Let X_1 and X_2 be two independent Gaussian random variables, with mean μ_1, μ_2 and standard deviation $\sigma_1^2 = 2\mu_1$, $\sigma_2^2 = 2\mu_2$. Let $Y = 3X_1 + X_2$, the mean of Y is $\mu_Y = 6$, the variance of Y is $\sigma_Y^2 = 24$. Then, the parameter μ_1=______, σ_2^2=______, and the probability density function of X_1 is $f_{X1}(x)$=________________.

$$\sigma_x^2 = D[X] = E[X - E(X)]^2$$

Mean and variance of Gaussian random variable (1.3)

$$D[aX] = E[aX - E(aX)]^2 = a^2 E[X - E(X)] = a^2 D[X]$$

The random process $X(t)$ is defined by $X(t) = A cos(2\pi f_0 t + \theta)$, where two independent random variables A and θ are uniformly distributed on [-1, 1] and [0, π]. Then, the autocorrelation function of $X(t)$ is ____________ and its power spectral density is $S_x(f)$=_______.

Mean, autocorrelation and power spectral density (1.3, 1.5 & 1.12)

The autocorrelation function of a wide-sense stationary random process $X(t)$ is $\sin^2(4\pi f_c t) + 3\delta(t)$, then its power spectral density is ____________________________.

Autocorrelation and power spectral density (1.12)

A noise signal $n_i(t)$ with PSD, $S_n(f) = K$, $|f| \leq B$, is applied at the input of an ideal differentiator (dx/dt). The PSD of the output noise signal $n_o(t) = dn_i(t)/dt$ is ______ and the average power of the $n_o(t)$ is ______.

Output power spectral density of a filter (1.15)

In a FM system, the highest frequency component of modulating signal is f_H=15k(Hz), and the maximum frequency deviation is Δf=____(Hz). Using Carson's rule, the bandwidth of the FM signal is approximately B=180k (Hz).

Bandwidth of FM using Carson's rule

For the DSB-SC signal $m(t)\cos(20000\pi t)$ and the baseband signal $m(t)=\cos(2000\pi t)+\cos(6000\pi t)$, the frequency range of the USB spectra is [Hz ~ Hz], and frequency range of the LSB spectra is [Hz~ Hz].

Spectrum of DSB-SC and Sideband overlap (2.8&2.15)

A random process $X(t)=A\cos(2\pi f_c t)$ in which the frequency f_c is constant and the amplitude A is uniformly distributed over the interval [0, 1] is stationary. ( )

wide-sense stationary process (1.1)

An FM signal is modulated by $m(t)=\sin(2000\pi t)$, $k_f=1\times 10^5$. The bandwidth B of the baseband signal m(t) is _____ Hz, the frequency deviation Δf of the modulated signal is _____ Hz, and the modulation index β is _____.

Modulation index and frequency deviation of FM signal (2.32)

A narrowband signal is $X(t) = A\cos(2\pi f_c t + m(t))$, where f_c is carrier frequency and f_c = 10 MHz, the average power of the narrowband signal is P=18, the quadrature component of X(t) is $A\sin(2\pi t)$. Then, the amplitude value A=_____, the in-phase component of X(t) is _______, and the signal m(t) = ______.

Narrowband signal (1.28)

An single tone frequency modulation (FM) signal with carrier frequency $f_c = 1(MHz)$ is described by the equation $s(t)=A\cos(2\pi f_c t + 8\sin(2f_m\pi t))$. The power of the modulated signal is 800, the amplitude value A= _____ , the single-tone frequency f_m= ____(Hz) , the frequency deviation Δf is ____(Hz), the modulation index β is ____, and the approximate value of the transmission bandwidth B_T is 4500(Hz).

Modulation index and frequency deviation of single tone FM signal (2.28&2.32)

The random process $X(t)=A\cos(2\pi ft+\phi)$, is not a wide-sense stationary process, where f is a constant and A and ϕ are independent random variables uniformly distributed in the ranges (-2, 2) and $(0, \pi)$, respectively.

wide-sense stationary process (1.1)

An single tone FM signal with carrier frequency f_c=1MHz is described by the equation $s(t)=50\cos(2\pi f_c t+5\sin(2000\pi t))$. The frequency deviation Δf is______(Hz) , the modulation index β is___, and the approximate value of the transmission bandwidth B_T is 12000Hz

Modulation index and frequency deviation of single tone FM signal (2.28&2.32)

An angel modulated signal with carrier frequency $f_c=10^5$ is described by $s(t)=10\cos(2\pi f_c t+5\sin 1000\pi t+10\sin 2000\pi t)$,

a) Find the power P of the modulated signal;

b) Find the frequency deviation Δf of the signal;

c) Find the deviation ratio β;

d) Estimate the bandwidth of s(t), by Carson' rule.

Modulation index and frequency deviation of multiple tones FM signal

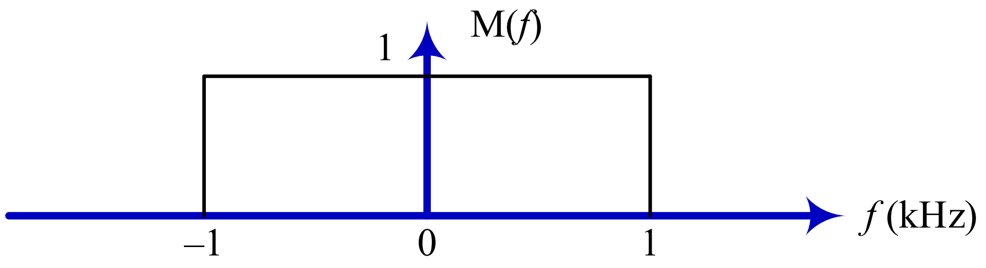
Given a random process $X(t) = k$, where k is a random variable uniformly distributed in the range (-1, 1). This process is wide-sense stationary and ergodic.

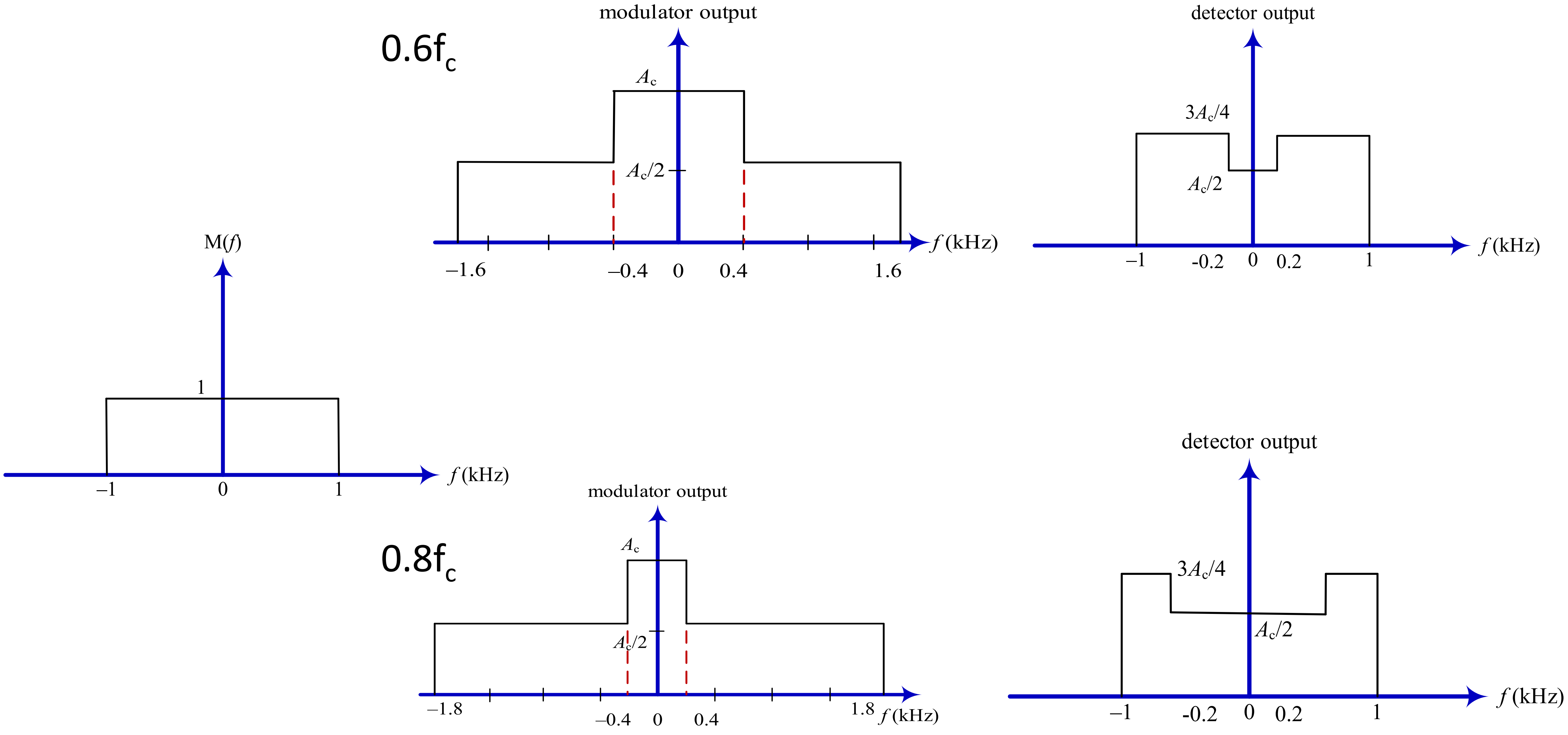
wide-sense stationary process and ergodic (1.1)

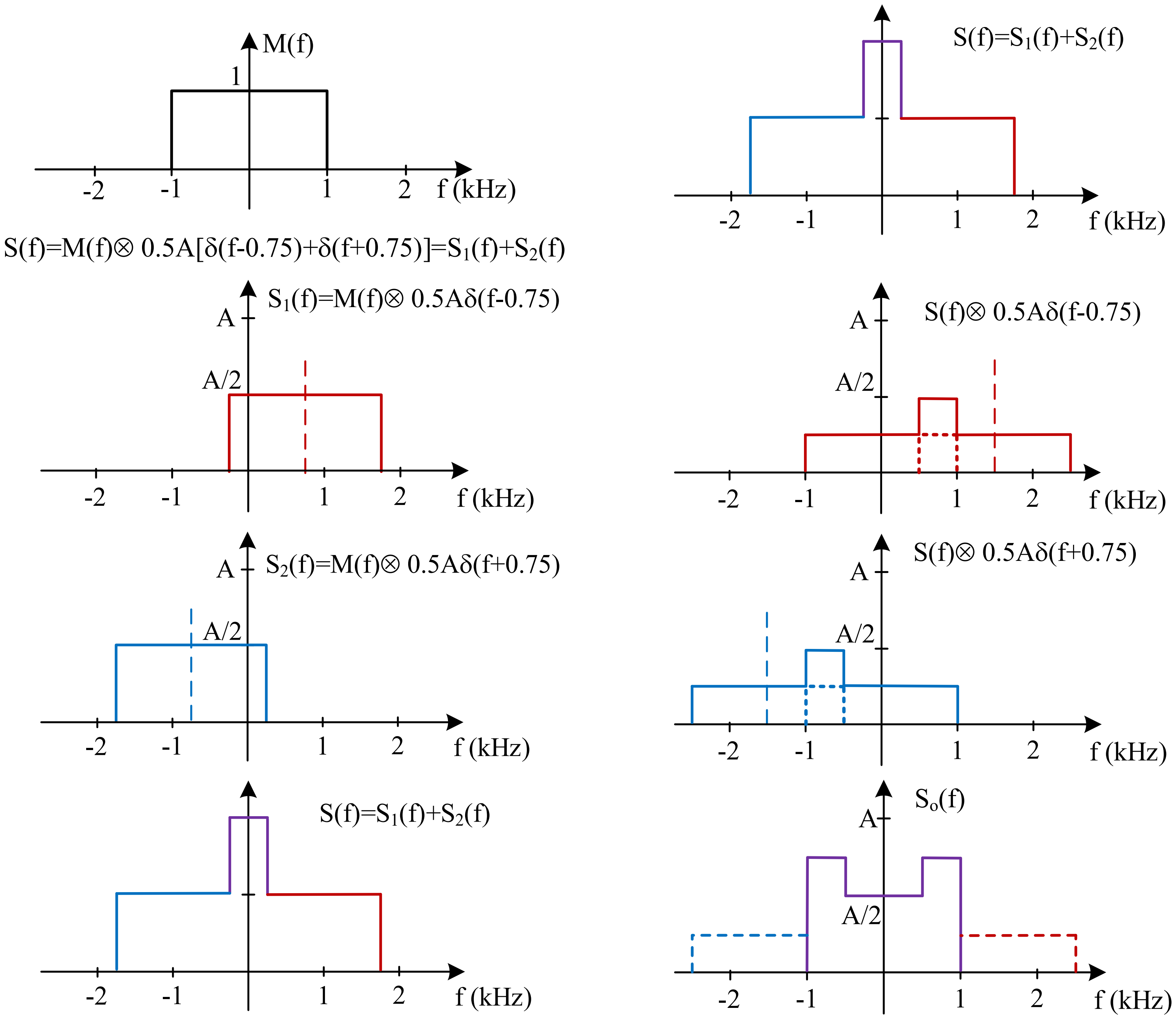
Spectrum of a message signal m(t) is shown in the following Figure. This message signal is DSB-SC modulated with a carrier wave $A_c\cos(2\pi f_c t)$. The modulated signal is next applied to a coherent detector with a carrier wave of unit amplitude.

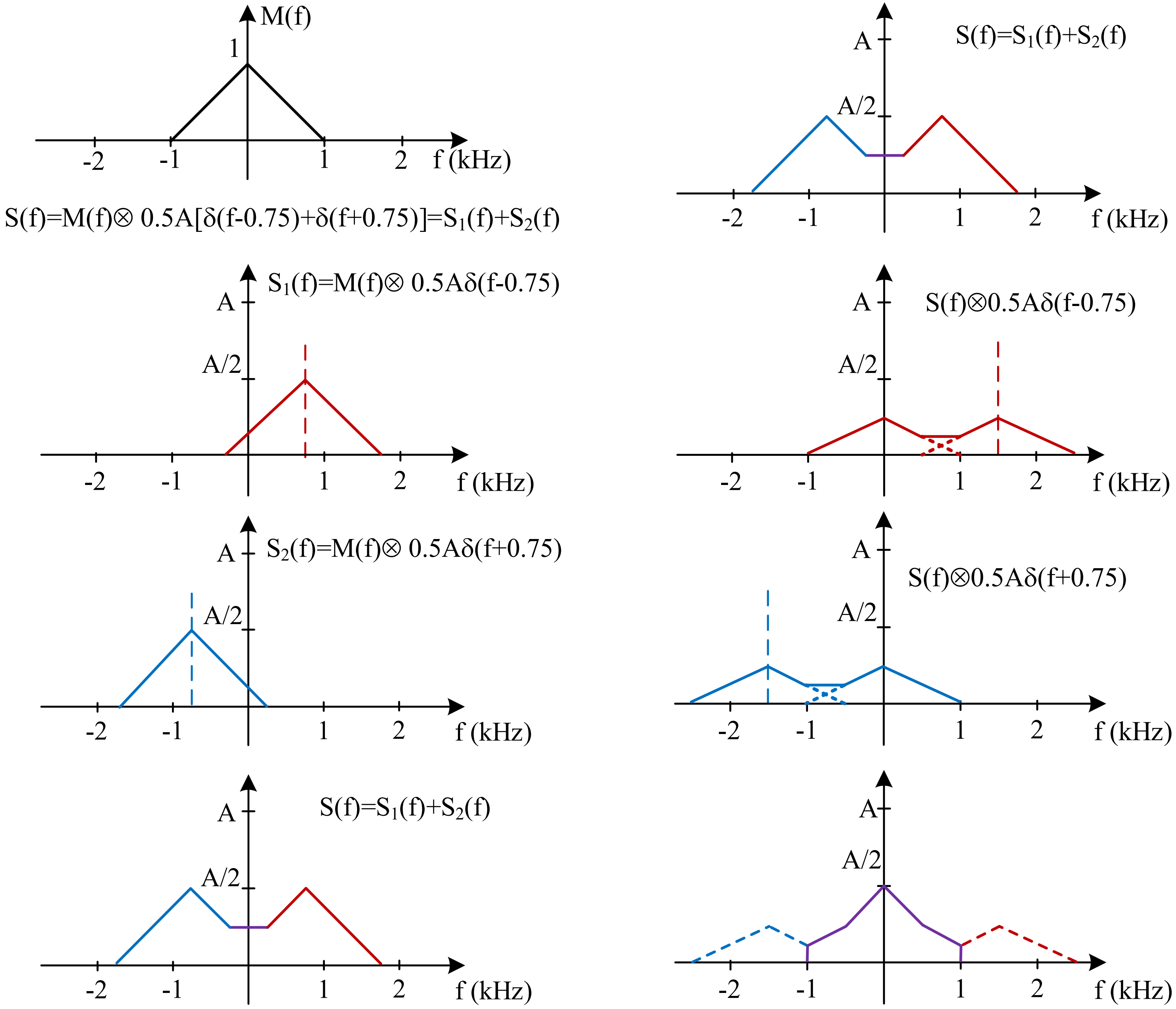
- ☐ If $0.6f_c$ and $0.8f_c$, plot the spectrum of the modulator output and the detector output;
- ☐ What is the lowest value of fc that keeps the DSB-SC modulation from sideband overlap?

Spectrum of DSB-SC and Sideband overlap (2.8)




0.6f_c
modulator output
A_c
A_c/2
−1.6
−0.4
0
0.4
1.6
f (kHz)
detector output
3A_c/4
A_c/2
−1
-0.2
0
0.2
1
f (kHz)
M(f)
1
−1
0
1
f (kHz)
0.8f_c
modulator output
A_c
A_c/2
−1.8
−0.4
0
0.4
1.8
f (kHz)
detector output
3A_c/4
A_c/2
−1
-0.2
0
0.2
1
f (kHz)


M(f)
1
-2
-1
1
2
f (kHz)
S(f)=M(f)⊗ 0.5A[δ(f-0.75)+δ(f+0.75)]=S1(f)+S2(f)
S1(f)=M(f)⊗ 0.5Aδ(f-0.75)
A
A/2
S2(f)=M(f)⊗ 0.5Aδ(f+0.75)
S(f)=S1(f)+S2(f)
S(f)⊗ 0.5Aδ(f-0.75)
S(f)⊗ 0.5Aδ(f+0.75)
So(f)


M(f)
1
-2
-1
1
2
f (kHz)
S(f)=M(f)⊗ 0.5A[δ(f-0.75)+δ(f+0.75)]=S1(f)+S2(f)
S1(f)=M(f)⊗ 0.5Aδ(f-0.75)
A
A/2
f (kHz)
S2(f)=M(f)⊗ 0.5Aδ(f+0.75)
A
A/2
f (kHz)
S(f)=S1(f)+S2(f)
A
A/2
f (kHz)
S(f)=S1(f)+S2(f)
A
A/2
f (kHz)
S(f)⊗0.5Aδ(f-0.75)
A
A/2
f (kHz)
S(f)⊗0.5Aδ(f+0.75)
A
A/2
f (kHz)
A
A/2
f (kHz)

A carrier wave of frequency 1 MHz is frequency modulated by a message signal $m(t) = 2\cos(30\pi t)$ (V), the frequency sensitivity of the modulator is 10 Hz/V, by using Carson's rule, bandwidth of the modulated wave is _____Hz.

Modulation index and frequency deviation of single tone FM signal (2.28&2.32)

If the bandwidth of a signal g(t) is B Hz, then bandwidth of the signal $y(t) = a_0 + a_1 g(t) + a_2 g^2(t) + \ldots + a_k g^k(t)$ is $4kB$ Hz. ????

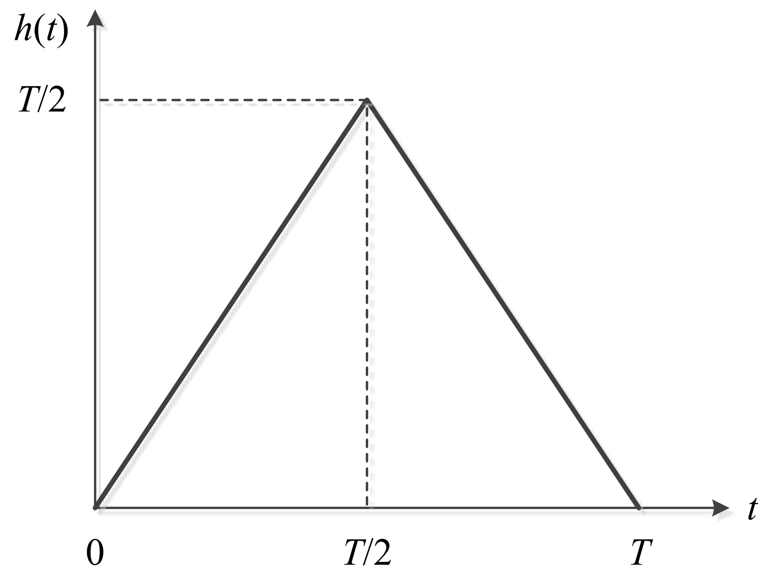
If the bandwidth of a signal $y(t)$ is $n \cdot a_n \cdot B$ Hz, $y(t)=a_0+a_1 g(t)+\ldots+a_n g^n(t)$, then bandwidth of the signal $g(t)$ is B Hz. ????

Nonlinear effect

A stationary Gaussian process $X(t)$ with zero mean and power spectral density $S_X(f)$ is applied to a linear filter whose impulse response $h(t)$ is show in Fig. C-1. A sample Y is taken of the random process at the filter output at time T.

Determine the mean and variance of Y.

What is the probability density function of Y?



$$H(f)=\int_{-\infty}^{\infty} h(t)\exp(-j2\pi ft)\,dt=\frac{T}{2}\mathrm{sinc}^2\left(\frac{fT}{2}\right)\exp(-j\pi fT)$$

$$Y(t)=\int_{-\infty}^{\infty} h(\tau)X(t-\tau)\,d\tau=\int_0^{\frac{T}{2}}\tau X(t-\tau)\,d\tau+\int_{\frac{T}{2}}^{T}(T-\tau)X(t-\tau)\,d\tau$$

$$Y=\int_0^{\frac{T}{2}}\tau X(T-\tau)\,d\tau+\int_{\frac{T}{2}}^{T}(T-\tau)X(T-\tau)\,d\tau$$

$$E[Y]=E\left[\int_0^{\frac{T}{2}}\tau X(T-\tau)\,d\tau+\int_{\frac{T}{2}}^{T}(T-\tau)X(T-\tau)\,d\tau\right]$$

$$=\int_0^{\frac{T}{2}}\tau E\left[X(T-\tau)\right]d\tau+\int_{\frac{T}{2}}^{T}(T-\tau)E\left[X(T-\tau)\right]d\tau=0$$

PDF of filter output (1.3)

$$f_Y(y)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma_Y^2}\right)$$

$$\sigma_Y^2=E\left[Y^2\right]-\{E[Y]\}^2=R_Y(0)=\int_{-\infty}^{\infty}S_Y(f)\,df=\int_{-\infty}^{\infty}S_X(f)|H(f)|^2\,df$$

For an amplitude-modulated (AM) signal, $s(t) = A_c[1+k_a m(t)]\cos 2\pi f_c t$, $m(t)= \cos 2\pi f_m t$. If $k_a=0.5$, the percentage of the total power carried by the sidebands is ______.

For an AM signal $s(t)=Ac[1+k_a m(t)]\cos 2\pi fct$ where $m(t)=2\cos 2\pi fmt$, if the percentage of the total power carried by the sidebands is 33.33% , then the parameter ka =________.

For an AM signal with 50 percent modulation $s(t)=A_c[1+k_a m(t)]\cos 2\pi f_c t$, where $m(t)=2\cos 2\pi f_m t$. Then, the percentage of the total power carried by the sidebands is_______, and the parameter k_a =________.

Average power of AM signals (2.7, 2.46&2.52)

Let X_1 and X_2 be two independent Gaussian random variables, with mean 2 and standard deviation 4. Let $Y = 2X_1 + X_2$, the mean of Y is _____， the variance of Y is______ ， and the probability density function of Y is???

Mean and variance (1.3&1.5)
Gaussian distribution

An angle modulated signal with carrier frequency f_c=2 MHz is $x_c(t) = 10\cos(2\pi f_c t+5\sin(4000\pi t))$, the peak phase deviation in radians is____radians, the peak frequency deviation is_____KHz

Frequency and phase modulation (2.28, 2.32&2.54)

Define the random variables $X=\cos\theta$ and $Y=\sin\theta$, where θ is uniformly distributed over the interval $[-\pi/4, \pi/4]$. Then, X and Y are uncorrelated, but are not independent. ???????

Correlation and independent (1.12&1.26)

The random process $X(t)$ is defined by $X(t)=A\cos(2\pi f_0 t+\theta)$, where two independent random variables A and θ are uniformly distributed on $[-1, 2]$ and $[0, \pi]$. Then, the autocorrelation function of $X(t)$ is ________________ and its power spectral density is __________________.

autocorrelation and power spectral density (1.12)

The message signal $m(t)=10\text{sinc}^2(200t)$ frequency modulates the carrier $c(t)=100\cos(2\pi f_c t)$. The modulation index is $\beta=5$. Calculate the maximum frequency deviation of the modulated signal $\Delta f=$______(Hz) . Find the bandwidth of the modulated signal $B_T=$_______ (Hz) .

Modulation index and frequency deviation of general FM signal