Exp 2

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编写用追赶法解三对角线性方程的程序，并解下列方程组：

$Ax = b$，其中

$$A_{10\times10} = \begin{bmatrix} -4 & 1 & & & & & \\ 1 & -4 & 1 & & & & \\ & 1 & -4 & 1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & 1 & -4 & 1 & \\ & & & & 1 & -4 \end{bmatrix}, \qquad b = \begin{bmatrix} -27 \\ -15 \\ -15 \\ \vdots \\ -15 \\ -15 \end{bmatrix}$$

In [1]:
```
import numpy as np
```

In [2]:
```
# 追赶法求三对角线性方程组
# a 为主对角线元素，bb，c分别为次对角线元素，x为解
def solution(A,b):
    n = A.shape[0]
    a = np.array([])
    bb = np.array([])
    c = np.array([])

    a = np.append(a,A[0,0])
    bb = np.append(bb,0)
    c = np.append(c,A[0,1])
    for i in range(n-2):
        a = np.append(a,A[i+1,i+1])
        bb = np.append(bb,A[i+1,i])
        c = np.append(c,A[i+1,i+2])
    a = np.append(a,A[n-1,n-1])
    bb = np.append(bb,A[n-1,n-2])
    c = np.append(c,0)

    l = np.array([])
    beta = np.array([])
    y = np.array([])
    beta = np.append(beta,a[0])
    y = np.append(y,b[0])
    l = np.append(l,0)
    for i in range(n-1):
        l = np.append(l,bb[i+1]/beta[i])
        beta = np.append(beta,a[i+1]-l[i+1]*c[i])
        y = np.append(y,b[i+1]-l[i+1]*y[i])

    x = np.array([])
    for i in range(n):
        x = np.append(x,0)

    x[n-1] = y[n-1]/beta[n-1]
    for i in range(n-1):
        x[n-2-i] = (y[n-2-i] - c[n-2-i]*x[n-2-i+1])/beta[n-2-i]
    return x
```

In [3]:
```
A = np.array([[-4,1,0,0,0,0,0,0,0,0],
              [1,-4,1,0,0,0,0,0,0,0],
              [0,1,-4,1,0,0,0,0,0,0],
              [0,0,1,-4,1,0,0,0,0,0],
              [0,0,0,1,-4,1,0,0,0,0],
              [0,0,0,0,1,-4,1,0,0,0],
              [0,0,0,0,0,1,-4,1,0,0],
              [0,0,0,0,0,0,1,-4,1,0],
              [0,0,0,0,0,0,0,1,-4,1],
              [0,0,0,0,0,0,0,0,1,-4]])
```

In [4]:
```
A.shape
```

Out[4]: (10, 10)

In [5]:
```
A
```

Out[5]:
```
array([[-4,  1,  0,  0,  0,  0,  0,  0,  0,  0],
       [ 1, -4,  1,  0,  0,  0,  0,  0,  0,  0],
       [ 0,  1, -4,  1,  0,  0,  0,  0,  0,  0],
       [ 0,  0,  1, -4,  1,  0,  0,  0,  0,  0],
       [ 0,  0,  0,  1, -4,  1,  0,  0,  0,  0],
       [ 0,  0,  0,  0,  1, -4,  1,  0,  0,  0],
       [ 0,  0,  0,  0,  0,  1, -4,  1,  0,  0],
       [ 0,  0,  0,  0,  0,  0,  1, -4,  1,  0],
       [ 0,  0,  0,  0,  0,  0,  0,  1, -4,  1],
       [ 0,  0,  0,  0,  0,  0,  0,  0,  1, -4]])
```

In [6]:
```
b = np.array([-27,-15,-15,-15,-15,-15,-15,-15,-15,-15])
```

In [7]:
```
b.shape
```

Out[7]: (10,)

In [8]:
```
b
```

Out[8]:
```
array([-27, -15, -15, -15, -15, -15, -15, -15, -15, -15])
```

In [9]:
```
x = solution(A,b)
```

In [10]:
```
x
```

Out[10]:
```
array([8.70575808, 7.82303234, 7.58637127, 7.52245276, 7.50343976,
       7.4913063 , 7.46178542, 7.35583538, 6.9615561 , 5.49038903])
```

Conclusion

所以解得

$$x = \begin{bmatrix} 8.70575808 \\ 7.82303234 \\ 7.58637127 \\ 7.52245276 \\ 7.50343976 \\ 7.4913063 \\ 7.46178542 \\ 7.35583538 \\ 6.9615561 \\ 5.49038903 \end{bmatrix}$$