Design and Analysis of Algorithms

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Contents

1 Introduction to Algorithms

An algorithm is a sequence of unambiguous instructions for solving a problem i.e. for obtaining a required output for any legitimate(valid) input in finite time.

1.1 Performance/Analysis of Algorithms

It refers to the memory and time representation of the program.

Methods of Analysis:

- Analytical
- Experimental

Any algorithm is analysed on the following criteria:

- Space Complexity
- Time Complexity

1.1.1 Space Complexity

It is the amount of memory required for a program to completion. It has 3 categories:

- Instruction Space (Compiled Program)
- Data Space (Space needed by var/const)
- Environment Stack Space (Recursive calls)

Denoted by: $C + S_p$

Sample Questions

1. Sum of array without recursion

```
int sum(int a[],int n)                    1
{                                         2
    int sum = 0;                          3
    for(int i = 0;i < n;i++)              4
        sum = sum + a[i];                 5
    return sum;                           6
}                                         7
```

Space Complexity: $6x$ bytes[a]

Reason: Line 1 occupies x bytes for pointer a and x bytes for integer n. Line 3 occupies x bytes for sum and x bytes for allocating 0. Line 4 will occupy x bytes for allocating integer i. In Line 6 space will reserved for returning data.

[a] where x is bytes occupied by int

2. Sum of array with recursion

```
int sum(int a[],int n)                    1
{                                         2
    if(n > 0)                             3
        return sum(a,n-1) + a[n-1];       4
    return 0;                             5
}                                         6
```

Space Complexity: $3x \times (n+1)$ bytes[a]

Reason: Line 1 occupies x bytes for pointer a and x bytes for integer n. Line 4 will execute n times and each time space is reserved for pointer a and $n-1$ thus giving $3x \times n$. During the last case of $n = 0$, Line 5 will be executed returing 0 thus occupying x bytes.

[a] where x is bytes occupied by int and n is the size of array

3. Linear Search using Recursion

```
int search(int a[],int n,int n)                    1
{                                                  2
      if(n < 1) return -1;                         3
      if(a[n-1] == x) return x-1;                  4
      search(a,n-1,x);                             5
}                                                  6
```

Space Complexity: $8x \times (n+1)$ bytes[a]

Reason: Line 5 occupies 8 bytes for everytime it's executed and similar to previous question we get $n+1$ total executions.

[a] where x is bytes occupied by int and n is the size of array

1.1.2 Time Complexity

There are 2 approaches to estimate time:

- Operation Counter
- Step Counter

Sample Questions

1. Finding max element position

```
int maxpos(int a[],int n) {                        1
    int pos = 0;                                   2
    for(int i = 0;i < n;i++)                       3
        if(a[pos] > a[i]) pos = i; return pos;     4
```

Time Complexity: $n-1$

Reason: Line 4 is executed $n-1$ times.

2. Polynomial Evaluation

```
int polyeval(int coeff[],int n,int x) {            1
    int y = 1, value = coeff[0];                   2
    for(int i = 1;i <= n;i++) {                    3
        y = y * x;                                 4
```

```
        value += y * coeff[i];                    5
    }                                             6
    return value;                                 7
}                                                 8
```

Time Complexity: $2n$ Multiplication & n Addition

Reason: Line 4 & 5 shows multiplication is done $2n$ times and addition operation is done n times.

3. Polynomial Evaluation using Horner's Algorithm

```
int horner(int coeff[],int n,int x) {             1
    int value = coeff[n];                         2
    for(int i = 1;i <= n;i++)                     3
        value = value * x + coeff[n-i];           4
    return value;                                 5
}                                                 6
```

Time Complexity: n Multiplication & n Addition

Reason: Let's assume a polynomial $3x^2 + 3x + 1$. By previous method we were simply substituting x into the equation. In case of Horner's Algorithm we simplified the expression i.e. $3x^2+3x+1 = x \underbrace{(3x+3)}_{\text{evaluated first}} +1$.

4. Rank Sorting

```
void rank(int a[],int n,int r[]) {                1
    for(int i=0;i < n;i++) r[i] = 0;              2
    for(int i=1;i < n;i++)                        3
        for(int j = 0;j < i;j++)                  4
            if(a[j] <= api])                      5
                r[i]++;                           6
            else                                  7
                r[j]++;                           8
}                                                 9
                                                  10
void rearrange(int a[],int n,int r[]) {           11
    int *n = new int[n];                          12
```

```
    for(int i = 0;i < n;i++) u[r[i]] = a[i];          13
    for(int i = 0;i < n;i++) a[i] = u[i];             14
    delete u[];                                       15
}                                                     16
```

Time Complexity: $\frac{n(n-1)}{2} + 2n$

Reason: As in Line 3 iterable i goes from 1 to $n-1$ the iterable j in Line 4 goes from 0 to $i-1$ for every value of i i.e. if $i = 1$ then $j : 0 \to 0$, if $i = 2$ then $j : 0 \to 1 \ldots$ if $i = n-1$ then $j : 0 \to n-2$ which can be simplified as sum of number of iterations of j:

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

And from Line 13 & 14 we get the $2n$ operations.

5. Selection Sort

```
void SelectionSort(int a[],int n) {                   1
    for(int size = n;size > 1;size--) {               2
        int j = max(a,size);                          3
        std::swap(a[j],a[size-1]);                    4
    }                                                 5
}                                                     6
                                                      7
int max(int a[],int n) {                              8
    int pos = 0;                                      9
    for(inr i = 1;i < n;i++)                          10
        if(a[pos] < a[i])                             11
            pos = i;                                  12
    return pos;                                       13
}                                                     14
```

Time Complexity: $\frac{n(n-1)}{2} + 3(n-1)$

Reason: The $3(n-1)$ factor comes from swapping $n-1$ times while the former follows same pattern as precious Q.4.

6. Transpose with Step-Counter method

```
void transpose(int **n,int r) {                    1
    for(int i = 0;i < r;i++)                       2
        for(int j = i+1;j < r;j++)                 3
            std::swap(a[i][j],a[j][i]);            4
}                                                  5
```

Time Complexity: $\frac{(r-1)r}{2} + \frac{r(r+1)}{2} + (r+1)$

Reason: Let's count the steps, time taken, total execution time with a table.

Line	t	ν	$\nu \times t$
1	0	0	0
2	0	0	0
3	1	$r+1$	$r+1$
4	1	$\frac{r(r+1)}{2}$	$\frac{r(r+1)}{2}$ [a]
5	1	$\frac{(r-1)r}{2}$	$\frac{(r-1)r}{2}$
Total Time			r^2+r+1

[a] r+1 because of exit loop condition