Sunflower conjecture: We have some family of sets of size r. Let's call (k, r) sunflower the following thing: k sets, which have common intersection (core), and other parts for every 2 sets (petails) are disjoint. Famous conjecture says that size of family without sunflowers can'e be no more then c_k^r for some constant c_k depending only on k.

Let k be fixed now ($k = 3$, in fact is doesn't matter much).

Old result of Erdos says that size of sunflower-free set is no more then $w^{O(w)}$. In the work is proven that size of sunflower-free set is no more then $(logw)^{O(w)}$. Brief idea:

1)We will make definion of sunflower more flexible. So definition:

Set system F is (α, β) **satisfying** , if $Pr_Y(\exists S \in F, S \subset Y) > 1 - \beta$.

Robust sunflower - Let F be set system, K - common intersection, K not if F. F is (α, β) sunflower, if F_K is α, β satisfying, where F_K is the following - we consider only sets which contain K, and F_K is image of this sets after cutting K.

Why it is useful - $(\frac{1}{r}, \frac{1}{r})$ robust sunflower contains usual r-sunflower. Proof is simple randomness argument.

Now we want to find big robust sunflower. Our idea is consider elements with weights. Let's define **weight profile** - vector $s = (1 \geq s_0 \geq s_1 \geq \ldots \geq s_k)$ of rational numbers. Then we define **weighted set system (WSS)** as set of sets with some weights.

$o(F) = \sum_{F' \subset F} w(F')$. WSS is s-bounded, if $o(F) \geq s_0$, $O(F') \leq s_{|W'|}$ for all smaller sets. Finally, weight profine is α, β **satisfying** , if any s-bounded set system is (α, β) satisfying.

Then if weight profile $(1, k, k^2, \ldots, k^l)$ is (α, β) satisfying for every l less the w, then any w-set system of size k^{-w} is α, β satisfying.

In work is proven that for $k = logw(\frac{loglogwlog(\frac{1}{\beta})}{\alpha})^{O(1)}$ this weight profile is (α, β) satisfying.

Proof is complicated induction with rebuilding weights. Then we use it with $\alpha = \beta = \frac{1}{r}$