1 Set Theory

A Set is a collection of well defined objects which is denoted by a capital letter and it's elements are described by small letters or numbers.

Types of Sets

- Universal Set (ξ or U)
- Null Set (ϕ)
- Subset ($\subset$)
- Superset ($\supset$)
- Compliment of a set (A^c or $\bar{A}$)
- Equal Sets ($=$)

Operations on Sets

- Union ($\cap$)
- Intersection ($\cup$)
- De Morgans
- Laws - Associative, Distributive

1.1 Random Experiments, Events and more

If the repetition of an experiment under identical condition results in different possible outcomes, then such an experiment is called Randome Experiment or Stochastic Experiment.

Sample Space (S) is a set of all possible outcomes of a random experiment.

Event (E) is a subset of Sample Space **S**

$\boxed{\text{Example}}$ Tossing of coin: $\mathbf{S} = \{\text{H,T}\}$

Types of Events

- Mutually Exclusive
- Equally Likely

1.2 Probability

Let **A** be an event of **S**. If **A** occurs m different ways out of a total of n, then probability of **A** is denoted by

$$P(A) = \frac{Favorable\ Cases}{Total\ Outcomes} = \frac{m}{n}$$

1.2.1 Kalmogorov's Axioms

Let **E** be an experiment with sample space **S**. Let **A** be an event of **S**, then:

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- Given A & B are mutually exclusive then, $P(A \cup B) = P(A) + P(B)$
- If $A_1, A_2, A_3 ... A_n$ are mutually exclusive then, $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

Theorem 1.1. *If A is an event of S then,*

i $P(\phi) = 0$

ii $P(A) + P(\bar{A}) = 1$

Proof. i) Let $A \cup \phi = \phi$

$$A \cap \phi = \phi \tag{1a}$$
$$P(A \cap \phi) = P(\phi)$$

$$A \cup \phi = \phi \tag{1b}$$
$$P(A \cup \phi) = P(\phi)$$

Using axiom from 1.2.1 & equation.(1b) we get,

$$P(A) + P(\phi) = P(A)$$

$$P(\phi) = 0$$

ii) Let $S = A \cup \bar{A}$

$$P(S) = P(A \cup \bar{A}) \qquad \text{[Mutually Exclusive]}$$

$$1 = P(A) + P(\bar{A})$$

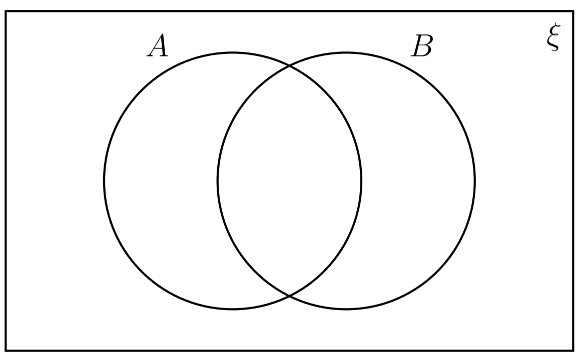
$$P(A) + P(\bar{A}) = 1$$

□

1.2.2 Addition Rule

If A & B are two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by addition rule.

Proof. Consider the following venn diagram having sets A and B.



$$A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$$

Consider, $\quad B = (A \cap B) \cup (\bar{A} \cap B)$

$$P(B) = P((A \cap B) \cup (\bar{A} \cap B)) \qquad \text{[Mutually Exclusive]}$$

$$P(B) = P(A \cap B) \cup P(\bar{A} \cap B) \tag{3a}$$

Consider, $\quad A = (A \cap B) \cup (A \cap \bar{B})$

$$P(B) = P((A \cap B) \cup (A \cap \bar{B})) \qquad \text{[Mutually Exclusive]}$$

$$P(B) = P(A \cap B) \cup P(A \cap \bar{B}) \tag{3b}$$

Thus from (3a) and (3b) we get,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$\square$