

1 Probability

Let **A** be an event of **S**. If **A** occurs m different ways out of a total of n , then probability of **A** is denoted by

$$P(A) = \frac{\text{Favorable Cases}}{\text{Total Outcomes}} = \frac{m}{n}$$

Similarly we have a thing called odds in favor of A which is defined as the ratio of favorable cases to unfavorable cases

$$\text{Odds in favor of } A = \frac{\text{Favorable Cases}}{\text{Unfavorable Cases}} = \frac{m}{n-m}$$

1.1 Kalmogorov's Axioms

Let **E** be an experiment with sample space **S**. Let **A** be an event of **S**, then:

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- Given A & B are mutually exclusive then, $P(A \cup B) = P(A) + P(B)$
- If $A_1, A_2, A_3 \dots A_n$ are mutually exclusive then, $P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$

Theorem 1.1. *If A is an event of S then,*

i $P(\phi) = 0$

ii $P(A) + P(\bar{A}) = 1$

Proof. i) Let $A \cup \phi = \phi$

$$A \cap \phi = \phi \tag{1a}$$

$$P(A \cap \phi) = P(\phi)$$

$$A \cup \phi = \phi \tag{1b}$$

$$P(A \cup \phi) = P(\phi)$$

Using axiom from 1.1 & equation.(1b) we get,

$$P(A) + P(\phi) = P(A)$$

$$P(\phi) = 0$$

ii) Let $S = A \cup \bar{A}$

$$P(S) = P(A \cup \bar{A}) \quad [\text{Mutually Exclusive}]$$

$$1 = P(A) + P(\bar{A})$$

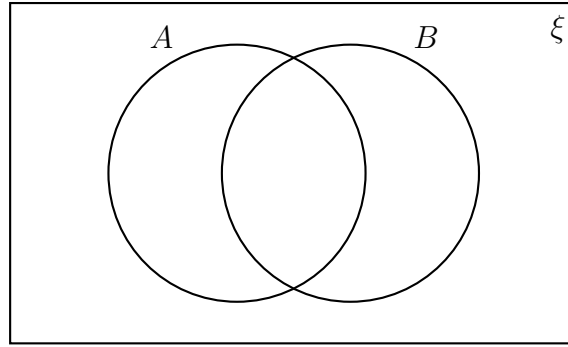
$$P(A) + P(\bar{A}) = 1$$

□

1.2 Addition Rule

If A & B are two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by addition rule.

Proof. Consider the following venn diagram having sets A and B.



$$A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$$

Consider, $B = (A \cap B) \cup (\bar{A} \cap B)$

$$P(B) = P((A \cap B) \cup (\bar{A} \cap B)) \quad [\text{Mutually Exclusive}]$$

$$P(B) = P(A \cap B) \cup P(\bar{A} \cap B) \quad (3a)$$

Consider, $A = (A \cap B) \cup (A \cap \bar{B})$

$$P(A) = P((A \cap B) \cup (A \cap \bar{B})) \quad [\text{Mutually Exclusive}]$$

$$P(A) = P(A \cap B) \cup P(A \cap \bar{B}) \quad (3b)$$

Thus from (3a) and (3b) we get,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

□

Generalised Addition Rule

If $A_1, A_2, A_3 \dots A_n$ are n events in a given sample space S .

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i) - \sum_{i=1}^n P(A_i \cap A_j) \dots (-1)^n P\left(\bigcap_{i=1}^n A_i\right)$$

1.3 Conditional Probability

Conditional Probability defines the probability of an event \mathbb{A} under a given circumstance say \mathbb{B} as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If the event is independent of the circumstance, then:

$$P(A) \cap B = P(A) \times P(B)$$

Total Probability Theorem

If $B_1, B_2, B_3 \dots B_k$ are partitions of S with $P(B_i) \neq 0$ & A is an arbitrary event of S , then

$$P(A) = \sum_{i=1}^k P(A|B_i) \times P(B_i)$$

1.4 Bayes' Theorem

Let $B_1, B_2, B_3 \dots B_k$ be events of S and are said to be partitions of S if:

- $\bigcup_{i=1}^k B_i = S$
- $B_i \cap B_j = \phi$

Bayes' Theorem:

If $B_1, B_2, B_3 \dots B_k$ are partitions of S with $P(B_i) \neq 0$ & A is an arbitrary event of S , then

$$P(B_i|A) = \frac{P(A|B_i) \times P(B_i)}{\sum_{i=1}^k P(A|B_i) \times P(B_i)}$$