Conflict Analysis with Minimal Cuts

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Technion

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The problem

- The goal is study how using different conflict analysis schemes affect the performance of a CDCL solver.
- The work is based on the paper:
  Lintao Zhang et al. "Efficient conflict driven learning in a Boolean satisfiability solver". In: *IEEE/ACM International Conference on Computer Aided Design. ICCAD 2001. IEEE/ACM Digest of Technical Papers (Cat. No.01CH37281)*. 2001, pp. 279–285. DOI: 10.1109/ICCAD.2001.968634

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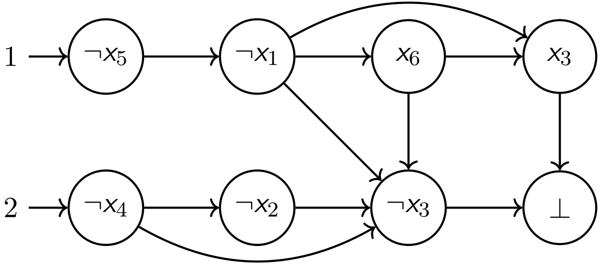

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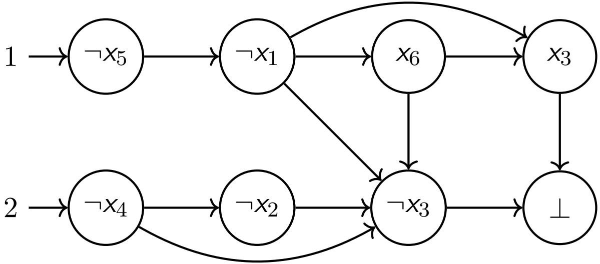

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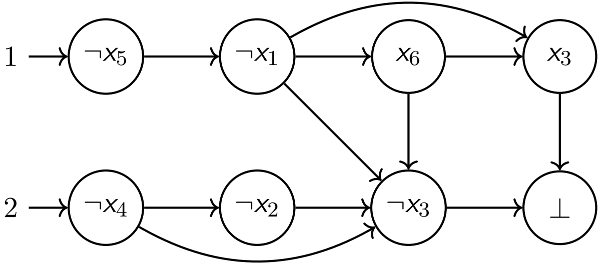

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- Among all suitable cuts, we should prioritize closest to the contradiction.
- What about minimality?

Previous results

Decision Strategy		Microprocessor Formal Verification[19]			Bounded Model Checking [18]			
		fvp-unsat.1.0(4)	sss.1.0(48)	sss.1.0a(9)	barrel (8)	longmult(16)	queueinvar(10)	satplan(20)
VSIDS	1uip	532.8	24.56	10.63	1012.62	2887.11	6.58	39.34
	2uip	746.87	27.32	16.96	641.64	2734.57	16.37	41.37
	3uip	2151.26	69.12	47.66	656.56	2946.73	19.44	57.16
	alluip	0.68(3)	1746.27(2)	79.09	1081.57(1)	11160.25	18.07	71.86
	rel_sat	2034.09	193.93	82.51	292.33(1)	5719.73	14.4	96.61
	mincut	1612.74(2)	2293.18	11.15	4119.34(1)	7321.69(5)	100.94	43.84
	grasp	2224.44	94.64	33.99	654.54	6196.82	97.82	309.03
	decision*	0(4)	1022.57(17)	227.37(3)	541.96(2)	1421.35(4)	334.39(4)	193.36(3)
FIXED	1uip_f	11.36(3)	15307.13(3)	2997.37	281.48(1)	3141.8	817.07(5)	18(2)
	2uip_f	23.07(3)	18844.51(3)	2646.99	344.34(1)	4279.07	777.2(5)	29.51(2)
	3uip_f	40.75(3)	3985.23(9)	4109.86	432.77(1)	4440.49	860.3(5)	37.62(2)
	alluip_f	0(4)	4063.44(25)	0.28(8)	699.42(1)	11375.32	2025.08(5)	1136.44(2)
	relsat_f	80.94(3)	3114.83(16)	4261.25(4)	293.09(1)	4396.73	478.37(6)	3323.71(3)
	mincut_f	0(4)	5619.4(15)	590.79(4)	3408.28(1)	5232.69(5)	3206.36(4)	373.78(2)
	grasp_f	22.51(3)	6497.99(8)	3382.47	326.46(1)	5597.1	792.12(5)	149.8(2)
	decision_f*	0(4)	415.76(42)	40.57(8)	479.61(1)	1006.58(6)	1.27(8)	600.87(10)

* timeout set to 600s instead of 3600s

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- The flow algorithm used in the paper is $\mathcal{O}(VE\log(V^2/E))$, which is probably suboptimal.

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- Can reuse a part of the graph after backtracking*.

Our plan

1. Implement a CDCL solver. ✓
2. Implement the flow algorithm. ✓
3. Befriend them.
4. Simulate all interesting conflict analysis schemes in terms of the flow.
5. Run the benchmarks from the paper.
6. Publish to the repository.