电磁场作业 1

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已知直角坐标中
$$\vec{r}(x,y,z)$$
 位置矢量,求:
1) $\nabla\cdot\vec{r}$; 2) $\nabla\times\vec{r}$; 3) ∇r ; 4) $\nabla(\vec{k}\cdot\vec{r})$ 【 \vec{k} 为常矢量】

Solution:

1)

$$abla \cdot ec{r} = (rac{\partial}{\partial x}ec{i} + rac{\partial}{\partial y}ec{j} + rac{\partial}{\partial z}ec{k}) \cdot (r_xec{i} + r_yec{j} + r_zec{k}) = rac{\partial r_x}{\partial x} + rac{\partial r_y}{\partial y} + rac{\partial r_z}{\partial z}$$

2)

$$egin{align*}
abla imes ec{r} &= egin{align*} ec{i} & ec{j} & ec{k} \ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \ r_x & r_y & r_z \ \end{pmatrix} \ &= \left(\dfrac{\partial r_z}{\partial y} - \dfrac{\partial r_y}{\partial z}
ight) ec{i} + \left(\dfrac{\partial r_x}{\partial z} - \dfrac{\partial r_z}{\partial x}
ight) ec{j} + \left(\dfrac{\partial r_y}{\partial x} - \dfrac{\partial r_x}{\partial y}
ight) ec{k} \end{split}$$

3)

$$abla r = rac{\partial r}{\partial x}ec{i} + rac{\partial r}{\partial y}ec{j} + rac{\partial r}{\partial z}ec{k}$$

4) $ec{k}$ 为常矢量,可以设为 $ec{k}=k_xec{i}+k_yec{j}+k_zec{k}$,其中系数均为常数。

$$abla(ec{k}\cdotec{r})=
abla(k_xr_x+k_yr_y+k_zr_z)$$

记 $l=k_xr_x+k_yr_y+k_zr_z$,则

$$abla(ec{k}\cdotec{r})=rac{\partial l}{\partial x}ec{i}+rac{\partial l}{\partial y}ec{j}+rac{\partial l}{\partial z}ec{k}$$

注意

也可以利用公式

$$abla(ec{k}\cdotec{r}) = (ec{k}\cdot
abla)ec{r} + ec{k} imes (
abla imesec{r}) + (ec{r}\cdot
abla)ec{k} + ec{r} imes (
abla imesec{k})$$