Conjecture: For any integer n, the number $n^2 + n + 1$ is odd.

We assume the conjecture is true. Any arbitrary integer, n, can either be odd or even.

If n is even, then $n = 2k$, for some $k \in \mathcal{Z}$ therefore,

$$\begin{aligned}
n^2 + n + 1 &= (2k)^2 + 2k + 1 \\
&= 4k^2 + 2k + 1 && \text{Expanding equation} \\
&= 2(2k^2 + 1) + 1 && \text{Factoring to the form } 2r + 1
\end{aligned}$$

which is odd.

If n is odd, then $n = 2k + 1$, for some $k \in \mathcal{Z}$ therefore,

$$\begin{aligned}
n^2 + n + 1 &= (2k + 1)^2 + (2k + 1) + 1 \\
&= 4k^2 + 6k + 3 && \text{Expanding equation} \\
&= 2(2k^2 + 3k + 1) + 1 && \text{Factoring to the form } 2r + 1
\end{aligned}$$

Which is also odd.
Therefore, for any integer $n^2 + n + 1$ is odd. The conjecture is true.