东 南 大 学 考 试 卷

课程名称 高等数学AB（上）期中 考试学期 14-15-2 得分 _____

适用专业 双语班 考试形式 闭 卷 考试时间长度 120 分钟

题号	一	二	三	四	五	六	七
得分							
评阅人							

一、 填空题（本题共8小题，每小题4分，满分32分）

1. Let $y = x^5 e^{-x}$, then y is increasing on_____.

2. Let $y = \log_x e (x > 0, x \neq 1)$, then $dy =$ _____.

3. Suppose that f is continuous on $(-1, 1)$, then $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1 + f(x)\sin x} - 1}{3^x - 1} =$ _____.

4. The discontinuity points of $f(x) = \lim\limits_{n\to\infty} \dfrac{1 + x}{1 + x^{2n}}$ is $x =$ _____, and it is _____ discontinuity.

5. If $x =$ _____, then the tangent line to the curve $y = \arcsin \dfrac{x}{2}$ is perpendicular to the tangent line to $y = x^2$.

6. Let $\{a_n\}$ be the sequence with $\lim\limits_{n\to\infty} \dfrac{a_1 + a_2 + \cdots + a_n}{n} = a (a \text{ is finite})$, then $\lim\limits_{n\to\infty} \dfrac{a_n}{n} =$ ____.

7. If $y = \dfrac{x^2}{1 - x}$, then $y^{(5)}(x) =$ _____.

8. Let $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then $f''(0) =$ _____.

二、 计算下列各题（本题共4小题，每小题8分，满分32分）

1. Let $f(x) = a^x + x^a + x^x + a^a$ (where $x > 0, x \neq 1$, and $a > 0, a \neq 1$ is a constant), find $f'(x)$.

2. Let $y = y(x)$ be determined by the parametric equations $\begin{cases} x = 2 + t^2 \\ y = t \sin t \end{cases}$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

3. Suppose that $y = y(x)$ is determined by $y = x \ln(x^2 + y^2)$, and it has second order derivative, and $y(1) = 0$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ 及 $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}|_{x=1}$.

4. Find $\lim\limits_{x \to +\infty} x^{\frac{7}{4}}(\sqrt[4]{x+1} + \sqrt[4]{x-1} - 2\sqrt[4]{x})$.

三、 （本题满分7分） Let $f(x) = |x^2 - 4| \ln(3 + x)$.

(1) Discuss the differentiability of f on its domain;

(2)Find $f'(x)$ at the points where f is differentiable.

四、 （本题满分7分） Find the Maclaurin polynomial of $f(x) = \arcsin x$ with Peano reminder of order 5.

五、 （本题满分7分） Show that

$$1 + x \ln(x + \sqrt{1 + x^2}) > \sqrt{1 + x^2}, \quad x > 0$$

六、 （本题满分7分） Let $a_1 = 2, a_n = 2 - \dfrac{1}{a_{n-1}^2}, n = 2, 3, \cdots$. Show that $\{a_n\}$ converges, and find $\lim\limits_{n\to\infty} a_n$.

七、 （本题满分8分）

1. Show that the equation $x^n + nx - 2 = 0$ (n is positive integer) has only one positive real root a_n;

2. Calculate $\lim\limits_{n\to\infty} (1 + a_n)^n$.