λ-račun

abstrakcije

$M, N ::= x \mid \lambda x. M \mid M N \mid \underline{m} \mid M + N \mid \ldots$

$\mid$ if M then N_1 else N_2 $\mid \ldots$

spremenljivke — x je vezan v M — aplikacije

fun x -> x + 2
lambda x: x+2
\ x -> x + 2
$x \mapsto x + 2$
$\lambda x . x + 2$
^ x . x+2
$\hat{x} . x+2$

Primeri

$\lambda x . x$ .... identiteta

$\lambda x . y$ ... konstanta y

$\lambda f . \lambda g . \lambda x . f(g x)$ .... kompozitum

$((\lambda x.(\lambda y. x+y)) 2) 3$

$(\lambda x . x x)(\lambda y . y y)$ ... Ω

$\lambda f . (\lambda x . f(xx))(\lambda x . f(xx))$ ... fiksna točka f – Y-kombinator

Definicije

Substitucija $M[N/x]$ je izraz M, v katerem smo vse proste pojavitve spremenljivke x zamenjali z N.

$x[N/x] := N$

$y[N/x] := y \quad (y \neq x)$

$(\lambda y . M)[N/x] := \lambda y . M[N/x] \quad (y \neq x,\ y$ ni prost v $N)$

po potrebi preimenujemo y

$(M_1 M_2)[N/x] := (M_1[N/x])(M_2[N/x])$

~~$(\lambda y . x)[y/x] = \lambda x . x$~~

Operacijska semantika

$M \rightsquigarrow M'$

$$\frac{M \rightsquigarrow M'}{M N \rightsquigarrow M' N}$$

LENO IZVAJANJE

$$\frac{}{(\lambda x . M) N \rightsquigarrow M[N/x]}$$

$(\lambda x . x+x)(1+2)$
$\rightsquigarrow (1+2) + (1+2)$
$\rightsquigarrow 3 + (1+2)$
$\rightsquigarrow 3+3 \rightsquigarrow 6$

LAZY EVALUATION

CBN

CALL-BY-NAME

NEUČAKANO IZVAJANJE

$$\frac{N \rightsquigarrow N'}{(\lambda x . M) N \rightsquigarrow (\lambda x . M) N'}$$

$(\lambda x . x+x)(1+2)$
$\rightsquigarrow (\lambda x . x+x) 3$
$\rightsquigarrow 3+3 \rightsquigarrow 6$

$$\frac{}{(\lambda x . M) V \rightsquigarrow M[V/x]}$$

$V ::= \cancel{x} \mid \lambda x . M \mid \underline{m}$

CBV

EAGER EVALUATION

CALL-BY-VALUE

Churchevo kodiranje

$\underline{0} ::= \lambda f. \lambda x. x$

$\underline{1} ::= \lambda f. \lambda x. f x$

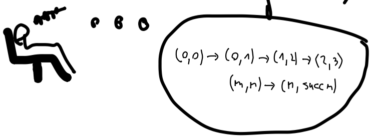
$\vdots$

$\underline{m} ::= \lambda f. \lambda x. \underbrace{f(\cdots(f x)\cdots)}_{m}$

$succ ::= \lambda m. \lambda f. \lambda x. f(m f x)$

$plus ::= \lambda m. \lambda m. \lambda f. \lambda x. m f (m f x)$

$times ::= \lambda m. \lambda n. \lambda f. \lambda x. m (m f) x$

$pred ::=$ 

$isZero ::= \lambda n. n(\lambda x. ff)\, tt$

$let\ x = M\ in\ N ::= (\lambda x. N) M$

$tt ::= \lambda x. \lambda y. x$

$ff ::= \lambda x. \lambda y. y$

$if\text{-}then\text{-}else ::= \lambda b. \lambda x. \lambda y. (b\, x)\, y$

$= \lambda b. \lambda x. b x$

$= \lambda b. b$

$pair ::= \lambda x. \lambda y. \lambda p. (p\, x)\, y$

$fst ::= \lambda g. g (\lambda x. \lambda y. x)$

$snd ::= \lambda g. g (\lambda x. \lambda y. y)$

$fact = fun\ n \to if\ n = 0\ then\ 1\ else\ n * fact\ (n-1)$

$\underbrace{(fun\ f \to fun\ n \to if\ n=0\ then\ 1\ else\ n * f(n-1))}_{\Psi}\ fact$

$\Psi(\lambda x. 0) = fun\ n \to if\ n=0\ then\ 1\ else\ n * 0$

$\Psi(fact) = fact$

$(\lambda f. (\lambda x. \cancel{f} (x x)) (\lambda x. f (x x)))\ \Psi$

$\rightsquigarrow (\lambda x. \cancel{\Psi} (x x)) (\lambda x. \Psi (x x))$

$\rightsquigarrow \cancel{\Psi} ((\lambda x. \Psi (x x)) (\lambda x. \Psi (x x)))$

$\overset{?}{\rightsquigarrow} \cancel{\Psi} (\Psi (\lambda x. \Psi (x x)) (\lambda x. \Psi (x x)))$

$f = \dots$

$(\Psi f)(0) = 0!$

$(\Psi(\Psi f))(0) = 0!$ $\qquad (\Psi(\Psi f))(1) = 1\cdot((\Psi f)(0)) = 1\cdot 0! = 1!$

$(\Psi^n f)(0) = 0!$ $\quad (\Psi^n f)(1) = 1!$ $\quad \dots \quad (\Psi^n f)(n-1) = (n-1)!$

<u>Primer</u> $\quad \Psi = \lambda f.\ \lambda n.\ \text{if } n=0 \text{ then } 1 \text{ else } n\cdot f(n-1)$

$$
\begin{aligned}
Y\,\Psi &\rightsquigarrow \Psi\,((\lambda x.\,\Psi(xx))(\lambda x.\,\Psi(xx))) \\
&= (\lambda f.\ \lambda n\ \dots)((\lambda x.\,\Psi(xx))(\lambda x.\,\Psi(xx))) \\
&\rightsquigarrow (\lambda f.\,\lambda n \dots)(\Psi(\lambda x.\,\Psi(xx))(\lambda x.\,\Psi(xx)))
\end{aligned}
$$

$$Y' = \lambda f.\,(\lambda x.\ xx)(\lambda x.\lambda y.f(xx)y)$$

$$
\begin{aligned}
Y'\,\Psi &\rightsquigarrow (\lambda x.\,xx)(\underline{\lambda x.\lambda y.\,\Psi(xx)y}) \\
&\rightsquigarrow (\underline{\lambda x.\lambda y.\,\Psi(xx)y})(\underline{\lambda x.\lambda y.\,\Psi(xx)y}) \\
&\rightsquigarrow \underline{\lambda y.\,\Psi}((\underline{\lambda x.\lambda y.\Psi(xx)})(\underline{\lambda x.\lambda y.\Psi(xx)})\,y
\end{aligned}
$$

$$
\begin{aligned}
(Y'\,\Psi_{fact})\,3 &\rightsquigarrow \Psi_{fact}((\underline{\dots})(\underline{\dots}))\,3 \\
&\rightsquigarrow \Psi_{fact}(\underline{\lambda y.\,\Psi}(\underline{\dots})(\underline{\dots}))\,3 \\
&\rightsquigarrow \text{if } 3=0 \text{ then } 1 \text{ else } 3\cdot(\underline{\lambda y.\Psi}(\underline{\dots})(\underline{\dots}))(3-1) \\
&\rightsquigarrow \dots \rightsquigarrow 3\cdot(\lambda y.\,\Psi(\dots)(\dots))(2)
\end{aligned}
$$

α-ekvivalenca: $\quad \lambda x.M = \lambda y.\,M[y/x]$

β-redukcija: $\quad (\lambda x.M)N \rightsquigarrow M[N/x]$ $\qquad fst\,(M_1, M_2) \rightsquigarrow M_1$

β-ekvivalenca: $\quad (\lambda x.M)N = M[N/x]$ $\qquad fst(M_1, M_2) = M_1$

η-ekvivalenca: $\quad \lambda x.Mx = M$ $\qquad (fst\,M, snd\,M) = M$