

$$(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12) \quad (1)$$

We will prove that (1) is false by showing that  $\forall n \in \mathcal{N}$  there  $\nexists m \in \mathcal{N}$  such that  $3m + 5n = 12$ .

Since  $n \in \mathcal{N}$ ,  $n \geq 1$  therefore  $5n \geq 5$ . This means that  $5 \leq 5n = 12 - 3m$ . If  $m > 2$  then  $12 - 3m < 5$  so  $m$  must be either 1 or 2.

If  $m = 1$ ,

$5n = 9$ . 9 is not divisible by 5 therefore  $n \notin \mathcal{N}$  which is a contradiction so  $m \neq 1$

If  $m = 2$ ,

$5n = 6$ . 6 is not divisible by 5 therefore  $n \notin \mathcal{N}$  which is also a contradiction so  $m \neq 2$

Thus,  $\nexists m \in \mathcal{N}$  such that (1) is true. so (1) must be false.