17-18-3高等数学A期中试卷参考答案及评分标准

一、 填空题（本题共5小题，每小题4分，满分20分）

1. 1;　2. $\dfrac{4}{3}$;　3. $\displaystyle\int_0^1 \mathrm{d}y \int_{y^2}^{\sqrt{y}} f(x,y)\mathrm{d}x$;　4. $\mathrm{e}^{-(\frac{2}{3}\pi+2k\pi)}\cdot\sin(\ln 2), k=0,\pm1,\pm2,\cdots$;　5. $30a$.

二、 填空题（本题共4小题，每小题4分，满分16分）

1. B;　2. B;　3. A;　4. C.

三、 计算下列各题（本题共4小题，每小题8分，满分32分）

1. $\dfrac{\partial z}{\partial x}=yf_1+f_2$(2分), $\dfrac{\partial^2 z}{\partial x\partial y}=f_1+y(xf_{11})+xf_{21}=f_1+xyf_{11}+xf_{21}$(6分).

2. 令$u=2x^2-y^2$, 则$z=f(u)$. $\dfrac{\partial z}{\partial x}\Big|_{(1,\,1)}=4xf'(u)\Big|_{(1,\,1)}=4$(2分+1 分), $\dfrac{\partial z}{\partial y}\Big|_{(1,\,1)}=(-2y)f'(u)\Big|_{(1,\,1)}=-2$(2分+1分). 所以$\mathrm{d}z\Big|_{(1,\,1)}=4\mathrm{d}x-2\mathrm{d}y$(2分).

3. 设曲面上的切点为 $P(x_0,\ y_0,\ z_0)$, 则 P 点处的法向量 $\vec{n}=\{x_0,2y_0,-1\}$(3分). 由已知条件得, $\dfrac{x_0}{2}=\dfrac{2y_0}{2}=\dfrac{-1}{-1}$(2分), 所以切点为 $(2,1,3)$(1分), 故切平面方程为$2(x-2)+2(y-1)-(z-3)=0$, 即$2x+2y-z-3=0$(2分).

4. $\displaystyle\iint\limits_D x\mathrm{d}x\mathrm{d}y+\iint\limits_D xy\mathrm{e}^{\frac{x^2+y^2}{2}}\mathrm{d}x\mathrm{d}y=\iint\limits_D x\mathrm{d}x\mathrm{d}y=\int_{-1}^1\mathrm{d}x\int_x^1 x\mathrm{d}y=-\frac{2}{3}$(4分+4分).

5. 由对称性知, $\bar{x}=\bar{y}=0$(2分). $\displaystyle\iiint\limits_\Omega \mathrm{d}V=\int_0^{2\pi}\mathrm{d}\varphi\int_0^2\rho\mathrm{d}\rho\int_{\rho^2}^4\mathrm{d}z=8\pi$, $\displaystyle\iiint\limits_\Omega z\mathrm{d}V=\int_0^{2\pi}\mathrm{d}\varphi\int_0^2\rho\mathrm{d}\rho\int_{\rho^2}^4 z\mathrm{d}z=\frac{64\pi}{3}$. 所以$\bar{z}=\dfrac{\iiint\limits_\Omega z\mathrm{d}V}{\iiint\limits_\Omega \mathrm{d}V}=\dfrac{8}{3}$. 从而质心为 $(0,\ 0,\ \dfrac{8}{3})$(2分+2分+2分).

四、 由 $C-R$ 方程知, $v_y=u_x=3x^2-3y^2$(2分), 故 $v=3x^2y-y^3+\varphi(x)$(1分). 又 $v_x=6xy+\varphi'(x)=-u_y=6xy$(2分), 得 $\varphi(x)=C$(1分). 所以 $f(z)=x^3-3xy^2+\mathrm{i}(3x^2y-y^3+C)=z^3+C\mathrm{i}$(2分). 而 $f(\mathrm{i})=\mathrm{i}^3+C\mathrm{i}=-\mathrm{i}+C\mathrm{i}=-\mathrm{i}$, 故 $C=0$(1分). 从而 $f(z)=z^3$(1分).

五、 令 $f(x,\ y,\ \lambda)=x^2+y^2+\lambda(3x^2+3y^2-2xy-1)$(2分).

则$\begin{cases}F_x=2x+6\lambda x-2\lambda y=0,\\ F_y=2y+6\lambda y-2\lambda x=0,\\ F_\lambda=3x^2+3y^2-2xy-1=0.\end{cases}$ (2分) 解得$(x,\ y)=(\dfrac{1}{2},\ \dfrac{1}{2})$, $(-\dfrac{1}{2},\ -\dfrac{1}{2})$, $(\dfrac{\sqrt{2}}{4},\ -\dfrac{\sqrt{2}}{4})$, $(-\dfrac{\sqrt{2}}{4},\ \dfrac{\sqrt{2}}{4})$(2分). 所以长半轴和短半轴分别为$\sqrt{(\pm\dfrac{1}{2})^2+(\pm\dfrac{1}{2})^2}=\dfrac{\sqrt{2}}{2}$, $\sqrt{(\pm\dfrac{\sqrt{2}}{4})^2+(\mp\dfrac{\sqrt{2}}{4})^2}=\dfrac{1}{2}$(1分). 从而椭圆的面积为$\pi\dfrac{\sqrt{2}}{2}\dfrac{1}{2}=\dfrac{\sqrt{2}}{4}\pi$(1分).

六、 $\int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^y \frac{\sin z}{(1-z)^2} \mathrm{d}z = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}z \int_z^x \frac{\sin z}{(1-z)^2} \mathrm{d}y = \int_0^1 \mathrm{d}x \int_0^x \frac{\sin z}{(1-z)^2}(x-z)\mathrm{d}z = \int_0^1 \mathrm{d}z \int_z^1 \frac{\sin z}{(1-z)^2}(x-z)\mathrm{d}x = \int_0^1 \frac{\sin z}{(1-z)^2}\mathrm{d}z \int_z^1 (x-z)\mathrm{d}x = \int_0^1 \frac{\sin z}{2}\mathrm{d}z = \frac{1}{2}(1-\cos 1)$(3分+3分).