Conjecture: Prove that for any natural number n, $2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$

Let $A(n) : 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$

Proof by induction,

$A(1) : 2 = 2^2 - 2$

Assuming $A(n)$ is true then,

$$\begin{aligned} A(n+1) : 2 + 2^2 + 2^3 + \cdots + 2^{n+1} &= 2^{n+1} - 2 + 2^{n+1} && \text{[Induction hypothesis]} \\ &= 2(2^{n+1}) - 2 && \text{[Algebra]} \\ &= 2^{n+2} - 2 && \text{[Algebra]} \end{aligned}$$

Therefore, the conjecture is true.