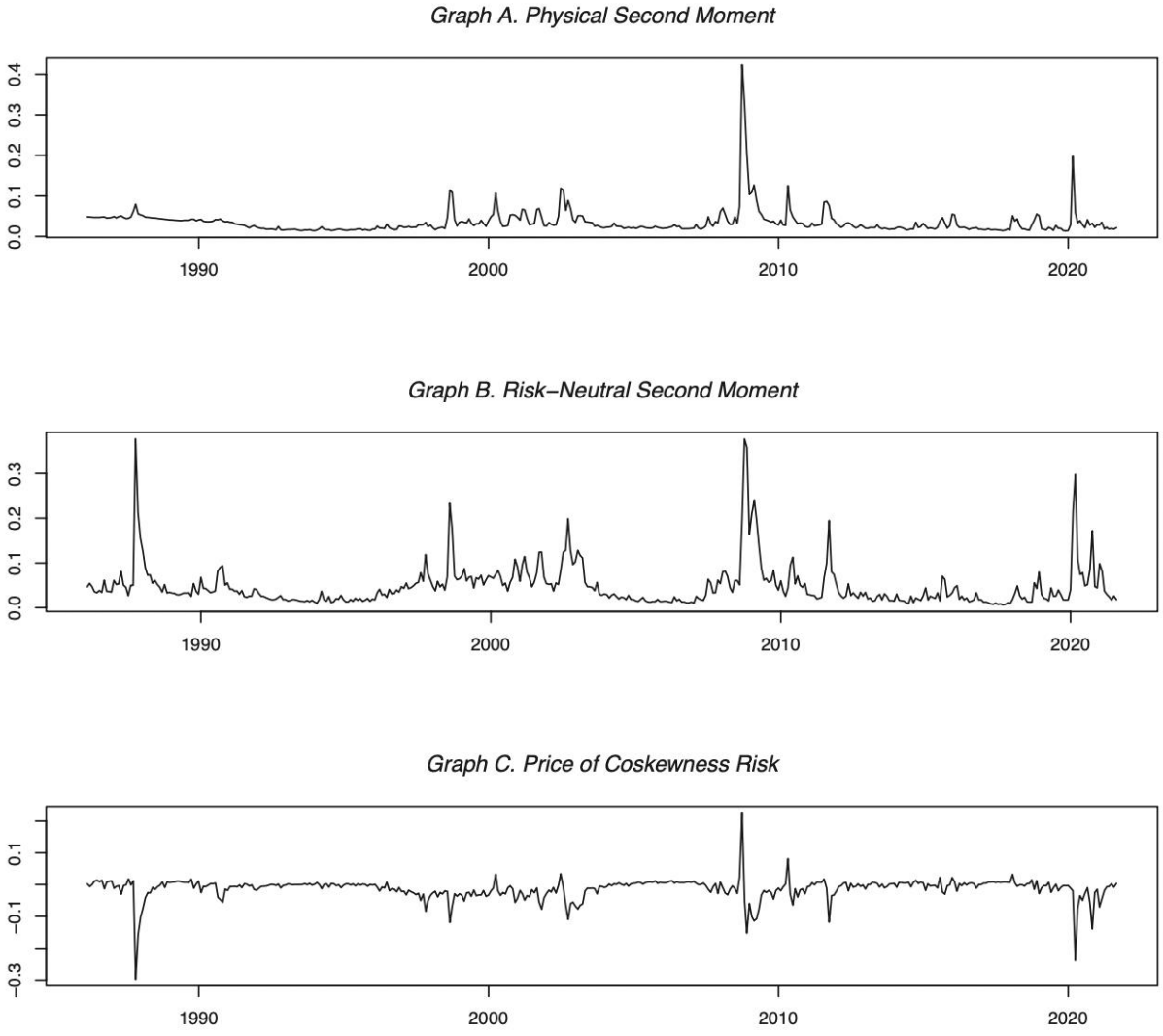
- For **Physical Variance**
  -> HAR of Corsi (2009)
  -> Henston (1993)
  -> AR
- For **Risk-Neutral Variance**
  -> VIX SQUARED (CBOE)
  -> Bakshi et al. (BKM) (2003)
- Estimates negative regardless of the model used

2. Main Findings

Descriptive Analysis & Regression Based Analysis

Test Assets	• Regression based estimates depend on test assets • Approach design is independent of the test asset
Large λ^{cosk}	• Unconditional estimate -0.146 • Large in absolute terms, magnitude compared to regression-based estimates
Negative Estimates	• Consistently negative estimates of the price of conditional co-skewness in their approach • Not the case in regression-based approaches

Our results – Figure 1



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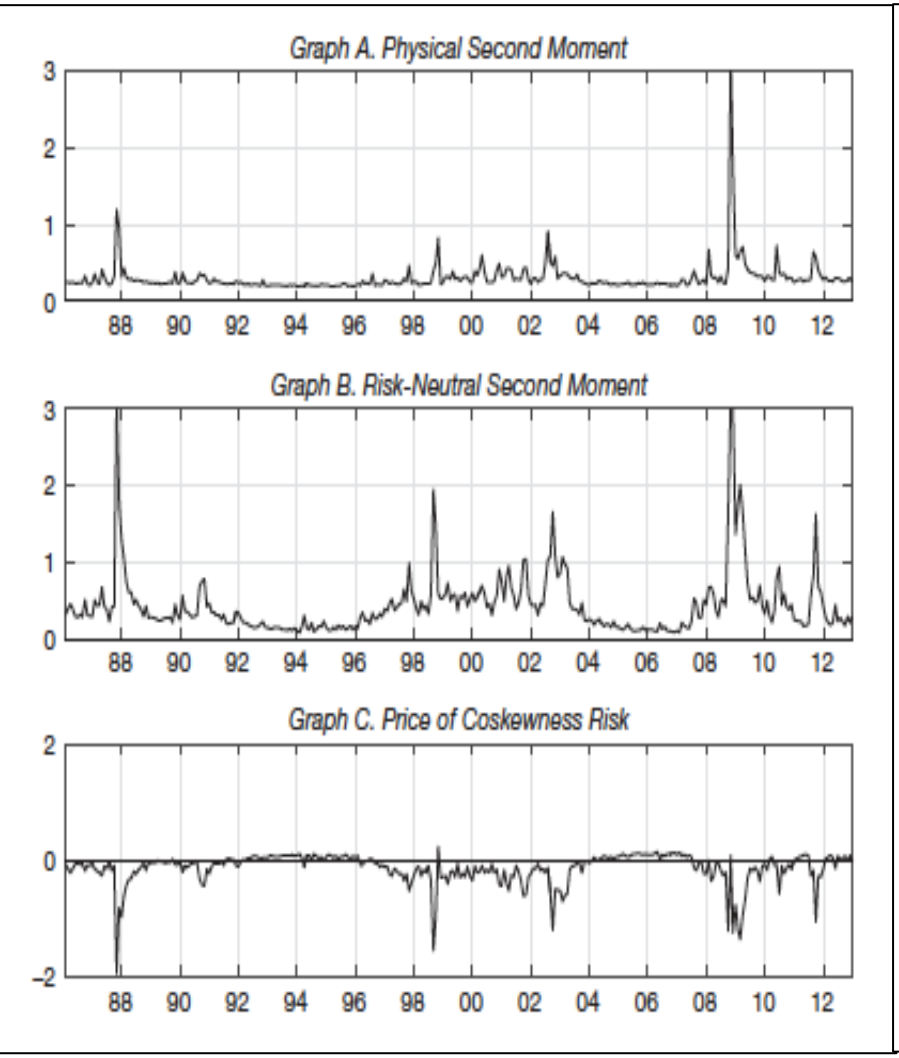
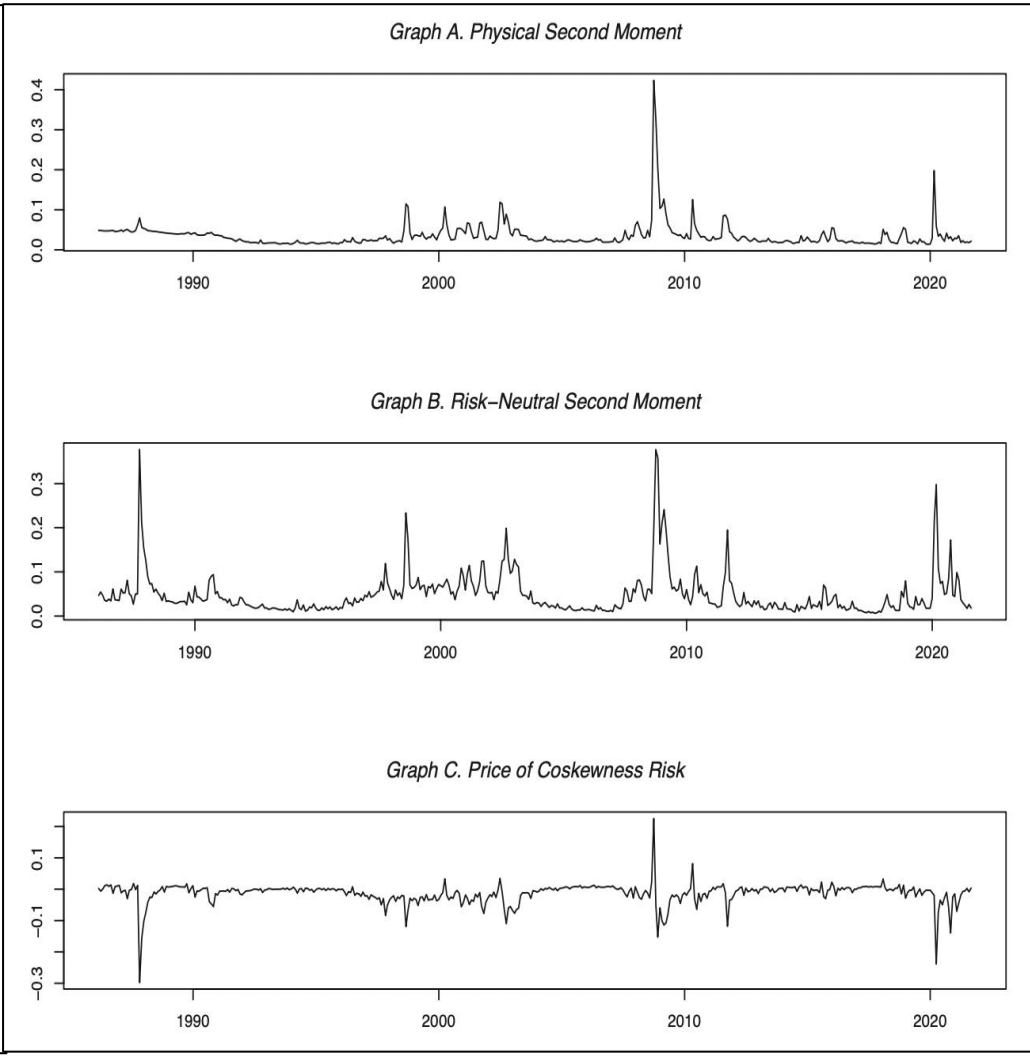
Comparison - Figure 1





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Left – paper, right - replication

Our results – Table 1

	Physical Second Moment	Risk–Neutral Second Moment	Price of Coskewness Risk
mean	0.035	0.0481	–0.0131
sd	0.0321	0.0482	0.0337
skew	6.8878	3.4817	–2.2494
kurtosis	67.0802	16.4571	23.1601
Autocorrelation	0.6893	0.7383	0.4764

	Physical Second Moment	Risk Neutral Second Moment	Price of Co-Skewness Risk
mean	0.3034	0.4499	-0.1464
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kurt	109.6172	19.1077	12.8347
ρ(1)	0.4996	0.7525	0.4352

- risk-neutral variance is > the physical variance
- price of co-skewness risk is negative

--> Bakshi and Madan (2006), Bollerslev, Tauchen & Zhou (2009), Carr and Wu (2009), Jackwerth and Rubinstein (1996)

Our results – Table 2

	Size–to–Book 1	Size–to–Book 2	Size–to–Book 3	Size–to–Momentum 1	Size–to–Momentum 2	Size–to–Momentum 3
l_0	1.435	1.412	1.3	1.093	0.9	1.178
l_MKT	–0.394	–0.439	–0.356	–0.034	0.084	–0.2
l_HML			0.063			–0.108
l_SMB			0.145			0.252
l_MOM			0.426			0.327
l_COSK		–3.564	–3.775		–9.53	–1.736
Adj.R.sq	0.155	0.266	0.478	0.181	0.287	0.591

	Size–to–STR 1	Size–to–STR 2	Size–to–STR 3	Size–to–LTR 1	Size–to–STR 2	Size–to–STR 3
l_0	0.846	0.917	0.884	1.045	1.021	0.625
l_MKT	0.173	0.091	0.032	0.045	0.032	0.299
l_HML			0.269			0.136
l_SMB			0.074			0.247
l_MOM			–0.201			0.486
l_COSK		–5.067	7.452		–7.598	–3.773
Adj.R.sq	0.16	0.294	0.519	0.143	0.255	0.48

5. Comparing predictive performance. Table 3.

$$\varepsilon_{t+1}^{p,k} = R_{p,t+1} - \lambda_{RB,t}^{MKT}\beta_{p,t}^{MKT} - \lambda_{k,t}^{COSK}\beta_{p,t}^{COSK},$$

$$\Delta MSE_t = \left(\frac{1}{25}\sum_{p=1}^{25}\left(\varepsilon_t^{p,RB}\right)^2 - \left(\varepsilon_t^{p,OI}\right)^2\right) \times 12 \times 100.$$

- ΔMSE - difference in mean squared error
- We multiply the cross-sectional average of the monthly difference in mean squared errors by 12 to annualize and by 100 to express it as a percentage.
- Positive ΔMSE indicates a superior forecast performance of the option-implied estimate versus the regression-based estimate.

	25 Size/BM	25 Size/MOM	25 Size/STR	25 Size/LTR	All
Panel A. Difference in Mean Squared Error					
ΔMSE	0.325	0.134	0.233	0.081	0.193
NW *p*-value	0.30%	5.35%	0.30%	15.61%	0.40%
BS *p*-value	0.15%	3.37%	0.14%	13.58%	0.16%
BS 5th-percentile bound	0.171	0.015	0.112	−0.040	0.091
BS 95th-percentile bound	0.492	0.255	0.359	0.202	0.298

MSE_SB	MSE_SM	MSE_SS	MSE_SL
−40.0341124507312	10.2708850921419	26.9686956644452	−24.0533985108649

6. Conclusion

1. Given the results of the replication **we can confirm the results** of original paper of Christoffersen:

- - price of co-skewness risk has the theoretically expected negative sign in most cases and price of co-kurtosis risk has theoretically expected positive sign in most cases.
- - alternative – option-based – strategy for estimating the price of co-skewness and co-kurtosis risk is a better estimation compared to the regression-based estimates.

2. Our results deviate from the paper`s ones and can provide different conclusions

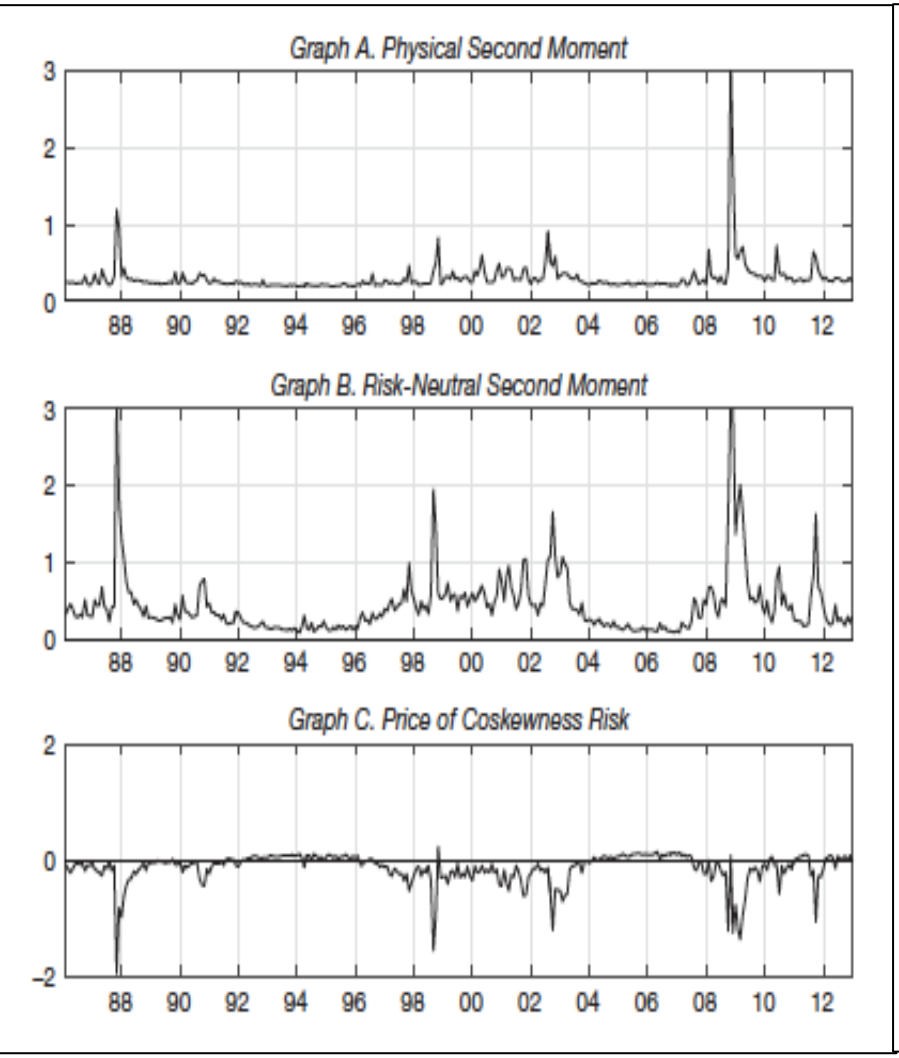
We assume it may be due to:

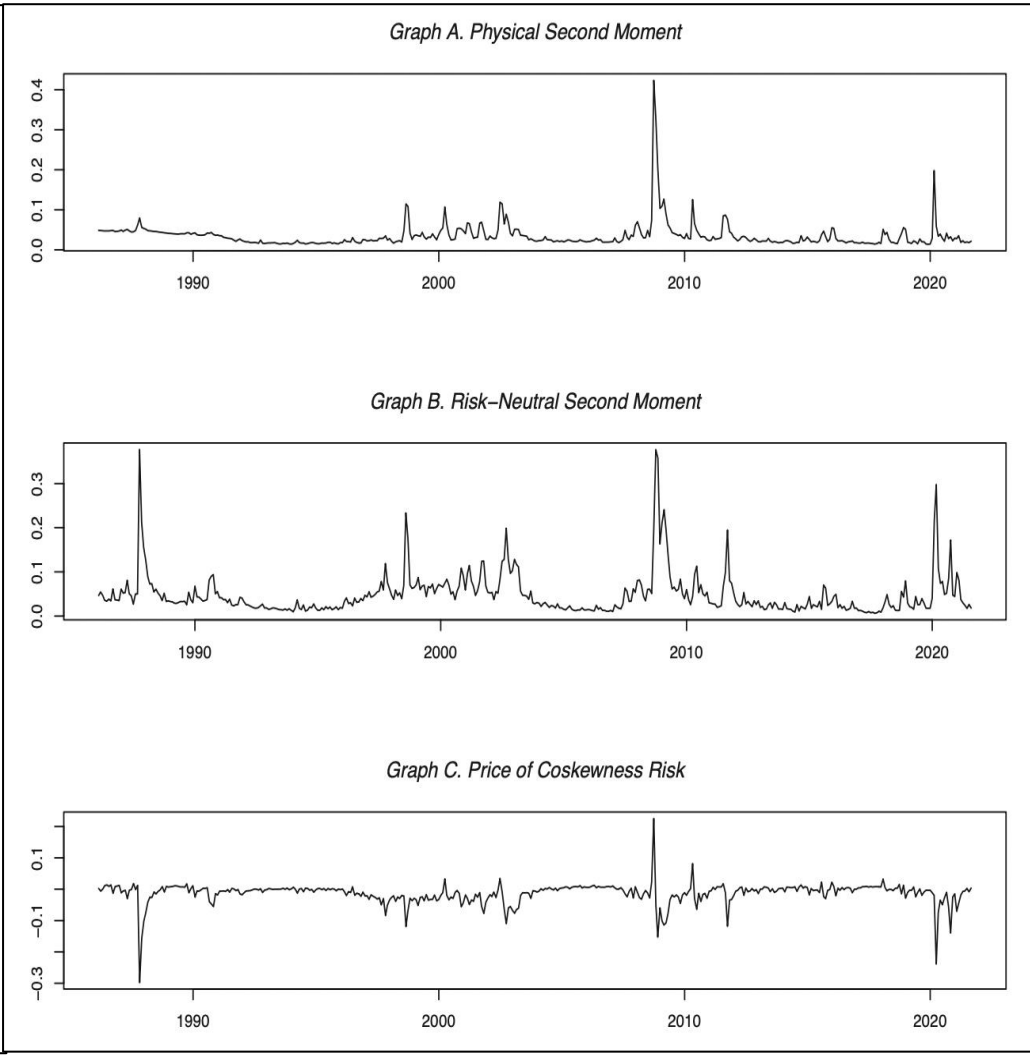
- Different data set /newer / larger time-frame (but only to have small deviations in the means)
- We observe some differences in the data of S&P500
- Mistake in HAR model to estimate the physical second moment
- Ordinary mistake

Thank you for your attention!

Now it is time for questions!

Comparison - Figure 1





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Down – paper, up - replication

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Adj. R^2	16.01	31.14	44.99	13.78	27.26	56.22	14.37	32.73	51.11	7.84	21.86	43.75

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