Conjecture: For any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.
Assume the conjecture is false. Then by the division theorem, $\exists q_1, q_2, q_3, r_1, r_2, r_3 \in \mathcal{Z}, 0 < r_1, r_2, r_3 < 3$ such that

$$n = 3q_1 + r_1 \tag{1}$$
$$n + 2 = 3q_2 + r_2 \tag{2}$$
$$n + 4 = 3q_3 + r_3 \tag{3}$$

given that r_1 can only be 1 or 2, when $r_1 = 1$, (2) becomes

$$\begin{aligned} 3q_1 + 3 &= 3q_2 + r_2 && \text{Substituting (1) and } r_1 = 1 \\ r_2 &= 3(q_1 - q_2 + 1) && \text{Re-arranging} \end{aligned}$$

Therefore, $n+2$ is divisible by 3 when $r_1 = 1$. Which is a contradiction.

When $r_1 = 2$, (3) becomes

$$\begin{aligned} 3q_1 + 6 &= 3q_3 + r_2 && \text{Substituting (1) and } r_1 = 2 \\ r_2 &= 3(q_1 - q_3 + 2) && \text{Re-arranging} \end{aligned}$$

Therefore, $n+4$ is divisible by 3 when $r_1 = 2$. Which is also a contradiction.

We have shown that for all possible values of r_1 such that n is not divisible by 3 either $n+2$ or $n+4$ is divisible by three. The only other case is that r_1 is zero and n is divisible by 3. Therefore, our initial assumption is false and the conjecture must be true.