

# Računski učinki

$$f\ x + g\ y$$

let  $z = f\ x$  in  
let  $w = g\ y$  in  
 $z + w$

Drobnozrnat. 'neučakani'  $\lambda$ -račun (fine-grained call-by-value)

vrednosti  $V ::= x \mid \underline{m} \mid \text{true} \mid \text{false} \mid \lambda x. M$

izračuni  $M, N ::= V_1 + V_2 \mid V_1 * V_2 \mid V_1 - V_2 \mid V_1 < V_2 \mid V_1 > V_2 \mid V_1 = V_2$   
if  $V$  then  $M_1$  else  $M_2 \mid V_1 V_2 \mid$   
let  $x = M$  in  $N \mid \text{return } V$

cbv  $(2 * 3) * 7 \rightsquigarrow 6 * 7 \rightsquigarrow 42$

fgcbv let  $x = 2 * 3$  in  $x * 7 \rightsquigarrow$  let  $x = \text{return } 6$  in  $x * 7 \rightsquigarrow 6 * 7 \rightsquigarrow \text{return } 42$

$$M \rightsquigarrow M'$$

(in podobno za  $+$ ,  $*$ ,  $-$ ,  $<$ ,  $>$ ,  $=$ )

$$\frac{}{\underline{m}_1 + \underline{m}_2 \rightsquigarrow \text{return } \underline{m}_1 + \underline{m}_2}$$

$$\frac{}{\text{if true then } M_1 \text{ else } M_2 \rightsquigarrow M_1}$$

$$\frac{}{\text{if false then } M_1 \text{ else } M_2 \rightsquigarrow M_2}$$

$$M \rightsquigarrow M'$$

$$\frac{}{(\lambda x. M) V \rightsquigarrow M[V/x]}$$

$$\frac{}{\text{let } x = M \text{ in } N \rightsquigarrow \text{let } x = M' \text{ in } N}$$

$$\frac{}{\text{let } x = \text{return } V \text{ in } N \rightsquigarrow N[V/x]}$$

$$\Gamma \vdash_v V : A$$

$$\Gamma \vdash_c M : A$$

$$V ::= x \mid \underline{m} \mid \text{true} \mid \text{false} \mid \lambda x. M$$

$$M, N ::= V_1 + V_2 \mid V_1 * V_2 \mid V_1 - V_2 \mid V_1 < V_2 \mid V_1 > V_2 \mid V_1 = V_2$$

$$\text{if } V \text{ then } M_1 \text{ else } M_2 \mid V_1 V_2 \mid$$

$$\text{let } x = M \text{ in } N \mid \text{return } V$$

$$\frac{(x:A) \in \Gamma}{\Gamma \vdash_v x : A}$$

$$\frac{\Gamma, x:A \vdash_c M : B}{\Gamma \vdash_v \lambda x. M : A \rightarrow B}$$

$$\frac{\Gamma \vdash_v V_1 : A \rightarrow B \quad \Gamma \vdash_v V_2 : A}{\Gamma \vdash_c V_1 V_2 : B}$$

$$\frac{\Gamma \vdash_c M : A \quad \Gamma, x:A \vdash_c N : B}{\Gamma \vdash_c \text{let } x = M \text{ in } N : B}$$

$$\frac{\Gamma \vdash_v V : A}{\Gamma \vdash_c \text{return } V : A}$$

Izrek o varnosti

Trditveni (napredni): Če velja  $\emptyset \vdash_c M : A$ , tedaj:  
- obstaja  $M'$ , da velja  $M \rightsquigarrow M'$ , ali  
- obstaja  $V$ , da je  $M = \text{return } V$

Trditveni (obvinitveni): Če velja  $\Gamma \vdash_c M : A$  in  $M \rightsquigarrow M'$ , tedaj velja  $\Gamma \vdash_c M' : A$ .

# Primeri učenikov

## Izjeme

$M ::= \dots \mid \text{raise } E \mid \text{try } M \text{ except } N$

$E_1 \rightarrow N_1 \mid E \rightarrow M_2$

$M \rightsquigarrow M'$

$\text{let } x = \text{raise } E \text{ in } M \rightsquigarrow \text{raise } E$

$\text{try } M \text{ except } N \rightsquigarrow \text{try } M' \text{ except } N$

$\Gamma \vdash_c \text{raise } E : A$

Alternativa

$\text{try } (\text{raise } E) \text{ except } N \rightsquigarrow N$

$\text{try } (\text{return } V) \text{ except } N \rightsquigarrow \text{return } V$

$\Gamma \vdash_c M : A \quad \Gamma \vdash_c N : A$

$\Gamma \vdash_c \text{try } M \text{ except } N : A$

$\Gamma \vdash_c \text{raise } E : 0$  in potem uporabimo abstrakcijo  $(\text{raise } E) : A$

## Izrek o varnosti

Napredek:  $\dots$

$\dots$  obstaja  $E$ , da je  $M = \text{raise } E$

Ohranitev:  $\dots$

## Delo s pomnilnikom

$M ::= \dots \mid !l \mid l := V$

$\Gamma \vdash_v V : \text{int}$

$\Gamma \vdash_c !l : \text{int}$

$\Gamma \vdash_c l := V : \text{int}$   
~~empty~~

$S, l := \underline{m} \rightsquigarrow S[l \mapsto \underline{m}], \text{return } \underline{m}$

C

$S, l := \underline{m} \rightsquigarrow S[l \mapsto \underline{m}], \text{return } ()$  OCamL

Ohranitev:

Napredek: Poskrbeti moramo, da so vse lokacije, ki jih beremo, inicializirane.

- lahko uporabimo privzete vrednosti
- lahko sledimo lokacijam v tipih  $\Gamma, L \vdash M : A$

$(l \mapsto m) \in S$

$S, !l \rightsquigarrow S, \text{return } \underline{n}$

$l \notin S$

$S, !l \rightsquigarrow S, \text{return } \underline{0}$

## Vhod / izhod

$M ::= \dots \mid \text{print } V \mid \text{read}$

$\Gamma \vdash_v V : \text{int}$

$\Gamma \vdash_c \text{print } V : \text{int}$

$\Gamma \vdash_c \text{read} : \text{int}$

$M, m_1 m_2 \dots m_k \rightsquigarrow M', m'_1 m'_2 \dots m'_k$

X ker ni omejitve glede tega, kaj lahko določimo z obstoječim zapisom

$M \rightsquigarrow M', m_1 m_2 \dots m_k$

X ne moremo kombinirati zapisov več korakov

$M \Downarrow V, m_1 \dots m_k$

✓ dela, ampak je semantika čisto drugačna.

$M \Downarrow V, m_1 \dots m_k \quad N[V/x] \Downarrow W, m'_1 \dots m'_m$

$\text{print } \underline{m} \Downarrow (), m$

$\text{return } V \Downarrow V, \emptyset$

$\text{let } x = M \text{ in } N \Downarrow W, m_1 \dots m_k m'_1 \dots m'_m$

$$M \rightsquigarrow M' \quad M \rightsquigarrow M'$$

$$\frac{}{\text{read } \rightsquigarrow \text{return } m}$$

$$M \stackrel{?m}{\rightsquigarrow} M'$$

$$\frac{}{\text{let } x = M \text{ in } N \rightsquigarrow \text{let } x = M' \text{ in } N}$$

Kako to skombiniramo skupaj s semantiko izhoda ??

## Nedeterminizem

$$M ::= \dots \mid \text{choose } M_1 M_2$$

$$\frac{\Gamma \vdash M_1 : A \quad \Gamma \vdash M_2 : A}{\Gamma \vdash \text{choose } M_1 M_2 : A}$$

$$\frac{}{\text{choose } M_1 M_2 \rightsquigarrow M_1}$$

$$\frac{}{\text{choose } M_1 M_2 \rightsquigarrow M_2}$$

$$M \Downarrow \{V_1, \dots, V_n\}$$

$$M_1 \Downarrow \{V_1, \dots, V_n\} \quad M_2 \Downarrow \{V'_1, \dots, V'_m\}$$

$$\frac{}{\text{choose } M_1 M_2 \Downarrow \{V_1, \dots, V_n, V'_1, \dots, V'_m\}}$$

$$\frac{}{\text{return } V \Downarrow \{V\}}$$

$$\frac{}{\underline{m}_1 + \underline{m}_2 \Downarrow \{\underline{m}_1 + \underline{m}_2\}}$$

$$\frac{M \Downarrow \mathcal{V} \quad (N[V/x] \Downarrow \mathcal{W}_V)_{V \in \mathcal{V}}}{\text{let } x = M \text{ in } N \Downarrow \bigcup_{V \in \mathcal{V}} \mathcal{W}_V}$$