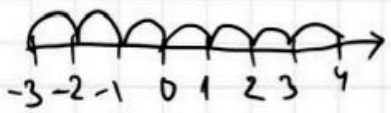
# МОДУЛНА ФУНКЦИЯ

Модул (абсолютна стойност) = разстояние от 1 число до 0




$|4|=4$

$|-3|=3$

$|0|=0$

ДС $|a|\geq 0$ винаги

# ГРАФИКА НА ЛИНЕЙНА ФУНКЦИЯ

$y=2x-3$

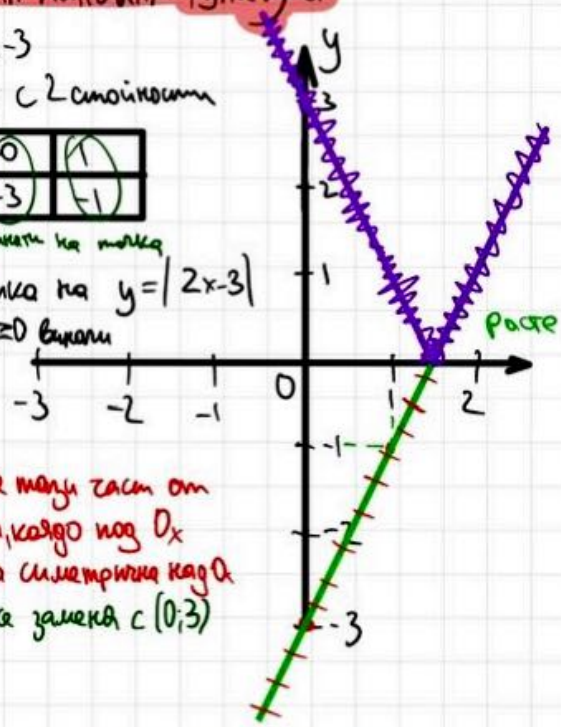
Таблица с 2 стойности

| x | 0 | 1 |
|---|---|---|
| y | -3 | -1 |

координати на точка

Зад. Графика на $y=|2x-3|$

$\Rightarrow y\geq 0$ винаги




Заменяме тази част от графиката, която под $O_x$ с нейната симетрична над $O_x$

т. $(0;-3)$ се замена с $(0;3)$

# ГРАФИКА НА КВАДРАТНА ФУНКЦИЯ

Зад.3 За коя стойност на параметъра $a$, уравнението $|x^2+4x+3|=a$ има точно 3 корена.

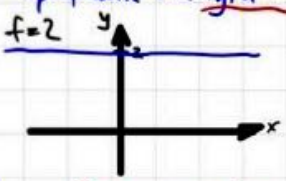
$|y|$ $f$

Построяваме графика на $y=x^2+4x+3$

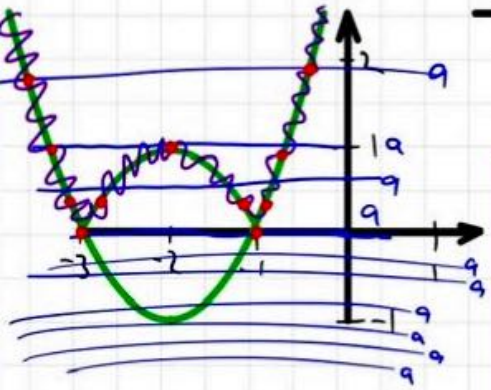
$x_v=\frac{-b}{2a}=\frac{-4}{2\cdot 1}=-2$

| x | -3 | -2 | -1 |
|---|---|---|---|
| y | 0 | -1 | 0 |

Търсим пресечни т. на графиките на $|y|$ и $f=a$




Графиката на $f=a$ ≡ права линия $\parallel O_x$



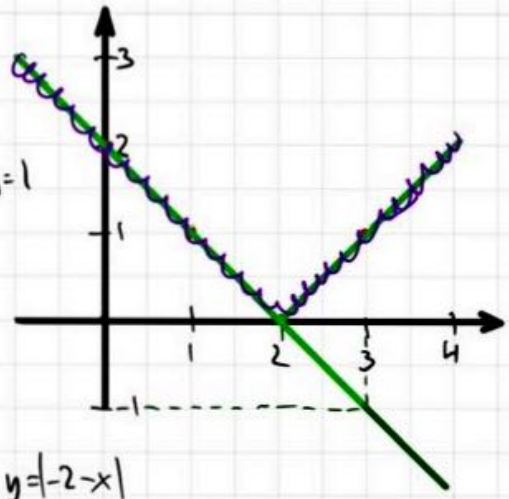

① При $a<0$, няма решение
② При $a=0$, има 2 решения
③ При $a\in(0;1)$, има 4 решения
④ При $a=1$, има 3 решения
⑤ При $a>1$, има 2 решения

Зад.1 Постр. графика на $y=|2-x|$

| x | 0 | 1 |
|---|---|---|
| y | 2 | 1 |

При $x=3$ $y=-1$

$\Rightarrow$ постр. $x=3$ $y=1$

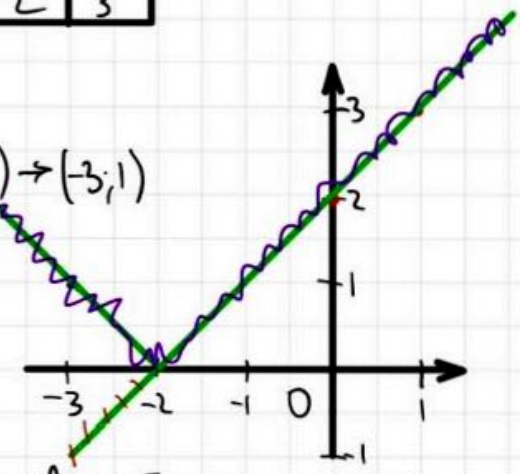



Зад.2 $y=|-2-x|$

$y=|2+x|$

| x | 0 | 1 |
|---|---|---|
| y | 2 | 3 |

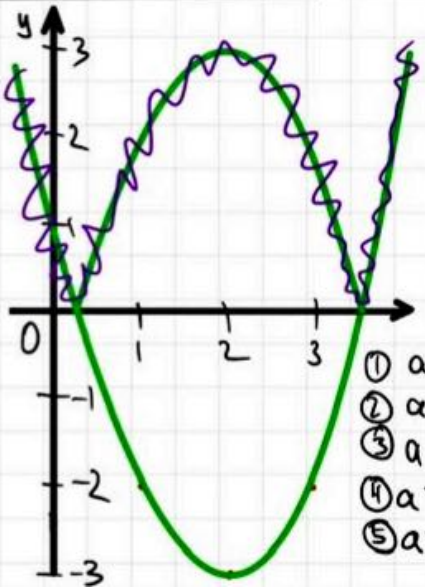
$(-3;-1)\to(-3;1)$




Зад.4 Изследвайте броя на решенията на $|x^2-4x+1|=a$, $a$ = реален параметър

$x_v=\frac{b}{2a}=\frac{4}{2}=2$

| x | 1 | 2 | 3 |
|---|---|---|---|
| y | -2 | -3 | -2 |




① $a<0$ н.р-е
② $a=0$ - 2 р-я
③ $a\in(0;3)$ - 4 р-я
④ $a=3$ - 3 р-я
⑤ $a>3$ - 2 р-я

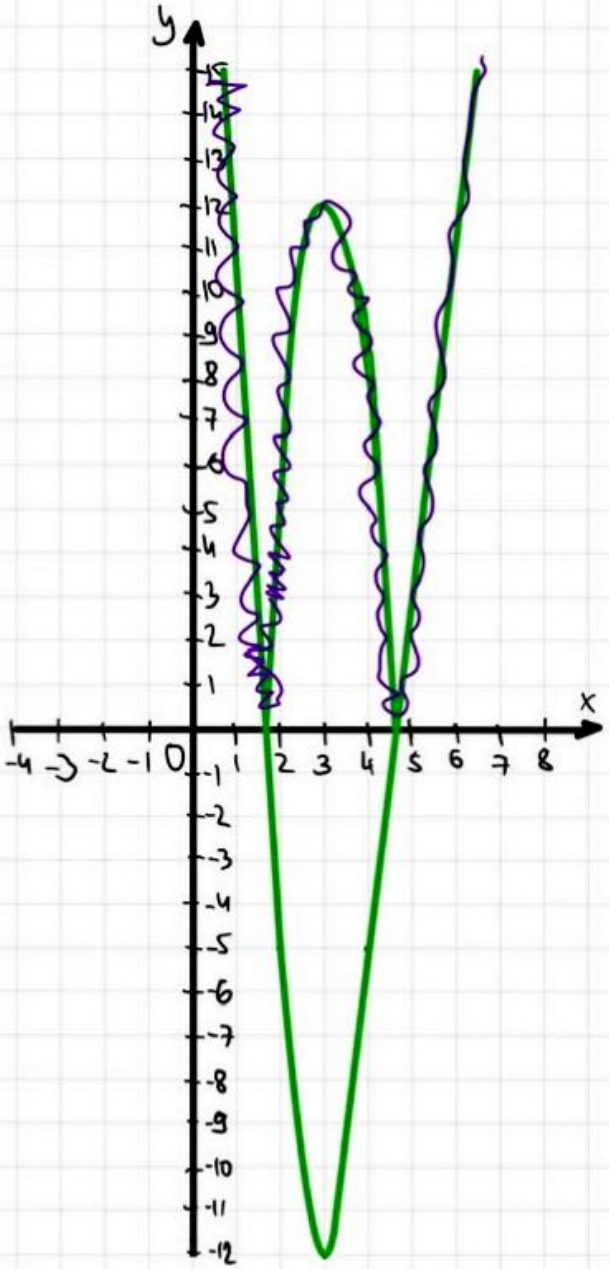
Домашна работа

Изследвайте броя на решенията на

$|x^2-6x+4|=a$

$x_v=\frac{6}{2}=3$

| x | 2 | 3 | 4 |
|---|---|---|---|
| y | -12 | -5 | -12 |




① $a<0$ няма решения

② $a=0$ - 2 р-я

③ $a\in(0;12)$ - 4 р-я

④ $a=12$ - 3 р-я

⑤ $a>12$ - 2 р-я

Кандидат-студентски изпит за ТУ

Зад. 3 $x^2-3ax+a^2=0$

$x_1^2+x_2^2=28$

$28=(x_1+x_2)^2-2x_1x_2=9a^2-2a^2=7a^2$

$a^2=4$

$a_{1,2}=\pm 2$ -а)

Зад. 4 $\frac{x^2-8}{x}\le -x$

$\frac{x^2-8}{x}+x\le 0 \quad x\ne 0$

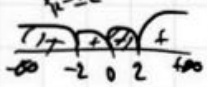
$\frac{x^2-8+x^2}{x}\le 0$ ✗

$\frac{2x^2-8}{x}\le 0$

$2x^2-8=0 \quad x\ne 0$

$x^2=4$

$x_{1,2}=\pm 2$




$x\in(-\infty;-2]\cup(0;2]$ -А)

Зад. 5 $f(1)=4$ $f(x)=x^2+2ax+b$

$f(-1)=0$ $f(-2)=?$

$\begin{cases} 4=1+2a+b \\ 0=1-2a+b \end{cases}$

$+\begin{cases} 3=2a+b \\ -1=-2a+b \end{cases}$ $a=1$ $b=1$

$2=2b \Rightarrow f(x)=x^2+2x+1$

$b=1$ $f(-2)=4-4+1$

$-1=-2a+1$ $f(-2)=1$ -г)

$-2=-2a \ /:(-2)$

$a=1$

Зад. 6 $\begin{cases} \frac{3}{x-2}-\frac{1}{y+1}=1 \\ \frac{4}{y+1}+\frac{2}{x-2}=3 \end{cases}$

Полагаме $\frac{1}{x-2}=u$ $\frac{1}{y+1}=v$

$\begin{cases} 3u-v=1 \ /\cdot 4 \\ 4v+2u=3 \end{cases}$

$+\begin{cases} 12u-4v=4 \\ 4v+2u=3 \end{cases}$

$14u=7$

$u=\frac{1}{2}$

$6-4v=4$

$-4v=-2$

$v=\frac{1}{2}$

$\frac{1}{x-2}=\frac{1}{2}$ $\frac{1}{y+1}=\frac{1}{2}$

$x-2=2$ $y+1=2$

$x=4$ $y=1$

$(4;1)$

# УРАВНЕНИЯ И НЕРАВЕНСТВА С ДВА МОДУЛА

## УРАВНЕНИЯ

① Решете уравнението

$|x+1|+|2x+7|=10$

$|a|=\begin{cases} a, \text{ когато } a\geq 0 \\ -a, \text{ когато } a<0 \end{cases}$

$|5|=5 \quad |-7|=7=-(-7)$

$x+1=0 \quad 2x+7=0$

$x=-1 \quad x=-\frac{7}{2}$

Правим таблица с 3 интервала. Във всеки интервал попълваме знака на вътрешността на модула

Пример

| $-\infty$ | $-6$ | $-3{,}5$ | $-2$ | $-1$ | $0$ | $+\infty$ |
|---|---|---|---|---|---|---|
| $x+1$ | $-$ | | $-$ | | $+$ | |
| $2x+7$ | $-$ | | $+$ | | $+$ | |

I случай $x\in(-\infty;-3{,}5]$

$|x+1|=-(x+1)=-x-1$

$|2x+7|=-(2x+7)=-2x-7$

$\Rightarrow -x-1-2x-7=10$

$-3x=18 \;|:(-3)$

$x=-6$

е решение

II случай $x\in(-3{,}5;-1]$

$|x+1|=-(x+1)=-x-1$

$|2x+7|=2x+7$

$-x-1+2x+7=10$

$x=4$

не е решение

III случай $x\in(-1;+\infty)$

$|x+1|=x+1$

$|2x+7|=2x+7$

$x+1+2x+7=10$

$3x=2$

$x=\frac{2}{3}$

е решение

$\Rightarrow x_1=-6 \quad x_2=\frac{2}{3}$

② $|x-2|+|3x-5|=6$

$x-2=0 \quad 3x-5=0$

$x=2 \quad x=\frac{5}{3}=1\frac{2}{3}$

| $-\infty$ | $1\frac{2}{3}$ | $2$ | $+\infty$ |
|---|---|---|---|
| $x-2$ | $-$ | $-$ | $+$ |
| $3x-5$ | $-$ | $+$ | $+$ |

I сл. $x\in(-\infty;\frac{5}{3}]$

$|x-2|=-x+2$

$|3x-5|=-3x+5$

$-x+2-3x+5=6$

$-4x=-1$

$x=\frac{1}{4}$

е решение

II сл. $x\in(1\frac{2}{3};2]$

$|x-2|=-x+2$

$|3x-5|=3x-5$

$-x+2+3x-5=6$

$2x=9$

$x=4{,}5$

не е решение

III сл. $x\in(2;+\infty)$

$|x-2|=x-2$

$|3x-5|=3x-5$

$x-2+3x-5=6$

$4x=13$

$x=\frac{13}{4}=3{,}25$

е решение

ТУ Намерете най-малкото цяло (x), за което

$\frac{|2x-3|-1}{6x^2-5x+2}\leq 0$ ДС $6x^2-5x+2\neq 0$

$D<0 \Rightarrow$ няма ограничения

I случай $2x-3\geq 0$

!!! $x\geq\frac{3}{2}$

$\Rightarrow |2x-3|=2x-3$

$\frac{2x-3-1}{6x^2-5x+2}\leq 0$

$2x-4=0 \quad 6x^2-5x+2=0$

$x_1=2 \quad D<0$ н.р.к.

II случай $2x-3<0$

!!! $x<\frac{3}{2}$

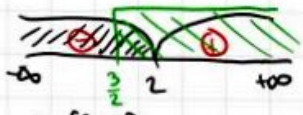
$\Rightarrow |2x-3|=-(2x-3)=-2x+3$

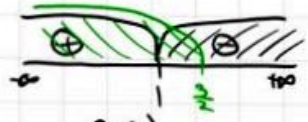
$\frac{-2x+3-1}{6x^2-5x+2}\leq 0$

$-2x+3-1=0 \quad 6x^2-5x+2=0$

$-2x=-2 \quad D<0 \Rightarrow$ н.р.к.

$x=1$





Отг. $x\in[\frac{3}{2};2]$

Отг. $x\in[1;\frac{3}{2})$

Краен отговор $x\in[1;\frac{3}{2})\cup[\frac{3}{2};2] \Rightarrow x\in[1;2]$

$\Rightarrow$ Най-малкото цяло р-е е $x=1$

ТУ Намерете най-малкото цяло (x), за което

$\frac{|3x-5|-2}{2x^2+x+7}\leq 0$ ДС $2x^2+x+7\neq 0$

$D<0 \Rightarrow$ няма ограничение

I сл. $3x-5\geq 0$

$x\geq\frac{5}{3}$

$\Rightarrow |3x-5|=3x-5$

$\frac{3x-5-2}{2x^2+x+7}\leq 0$

$3x-7=0 \quad 2x^2+x+7=0$

$x=\frac{7}{3} \quad D<0 \Rightarrow$ н.р.к.

II сл. $3x-5<0$

$x<\frac{5}{3}$

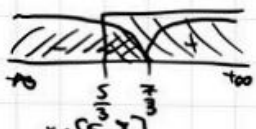
$\Rightarrow |3x-5|=-3x+5$

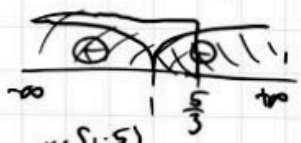
$\frac{-3x+5-2}{2x^2+x+7}\leq 0$

$-3x=-3$

$x=1 \quad 2x^2+x+7=0$

$D<0 \Rightarrow$ н.р.к.





$x\in[\frac{5}{3};\frac{7}{3}]$

$x\in[1;\frac{5}{3})$

Краен отг. $x\in[1;\frac{5}{3})\cup[\frac{5}{3};\frac{7}{3}] \Rightarrow x\in[1;\frac{7}{3}]$

$\Rightarrow$ Най-малкото $x=1$



ДР ИВМ зад-за оц.

# НЕРАВЕНСТВА

ТУ ① $|x-1|+|x-4|<5$

$x-1=0 \quad x-4=0$

$x=1 \quad x=4$

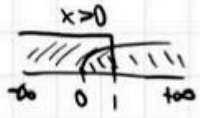
| | $-\infty$ … 1 | 1 … 4 | 4 … $+\infty$ |
|---|---|---|---|
| $x-1$ | ⊖ | ⊕ | ⊕ |
| $x-4$ | ⊖ | ⊖ | ⊕ |

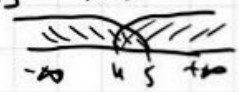
I сл. $x\in(-\infty;1]$ II сл. $x\in(1;4]$ III сл. $x\in(4;+\infty)$

$-x+1-x+4<5$ $\quad$ $\cancel{x}-1-\cancel{x}+4<5$ $\quad$ $x-1+x-4<5$

$-2x<0 /:(-2)$ $\quad$ $3<5$ $\quad$ $2x<10 /:2$

$x>0$ $\quad$ $\forall x\in(1;4]$ $\quad$ $x<5$





$x\in(0;1]$ $\quad$ $x\in(4;5)$

$x\in(0;1]\cup(1;4]\cup(4;5)$

$\Rightarrow x\in(0;5)$ ⊕ – в таки задача моща да пропуснем да разглеждаме

$\geq 0 \quad +1$

СУ 04.2022г. $|\sqrt{x+1}-1|+|\sqrt{x+1}+1|=4$ $\quad |\sqrt{x+1}+1|$ ⊖

Полагаме $\sqrt{x+1}=y$

$|y-1|+|y+1|=4$

$y-1=0 \quad y+1=0$

$y=1 \quad y=-1$

| | $-\infty$ … $-1$ | $-1$ … 1 | 1 … $+\infty$ |
|---|---|---|---|
| $y-1$ | ⊖ | ⊖ | ⊕ |
| $y+1$ | ⊖ | ⊕ | ⊕ |

I сл. $y\in(-\infty;-1]$ II сл. $y\in(-1;1]$ III $y\in(1;+\infty)$

$|y-1|=-y+1$ $\quad$ $|y-1|=-y+1$ $\quad$ $|y-1|=y-1$

$|y+1|=-y-1$ $\quad$ $|y+1|=y+1$ $\quad$ $|y+1|=y+1$

$-y+\cancel{1}-y-\cancel{1}=4$ $\quad$ $\cancel{-y}+1+\cancel{y}+1=4$ $\quad$ $y-\cancel{1}+y+\cancel{1}=4$

$-2y=4 /:(-2)$ $\quad$ $2=4$ $\quad$ $2y=4$

$y=-2$ е решение $\quad$ $\Rightarrow$ н.р.к. $\quad$ $y=2$ е решение

$\Rightarrow \sqrt{x+1}=-2$ $\quad$ $\Rightarrow (\sqrt{x+1})^2=2^2$

$\Rightarrow$ н.р.к. $\quad$ $x+1=4$

$x=3$

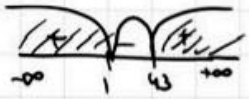
СУ 2021 $|x^2-44x+43|\leq x^2-90x+2021$

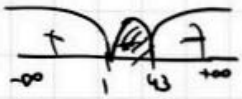
Да го подредим

I сл. $x^2-44x+43\geq 0$ ⊕ $\quad$ II сл. $x^2-44x+43<0$ ⊖

$x^2-44x+43=0$ $\quad$ ⊕ $x^2-44x+43=0$

$x_1=43 \quad x_2=1$ $\quad$ $x_1=43 \quad x_2=1$





$x\in(-\infty;1]\cup[43;+\infty)$ $\quad$ $x\in(1;43)$

$\cancel{x^2}-44x+43\leq \cancel{x^2}-90x+2021$ $\quad$ $-x^2+44x-43\leq x^2-90x+2021$

$46x\leq 1978 /:46$ $\quad$ $-2x^2+134x-2064\leq 0 /:(-2)$

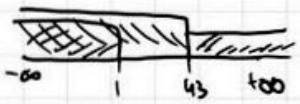
$x\leq 43$ $\quad$ ⊕ $x^2-67x+1032\geq 0$ ⊕

$x^2-67x+1032=0$

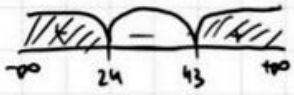
$D=4489-4128=361$

$x_1=\frac{67+19}{2}=43$

$x_2=\frac{67-19}{2}=24$



$x\in(-\infty;1]\cup\{43\}$



$x\in(-\infty;24]\cup\{43;+\infty)$

$x\in(1;24)$

$\Rightarrow$ Отг. $x\in\{43\} / x=43$

Зад.1 $|3x-2|+|x+6|<8$

$x+6=0 \quad 3x-2=0$

$x=-6 \quad x=\frac{2}{3}$

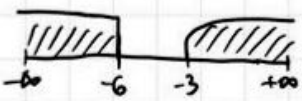
| | $-\infty$ … $-6$ | $-6$ … $\frac{2}{3}$ | $\frac{2}{3}$ … $+\infty$ |
|---|---|---|---|
| $x+6$ | ⊖ | ⊕ | ⊕ |
| $3x-2$ | ⊖ | ⊖ | ⊕ |

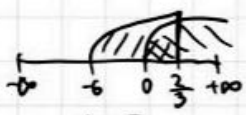
I сл. $x\in(-\infty;-6]$ II сл. $x\in(-6;\frac{2}{3}]$ III сл. $x\in(\frac{2}{3};+\infty)$

$-3x+2-x-6<8$ $\quad$ $-3x+\cancel{2}+x+\cancel{6}<\cancel{8}$ $\quad$ $3x-2+x+6<8$

$-4x<12 /:(-4)$ $\quad$ $-2x<0 /:(-2)$ $\quad$ $4x<4 /:4$

$x>-3$ $\quad$ $x>0$ $\quad$ $x<1$



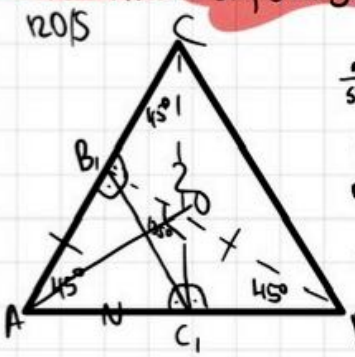


$\Rightarrow$ н.р.к. $\quad$ $\Rightarrow x\in(0;\frac{2}{3}]$ $\quad$ $x\in(\frac{2}{3};1)$

$x\in(0;\frac{2}{3}]\cup(\frac{2}{3};1)\Rightarrow x\in(0;1)$

# ТЕОРЕМА ПІТАГОРОВА, СИНУСОВА, КОСИНУСОВА

1205




$\frac{a}{\sin\alpha} = 2R$ $\Delta ABB_1 \sim \Delta ACC_1$ (по I пр.

$\frac{b}{\frac{\sqrt{2}}{2}} = 2R$ 1) $\angle A$ – общий

$\frac{R\sqrt{2}}{2} = 2R$ 2) $\angle B_1 = \angle C_1$

$R = 4\sqrt{5}$ см $\Rightarrow \frac{BB_1}{CC_1} = \frac{AB}{AC} = \frac{AB_1}{AC_1}$

$\Delta AC_1B_1 \sim \Delta ABC$ (по II пр.)

1) $\angle A$ – общий

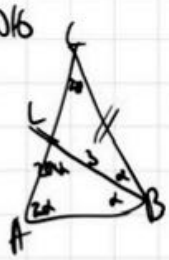
2) $\frac{AB_1}{AB} = \frac{AC_1}{AC} = \frac{B_1C_1}{BC} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{R_{\Delta AB_1C_1}}{R_{\Delta ABC}} = \frac{1}{\sqrt{2}}$

$\frac{R_{\Delta AB_1C_1}}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow R_{\Delta AB_1C_1} = 4$ см – т)

1206




$AC = BC$

$\Rightarrow \angle A = \angle ABC = \frac{180° - 2\beta}{2} = 90° - \beta$

$\angle LBC = \frac{90° - \beta}{2} = 45° - \frac{\beta}{2}$

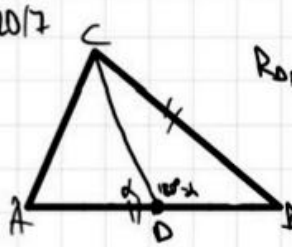
В $\Delta BLC$

$\angle CLB = 180° - 2\beta - 45° + \frac{\beta}{2} = 135° - \frac{3}{2}\beta$

по sin T

$\frac{3}{\sin 2\beta} = BC$

1207




$R_{\Delta ADC} = 6$ см

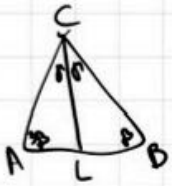
$\frac{AC}{\sin\alpha} = 2R_{\Delta ADC} \to 2 \cdot 6 = 12$

$\sin\alpha = \frac{AC}{12}$

$\frac{BC}{\sin(180° - \alpha)} = 2R_{\Delta ABC}$

$\frac{BC}{\frac{AC}{12}} = 2R_{\Delta ABC} \to 6\frac{BC}{AC} = R_{\Delta ABC}$

1208 $\Delta ABC$

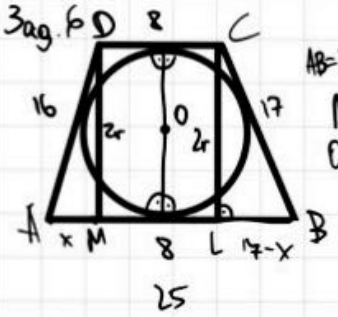



$S_{ACL} = \frac{1}{2} AC \cdot AL \cdot \sin 3\beta$

$S_{BCL} = \frac{1}{2}$

(СУ) 2022

3ад.6




$AB = 33 - 8 = 25$ см

$ML = DC = 8$ см

Озн. $AM$ с $x \Rightarrow LB = 25 - 8 - x = 17 - x$

$DM = CL$

$DM^2 = 16^2 - x^2$

$CL^2 = 17^2 - (17-x)^2$

$16^2 - \cancel{x^2} = \cancel{17^2} - \cancel{17^2} + 34x - \cancel{x^2}$

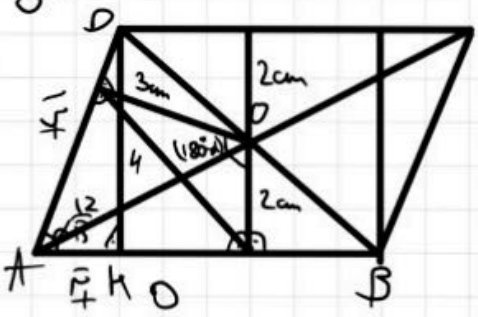
$16^2 = 34x$

$x = \frac{16 \cdot \cancel{16}^{8}}{8 \cdot 17} = \frac{128}{17}$

$DM = \sqrt{256^2 - \left(\frac{128}{17}\right)^2} = \sqrt{256^{289} - \frac{16384}{289}} = \frac{\sqrt{73984 - 16384}}{17} = \frac{\sqrt{57600}}{17} = \frac{240}{17}$

$r = \frac{\frac{240}{17}}{2} = \frac{\cancel{240}^{120}}{17} \cdot \frac{1}{\cancel{2}} = \frac{120}{17}$ — г)

3ад.7




$\cos\alpha = \frac{12}{13}$ $OM = 2$ см $\Rightarrow h_a = 4$ см

$h_b = 6$ см

$S_{ABCD} = a h_a$

$S_{ABCD} = 2S_{\triangle AOB} + 2S_{\triangle AOD} = 2 \cdot \frac{2a}{2} + 2 \cdot \frac{3b}{2}$

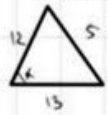
$a \cdot 4a = 2a + 3b$

$4a = 2a + 3b$

$2a = 3b$

$a = \frac{3b}{2}$

В $\triangle AHD$




$144 + y^2 = 169 \Rightarrow 5x = 4$

$y^2 = 25$ $x = \frac{4}{5}$

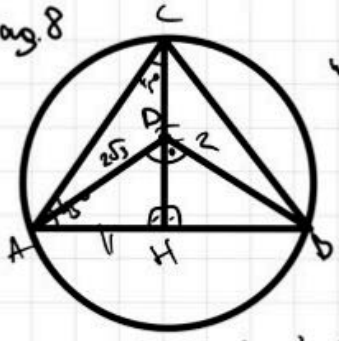
$y = 5$

$\Rightarrow b = 13x = \frac{52}{5} = 10{,}4$

$a = \frac{3 \cdot \cancel{10{,}4}^{5{,}2}}{2} = 15{,}6$

$P = 2a + 2b = 20{,}8 + 31{,}2 = 52$ — в)

3ад.8




по П.Т. в $\triangle ABD$

$AB = 4$

$\frac{2}{\sin\angle HAD} = 4$

$\sin\angle HAD = \frac{1}{2} \Rightarrow \angle HAD = 30° \Rightarrow \angle ABD = 60°$

} или $ABD \equiv \triangle$ кат. = 2 × и пот. 4 $\Rightarrow \angle A = 30°$

в $\triangle ADC$

по $\sin P$

$\frac{AC}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{\frac{\sqrt{2}}{2}}$

$\angle A = 15°$ $\Rightarrow \angle B = 120°$ } $\sin 120° = \frac{\sqrt{3}}{2}$

$2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2} AC$

$3 = \frac{\sqrt{2}}{2} AC$

$AC = \frac{3 \cdot 2}{\sqrt{2}} = \frac{\cancel{6}^{3}\sqrt{2}}{\cancel{2}} = 3\sqrt{2}$ см

3ад.9 т. $P(10; 22)$

$g = 2x - y + 9 = 0$

$h: x + 3y + 1 = 0$

$g \cap h =$ т. $M$

т. $Q \equiv MP \rightarrow MQ : QP = 3 : 4$

3ад 9

# ПОДГОТОВКА НА ОЛИМПИАДАТА

① Б)

$$C=\frac{\log_2 24}{\log_{96}2}-\frac{\log_2 192}{\log_{12}2}$$

$\log_a b=\frac{1}{\log_b a}$

$C=\log_2 24\cdot\log_2 96-\log_2 192\cdot\log_2 12=$

$C=\log_2(8\cdot3)\cdot\log_2(32\cdot3)-\log_2(64\cdot3)\cdot\log_2(4\cdot3)$

$C=(\log_2 8+\log_2 3)(\log_2 32+\log_2 3)-(\log_2 64+\log_2 3)(\log_2 4+\log_2 3)$

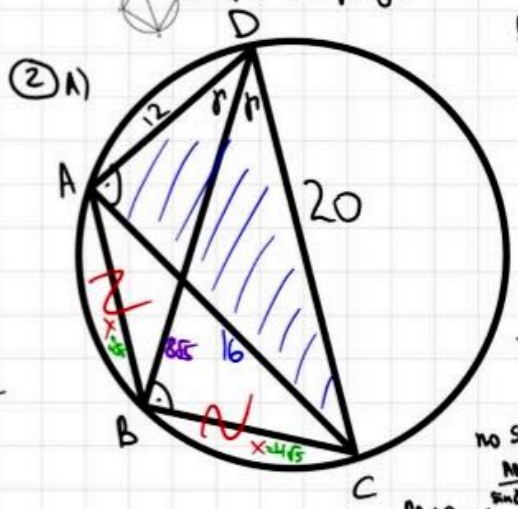
$C=(3+\log_2 3)(5+\log_2 3)-(6+\log_2 3)(2+\log_2 3)$

$C=15+3\log_2 3+5\log_2 3+\log_2 3\cdot\log_2 3-12-6\log_2 3-2\log_2 3-\log_2 3\cdot\log_2 3$

$C=3$

$C^B=3^3=27$

② А)




- първоначален чертеж.

$R=10\text{cm}$

$\frac{20}{\sin\angle DBC}=2R$

$\sin\angle DBC=1$

$\Rightarrow\angle DBC=90°$

$\Rightarrow DC\equiv d$

$\angle CAD=\frac{DC}{2}=90°$

$\Rightarrow$ в $\triangle CAD$ по П.Т.

$AC=16\text{cm}$

по Sin Т. за окр. $\angle ADB=\angle ABC$

$\frac{AB}{\sin\theta}=\frac{BC}{\sin\theta}=2R\Rightarrow AB=BC=x$

$ABCD$ е вписан четириъгълник

$\Rightarrow\angle ADC+\angle ABC=180°$

$\cos\angle ADC=\frac{AD}{DC}=\frac{12}{20}=\frac{3}{5}$

$\Rightarrow\cos\angle ABC=-\frac{3}{5}$

по Cos Т в $\triangle ABC$

$16^2=x^2+x^2-2x\cdot x\left(-\frac{3}{5}\right)$

$2x^2+\frac{6}{5}x^2=256/5$

$10x^2+6x^2=16^2\cdot5$

$16x^2=16^2\cdot5\ /:16$

$x^2=16\cdot5$

$x^2=80$

$x=4\sqrt5=AB=BC$

по П.Т. в $\triangle BDC$

$BD=\sqrt{400-80}$

$BD=\sqrt{320}=\sqrt{32\cdot10}=\sqrt{64\cdot5}=8\sqrt5$

③ Вариант 2

$\frac{2x^4}{x^2}+\frac{5x^3}{x^2}-\frac{7x^2}{x^2}-\frac{5x}{x^2}+\frac{2}{x^2}=0$

$2\left(x^2+\frac{1}{x^2}\right)+5\left(x-\frac{1}{x}\right)-7=0$

Полагаме $x-\frac{1}{x}=t$

$\left(x-\frac{1}{x}\right)^2=x^2-2+\frac{1}{x^2}$

$\Rightarrow x^2+\frac{1}{x^2}=t^2+2$

$2t^2+4+5t-7=0$

$2t^2+5t-3=0$

$D=25+24=49$

$t_1=\frac{-5+7}{4}=\frac{1}{2}$

$t_2=\frac{-5-7}{4}=-3$

$x-\frac{1}{x}=\frac{1}{2}\ /\cdot2x$

$2x^2-x-2=0$

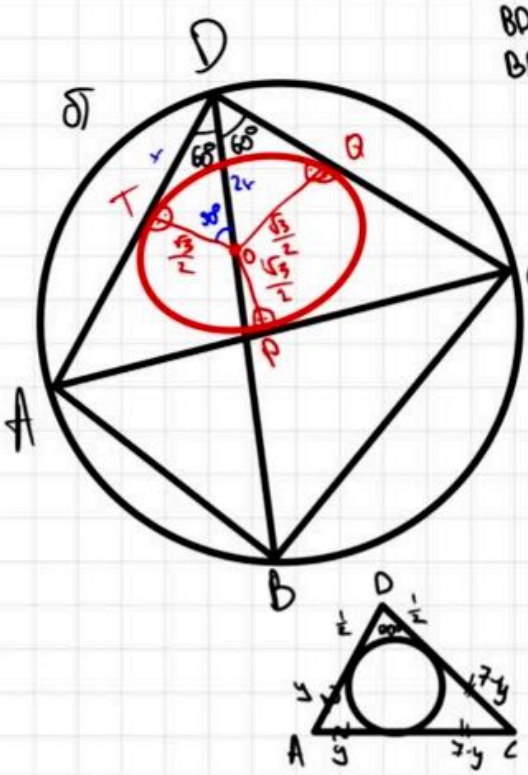
$D=17$

$x_{1,2}=\frac{1\pm\sqrt{17}}{4}$

$x-\frac{1}{x}=-3$

$x^2+3x-1$

③ Б)




$T.G\left(\frac{x_A+x_B+x_C}{3};\frac{y_A+y_B+y_C}{3}\right)$

$T.G\left(\frac{2-4+\sqrt2-1}{3};\frac{-2+4+5}{3}\right)$

$T.G\left(\frac{\sqrt2-3}{3};\frac{7}{3}\right)$

$\frac{AC}{\sin120°}=2R$

$R=\frac{2\sqrt3}{3}$ $\sin120°=\frac{\sqrt3}{2}$

$AC=2$

$r_{\text{вп.кр.}}=\frac{\sqrt3}{2}$ $\angle ADC=120°$

в $\triangle TDO_1$

$\angle TO_1D=30°$

$\Rightarrow TD=x\Rightarrow O_1D=2x$

$\Rightarrow$ по П.Т.

$x^2+\left(\frac{\sqrt3}{2}\right)^2=4x^2$

$x=\frac{1}{2}=TD=DO$




по Cos Т

$7^2=\left(y+\frac{1}{2}\right)^2+\left(7-y+\frac{1}{2}\right)^2$

$-2\left(y+\frac{1}{2}\right)\left(\frac{15}{2}-y\right)\cos120°$

$y_1=\frac{9}{2}$ $y_2=\frac{5}{2}$

# ПРИМЕРНА КОНТРОЛНА РАБОТА

**Зад. 1**

$a=b$

$m_a^2=\frac{1}{4}(2b^2+2c^2-a^2)$

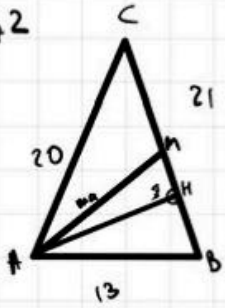
$100=b^2+2.32$

$b^2=36$

$b=6$ ($b=-6$ не е р-е)

$a=b=$ дадро $=6$ cm – В)

**Зад. 2**




$P_{ABC}=27$

$S_{ABC}=\sqrt{27.6.7.14}$

$S_{ABC}=\sqrt{3.3.3.3.2.7.2.7}$

$S_{ABC}=126\,cm^2$

$S_{\triangle ABC}=S_{\triangle AMB}=\frac{S_{ABC}}{2}=\frac{126}{2}=63\,cm^2$

$S_{ABC}=\frac{a.h_a}{2}$

$126=\frac{21}{2}.h_a$

$h_a=\frac{252}{21}=12$ cm

по П.Т. в $\triangle AHC$: $AH^2+CH^2=AC^2$

$144+CH^2=400$

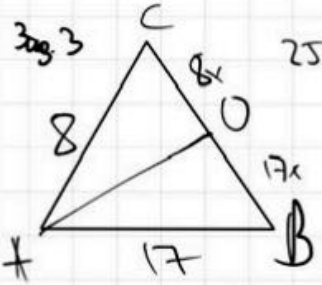
$CH=\sqrt{256}$

$CH=16$ cm

$MH=5,5$ cm

$S_{\triangle AMH}=\frac{AH.MH}{2}=\frac{5,5.12}{2}=33\,cm^2$ – г)

**Зад. 3**




$p=20$ cm

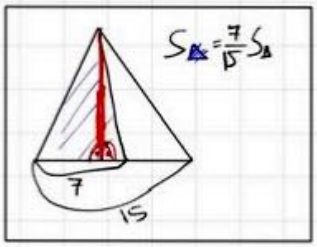
$S_{ABC}=\sqrt{20.12.3.5}$

$S_{ABC}=\sqrt{2.2.5.2.2.3.5}$

$S_{ABC}=60\,cm^2$

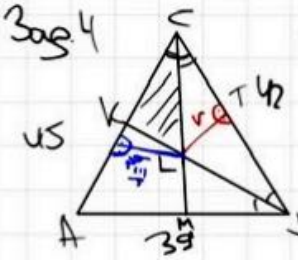
$S_{AOB}=\frac{17}{25}.S_{ABC}=\frac{17}{25}.60$

$S_{AOB}=\frac{204}{5}=40,8\,cm^2$ – А)




$\frac{S}{25}-\frac{S}{17}$

**Зад. 4**




$P_{ABC}=\frac{126}{2}=63$

$S_{ABC}=\sqrt{63.24.21.18}$

$S_{ABC}=\sqrt{7.3.3.2.2.2.3.7.3.3.2}$

$S_{ABC}=27.4.7=756\,cm^2$

$\frac{CK}{AK}=\frac{BC}{AB}=\frac{42}{39}=\frac{14}{13}\Rightarrow$ ($CK=14x$, $AK=13x$)

$14x+13x=45$

$27x=45$

$x=\frac{45}{27}=\frac{5}{3}$

$CK=\frac{70}{3}$ cm

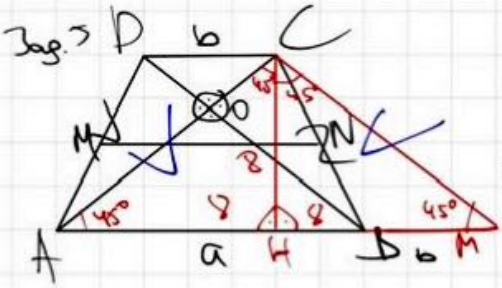
$S=p.r$

$756=63.r$

$r=12$ cm

$S_{OCK}=\frac{1}{2}.\frac{70}{3}.12=140\,cm^2$ – г)

**Зад. 5**




$MN=8$ cm

$MN=\frac{a+b}{2}\Rightarrow a+b=16$ cm

Допълнително $CM\parallel BD$

$\angle AMC=180°-90°=90°$ (външен към $\triangle BDC$)

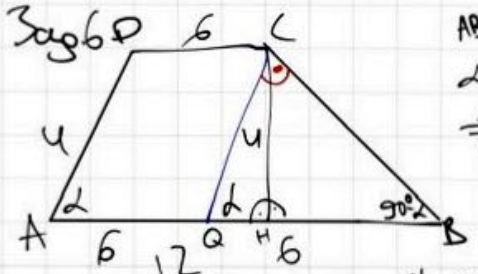
$\triangle ACM$ е $\triangle$ и равнобедрен $\Rightarrow$ ($H\equiv M\equiv L\equiv S=\frac{1}{2}AM$)

$BM\equiv CD\equiv$ усн.; $BM=DC=b$

$\Rightarrow AM=a+b=16$ cm

$S_{ABCD}=\frac{a+b}{2}.h=\frac{16}{2}.8=64\,cm^2$

**Зад. 6**




$AB\parallel CD$ $AD=4$ cm

$\alpha+\beta=90°$ (по усл.)

$\Rightarrow \beta=90°-\alpha$

Построим $CQ\parallel AD$ $\Rightarrow AD=BC=CQ=4$ cm

$\alpha+90°-\alpha+x=180$

$x=90°$

по П.Т. в $\triangle QBC$

$CQ^2+BC^2=QB^2$

$4^2+BC^2=36$

$BC^2=20$

$BC=2\sqrt{5}$ cm (–2$\sqrt{5}$ не е р-е)

$S_{ABC}=\frac{ab}{2}=\frac{4.2\sqrt{5}}{2}=4\sqrt{5}\,cm^2$

Построим $CH\equiv h$

$S_{ABC}=\frac{c.h_c}{2}$

$8\sqrt{5}=6h_c$

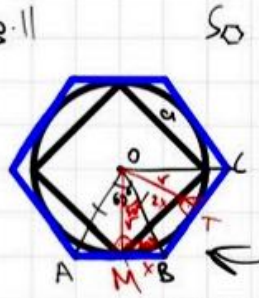
$h_c=\frac{4.2\sqrt{5}}{6}=\frac{4\sqrt{5}}{3}$ cm

$S_{ABCD}=\frac{a+b}{2}.h$

$S_{ABCD}=\frac{8+4}{2}.\frac{4\sqrt{5}}{3}$

$S_{ABCD}=12\sqrt{5}\,cm^2$

**Зад. 11**



$QF=d(c,y)$

по П.Т. $QF=a^2+a^2$

$QF=a\sqrt{2}$

$\Rightarrow r=\frac{1}{2}d=\frac{a\sqrt{2}}{2}$

$\Rightarrow MB=\frac{1}{2}OB=x\Rightarrow OB=2x$

по П.Т. в $\triangle AMO$: $x^2+\left(\frac{a\sqrt{2}}{2}\right)^2=(2x)^2$

$x^2+\frac{a^2.2}{4}=4x^2/.4$

$4x^2+2a^2=16x^2$

$12a^2=12x^2$

$x^2=a^2$

$x=\frac{a\sqrt{6}}{\sqrt{6}}=\frac{a\sqrt{6}}{6}$ cm

$\Rightarrow AB=\frac{2a}{\sqrt{6}}$

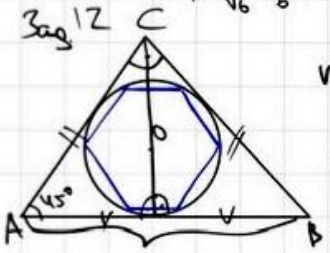
$S_{\triangle AOB}=\frac{1}{2}.\frac{2a}{\sqrt{6}}.\frac{a\sqrt{2}}{2}=\frac{a^2\sqrt{2}.\sqrt{6}}{2\sqrt{6}.\sqrt{6}}$

$S_{\triangle AOB}=\frac{a^2\sqrt{12}}{12}=\frac{2a^2\sqrt{3}}{12}=\frac{a^2\sqrt{3}}{6}$

$S_{\bigcirc}=6S_{\triangle AOB}=6.\frac{a^2\sqrt{3}}{6}$

$S_{\bigcirc}=a^2\sqrt{3}\,cm^2$

**Зад. 12**




$r=\frac{a+b-c}{2}$

$a=b=r$

по П.Т. $x^2+x^2=c^2$

$2x^2=c^2$

$x=\frac{c}{\sqrt{2}}=\frac{c\sqrt{2}}{2}$

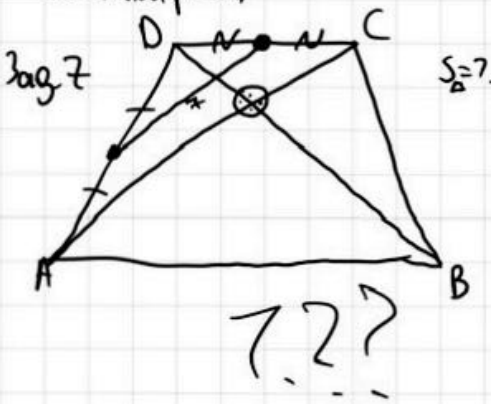
$\Rightarrow r=\frac{\frac{c\sqrt{2}}{2}+\frac{c\sqrt{2}}{2}-c}{2}=\frac{c(\sqrt{2}-1)}{2}$

$m\equiv$ страна на 6-ъгъл

$\Rightarrow m=r=\frac{c(\sqrt{2}-1)}{2}$

Домашна работа

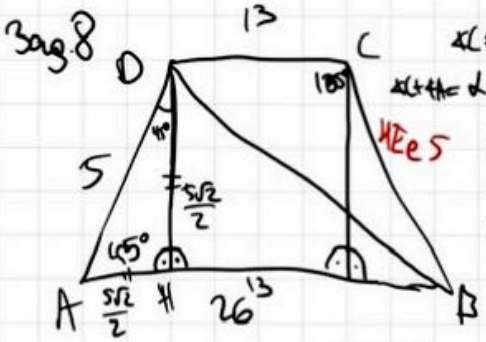
Зад.7




$S_\triangle = ?$

???

Зад.8




НЕе 5

$\angle C = \alpha \quad \angle C - \angle A = 90^\circ \Rightarrow \angle A = \alpha - 90^\circ$

$\angle C + \angle A = \alpha + \alpha - 90^\circ = 180^\circ$

$2\alpha = 270^\circ$

$\alpha = 135$

$\Rightarrow \angle A = 45^\circ$

по п.т. в $\triangle ADH$

$2DH^2 = 25$

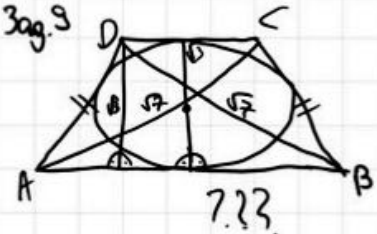
$DH = \frac{5\sqrt{2}}{2} \equiv h$

$\Rightarrow S_\triangle = \frac{39}{2} \cdot \frac{5\sqrt{2}}{2}$

$S_\triangle = \frac{19 \cdot 5\sqrt{2}}{4}$

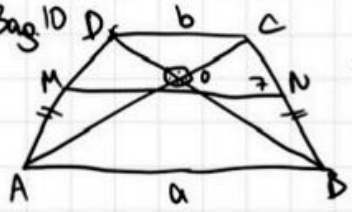
???

Зад.9




???

Зад.10




$a + b = 14 \quad a = 14 - b$

$\triangle ABO \sim \triangle CDO$

$\frac{AB}{CD} = \frac{AO}{CO} = \frac{BO}{DO} = \frac{14-b}{b}$

$AO = BO$

$CO = DO$

$\Rightarrow (14-b)x + bx = AC$

$14x - bx + bx = AC$

$S_\triangle = \frac{1}{2} d_1 d_2 \sin 90^\circ$

$S_\triangle = \frac{1}{2} \cdot 14x \cdot 14x$

$S_\triangle = 98x^2$

???

$10517 \div k-1; k; k+1$

$P = 3k$

$p = \frac{3k}{2}$

$S_\triangle = \sqrt{\frac{3k}{2}\left(\frac{3k}{2} - k + 1\right)\left(\frac{3k}{2} - k\right)\left(\frac{3k}{2} - k - 1\right)}$

$S_\triangle = \sqrt{\frac{3k}{2} \cdot \frac{k+2}{2} \cdot \frac{k}{2} \cdot \frac{k-2}{2}}$

$S_\triangle = \frac{\sqrt{3k^2(k^2-4)}}{4}$

$S_\triangle = \frac{\sqrt{3k^4 - 12k^2}}{4}$

$84 = \frac{k\sqrt{3k^2-12}}{4}$

$336^2 = \sqrt{3k^4 - 12k^2}^2$

$112896 = 3k^4 - 12k^2 \quad /:3$

$k^4 - 4k^2 - 37632 = 0$

Пол. $k^2 = n$

$n^2 - 4n - 37632 = 0$

$D = 16 + 4 \cdot 37632 = 4(4 + 37632)$

$\sqrt{D} = 2\sqrt{37636} = 2 \cdot 194 = 388$

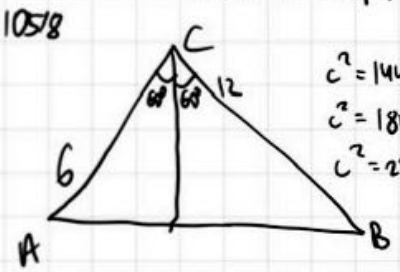
$n_1 = \frac{4+388}{2} = \frac{392}{2} = 196$

$n_2 < 0 \Rightarrow$ не е р-е

$k^2 = n$

$\Rightarrow k = 14$ ($k = -14$ не е р-е) $\Rightarrow$ Страните са 13, 14 и 15 см

10518




$c^2 = 144 + 36 - 2 \cdot 12 \cdot 6 \cdot \left(-\frac{1}{2}\right)$

$c^2 = 180 + 72$

$c^2 = 252$

$c = 6\sqrt{7}$

$l_c^2 = ab \cdot mn$

$\frac{m}{n} = \frac{1}{2}$

$3x = 6\sqrt{7}$

$x = 2\sqrt{7}$

$\Rightarrow m = 2\sqrt{7}$

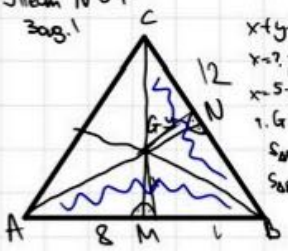
$n = 4\sqrt{7}$

$l_c^2 = 72 - 56$

$l_c = 4$ см

Тест №57

Зад.1




$x + y = 5$ см

$x = ?; y = ?$

$x = 5 - y$

т. G = медицентор

$S_{\triangle ABM} = \frac{AB \cdot MG}{2} = 4 \cdot (5-y) = 20 - 4y$

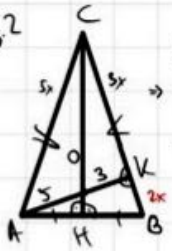
$S_{\triangle BCM} = \frac{BC \cdot MG}{2} = 6y$

$6y = 20 - 4y$

$10y = 20$

$\Rightarrow y = 2$ см; $x = 3$ см – в)

Зад.2




$CO \equiv l$

$\Rightarrow \frac{CA}{CK} = \frac{OA}{OK} = \frac{5}{3}$

$\Rightarrow CA = 5x; \; CK = 3x$

По п.т. в $\triangle AKC$

$(5x)^2 = 8^2 + 13x^2$

$16x^2 = 64$

$x = 2 \Rightarrow AC = BC = 10$ см

По п.т. в $\triangle ABK$, $AB = 4\sqrt{5}$

$S_\triangle = \frac{1}{2} AB \cdot CH = \frac{1}{2} BC \cdot AK$

$2\sqrt{5}\, CH = 5 \cdot 8$

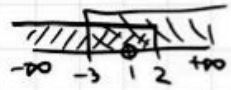
$CH = \frac{40}{2\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$ – б)

Учив. тест

Зад.3 ДС за $a$ $\frac{\sqrt{2-a}}{2-\sqrt{a+3}}$

ДС $\left|\begin{array}{l} 2-a\geq 0 \\ 2-\sqrt{a+3}\neq 0 \\ a+3\geq 0 \end{array}\right.$

$\left|\begin{array}{l} a\leq 2 \\ a+3\neq 4 \Rightarrow a\neq 1 \\ a\geq -3 \end{array}\right.$




$a\in[-3;1)\cup(1;2]$ – в)

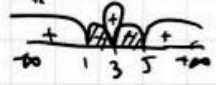
Зад.4 $\frac{1}{1-x}<\frac{x-4}{5-x}$ $\quad 5-x \quad 1-x$

$\frac{5-x-(5x-x^2-4)}{(5-x)(1-x)}<0$ ⊖

$\frac{x^2-6x+9}{(5-x)(1-x)}<0$

⊕ $x^2-6x+9=0$ $\quad$ ⊖⊖=⊕ $5-x=0$ $\quad 1-x=0$

$\ldots x_{1,2}=3$ $\quad x=5$ $\quad x=1$




$x\in(1;3)\cup(3;5)$

Зад.5 $p=\log_6 3$; $q=\log_6 5$; $\log_{45}12=?$

$\log_{45}12=\frac{\log_6 12}{\log_6 45}=\frac{\log_6\frac{36}{3}}{\log_6 9+\log_6 5}=\frac{\overset{2}{\log_6 36}+\overset{p}{\log_6 3}}{2\underset{p}{\log_6 3}+\underset{q}{\log_6 5}}=\frac{2+p}{2p+q}$ – г)

Зад.6 $+\left|\begin{array}{l} xy+4x+y=0 \;/\cdot(-2) \\ xy+3x+2y=0 \end{array}\right.$

$-2xy-8x-2y$

$x(5-y)=0$

$-x=\frac{0}{5-y}$

???

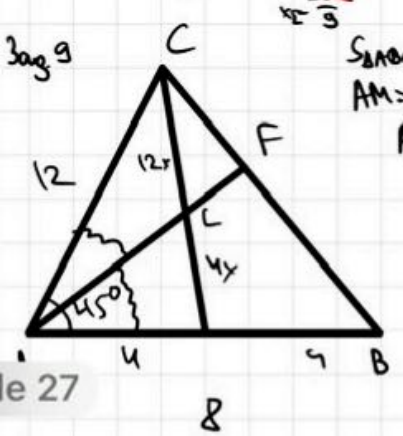
$x_1=0 \Rightarrow y_1=0$

~~$y_2=5$ $\quad 5x+4x+5=0$~~

~~$9x=-5$~~

~~$x_2=\frac{-5}{9}$~~

Зад.9



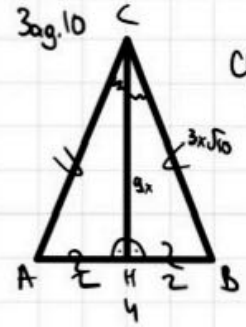

$S_{\Delta ABC}=\frac{8\cdot 12\cdot\frac{\sqrt{2}}{2}}{2}=24\sqrt{2}\,cm^2$

$AM=\frac{1}{2}AB \Rightarrow S_{\Delta AMC}=\frac{1}{2}S_{\Delta ABC}=12\sqrt{2}\,cm^2$

$AL\equiv l \Rightarrow ML=4x$; $LC=12x$

$LM=\frac{4}{16}MC \Rightarrow S_{\Delta AML}=\frac{1}{4}S_{\Delta AMC}=3\sqrt{2}\,cm^2$

Зад.10




$CH\equiv h\equiv l\equiv m\equiv s$

$R:CH=5:9 \Rightarrow R=5x$; $h_c=9x$

$S_{\Delta ABC}=\frac{ch_c}{2}=2\cdot 9x=18x$

$S_{\Delta ABC}=\frac{abc}{4R} \Rightarrow 5x=\frac{a^2\cdot 4}{4\cdot 18x}$

$90x^2=a^2$

$a=3x\sqrt{10}$

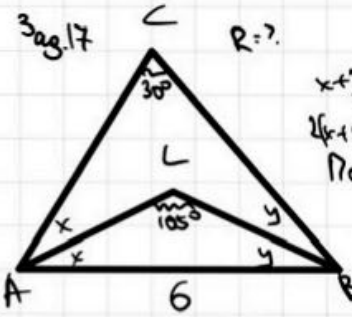
по П.Т. в $\Delta BCH$ $\quad 90x^2=4+81x^2$

$9x^2=4$

$x=\frac{2}{3} \Rightarrow h_c=6\,cm$

$\Rightarrow S_{\Delta ABC}=\frac{2\cdot 6}{2}=12\,cm^2$ – г)

Зад.17 $\quad R=?$




$x+y=75°$

$2(x+y)=150 \Rightarrow \angle C=30°$

По $\sin\Gamma$ $\quad \frac{c}{\sin\gamma}=2R$

$\frac{6}{\frac{1}{2}}=2R \Rightarrow R=6\,cm^2$

$CH\equiv h\equiv m\equiv s\equiv l$

Зад.18 $\quad CD=?$

по П.Т. в $\Delta CHB$

$\frac{121}{4}+CH^2=49/\cdot 4$

$4CH^2=196-121$

$CH^2=\frac{75}{4}$

$CH=\frac{5\sqrt{3}}{2}\,cm$

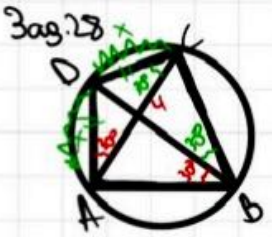
по П.Т. $\Delta CHD$

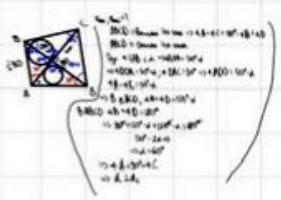
$\frac{25}{4}+\frac{75}{4}=CD^2$

$CD=\frac{10}{2}=5\,cm$

# ДЗИ 2014г.

Зад.28



} това направих аз

$\measuredangle CAD = 30^\circ \Rightarrow \overset{\frown}{DC} = 60^\circ$

и $\measuredangle DBC = 30^\circ = \frac{1}{2}\overset{\frown}{DC}$

аналог. $\measuredangle DBA = 30^\circ \Rightarrow \overset{\frown}{DA} = 60^\circ$

$\Rightarrow \measuredangle DCA = 30^\circ \Rightarrow AD = DC$; $\Rightarrow \measuredangle ADC = 120^\circ$

Озн. $AD = DC = x$

По cos Т $\triangle DCA$

$16 = x^2 + x^2 - 2x^2 \cdot \left(-\frac{1}{2}\right)$

$16 = 2x^2 + x^2$

$3x^2 = 16$

$x = \frac{4\sqrt{3}}{3} \Rightarrow AD = DC = \frac{4\sqrt{3}}{3}$

$ABCD \equiv$ описан 4$\measuredangle$-ник

$AB + DC = AD + BC$

$AB - BC = AD - DC$

, но $AD = DC$

$\Rightarrow AB \cdot BC = 0$

$\Rightarrow AB = BC$; $\measuredangle ABC = 60^\circ$

$\Rightarrow \triangle ABC \equiv$ равностранен $\triangle$

$p_{\triangle ABC} = \frac{3 \cdot 4}{2} = 6$ cm

$\Rightarrow S_{\triangle ABC} = \sqrt{6 \cdot 2^3} = 4\sqrt{3}$ cm$^2$

$r = \frac{S}{p} = \frac{{}^{2}4\sqrt{3}}{\cancel{6}_{3}} = \frac{2\sqrt{3}}{3}$ cm

$p_{\triangle DAC} = \frac{\frac{4\sqrt{3}}{3} \cdot 2 + 4}{2} = \frac{\frac{8\sqrt{3}+12}{3}}{2} = \frac{{}^{2}\cancel{4}(2\sqrt{3}+3)}{\cancel{6}_{3}} = \frac{4\sqrt{3}+6}{3}$ cm

$S_{\triangle DAC} = \frac{1}{\cancel{2}} \cdot \frac{4\sqrt{3}}{3} \cdot \overset{\frac{1}{2}}{\cancel{4}} \cdot \frac{1}{\cancel{2}} = \frac{4\sqrt{3}}{3}$ cm$^2$ ($\sin 30^\circ$)

$r = \frac{S_{\triangle DAC}}{p_{\triangle DAC}} = \frac{4\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{4\sqrt{3}+6}$

$r = \frac{4\sqrt{3}}{4\sqrt{3}+6} \cdot \frac{4\sqrt{3}-6}{4\sqrt{3}-6} = \frac{48 - 24\sqrt{3}}{12} = \frac{-\cancel{12}(4-2\sqrt{3})}{\cancel{12}} = 4 - 2\sqrt{3}$ cm

Зад.27 червени – 12 | усълти – $\frac{1}{3}$

сини – $x$ | сини – $\frac{2}{5}$

усълти – $y$

$\frac{y}{12+x+y} = \frac{1}{3}$ $\quad \frac{x}{12+x+y} = \frac{2}{5}$

$3y = 12 + x + y$

$x = 2y - 12$

$5x = 24 + 2x + 2y$

$5(2y-12) = 24 + 2(2y-12) + 2y$

$10y - 60 = \cancel{24} + 4y - \cancel{24} + 2y$

$4y = 60$

$y = 15$ усълти топчета

$x = 2y - 12 = 30 - 12 = 18$ сини топчета

$p(A) = \frac{m}{n}$

$n = C_{45}^{3} = \frac{45 \cdot 44 \cdot 43}{3 \cdot 2 \cdot 1}$

$m = 12 \cdot 15 \cdot 18$

$p(A) = \frac{{}^{3}\cancel{12} \cdot \cancel{15} \cdot 18 \cdot \cancel{3} \cdot 2 \cdot 1}{\cancel{45} \cdot \underset{11}{\cancel{44}} \cdot 43} = \frac{36 \cdot 3}{11 \cdot 43} = \frac{108}{473}$

Зад.26 $(x^2 - 3x + 1)^2 - 4(x^2 - 3x) = 9$

Полагаме $x^2 - 3x = t$

$t^2 + 2t + 1 - 4t - 9 = 0$

$t^2 - 2t - 8 = 0$

$D = 4 + 32 = 36$

$t_1 = \frac{2+6}{2} = 4$

$t_2 = -2$

$\Rightarrow x^2 - 3x - 4 = 0 \quad \Rightarrow x^2 - 3x + 2 = 0$

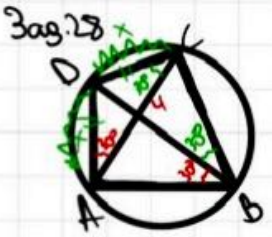
$D = 9 + 16 = 25 \quad D = 9 - 8 = 1$

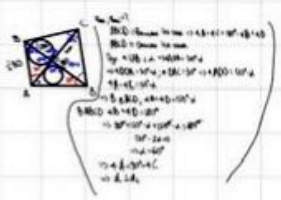
$x_1 = \frac{3+5}{2} = 4 \quad x_3 = \frac{3+1}{2} = 2$

$x_2 = \frac{3-5}{2} = -1 \quad x_4 = \frac{3-1}{2} = 1$

# ДЗИ 2014г.

Зад.28



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Зад.27 червени – 12 | усълти – $\frac{1}{3}$

сини – $x$ | сини – $\frac{2}{5}$

усълти – $y$

$\frac{y}{12+x+y} = \frac{1}{3}$ $\quad \frac{x}{12+x+y} = \frac{2}{5}$

$3y = 12 + x + y$

$x = 2y - 12$

$5x = 24 + 2x + 2y$

$5(2y-12) = 24 + 2(2y-12) + 2y$

$10y - 60 = \cancel{24} + 4y - \cancel{24} + 2y$

$4y = 60$

$y = 15$ усълти топчета

$x = 2y - 12 = 30 - 12 = 18$ сини топчета

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$p(A) = \frac{{}^{3}\cancel{12} \cdot \cancel{15} \cdot 18 \cdot \cancel{3} \cdot 2 \cdot 1}{\cancel{45} \cdot \underset{11}{\cancel{44}} \cdot 43} = \frac{36 \cdot 3}{11 \cdot 43} = \frac{108}{473}$

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$D = 9 + 16 = 25 \quad D = 9 - 8 = 1$

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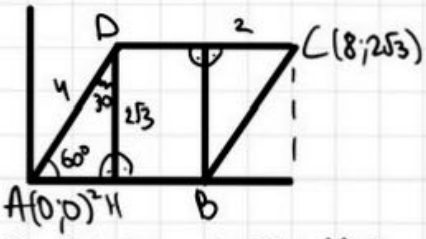
Зад. 20




Озн. $AH = x$ ; $\sphericalangle ADH = 30° \Rightarrow AD = 2x$

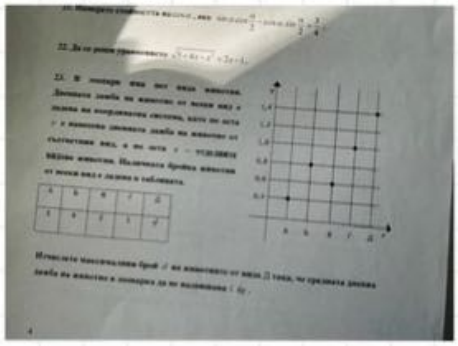
$3x^2 = 12$

$x = 2$ см ($x = -2$ не е р-е)

$\Rightarrow x_B = 8 - 2 = 6 \Rightarrow AB = 6$ см. ед.

$S_{ABCD} = a.h_a = 6.2\sqrt{3} = 12\sqrt{3}$ см$^2$

Зад. 23



$$\frac{3.0{,}4 + 4.0{,}8 + 2.0{,}6 + 1.1 + d.1{,}4}{3+4+2+1+d} \leq 1$$

$$\frac{1{,}2 + 3{,}2 + 1{,}2 + 1 + 1{,}4d}{10+d} \leq 1$$

$$\frac{6{,}6 + 1{,}4d}{10+d} \leq 1 \quad / .(10+d)$$

$6{,}6 + 1{,}4d \leq 10 + d$

$0{,}4d \leq 3{,}4 \quad / .10$

$4d \leq 34$

$d \leq \frac{34}{4}$

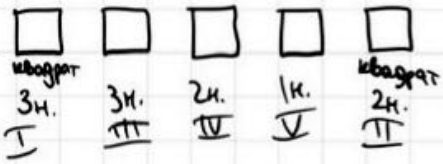
$d \leq 8{,}5$

$\Rightarrow$ максималният брой е 8

Зад. 27 5 фиг. в редица → (ляво дясно)

квадратни – 3 (с равн. страни)

кръгли – 2 (с равн. радиуси)



I, II, III, IV, V → поредност по която разбр. на картини

$\Rightarrow 3.3.2.1.2 = 6.6 = 36$ начина

# УРАВНЕНИЯ С ПОЛАГАНЕ

① $\frac{2x}{2x^2-3x-5}+\frac{6x}{2x^2+2x-5}=3$

Делим числителя и знаменателя на $x$

$\frac{\frac{2x}{x}}{\frac{2x^2}{x}-\frac{3x}{x}-\frac{5}{x}}+\frac{\frac{6x}{x}}{\frac{2x^2}{x}+\frac{2x}{x}-\frac{5}{x}}=3$

$\frac{2}{2x-3-\frac{5}{x}}+\frac{6}{2x+2-\frac{5}{x}}=3$

Полагаме $2x-\frac{5}{x}=t$

$\frac{2(t+2)}{t-3}+\frac{6}{t+2}=3 \quad /\cdot(t-3)(t+2)$

$2t+4+6t+18=3t^2-3t+18$

$3t^2-11t-4=0$

$D=121+48=169$

$t_1=\frac{11+13}{6}=4$

$t_2=-\frac{1}{3}$

$2x-\frac{5}{x}=4 \quad /\cdot x$ $\qquad 2x-\frac{5}{x}=-\frac{1}{3} \quad /\cdot 3x$

$2x^2-4x-5=0$ $\qquad 6x^2+x-15=0$

$D=16+40=56$ $\qquad D=1+360=361$

$x_{1,2}=\frac{4\pm\sqrt{56}}{4}=\frac{2\pm\sqrt{14}}{2}$ $\qquad x_3=\frac{-1+19}{12}=\frac{3}{2}$

$x_4=\frac{-1-19}{12}=-\frac{5}{3}$

② $\frac{x^2-10x+15}{x^2-6x+15}=\frac{3x}{x^2-8x+15}$

Делим на $x$ и числителя и знаменателя

$\frac{x-10+\frac{15}{x}}{x-6+\frac{15}{x}}=\frac{3}{x-8+\frac{15}{x}}$

Полагаме $x+\frac{15}{x}=t$

$\frac{t-10}{t-6}=\frac{3}{t-8} \quad /(t-6)(t-8)$

$t^2-18t+80=3t-18$

$t^2-21t+98=0$

$D=441-392=49$

$t_1=\frac{21+7}{2}=14$

$t_2=\frac{21-7}{2}=7$

$x+\frac{15}{x}=14 \quad /\cdot x$ $\qquad x+\frac{15}{x}=7 \quad /\cdot x$

$x^2-14x+15=0$ $\qquad x^2-7x+15=0$

$D=196-60=136$ $\qquad D=49-60<0$

$x_{1,2}=7\pm\sqrt{34}$ $\qquad \Rightarrow$ н.р-е

③ $(x+2)(x+3)(x+8)(x+12)=4x^2$

$(x^2+14x+24)(x^2+11x+24)=4x^2 \quad /:x^2$

$\left(x+14+\frac{24}{x}\right)\left(x+11+\frac{24}{x}\right)=4$

Полагаме $x+\frac{24}{x}=t$

$(t+14)(t+11)=4$

$t^2+25t+154=4$

$t^2+25t+150=0$

$D=625-600=25$

$t_1=\frac{-25+5}{2}=-10$

$t_2=-15$

$x+\frac{24}{x}=-10 \quad /\cdot x$ $\qquad x+\frac{24}{x}=-15 \quad /\cdot x$

$x^2+10x+24=0$ $\qquad x^2+15x+24=0$

$D=100-96=4$ $\qquad D=225-96=129$

$x_1=\frac{-10+2}{2}=-4$ $\qquad x_{3,4}=\frac{-15\pm\sqrt{129}}{2}$

$x_2=-6$

# НЕРАВЕНСТВА С ПОЛАГАНЕ

$\frac{x^2+2x+1}{x^2+2x+2} + \frac{x^2+2x+2}{x^2+2x+3} < \frac{7}{6}$

Полагаме $x^2+2x=t$

$\frac{t+1}{t+2} + \frac{t+2}{t+3} < \frac{7}{6}$

$\frac{t+1}{t+2} + \frac{t+2}{t+3} - \frac{7}{6} < 0$ $\quad (t+3)(t+2)\ (t+2)(t+3)\ 6(t+3)(t+2)$

$\frac{6(t+3)(t+1)+6(t+2)(t+1)-7(t+2)(t+3)}{6(t+3)(t+2)} < 0$

$\frac{6(t^2+4t+3)+6(t^2+3t+2)-7(t^2+5t+6)}{6(t+3)(t+2)} < 0$

$\frac{6t^2+24t+18+6t^2+24t+12-7t^2-35t-42}{6(t+3)(t+2)} < 0$

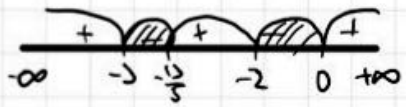
$\frac{5t^2+13t}{6(t+2)(t+3)} \le 0$ $\ominus$

$\oplus$ $5t^2+13t=0$

$t(5t+13)=0$ $\quad t+2=0$ $\quad t+3=0$

$t_1=0$ ; $5t+13=0$ $\quad t_3=-2$ $\quad t_4=-3$

$t_2=-\frac{13}{5}$




$t \in (-3; -\frac{13}{5}) \cup (-2; 0)$

$\Downarrow$

| | |
|---|---|
| $t > -3$ | $t > -2$ |
| $t < -\frac{13}{5}$ | $t < 0$ |
| $x^2+2x > -3$ | $x^2+2x > 2$ |
| $x^2+2x < -\frac{13}{5}$ | $x^2+2x < 0$ |
| $x^2+2x+3 > 0$ | $x^2+2x+2 > 0$ |
| $5x^2+10x+13 < 0$ | $x^2+2x < 0$ |

$x^2+2x+3>0$ $\oplus$ $\quad 5x^2+10x+13<0$ $\ominus$

$\oplus$ $x^2+2x+3=0$ $\quad \oplus 5x^2+10x+13=0$

$\mathcal{D}<0$ $\quad \mathcal{D}<0$

$\Rightarrow \forall x \in \mathbb{R}$ $\quad x \in \emptyset$

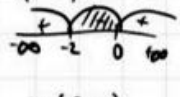
$\Rightarrow$ Няма решения

$x^2+2x+2>0$ $\oplus$ $\quad x^2+2x<0$ $\ominus$

$\oplus$ $x^2+2x+2=0$ $\quad x^2+2x=0$

$\mathcal{D}<0$ $\quad x(x+2)=0$

$x_1=0$ ; $x_2=-2$




$\forall x \in \mathbb{R}$ $\quad x \in (-2; 0)$

$\Rightarrow$ Отг. $x \in (-2; 0)$

# УРАВНЕНИЕ С ПАРАМЕТЪР

параметър – величина, постоянна само за дадена задача
↳ трябва изрично да е написано в условието

Решете уравнението, ако $k$ е параметър

① $(k-4)x=0$

I сл. Ако $k-4=0$ $\quad k=4$ $\quad 0.x=0$ $\Rightarrow \forall x \in \mathbb{R}$

II сл. Ако $k-4 \neq 0$ $\quad (k-4).x=0$ $\quad \rightarrow x=\frac{0}{k-4}$ $\quad x=0$

Отговор: При $k=4$, $\forall x \in \mathbb{R}$
При $k \neq 4$, $x=0$

② $(k-2)x=k$

I сл. Ако $k-2=0$ $\quad k=2$ $\quad 0.x=k$ $\quad x \in \emptyset$

II сл. Ако $k-2 \neq 0$ $\quad k \neq 2$ $\quad x=\frac{k}{k-2}$

Отг. При $k=2$ $x \in \emptyset$
При $k \neq 2$ $x=\frac{k}{k-2}$

③ За $k=?$, уравнението има реални корени?

а) $x^2+kx+2=0$

$\mathcal{D}=k^2-8$

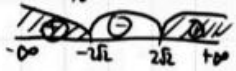
Искаме $\mathcal{D} \ge 0$

$\Rightarrow k^2-8 \ge 0$ $\oplus$

$\oplus$ $k^2-8=0$

$k^2=8$

$k_{1,2}=\pm 2\sqrt{2}$




$k \in (-\infty; -2\sqrt{2}] \cup [2\sqrt{2}; +\infty)$

б) уравнението има корени, за които единият корен е $5x$ по-голям от другия ↳ Виета

$x^2+54x+5k^2=0$

ДС Искаме $\mathcal{D} \ge 0$

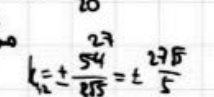
$\mathcal{D}=54^2-20k^2 \ge 0$

$\mathcal{D}=0$ $\Rightarrow x_1=x_2$ няма противоречие с условието

$54^2-20k^2=0$

$k^2=\frac{54^2}{20}$

$k_{1,2}=\pm\frac{54}{2\sqrt{5}}=\pm\frac{27\sqrt{5}}{5}$




$x_1+x_2=-54$

$x_1 \cdot x_2=5k^2$

$x_1=3x_2$

$5x_2+x_2=-54$

$6x_2=-54$

$x_2=-9$; $x_1=-45$

$-9.(-9).5=5k^2$

$k^2=81$

$k_{1,2}=\pm 9$

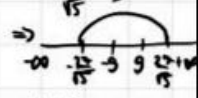
$k \in \mathbb{Z}$

$\frac{27}{\sqrt{5}}$ и $9$

$\sqrt{\frac{729}{5}}$ $\sqrt{81}$

$\frac{729}{5}$ $81$

$\Rightarrow \frac{27}{\sqrt{5}} > 9$




$\Rightarrow k \in \mathbb{Z} \Rightarrow$ са р-е

в) единият корен е с 3 по-голям от другия

$x^2 - 15x + k - 10 = 0$

Искаме $D \geq 0$

$D = 225 - 4k + 40 = 265 - 4k \geq 0$

$4k \leq 265$

$k \leq \frac{265}{4}$

$x_1 = 3 + x_2$

$3 + 2x_2 = 15$

$2x_2 = 12$

$x_2 = 6$

$\Rightarrow x_1 = 9$

$x_1 x_2 = k - 10$

$54 = k - 10$

$k = 64$

$64$ и $\frac{265}{4}$

$\frac{256}{4} < \frac{265}{4}$

$\Rightarrow k \in ДС \Rightarrow$ е р-е

г) единият е реципрочен на другия

$x^2 + 5x + k^2 + 3k + 3 = 0$

Искаме $D \geq 0$ ↳ при $x_1 = 1$ и $x_2 = 1$, $x_1 = x_2$, но $x_1 x_2 = 1$ (реципрочни)

$D = 25 - 4k^2 - 12k - 12 = -4k^2 - 12k + 13 \geq 0$ ⊕

$\ominus -4k^2 - 12k + 13 = 0$

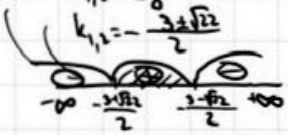
$D = 144 + 208 = 352$

$k_{1,2} = \frac{12 \pm 4\sqrt{22}}{-8}$

$k_{1,2} = -\frac{3 \pm \sqrt{22}}{2}$

$x_1 + x_2 = -\frac{b}{a} = -3$

$x_1 x_2 = \frac{c}{a} = 1 = c$




$k \in \left[\frac{3+\sqrt{22}}{2}; -\frac{3-\sqrt{22}}{2}\right]$

$x_1 \cdot x_2 = 1$ (по ус.)

$x_1 \cdot x_2 = k^2 + 3k + 3$ (Виет)

$k^2 + 3k + 3 = 1$

$k^2 + 3k + 2 = 0$

$D = 9 - 8 = 1$

$k_1 = \frac{-3+1}{2} = -1$

$k_2 = -2$

$k_1 \in ДС$

Сравняване $-1$ и $-\frac{3+\sqrt{22}}{2}$ ; $-1$ и $-\frac{3-\sqrt{22}}{2}$

$-\frac{2}{2}$ и $-\frac{3+\sqrt{22}}{2}$ ; $-\frac{2}{2}$ и $-\frac{3-\sqrt{22}}{2}$

$\Rightarrow$ Сравняване $2$ и $3+\sqrt{22}$ ; $2$ и $3-\sqrt{22}$

$2 < 3 + \sqrt{22}$ ; $4 < \sqrt{22} <$

понеже са отрицателни $\Rightarrow -1 > -\frac{3+\sqrt{22}}{2}$ ; $\Rightarrow 2 > 3 - \sqrt{22}$

понеже са отрицателни $\Rightarrow -1 < -\frac{3-\sqrt{22}}{2}$

$\Rightarrow k_1 \in ДС \Rightarrow$ е р-е

$k_2 \in ДС$

Сравняване $-2$ и $-\frac{3+\sqrt{22}}{2}$ ; $-2$ и $-\frac{3-\sqrt{22}}{2}$

$-\frac{4}{2}$ и $-\frac{3+\sqrt{22}}{2}$ ; $-\frac{4}{2}$ и $-\frac{3-\sqrt{22}}{2}$

$\Rightarrow$ Сравняване $4$ и $3+\sqrt{22}$ ; $\Rightarrow$ Сравняване $4$ и $3-\sqrt{22}$

$4 < \sqrt{22} < 5$ ; $4 > 3 - \sqrt{22}$

$\Rightarrow 4 < 3 + \sqrt{22}$ ; понеже са отрицателни $\Rightarrow -2 < -\frac{3-\sqrt{22}}{2}$

понеже са отрицателни $-2 > -\frac{3+\sqrt{22}}{2}$

$\Rightarrow k_2 \in ДС \Rightarrow$ е р-е

# УРАВНЕНИЯ С ПОЛАГАНЕ

Зад.1 $\underbrace{\frac{4x}{4x^2-8x+7}+\frac{3x}{4x^2-10x+7}}_{:x}=1$

$\frac{4}{4x-8+\frac{7}{x}}+\frac{3}{4x-10+\frac{7}{x}}=1$

Полагаме $4x+\frac{7}{x}=t$

$\frac{4}{t-8}+\frac{3}{t-10}=1 \quad t\neq 10,\ t\neq 8,\ (t-10)(t-8)$

$4t-40+3t-24=t^2-18t+80$

$t^2-25t+144=0$

$D=625-576=49$

$t_1=\frac{25-7}{2}=9$

$t_2=16$

$4x+\frac{7}{x}=9 \mid \cdot x$

$4x^2-9x+7=0$

$D=81-112<0 \Rightarrow$ н.р.-е

$4x+\frac{7}{x}=16 \mid \cdot x$

$4x^2-16x+7=0$

$D=256-112=144$

$x_1=\frac{16+12}{8}=\frac{7}{2}$

$x_2=\frac{1}{2}$

Зад.2 $(x^2-6x)^2-2(x-3)^2<81$

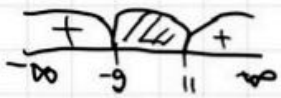
$(x^2-6x)^2-2(x^2-6x+9)<81$

Полагаме $x^2-6x=t$

$t^2-2t-18-81<0$

$t^2-2t-99<0$

$t_1=11 \quad t_2=-9$




$t\in(-9;11)$

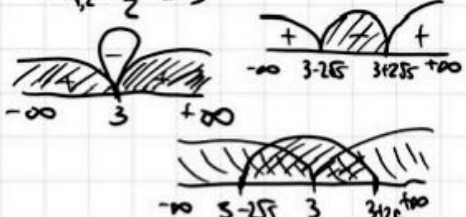
$\begin{cases} t>-9 \\ t<11 \end{cases}$

$\begin{cases} x^2-6x>-9 \\ x^2-6x<11 \end{cases}$

$x^2-6x+9>0 \quad D=36-36=0$

$\Rightarrow x_{1,2}=\frac{6}{2}=3$

$x^2-6x-11<0 \quad x_{1,2}=3\pm 2\sqrt{5}$




$x\in(3-2\sqrt{5};3)\cup(3;3+2\sqrt{5})$

Зад.3 $\frac{21}{x^2-4x+10}-x^2+4x>6$

Полагаме $x^2-4x=t$

$\frac{21}{10+t}-t-6>0 \quad t\neq -10$

$\frac{21-t^2-10t-6t-60}{10+t}>0$

$\frac{-t^2-16t-39}{10+t}>0 \mid \cdot(-1)$

$\frac{t^2+16t+39}{10+t}<0$

$t^2+16t+39=0 \quad 10+t=0$

$t_1=-3 \quad t_2=-13 \quad t_3=-10$

$x\in(-\infty;-13)\cup(-10;-3)$

$t<-13$

$x^2-4x<-13$

$x^2-4x+13<0$

$x^2-4x+13=0$

$D=16-52<0$

$\Rightarrow x\in\emptyset$

$t>-10$

$t<-3$

$x^2-4x>-10$

$x^2-4x+10>0$

$x^2-4x+10=0$

$D=16-40<0$

$x^2-4x<-3$

$x^2-4x+3<0$

$x^2-4x+3=0$

$D=16-12=4$

$x_1=1 \quad x_2=3$




$x\in(-\infty;+\infty)$

$x\in(1;3)$

$x\in(1;3)$

# НЕРАВЕНСТВА С ПАРАМЕТЪР

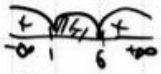
Зад.4 $k=?$ $x^2+2(k+1)x+9k-5>0$ е вярно

Искаме $D<0$ за $\forall x$

$D=4k^2+8k+4-36k+20=4k^2-28k+24<0$

$k^2-7k+6<0$

$k_1=1 \quad k_2=6$




$k\in(1;6)$

$\boxed{-}x^2+5kx+7\boxed{>0}$ $D<0 \to$ ⊖ $\Rightarrow x\in\emptyset$ → трябва ни ⊕

Зад.5 За $k=?$, вярно за $\forall x$

$(k+3)x^2-2(k+1)x+2k-4>0$

Искаме $a>0$; $D<0$

$$\begin{cases} k+3>0 \\ D<0 \end{cases}$$

$$\begin{cases} k>-3 \\ D=(-2(k+1))^2-4(k+3)(2k-4)<0 \end{cases}$$

$4k^2+\cancel{8k}+4-8k^2+\cancel{16k}-\cancel{24k}+48<0$

$-4k^2+52<0$

$-4(k^2-13)<0 \;/:(-4)$

$\oplus\; k^2-13>0 \;\oplus$

$k^2-13=0$

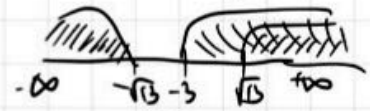
$k^2=13$

$k_1=\sqrt{13}$ $k_2=-\sqrt{13}$

$-\infty$ $-\sqrt{13}$ $\sqrt{13}$ $+\infty$

$k\in(-\infty;-\sqrt{13})\cup(\sqrt{13};+\infty)$ $3<\sqrt{13}<4$

$-\infty$ $-\sqrt{13}$ $-3$ $\sqrt{13}$ $+\infty$



$k\in(\sqrt{13};+\infty)$

Зад.6 $k=?$, положителни корени

$x^2-(k+1)x+k-7$

$$\begin{cases} D\geq 0 \\ x_1+x_2>0 \to -\frac{b}{a}>0 \\ x_1x_2>0 \to \frac{c}{a}>0 \end{cases}$$

$D=k^2+2k+1-4k+28\geq 0$

$\oplus\; k^2-2k+29\geq 0 \;\oplus$

$D=4-116<0$

$-\infty$ $+\infty$

$k+1>0$

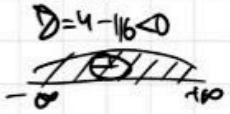
$k>-1$

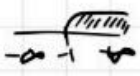
$-\infty$ $-1$ $+\infty$

$k-7>0$

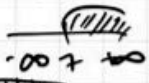
$k>7$

$-\infty$ $7$ $+\infty$





$-\infty$ $1$ $7$ $+\infty$



$x\in(7;+\infty)$

Зад.7 За $k=?$ има ⊕ корени

$(k-2)x^2-(3k+6)x+6k=0$

$$\begin{cases} D\geq 0 \\ x_1+x_2>0 \to \frac{b}{a}>0 \\ x_1\cdot x_2>0 \to \frac{c}{a}>0 \end{cases}$$

$D=9k^2+36k+36-24k^2+48k\geq 0$ $\oplus\; \frac{3k+6}{k-2}>0 \;\oplus$ $\oplus\; \frac{6k}{k-2}>0 \;\oplus$

$-15k^2+84k+36\geq 0$

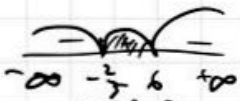
$-5k^2+28k+12\geq 0 \;\oplus$

$\ominus\; -5k^2+28k+12=0$

$D=784+240=1024$

$k_1=\frac{-28+32}{-10}=-\frac{2}{5}$

$k_2=\frac{-28-32}{-10}=6$



$k_1=-2$ $k_2=2$

$-\infty$ $-2$ $2$ $+\infty$

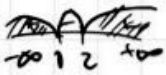
$k_1=0$ $k_2=2$

$-\infty$ $0$ $2$ $+\infty$

$-\infty$ $-\frac{2}{5}$ $6$ $+\infty$

$x\in[-\frac{2}{5};6]$

$-\infty$ $-2$ $-\frac{2}{5}$ $0$ $2$ $6$ $+\infty$



$x\in(2;6]$

# СИСТЕМИ УРАВНЕНИЯ

I начин - чрез събиране
II начин - чрез заместване

① $\begin{cases} x^2+y^2=10 \\ 2x+y=5 \Rightarrow y=5-2x \end{cases}$

$x^2+(5-2x)^2=10$
$x^2+25-20x+4x^2=10$
$5x^2-20x+15=0 \;/:5$
$x^2-4x+3=0$
$x_1=1 \quad x_2=3 \quad (1;3)$
$y_1=3 \quad y_2=-1 \quad (3;-1)$

② $\begin{cases} x-3xy+y=7 \\ x+3xy+7y=-5 \end{cases}$

$2x+8y=2 \;/:2$
$x+4y=1 \quad x=1-4y$

$1-4y-3y+12y^2+y=7$
$12y^2-6y-6=0 \;/:6$
$2y^2-y-1=0$
$D=1+8=9$
$y_1=\frac{1+3}{4}=1 \quad y_2=-\frac{1}{2}$
$x_1=-3 \quad x_2=3$
$(-3;1) \quad (3;-\frac{1}{2})$

③ $\begin{cases} x^2-2xy-3y^2=0 \\ x^2-xy-2x-3y=6 \end{cases}$

II степен, I степен, II степен, I степен
може да е навсякъде
↳ но искаме същия

всички събираеми са от едната страна на знака равенство
↗ хомогенно уравнение

$x^2-2xy-3y^2=0$
Делим на една от степените
↳ но-удобно е да е последната степен
Частно подредено.
$\frac{x^2}{y^2}-\frac{2xy}{y^2}-\frac{3y^2}{y^2}=\frac{0}{y^2}$

$\left(\frac{x}{y}\right)^2-2\left(\frac{x}{y}\right)-3=0$

Полагане $\frac{x}{y}=t$

$t^2-2t-3=0$
$t_1=3 \quad t_2=-1$
две системи

$\begin{cases} \frac{x}{y}=3 \quad x=3y \\ x^2-xy-2x-3y=6 \end{cases}$ $\qquad$ $\begin{cases} \frac{x}{y}=-1 \quad x=-y \\ x^2-xy-2x-3y=6 \end{cases}$

$9y^2-3y^2-6y-3y=6$
$6y^2-9y-6=0 \;/:3$
$2y^2-3y-2=0$
$D=9+16=25$
$y_1=\frac{3+5}{4}=2 \quad y_2=\frac{3-5}{4}=-\frac{1}{2}$
$x_1=6 \quad x_2=-\frac{3}{2}$
$(6;2) \quad (-\frac{3}{2};-\frac{1}{2})$

$y^2+y^2+2y-3y=6$
$2y^2-y-6=0$
$D=1+48=49$
$y_3=\frac{1+7}{4}=2 \quad y_4=\frac{1-7}{4}=-\frac{3}{2}$
$x_3=-2 \quad x_4=\frac{3}{2}$
$(-2;2) \quad (\frac{3}{2};-\frac{3}{2})$

④ $\begin{cases} 4x^2-3xy=10 \;/:3 \\ x^2+y^2-xy=3 \;/:(-10) \end{cases}$

$\begin{cases} 12x^2-9xy=30 \\ -10x^2-10y^2+10xy=-30 \end{cases}$ (+)

$2x^2+xy-10y^2=0$ ↦ хомогенно уравнение
$\frac{2x^2}{y^2}+\frac{xy}{y^2}-\frac{10y^2}{y^2}=\frac{0}{y^2}$

$2\left(\frac{x}{y}\right)^2+\left(\frac{x}{y}\right)-10=0$
Полагане $\frac{x}{y}=t$

$2t^2+t-10=0$
$D=1+80=81$
$t_1=\frac{-1+9}{4}=2 \quad t_2=\frac{-1-9}{4}=-\frac{5}{2}$
две системи

$\begin{cases} \frac{x}{y}=2 \Rightarrow x=2y \\ 4x^2-3xy=10 \end{cases}$ $\qquad$ $\begin{cases} \frac{x}{y}=-\frac{5}{2} \Rightarrow x=-\frac{5y}{2} \\ 4x^2-3xy=10 \end{cases}$

$16y^2-6y^2=10$
$10y^2=10$
$y^2=1$
$y_1=1 \quad y_2=-1$
$x_1=2 \quad x_2=-2$
$(2;1) \quad (-2;-1)$

$4\cdot\frac{25y^2}{4}+\frac{15y^2}{2}=10 \;/\cdot 2$

$65y^2=20 \;/:5$
$13y^2=4$
$y^2=\frac{4}{13}$
$y_3=\frac{2}{\sqrt{13}} \quad y_4=-\frac{2}{\sqrt{13}}$
$x_3=\frac{5}{\sqrt{13}} \quad x_4=\frac{5}{\sqrt{13}}$
$\left(\frac{5}{\sqrt{13}};\frac{2}{\sqrt{13}}\right) \quad \left(\frac{5}{\sqrt{13}};-\frac{2}{\sqrt{13}}\right)$

⑤ СУ 2023

$$\begin{cases} a^3-2b^3=2a^2b-ab^2 \\ (a+1)(b+1)=10 \end{cases}$$

$$\begin{cases} a^3-2b^3-2a^2b+ab^2=0 \\ ab+a+b=9 \end{cases}$$

$\frac{a^3}{b^3}-\frac{2b^3}{b^3}-\frac{2a^2b}{b^3}+\frac{ab^2}{b^3}=0$

$\left(\frac{a}{b}\right)^3-2-2\left(\frac{a}{b}\right)^2+\left(\frac{a}{b}\right)=0$

Полагаме $\frac{a}{b}=t$

$t^3-2t^2+t-2=0$

| | 1 | -2 | 1 | -2 |
|---|---|---|---|---|
| $t_1=2$ | 1 | 0 | 1 | 0 |

$t^2+1=0$

Няма решение

$\Rightarrow \frac{a}{b}=2 \Rightarrow a=2b$

$\Rightarrow 2b^2+2b+b=9$

$2b^2+3b-9=0$

$D=9+72=81$

$b_1=\frac{-3+9}{4}=\frac{3}{2}$ $b_2=\frac{-3-9}{4}=-3$

$a_1=3$ $a_2=-6$

$(3;\frac{3}{2})$ $(-6;-3)$

⑥ $\begin{cases} xy-3y=x^2+x-12 \\ y^2-xy+x+y-2=0 \end{cases}$ $\rightarrow x^2+x-12=0$ $x_1=-4$ $x_2=3$

$y(x-3)=(x+4)(x-3)$

$\begin{cases} x-3=0 \Rightarrow x=3 \\ y^2-xy+x+y-2=0 \end{cases}$ $\begin{cases} y-x-4=0 \quad x=y-4 \\ y^2-xy+x+y-2=0 \end{cases}$

$y^2-3y+3+y-2=0$

$y^2-2y+1=0$

$D=4-4=0$

$y_{1,2}=\frac{2}{2}=1$

$\Rightarrow (3;1);(3;1)$

$y^2-y^2+4y+y-4+y-2=0$

$6y-6=0$

$y=1$

$\Rightarrow x=-3$

$(-3;1)$

Задача за оценка

$$\begin{cases} x+xy+y=11 \\ x^2y+xy^2=30 \end{cases}$$

$$\begin{cases} (x+y)+xy=11 \\ xy(x+y)=30 \end{cases}$$

$x+y=11-xy$

$x+y=\frac{30}{xy}$

$x+y=x+y$

$11-xy=\frac{30}{xy}$

Полагаме $xy=t$

$11-t=\frac{30}{t}\ /.t$

$-t^2+11t=30\ /:(-1)$

$t^2-11t+30=0$

$D=121-120=1$

$t_1=\frac{11+1}{2}=6$ $t_2=\frac{11-1}{2}=5$

$\begin{cases} xy=6 \Rightarrow x=\frac{6}{y} \\ x+y+xy=11 \end{cases}$

$\frac{6}{y}+y=11-6\ /.y$

$y^2-5y+6=0$

$D=25-24=1$

$y_1=\frac{5+1}{2}=3$ $y_2=\frac{5-1}{2}=2$

$x_1=2$ $x_2=3$

$(2;3)$ $(3;2)$

$\begin{cases} xy=5 \Rightarrow x=\frac{5}{y} \\ x+y+xy=11 \end{cases}$

$\frac{5}{y}+y=11-5\ /.y$

$y^2-6y+5=0$

$D=36-20=16$

$y_1=\frac{6+4}{2}=5$ $y_2=\frac{6-4}{2}=1$

$x_1=1$ $x_2=5$

$(1;5)$ $(5;1)$

① $\frac{4}{x-1}+\frac{3x+1}{2-x}\le\frac{15x-37}{x^2-3x+2}$

$x^2-3x+2=0$

$D=9-8=1$

$x_1=\frac{3+1}{2}=2$

$x_2=\frac{3-1}{2}=1$

$\Rightarrow (x-2)(x-1)$

$\frac{4}{x-1}-\frac{3x+1}{x-2}\le\frac{15x-37}{(x-1)(x-2)}$

$\frac{4x-8-3x^2+3x-x+1-15x+37}{(x-1)(x-2)}\le 0$

$\frac{-3x^2-9x+30}{(x-1)(x-2)}\le 0 \;/:3$

$\frac{-x^2-3x+10}{(x-1)(x-2)}\le 0$ ⊖ ДС $x\ne1$ $x\ne2$

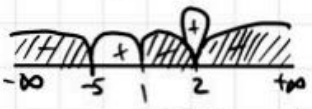
⊖ ⊕

$-x^2-3x+10=0$ $x-1=0$ $x-2=0$

$D=9+40=49$ $x_3=1$ $x_4=2$

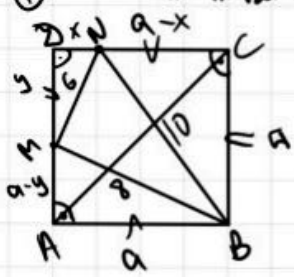
$x_1=\frac{3+7}{-2}=-5$

$x_2=\frac{3-7}{-2}=2$



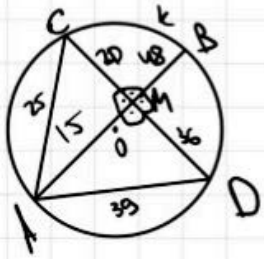

$x\in(-\infty;-5]\cup(1;2)\cup(2;+\infty)$

④ $a=?$ $AC=\sqrt{2}a=?$




② Окръжност k, радиус R. $(AB\perp CD)\equiv x$ орди

$AB\cap CD=\{M\}$; $AM=15$ $BM=48$ $CM=20$ $DM=?$; $R=?$




$MA\cdot MB=MC\cdot MD$

$\Rightarrow MD=\frac{\cancel{15}\cdot\cancel{48}^{12}}{\cancel{20}_{5}}=36$ cm

по П.Т. в $\triangle ACM$ $AC=25$ cm

по П.Т. в $\triangle ADM$ $AD=\sqrt{225+1296}=\sqrt{1521}=39$ cm

$S_{\triangle ACM}=\frac{15\cdot20}{2}=150\,cm^2$

$S_{\triangle ADM}=\frac{15\cdot\cancel{36}^{18}}{\cancel{2}}=270\,cm^2$

$\Rightarrow S_{\triangle ACD}=420\,cm^2$

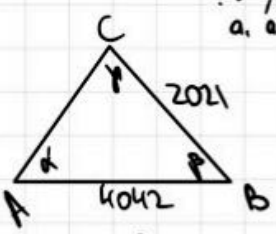
$420=\frac{25\cdot39\cdot\cancel{56}^{14}}{\cancel{4}R}$

$R=\frac{5\cdot5\cdot13\cdot3\cdot2\cdot7}{2\cdot3\cdot7\cdot2\cdot5}=\frac{65}{2}$ cm

③ В $\triangle ABC$, $\angle BAC=\alpha$; $\angle ABC=\beta$; $\angle ACB=\gamma$

$\div\alpha;\beta;\gamma$; $BC=2021$; $2\sin\alpha=\sin\left(\frac{2\pi}{3}-\alpha\right)$ ($\frac{2\pi}{3}=120^\circ$)

$a_1$ $a_1+d$ $a_1+2d$




$2\sin\alpha=\sin(120^\circ-120^\circ+\gamma)$

$S_3=\frac{\cancel{2}(a_1+d)}{\cancel{2}}\cdot3=180$

$2\sin\alpha=\sin\gamma$

$a_1+d=60^\circ=\beta$

по Sin T $\frac{2021}{\sin\alpha}=\frac{AB}{2\sin\alpha}$

$\alpha=180^\circ-\beta-\gamma=120^\circ-\gamma$

$AB=\frac{2021\cdot\cancel{2\sin\alpha}}{\cancel{\sin\alpha}}=4042$

$R_\triangle=?$

$AB=2BC\Rightarrow\alpha=30^\circ\Rightarrow\gamma=90^\circ\Rightarrow AB=d\Rightarrow r=\frac{4042}{2}=2021$ cm

Зад.1 Най-малкото число

× $3\sqrt{2}$ × $-3\sqrt{2}$ × $2\sqrt{5}$ $-2\sqrt{5}$

$\approx 3.1{,}4 \to 4{,}2$ $\approx 2.2{,}2 \to 4{,}4$

$\Rightarrow$ Отг: $-2\sqrt{5}$

Зад.2 $\frac{x^4-5x^2+4}{x^2-5x+4} = \frac{(x^2-1)(x^2-4)}{(x-1)(x-4)} = \frac{(x-1)(x+1)(x-2)(x+2)}{(x-1)(x-4)}$

$\Downarrow$

$x^4-5x^2+4=0$ $x^2-5x+4$ Отв. В) $\frac{(x+1)(x^2+4)}{x-4}$

Полагаме $x^2=t$ $x_1=4$

$t^2-5t+4=0$ $x_2=1$

$D=25-16=9$

$t_1=\frac{5+3}{2}=4$

$t_2=\frac{5-3}{2}=1$

Зад.3 $x^4-81=0$

$x^4-3^4=0$

$(x^2-3^2)(x^2+9)=0$

$(x-3)(x+3)(x^2+9)=0$

$x_1=3$ $x_2=-3$ $x^2=9$

$x_{3,4}=\pm 3$

$\Rightarrow$ ~~3+(-3)+3+(-3)~~ $=0$ -А)

Зад.4 $\frac{4x^2-x^4}{\sqrt{x+1}}>0$

ДС $x+1>0$

$x>-1$

$\frac{x^2(4-x^2)}{\sqrt{x+1}}>0$

$\frac{x^2(2-x)(2+x)}{\sqrt{x+1}}>0$ (+)

$x^2=0$ $2-x=0$ $2+x=0$ $\sqrt{x+1}^2=0^2$

$\Rightarrow x_{1,2}=0$ $x_3=2$ $x_4=-2$ $x_5=-1$

$\Rightarrow x\in(-1;0)\cup(0;2)$ -В)

Зад.5 $\begin{cases} x+y=10 \Rightarrow y=10-x \\ x\,y=25 \end{cases}$

$10x-x^2=25 \;/\cdot(-1)$

$x^2-10x+25=0$

$D=100-100=0$

$x_{1,2}=\frac{10}{2}=5$

$y_{1,2}=10-5=5$

$\Rightarrow$ Отг. $(5;5)$ -Б)

Зад.6 $\sqrt{\frac{9x-x^2-8}{4-4x+x^2}}$

ДС $\begin{cases} \frac{9x-x^2-8}{4-4x+x^2}\geq 0 \\ 4-4x+x^2\neq 0 \end{cases}$

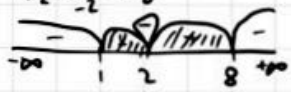
$\ominus[\oplus \frac{9x-x^2-8}{4-4x+x^2}\geq 0 \oplus]$; $4-4x+x^2\neq 0$

$9x-x^2-8=0$ $4-4x+x^2=0$ $D=16-16=0$

$D=81-32=49$ $D=16-16=0$ $x_{1,2}=\frac{4}{2}=2$

$x_1=\frac{-9+7}{-2}=1$ $x_{3,4}=\frac{4}{2}=2 \Rightarrow x\neq 2$

$x_2=\frac{-9-7}{-2}=8$




$x\in[1;2)\cup(2;8]$ -Г)

Зад.7 $\sqrt{x-1}+x^2+2=0$

$\sqrt{x-1}^2=(-x^2-2)^2$

$x-1=x^4+4x^2+4$

$x^4+4x^2-x+5=0$

| | 1 | 0 | 4 | -1 | 5 |
|---|---|---|---|---|---|
| $x_1=$ | 1 | | | | |

$\frac{\sin\alpha}{\cos\alpha}=\frac{\cos\beta}{\sin\beta}$

$\sin\alpha\sin\beta=\cos\alpha\cos\beta$