1 Probability

Let **A** be an event of **S**. If **A** occurs m different ways out of a total of n, then probability of **A** is denoted by

$$P(A) = \frac{Favorable\ Cases}{Total\ Outcomes} = \frac{m}{n}$$

1.1 Kalmogorov's Axioms

Let **E** be an experiment with sample space **S**. Let **A** be an event of **S**, then:

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- Given A & B are mutually exclusive then, $P(A \cup B) = P(A) + P(B)$
- If $A_1, A_2, A_3 ... A_n$ are mutually exclusive then, $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

Theorem 1.1. *If A is an event of S then,*

i $P(\phi) = 0$

ii $P(A) + P(\bar{A}) = 1$

Proof. i) Let $A \cup \phi = \phi$

$$A \cap \phi = \phi \tag{1a}$$
$$P(A \cap \phi) = P(\phi)$$

$$A \cup \phi = \phi \tag{1b}$$
$$P(A \cup \phi) = P(\phi)$$

Using axiom from 1.1 & equation.(1b) we get,

$$P(A) + P(\phi) = P(A)$$
$$P(\phi) = 0$$

ii) Let $S = A \cup \bar{A}$

$$P(S) = P(A \cup \bar{A}) \qquad \text{[Mutually Exclusive]}$$

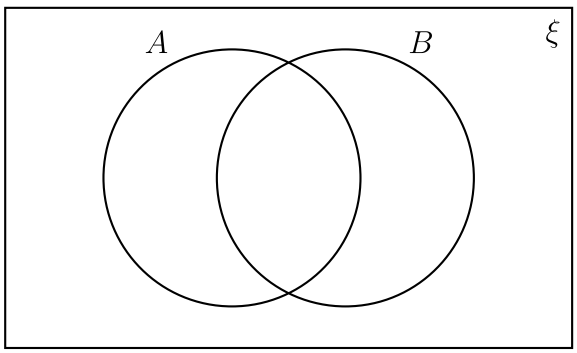
$$1 = P(A) + P(\bar{A})$$
$$P(A) + P(\bar{A}) = 1$$

$\square$

1.2 Addition Rule

If A & B are two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by addition rule.

Proof. Consider the following venn diagram having sets A and B.



$$A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$$

Consider, $\quad B = (A \cap B) \cup (\bar{A} \cap B)$

$$P(B) = P((A \cap B) \cup (\bar{A} \cap B)) \qquad \text{[Mutually Exclusive]}$$

$$P(B) = P(A \cap B) \cup P(\bar{A} \cap B) \tag{3a}$$

Consider, $\quad A = (A \cap B) \cup (A \cap \bar{B})$

$$P(B) = P((A \cap B) \cup (A \cap \bar{B})) \qquad \text{[Mutually Exclusive]}$$

$$P(B) = P(A \cap B) \cup P(A \cap \bar{B}) \tag{3b}$$

Thus from (3a) and (3b) we get,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$\square$

Generalised Addition Rule

If $A_1, A_2, A_3 \dots A_n$ are n events in a given sample space S.

$$\boxed{P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) \; - \; \sum_{i=1}^{n} P(A_i \bigcap A_j) \; \dots \; (-1)^n \, P(\bigcap \sum_{i=1}^{n} A_i)}$$

1.3 Condtional Probability

Conditional Probaility defines the probability of an event $\mathbb{A}$ under a given circumstance say $\mathbb{B}$ as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If the event is independent of the circumstance, then:

$$P(A) \cap B) = P(A) \times P(B)$$

Total Probability Theorem

If $B_1, B_2, B_3 \dots B_k$ are partitions of S with $P(B_i) \neq 0$ & A is an arbitrary event of S, then

$$\boxed{P(A) = \sum_{i=1}^{k} P(A|B_i) \times P(B_i)}$$

1.4 Bayes' Theorem

Let $B_1, B_2, B_3 \dots B_k$ be events of S and are said to be partitions of S if:

- $\bigcup_{i=1}^{k} B_i = S$
- $B_i \cup B_j = \phi$

Bayes' Theorem:

If $B_1, B_2, B_3 \ldots B_k$ are partitions of S with $P(B_i) \neq 0$ & A is an arbitrary event of S, then

$$P(B_i|A) = \frac{P(A|B_i) \times P(B_i)}{\sum_{i=1}^{k} P(A|B_i) \times P(B_i)}$$