Give an example of family of intervals $A_n, n = 1, 2, \dots,$ such that $A_{n+1} \subset A_n$ for all n and $\cap_{n=1}^{\infty} A_n = \emptyset$.

Let A_n be the interval $(\frac{n}{n+1}, 1)$, then A_{n+1} is the interval $(\frac{n+1}{n+2}, 1)$

The greatest lower bound of A_n is $\frac{n}{n+1}$ and the greatest lower bound of A_{n+1} is $\frac{n+1}{n+2}$. They have the same lowest upper bound, 1.

$$\begin{aligned}
\frac{n}{n+1} &= \frac{n}{n+1}\frac{n+2}{n+2} && \text{Multiply and divide by n+2} \\
&= \frac{n^2+n}{(n+1)(n+2)} && \text{Expand out numerator} \\
&< \frac{n^2+n+1}{(n+1)(n+2)} && \text{Add 1 to numerator} \\
&= \frac{(n+1)^2}{(n+1)(n+2)} && \text{Factor out } n+1 \text{ from numerator} \\
&= \frac{n+1}{n+2} && \text{Cancel out } n+1
\end{aligned}$$

Therefore $A_{n+1} \subset A_n$ since the greatest lower bound of A_n is strictly less than the greatest lower bound of A_{n+1}.

Now as $n \to \infty$, $\frac{n}{n+1} \to 1$, so $A_n \to (1, 1) = \emptyset$ since there are no values in this range. The intersection of the empty set with any set is the empty set. Therefore, $\cap_{n=1}^{\infty} A_n = \emptyset$. The family of intervals that lead to this conclusion was the interval $(\frac{n}{n+1}, 1)$, $n = 1, 2, \dots$