

Lambda račun (λ -račun)

$$e ::= m \mid e_1 + e_2 \mid e_1 - e_2 \mid e_1 * e_2 \mid$$

$$\text{true} \mid \text{false} \mid e_1 = e_2 \mid e_1 < e_2 \mid e_1 > e_2 \mid$$

$$\text{if } e \text{ then } e_1 \text{ else } e_2 \mid \lambda x. e \mid x \mid e_1 e_2$$

true - false
 true * 10
 if 2=true then ... else ...

$\frac{\text{fun } x \rightarrow e}{\lambda x. e}$
 $\frac{x \mapsto e}{e_1 e_2}$
 $\frac{\text{abstrakcija}}{\text{aplikacija}}$

 $n ::= m \mid \text{true} \mid \text{false} \mid \lambda x. e$

$e \Downarrow n \dots$ pravilno se evalvira v vrednost n

$$\frac{}{m \Downarrow m} \quad \frac{e_1 \Downarrow m_1 \quad e_2 \Downarrow m_2}{e_1 + e_2 \Downarrow m_1 + m_2} \quad \text{podobno za } -, *, =, <, > \quad \frac{\text{true} \Downarrow \text{true}}{\text{false} \Downarrow \text{false}}$$

$$\frac{e \Downarrow \text{true} \quad e_1 \Downarrow n}{\text{if } e \text{ then } e_1 \text{ else } e_2 \Downarrow n} \quad \frac{e \Downarrow \text{false} \quad e_2 \Downarrow n}{\text{if } e \text{ then } e_1 \text{ else } e_2 \Downarrow n} \quad \frac{}{\lambda x. e \Downarrow \lambda x. e}$$

~~$x \in ?$~~
 ni ustreznega
 povezave

$e_1 \Downarrow \lambda x. e \quad e_2 \Downarrow n$
 $\frac{\text{nečakana izvajenje}}{\text{call-by-value}}$
 $\frac{\text{substitucija}}{e[n/x] \Downarrow n'}$

$e_1 e_2 \Downarrow n'$
 ↑
 // e v katerem smislu
 vsi pojavitve x
 zamenjali z n

$$\frac{\lambda x. \lambda y. x > y \Downarrow \lambda x. \lambda y. x > y}{(\lambda x. \lambda y. x > y)(1+2) \Downarrow \lambda y. 3 > y} \quad \frac{\overline{1+2} \quad \overline{2+2}}{1+2 \Downarrow 3} \quad \frac{\lambda y. 3 > y \Downarrow \lambda y. 3 > y}{3 > 3 \Downarrow \text{false}} \quad \frac{}{3 \Downarrow 3} \quad \frac{\overline{3+3} \quad \overline{3+3}}{3 > 3 \Downarrow \text{false}}$$

$$\frac{}{((\lambda x. \lambda y. x > y)(1+2)) 3 \Downarrow \text{false}}$$

Možne težave pri substituciji:

$$\lambda x. \lambda x. x = \lambda z. \lambda y. y$$

$$(\lambda x. \lambda x. x) 10 = \cancel{\lambda 10. 10}$$

$$= \cancel{\lambda x. 10}$$

$$= \lambda x. x$$

Najemo zamanjati vseh spremenljivk,
temveč le proste

$$(\lambda x. y+x) [x/y] = ?$$

$$(\lambda y. \lambda x. y+x) x = \lambda x. x + x \quad //$$

ob substituciji,
smo ponesrečni
vezali prost
spremenljivko x

$$(\lambda y. \lambda z. y+z) x = \lambda y. y+x \quad \checkmark$$

$$\int_1^2 x dx = \int_1^2 t dt \quad \text{vezane}$$

$$\int_1^2 x dx = \int_1^2 x dx \quad \text{proste}$$

$$\sum_{i=a}^b i = \sum_{j=a}^b j \neq \sum_{l=a}^c l$$

$$e \rightsquigarrow e' \text{ izcat } e \text{ naredi korak } \sim e' \quad e ::= m \mid e_1 + e_2 \mid e_1 - e_2 \mid e_1 * e_2 \mid \\ \text{true} \mid \text{false} \mid e_1 = e_2 \mid e_1 < e_2 \mid e_1 > e_2 \mid \\ \text{if } e \text{ then } e_1 \text{ else } e_2 \mid \lambda x. e \mid x \mid e_1 e_2$$

$$\frac{e_1 \rightsquigarrow e'_1}{e_1 + e_2 \rightsquigarrow e'_1 + e_2}$$

$$\frac{e_2 \rightsquigarrow e'_2}{m_1 + e_2 \rightsquigarrow m_1 + e'_2}$$

$$\frac{m_1 + m_2 \rightsquigarrow m_1 + m_2}{+ \text{ iz sintakse} \qquad + \text{ na } \mathbb{Z}}$$

Podobno
za $-$, $*$, $=$, $<$, $>$

$$\begin{array}{c} (1+2)+(3+4) \\ 3+(3+4) \quad (1+2)+7 \\ 3+7 \end{array}$$

$$\frac{e \rightsquigarrow e'}{\text{if } e \text{ then } e_1 \text{ else } e_2 \rightsquigarrow \text{if } e' \text{ then } e_1 \text{ else } e_2}$$

$$\frac{}{\text{if true then } e_1 \text{ else } e_2 \rightsquigarrow e_1}$$

$$\frac{}{\text{if false then } e_1 \text{ else } e_2 \rightsquigarrow e_2}$$

$$\frac{e_1 \rightsquigarrow e'_1}{e_1 e_2 \rightsquigarrow e'_1 e_2}$$

$$\frac{e_2 \rightsquigarrow e'_2}{(\lambda x. e)e_2 \rightsquigarrow (\lambda x. e)e'_2}$$

$$\frac{}{(\lambda x. e)v \rightsquigarrow e[v/x]}$$

$$1+2 \rightsquigarrow 3$$

$$\frac{\frac{(x. \lambda y. x > y)(1+2) \rightsquigarrow (\lambda x. \lambda y. x > y)3}{((\lambda x. \lambda y. x > y)(1+2))3 \rightsquigarrow ((\lambda x. \lambda y. x > y)3)3}}{}$$

$$e ::= \dots \mid \text{rec } f x . e$$

$$n ::= \dots \mid \text{rec } f x . e$$

$$\frac{}{\text{rec } f x . e \Downarrow \text{rec } f x . e}$$

$$\frac{e_1 \Downarrow \text{rec } f x . e \quad e_2 \Downarrow n \quad e[n/x, (\text{rec } f x . e)/f] \Downarrow n'}{e_1 e_2 \Downarrow n'}$$

$$\frac{e_2 \rightsquigarrow e'_2}{(\text{rec } f x . e) e_2 \rightsquigarrow (\text{rec } f x . e) e'_2}$$

$$\frac{}{(\text{rec } f x . e) n \rightsquigarrow e[n/x, (\text{rec } f x . e)/f]}$$

Kaj želimo? $e \Downarrow n \Leftrightarrow \exists e_i \dots e = e_0 \rightsquigarrow e_1 \rightsquigarrow e_2 \rightsquigarrow \dots \rightsquigarrow e_{n-1} \rightsquigarrow e_n = n$

Definirajmo pomožno relacijo \rightsquigarrow^*

$$\frac{}{e \rightsquigarrow^* e} \quad \frac{e_1 \rightsquigarrow e_2 \quad e_2 \rightsquigarrow^* e_3}{e_1 \rightsquigarrow^* e_3}$$

Kako je videti indukcija na \rightsquigarrow^* ?

Če velje $\forall e. \Phi(e, e)$ in $\forall e_1, e_2, e_3. e_1 \rightsquigarrow e_2 \wedge \Phi((e_2, e_3)) \Rightarrow \Phi((e_1, e_3))$

$$\frac{}{e R e} \quad \frac{e_1 \rightsquigarrow e_2 \quad e_2 R e_3}{e_1 R e_3} \quad \boxed{\begin{array}{l} \text{Potem velja} \\ \forall e_1, e_2. e_1 \rightsquigarrow^* e_2 \Rightarrow \Phi((e_1, e_2)) \end{array}}$$

Lema 3 Če $e \rightsquigarrow e'$ in $e' \Downarrow n$, potem $e \Downarrow n$.

Posledica 4 Če $e \rightsquigarrow^* n$, potem $e \Downarrow n$

Dokaz posledice 4

Indukcija na $e \rightsquigarrow^* n$.

$$\Phi((e_1, e_2)) := \exists n. e_2 = n \Rightarrow e_1 \Downarrow n$$

• $n \rightsquigarrow^* n$. Ker je n vrednost, velja $n \Downarrow n$.

• $\frac{e \rightsquigarrow e' \quad e' \rightsquigarrow^* n}{e \rightsquigarrow^* n}$. Po ind. predp. velja $e' \Downarrow n$. Po lemi 3 dobimo $e \Downarrow n$. \blacksquare

Dokaz leme 3.

Indukcija na $e \rightarrow e'$:

- $\frac{e_0 \rightarrow e'_0}{\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \rightsquigarrow \text{if } e'_0 \text{ then } e_1 \text{ else } e_2}$. Kako smo dobili $e' \Downarrow N$?
Zašto je $\frac{\text{if } e'_0 \text{ then } e_1 \text{ else } e_2 \Downarrow N}{\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \Downarrow N}$ dobimo samo na dva načina.
 - $\frac{\frac{e'_0 \Downarrow \text{true } e_1 \Downarrow N}{\dots}}{\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \Downarrow N}$. Po IP. velja $e_0 \Downarrow \text{true}$, zato
- $\frac{e_0 \Downarrow \text{true } e_1 \Downarrow N}{\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \Downarrow N}$.
- za false podobno.
- $\frac{\text{if true then } e_1 \text{ else } e_2 \rightsquigarrow e_1}{\text{if true then } e_1 \text{ else } e_2 \Downarrow N}$ in $e_1 \Downarrow N$. Ker velja $\frac{\text{true} \Downarrow \text{true}}{\text{true} \Downarrow \text{true}}$,
velja tudi $\frac{\text{true} \Downarrow \text{true } e_1 \Downarrow N}{\text{if true then } e_1 \text{ else } e_2 \Downarrow N}$
- podobno za false.

