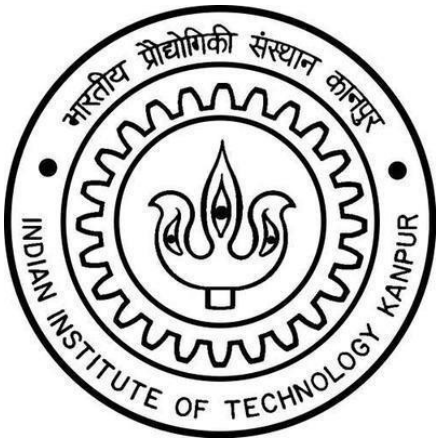

Indian Institute of Technology Kanpur

Course

AE-675: Introduction to Finite Element Method

Report On

One-Dimensional Beam Bending Problem

Submitted to

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Anand Kumar Singh (231120004)

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Project 2: One-Dimensional Code for Beam Bending Problem

Problem Statement

Write a one-dimensional finite element code using Hermite cubic shape functions with the following details for the beam bending problem.

1) Uniform cross-section: 1 cm X 1 cm
2) Length of the beam: 10 cm
3) E = 200 GPa
4) The code should be capable of handling the transverse loads of the type
    a. Concentrated/point load
    b. Uniformly distributed load
    c. Point moments at the center of the beam length only
5) Further, it should be capable of applying the appropriate combination of boundary conditions at either of the ends as:
    a. Specified transverse displacement
    b. Specified slope of the transverse displacement
    c. Shear force
    d. Bending moment

Now, take the appropriate values of loads as mentioned in Point # 4 above and perform the following finite element analysis using your code for 1, 4, 10, 50, and 100 elements.

1) Give continuous variation of transverse displacement and its slope
2) Give continuous variation of shear force and bending moment
3) Bending stress on the topmost line of the beam along its entire length.

Results and Discussion

Details for Beam Bending:

EI = 166.667 Nm^2

Shear force, V(x = L) = 600 N

Moment at center of the beam length M (x = 5 cm) = 0 N-m

Uniformly distributed load, Q = 240 N/m

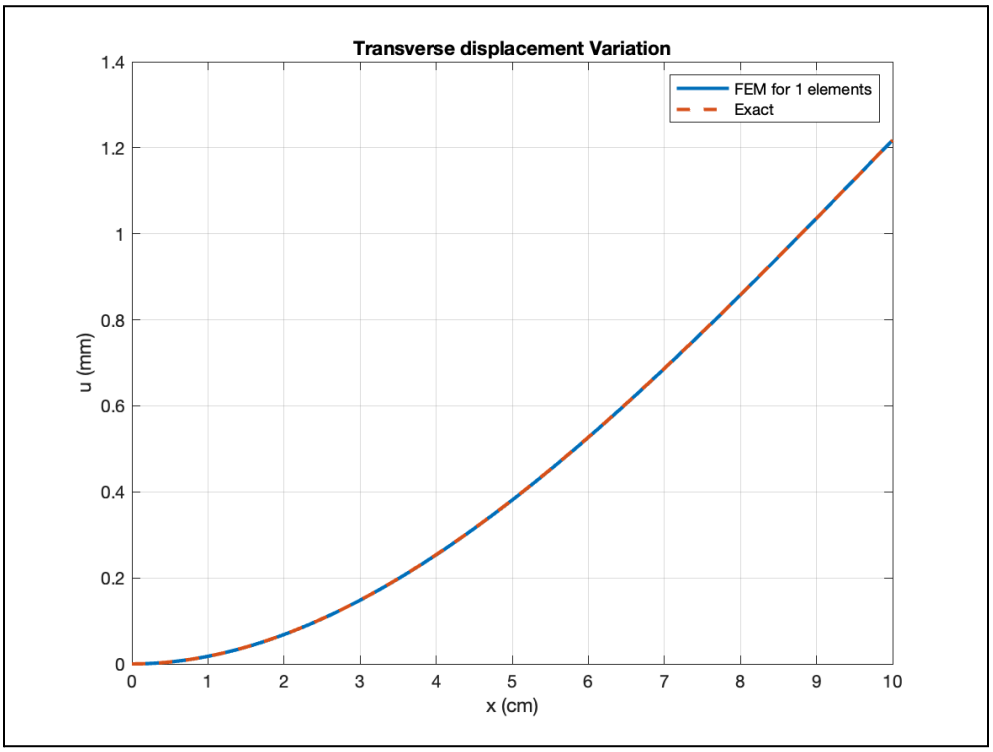
Transverse displacement, U(x = 0) = 0

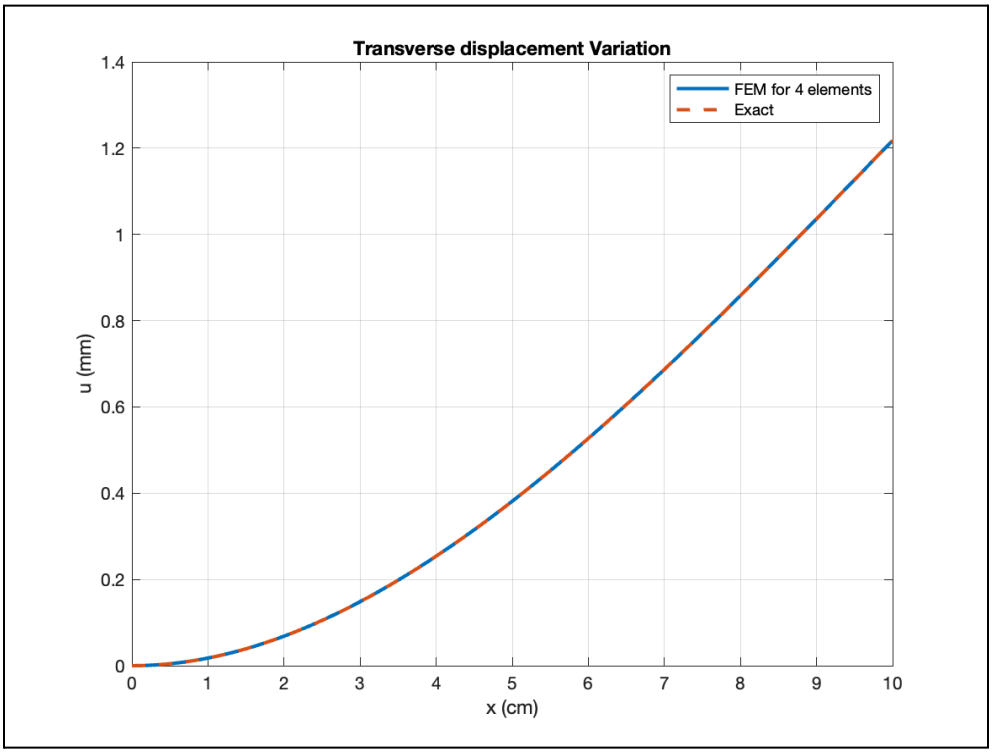
Slope of transverse displacement, du/dx(x = 0) = 0

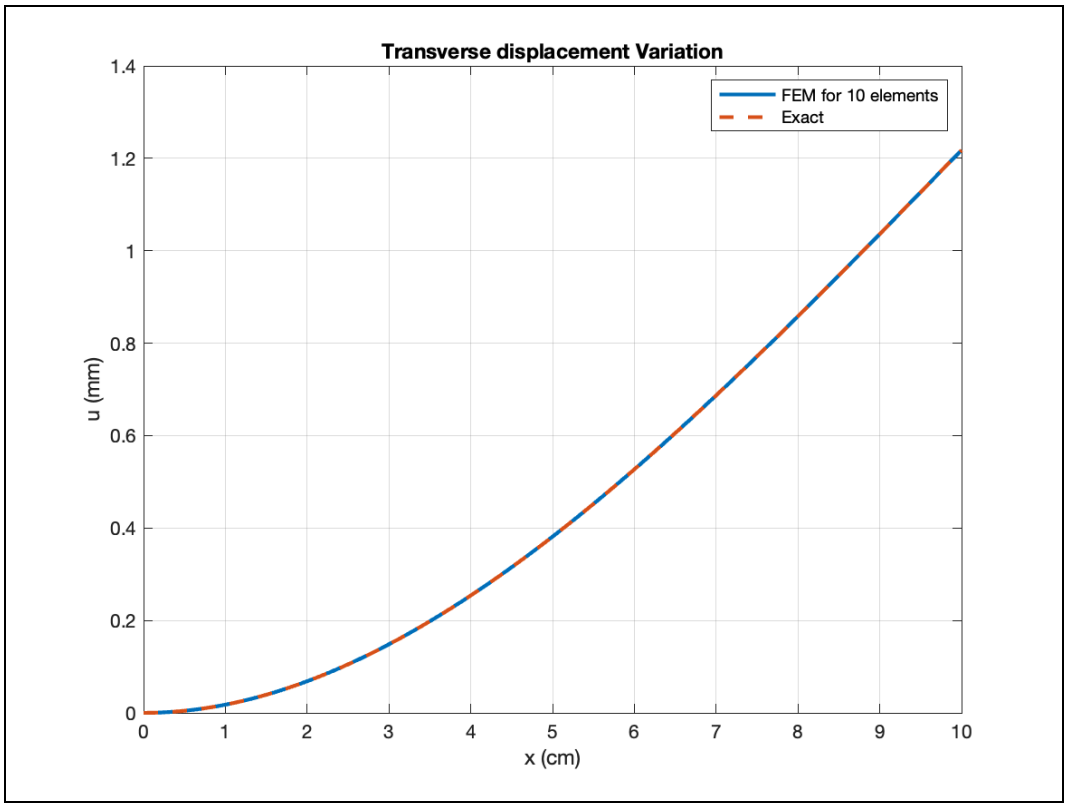
Problem 1

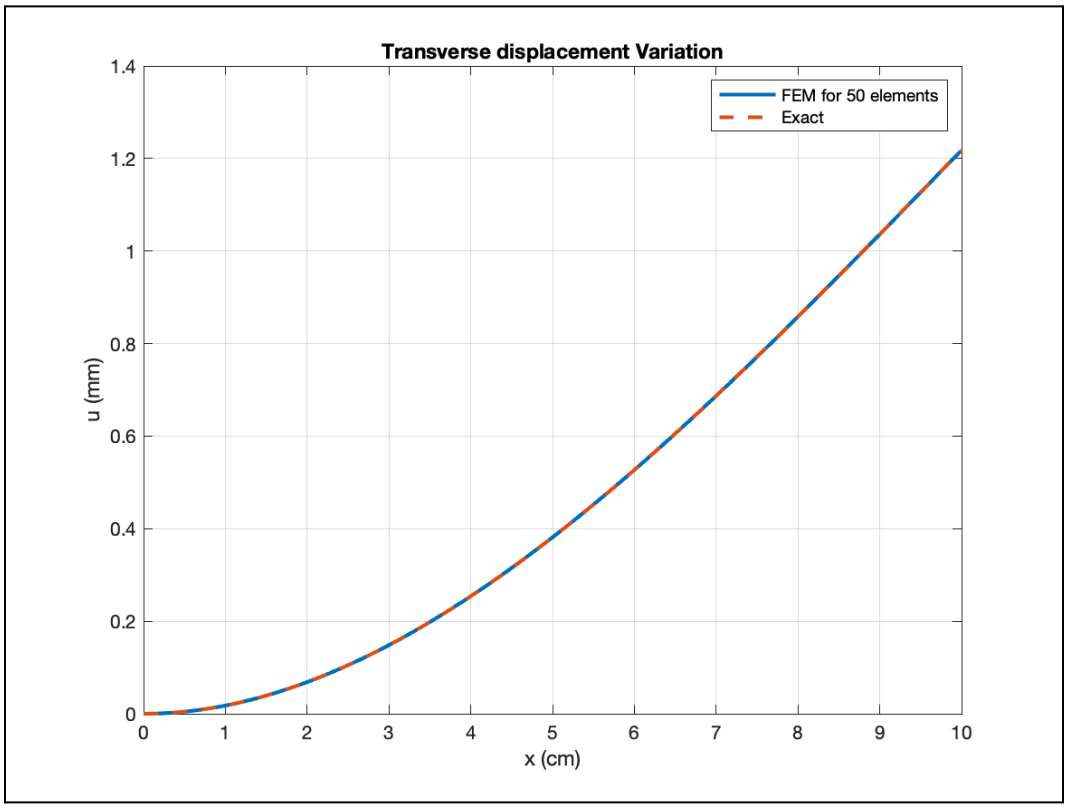
Give continuous variation of transverse displacement and its slope.

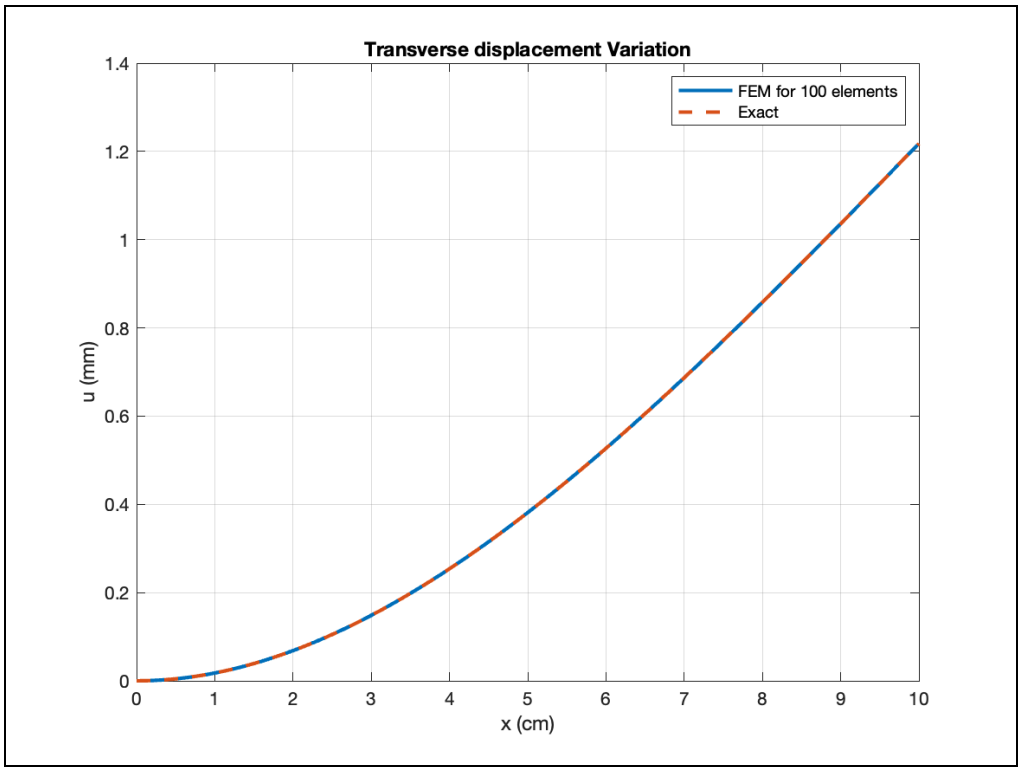
Plots for Continuous Variation of Transverse displacement for 1, 4, 10, 50 and 100 elements.



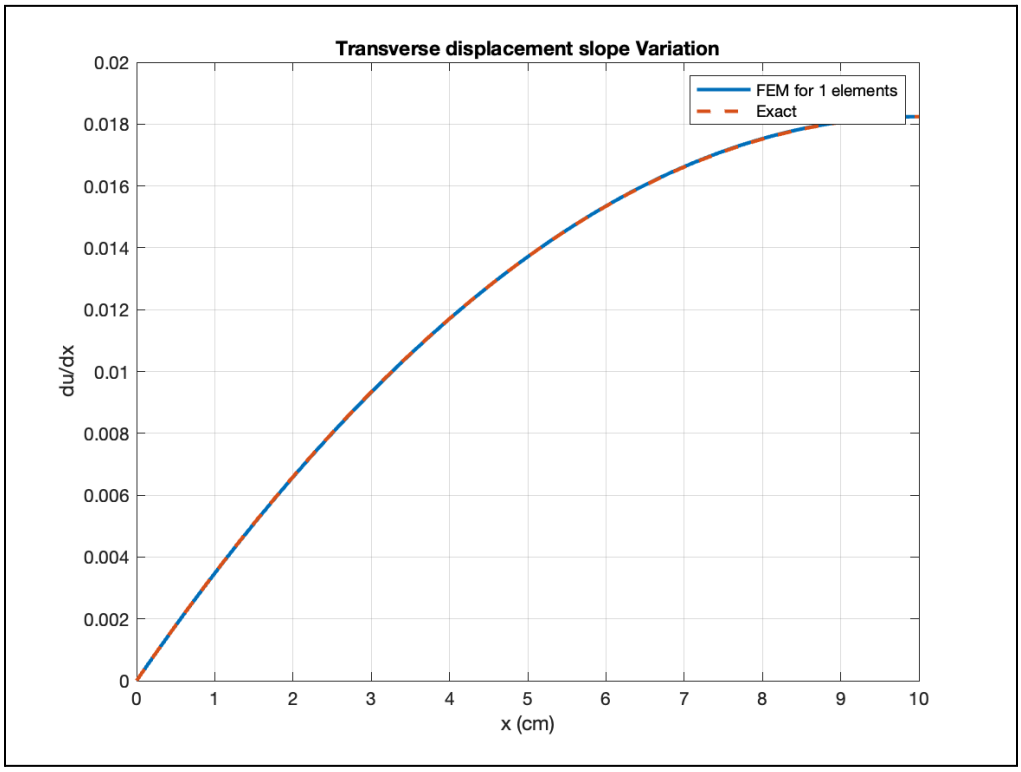


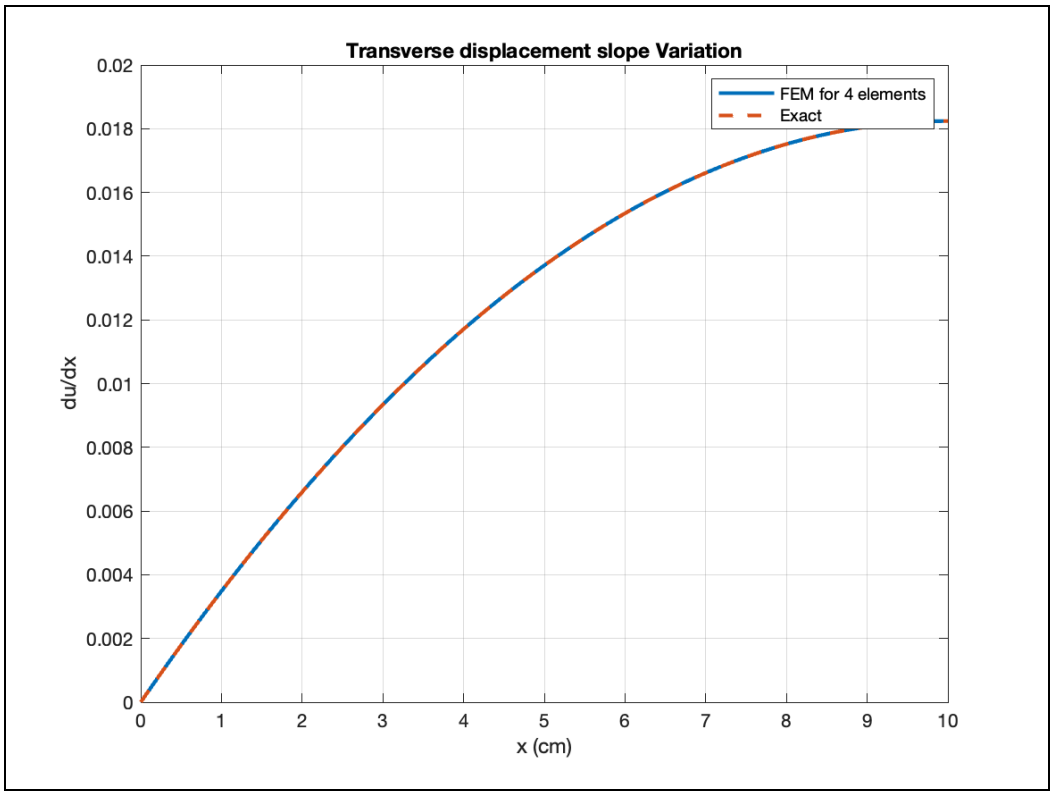

Transverse displacement Variation
FEM for 10 elements
Exact
u (mm)
x (cm)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
1
2
3
4
5
6
7
8
9
10

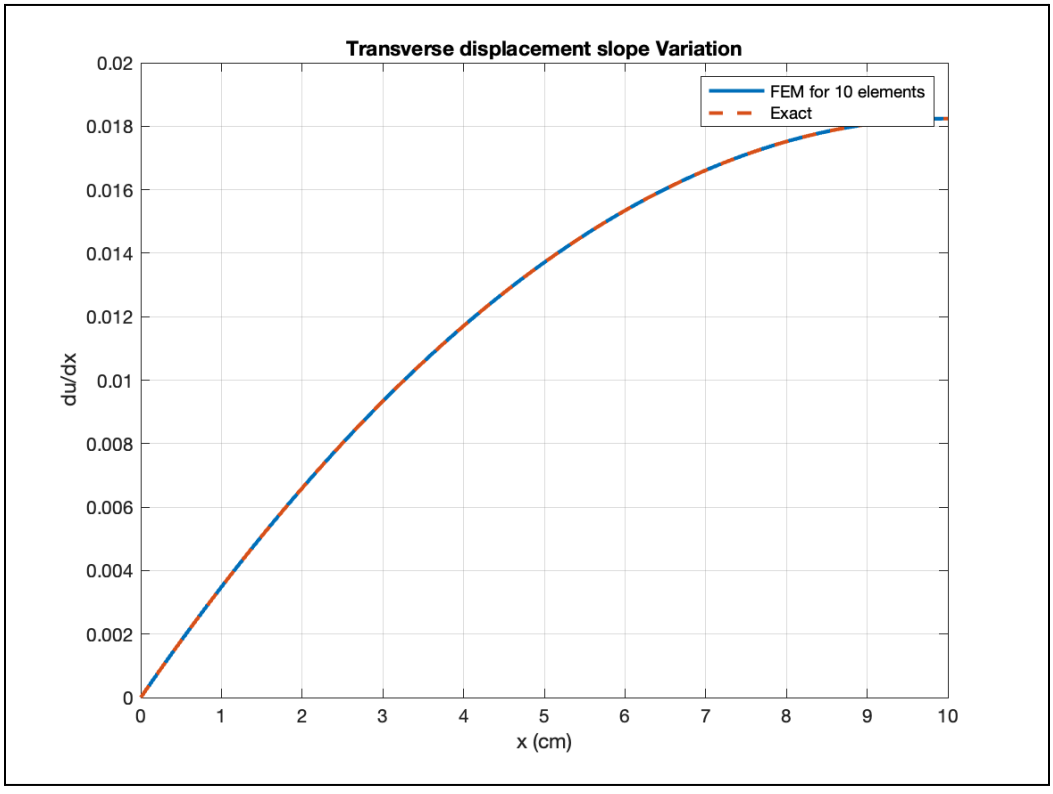

Transverse displacement Variation
FEM for 50 elements
Exact
u (mm)
x (cm)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
1
2
3
4
5
6
7
8
9
10

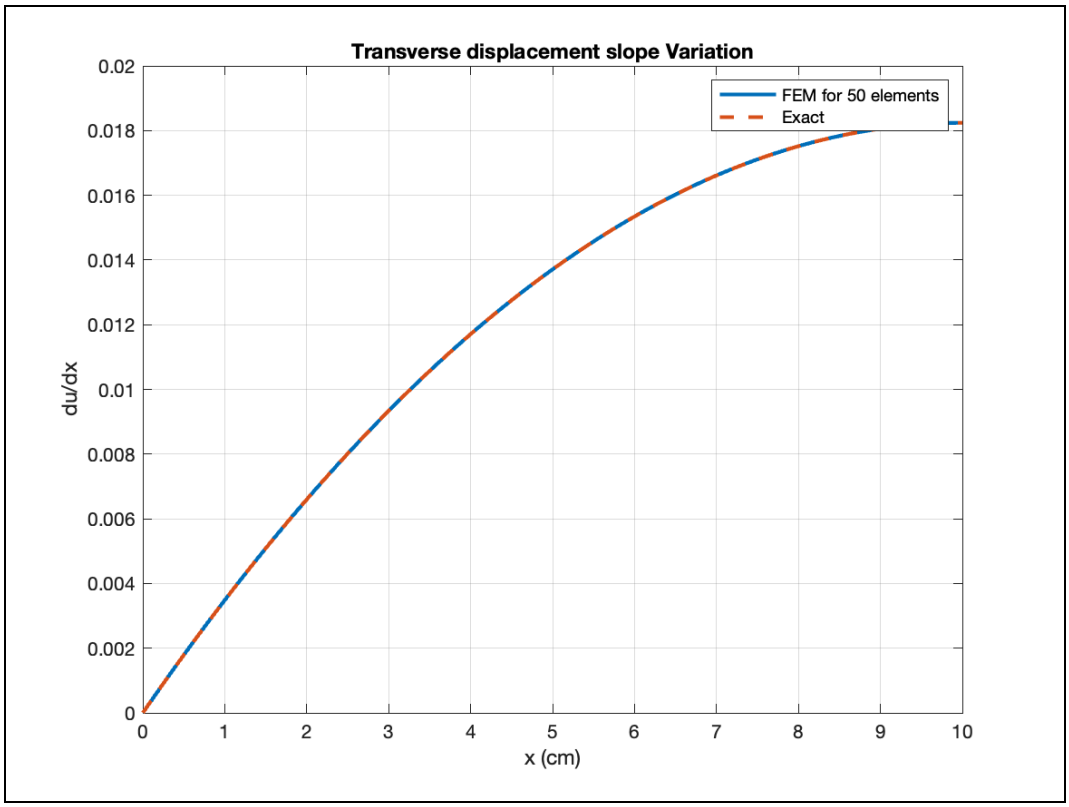


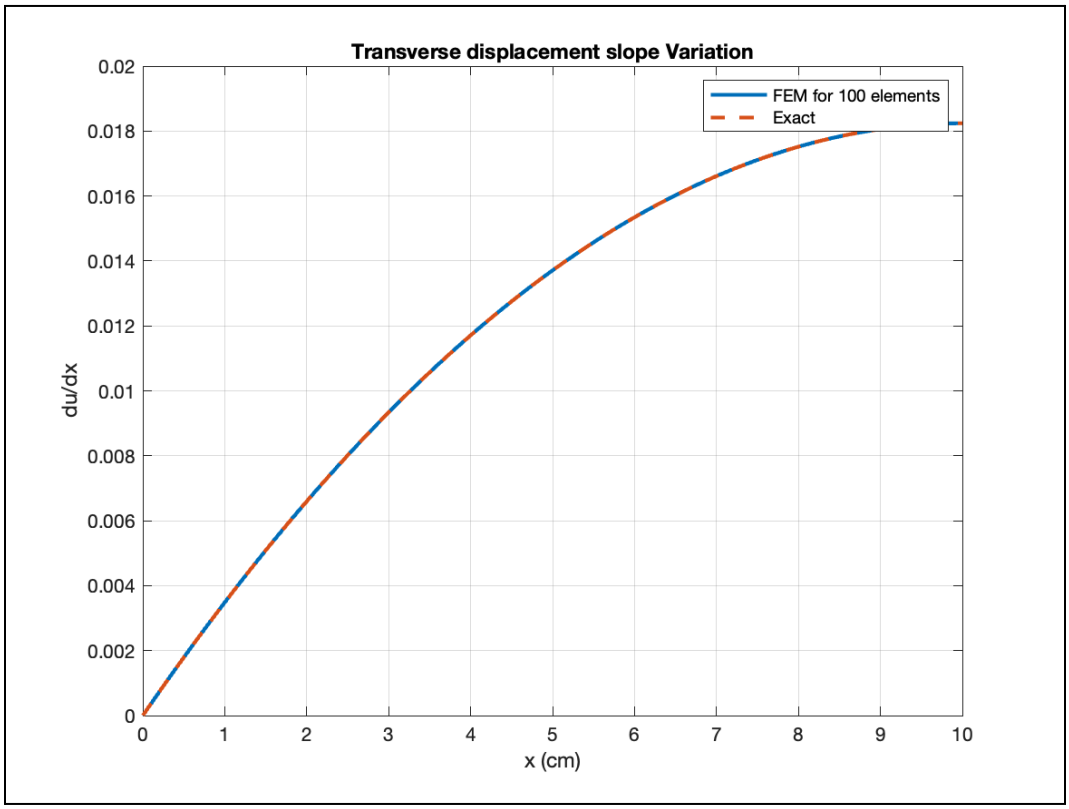
Plots for Continuous variation of the transverse displacement slope along the length for 1, 4, 10, 50, and 100 elements.




Transverse displacement slope Variation
FEM for 4 elements
Exact
du/dx
x (cm)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0 1 2 3 4 5 6 7 8 9 10


Transverse displacement slope Variation
FEM for 10 elements
Exact
du/dx
x (cm)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0 1 2 3 4 5 6 7 8 9 10

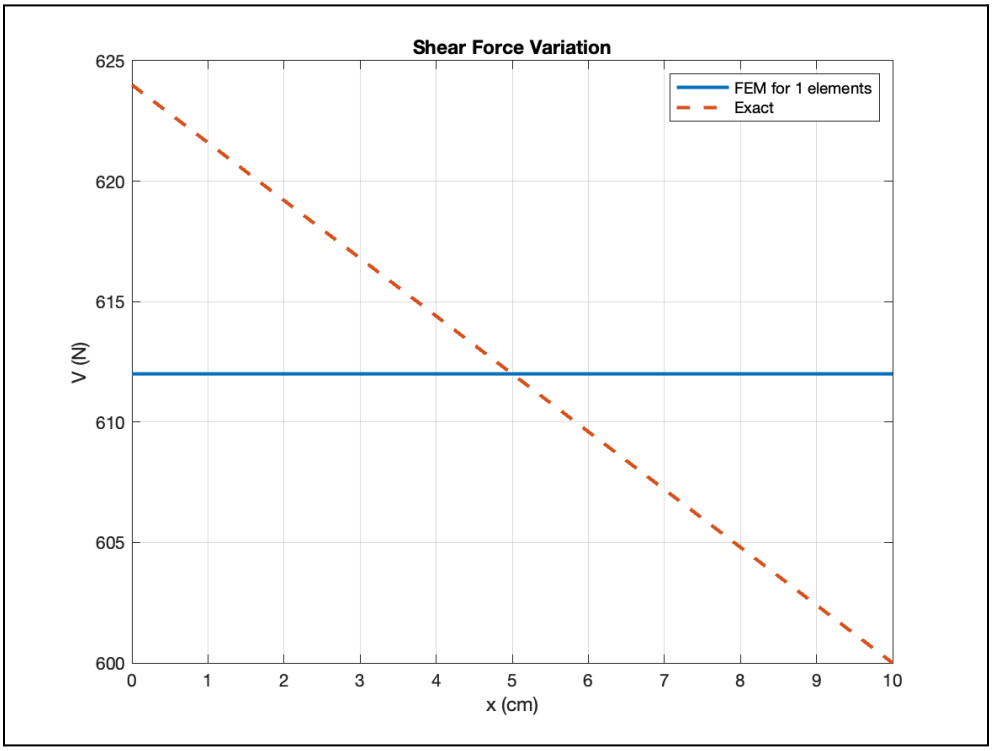

Transverse displacement slope Variation
FEM for 50 elements
Exact
du/dx
x (cm)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
1
2
3
4
5
6
7
8
9
10

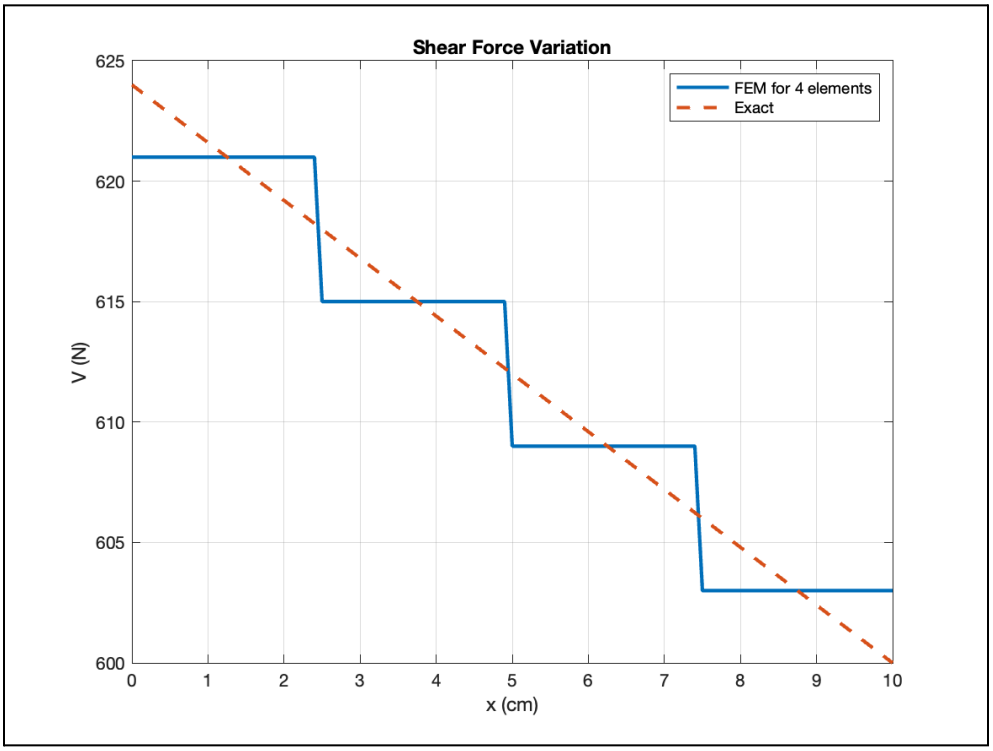

Transverse displacement slope Variation
FEM for 100 elements
Exact
du/dx
x (cm)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
1
2
3
4
5
6
7
8
9
10

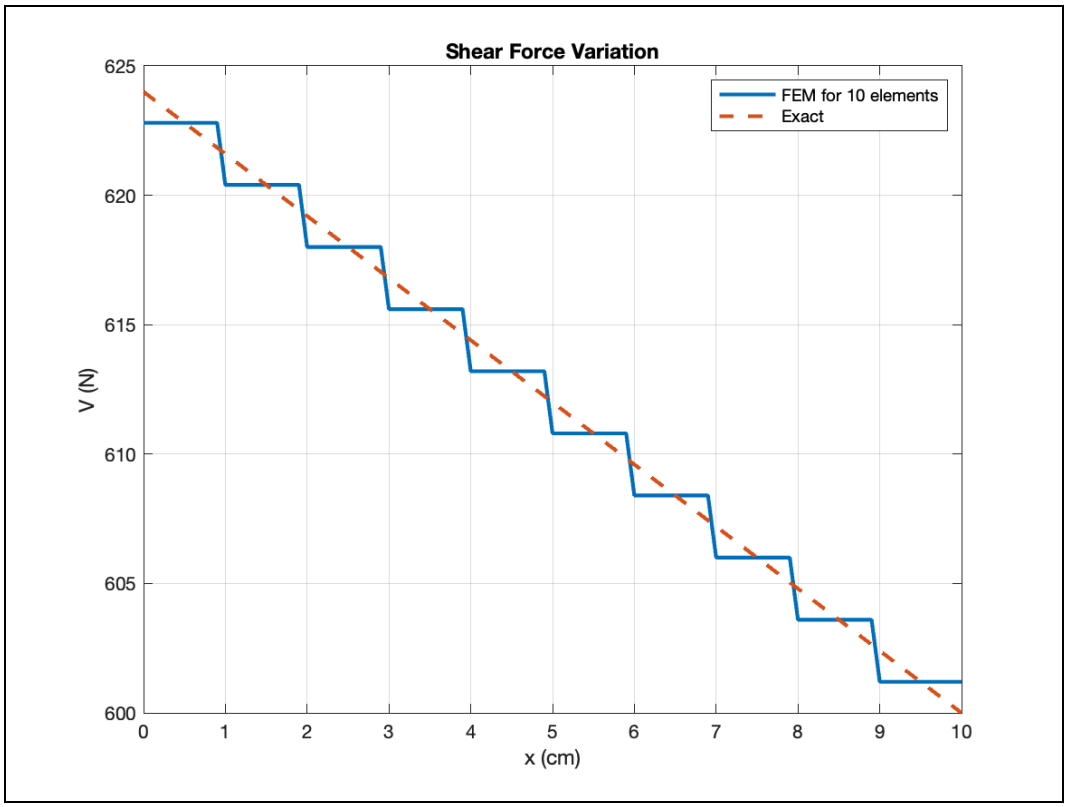
Problem 2

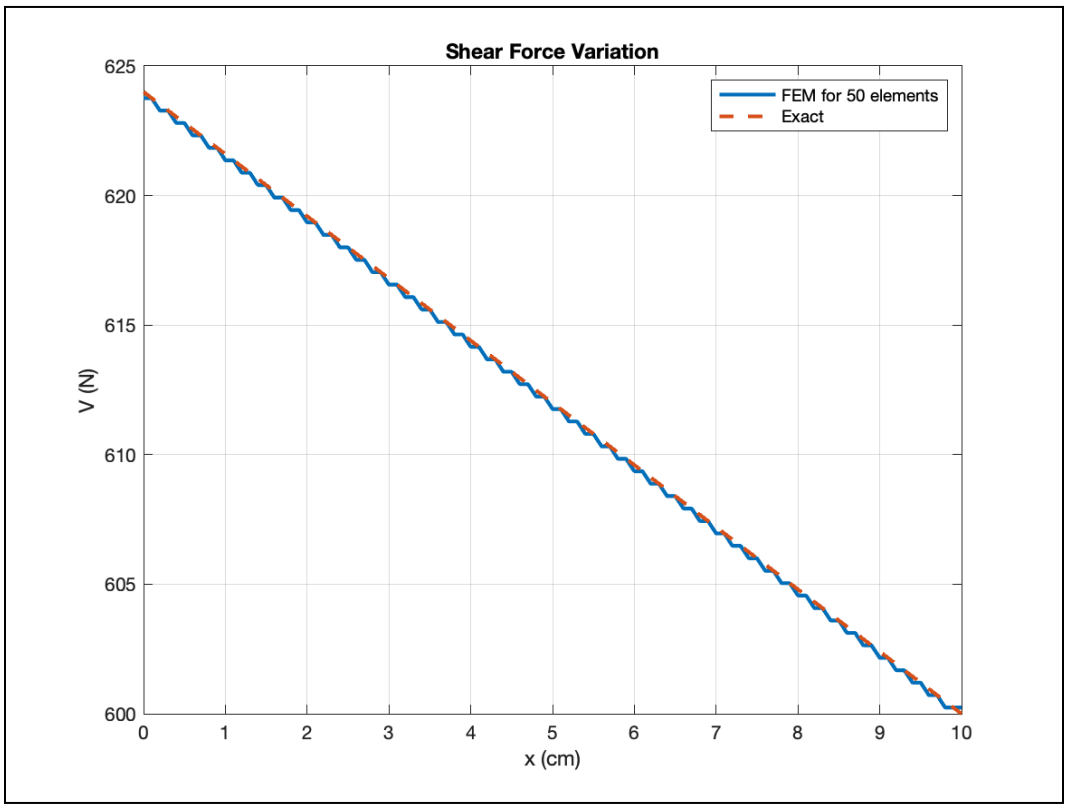
Give continuous variation of shear force and bending moment

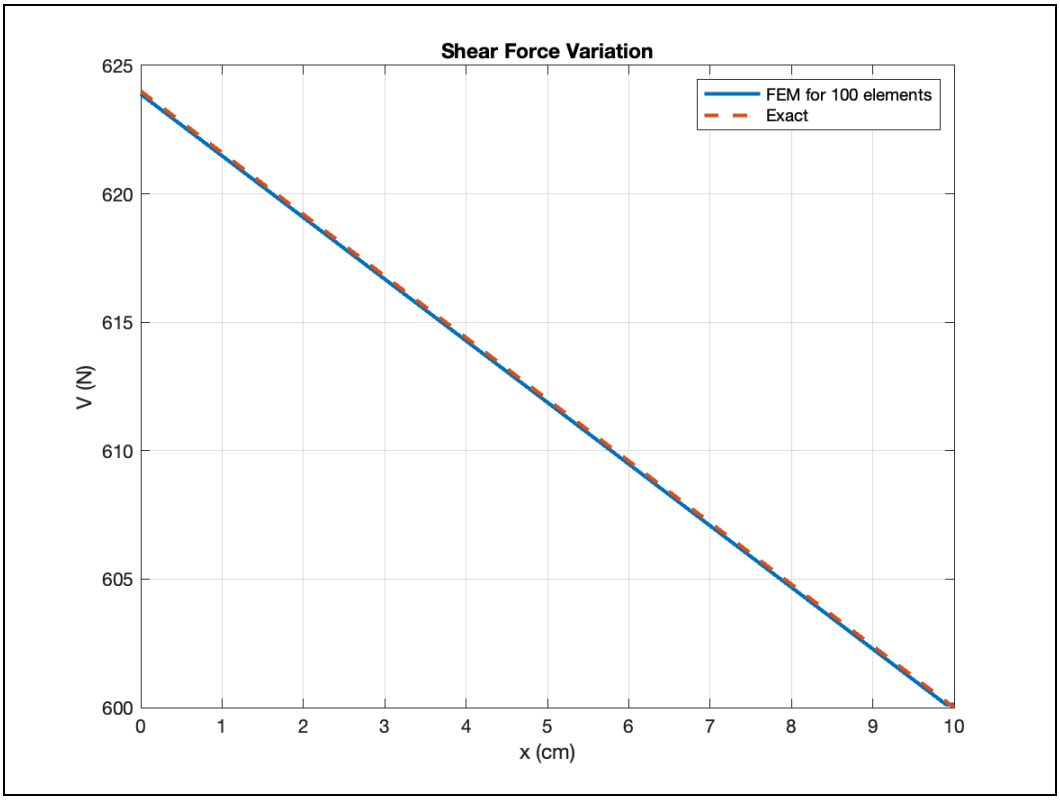
Plots for Continuous variation of shear force for 1,4,10,50 and 100 elements



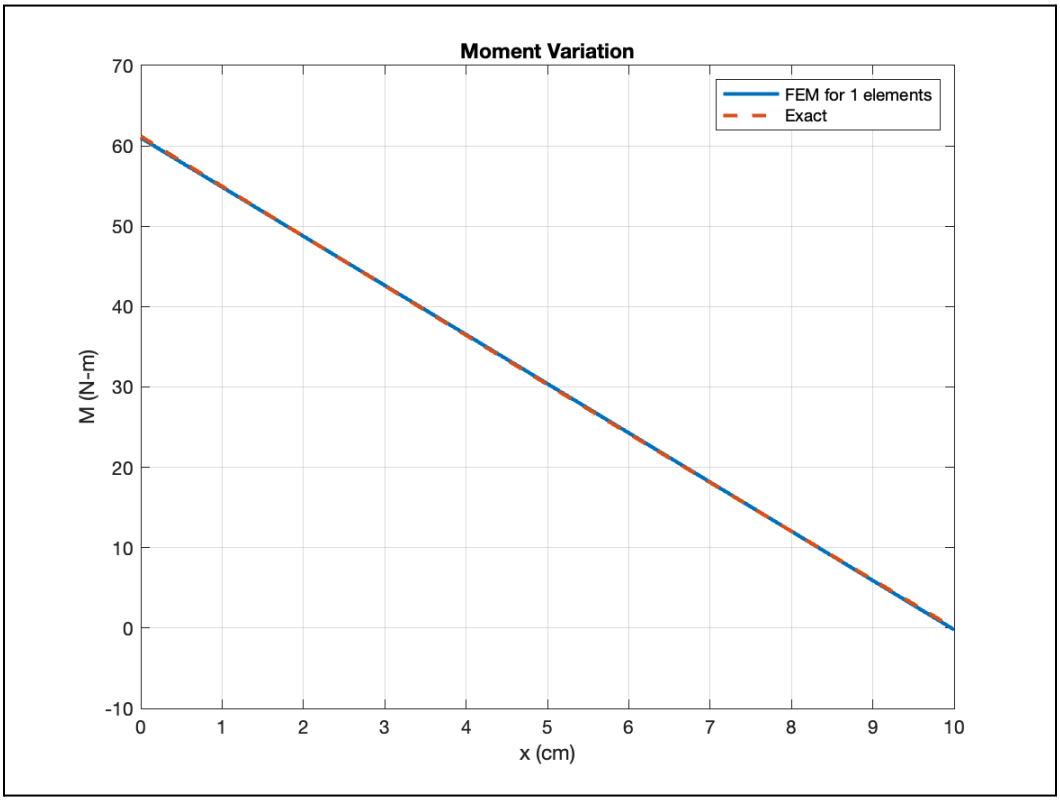


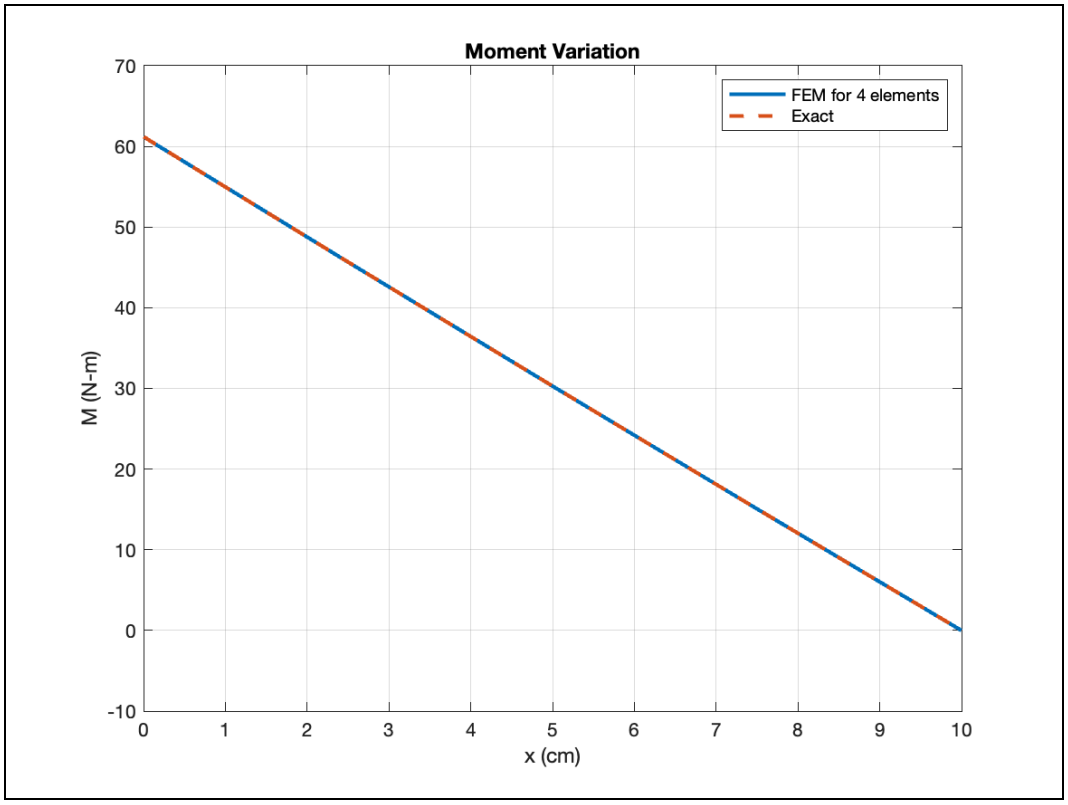

Shear Force Variation
FEM for 10 elements
Exact
V (N)
x (cm)
625
620
615
610
605
600
0
1
2
3
4
5
6
7
8
9
10

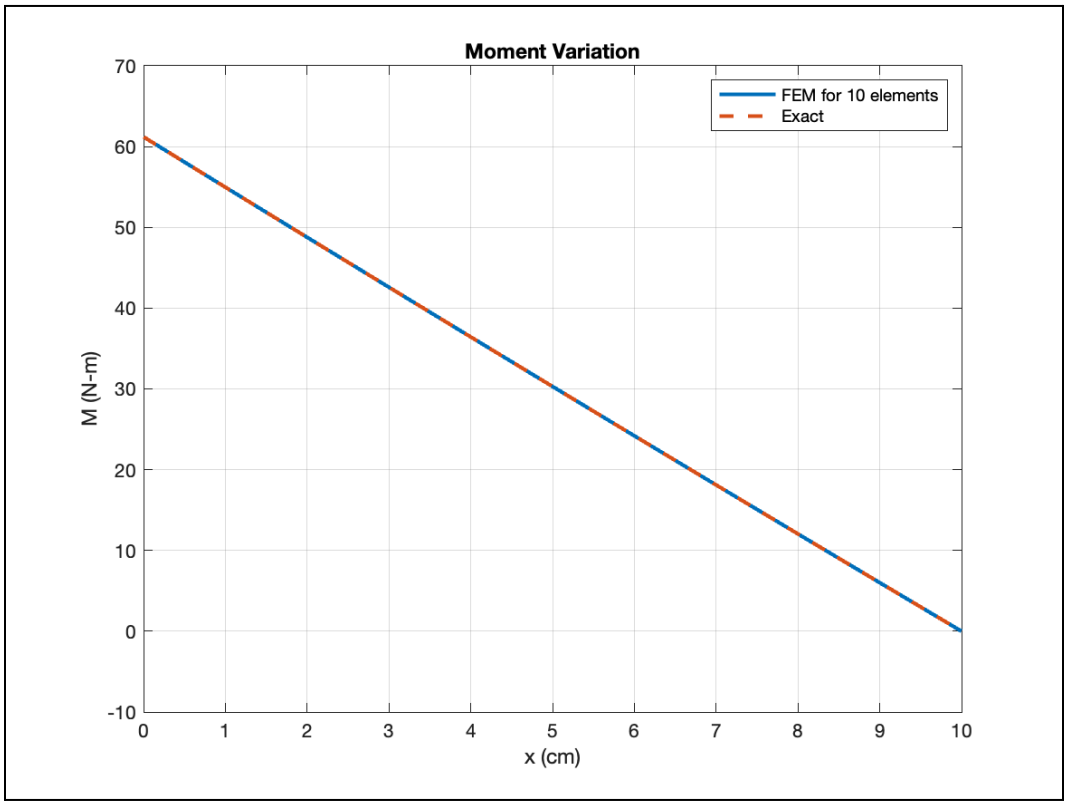

Shear Force Variation
FEM for 50 elements
Exact
V (N)
x (cm)
625
620
615
610
605
600
0
1
2
3
4
5
6
7
8
9
10

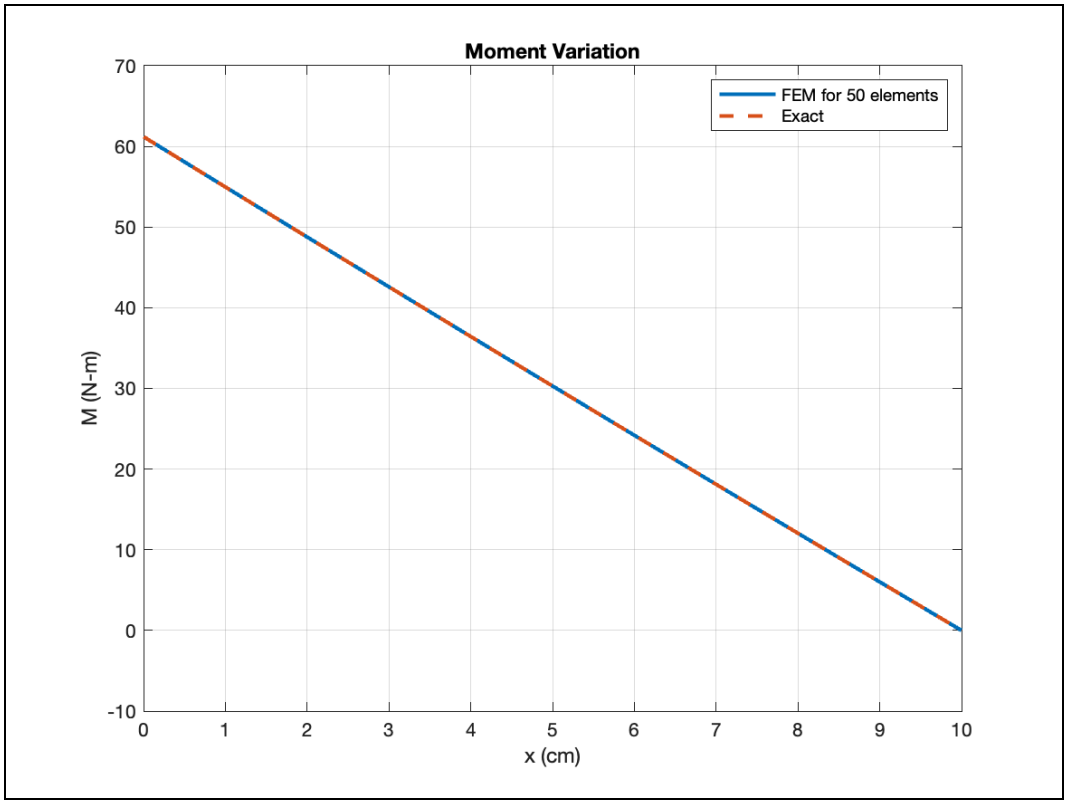


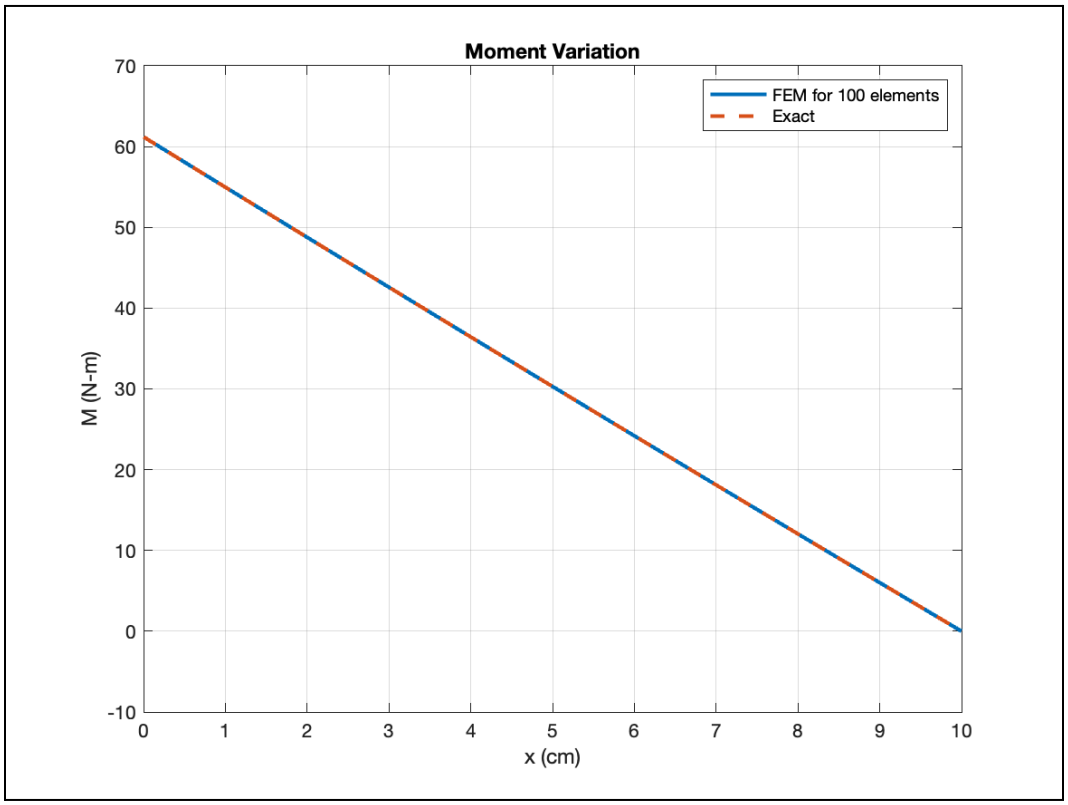
Plots for Continuous variation of the bending moment for 1,4,10,50 and 100 elements




Moment Variation
FEM for 4 elements
Exact
M (N-m)
x (cm)
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
10


Moment Variation
FEM for 10 elements
Exact
M (N-m)
x (cm)
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
10

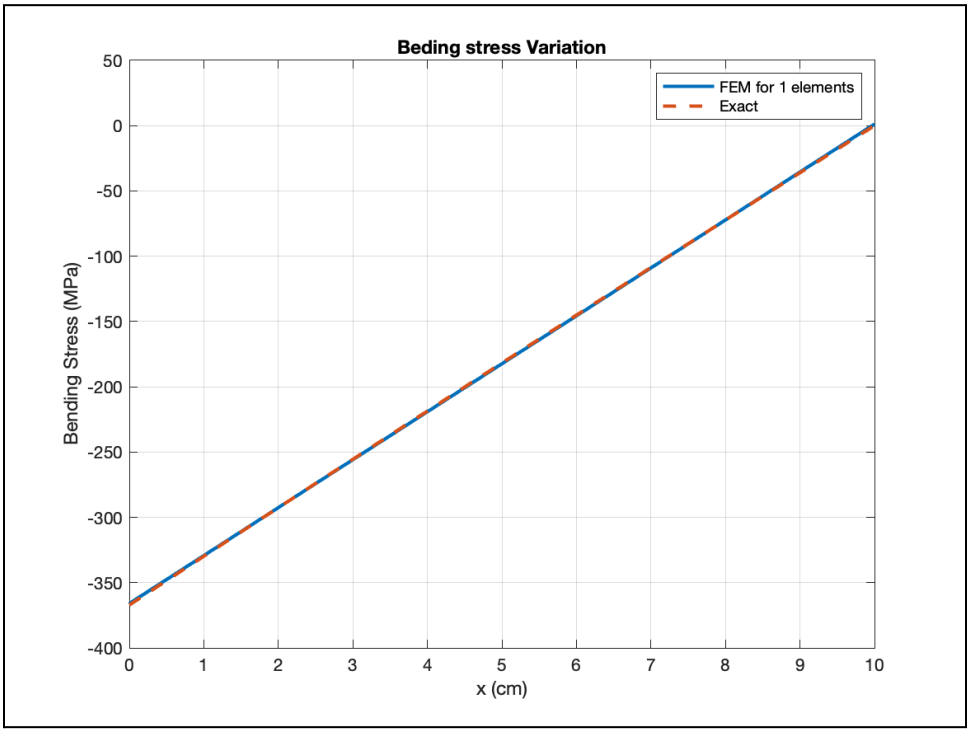

Moment Variation
FEM for 50 elements
Exact
M (N-m)
x (cm)
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
10

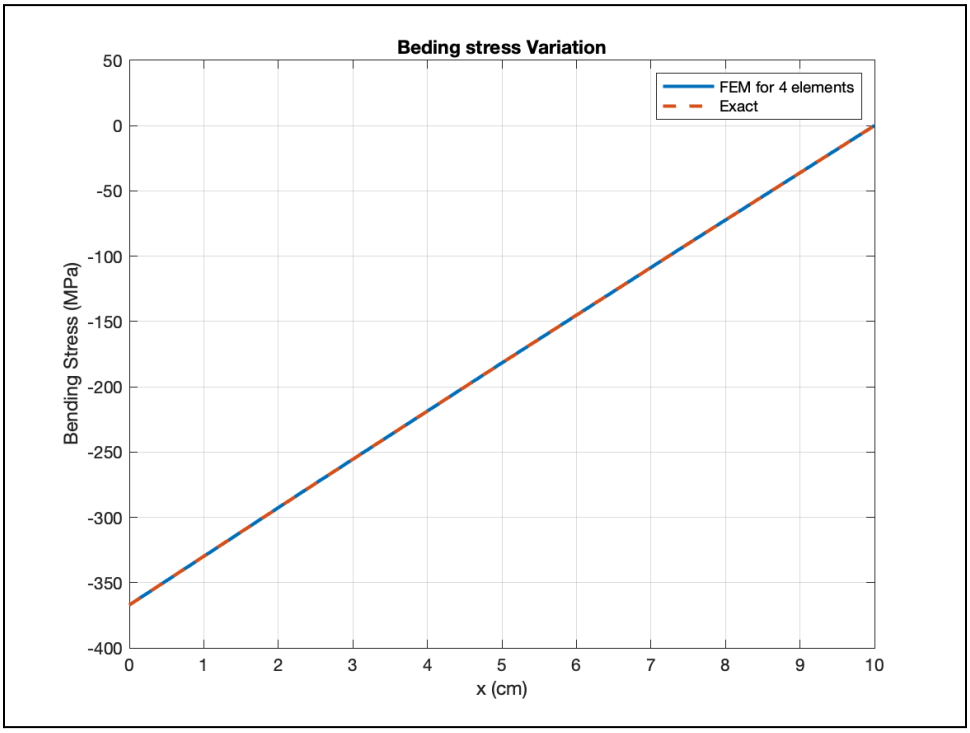

Moment Variation
FEM for 100 elements
Exact
M (N-m)
x (cm)
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
10

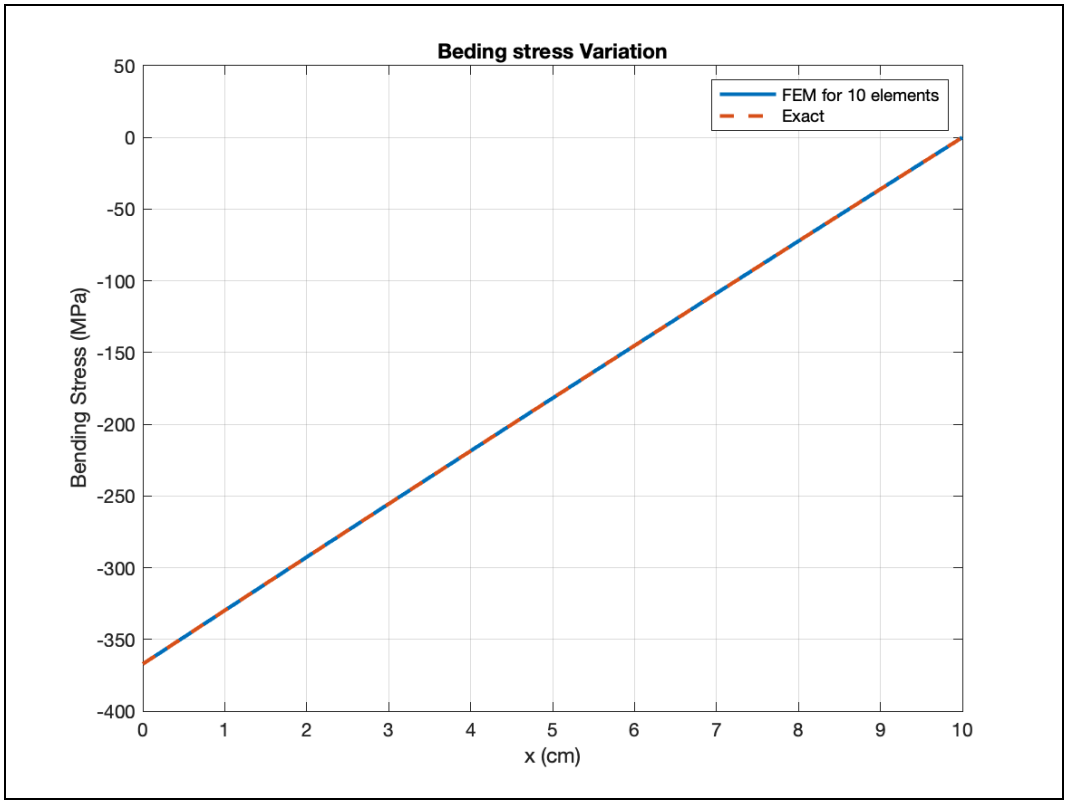
Problem 3

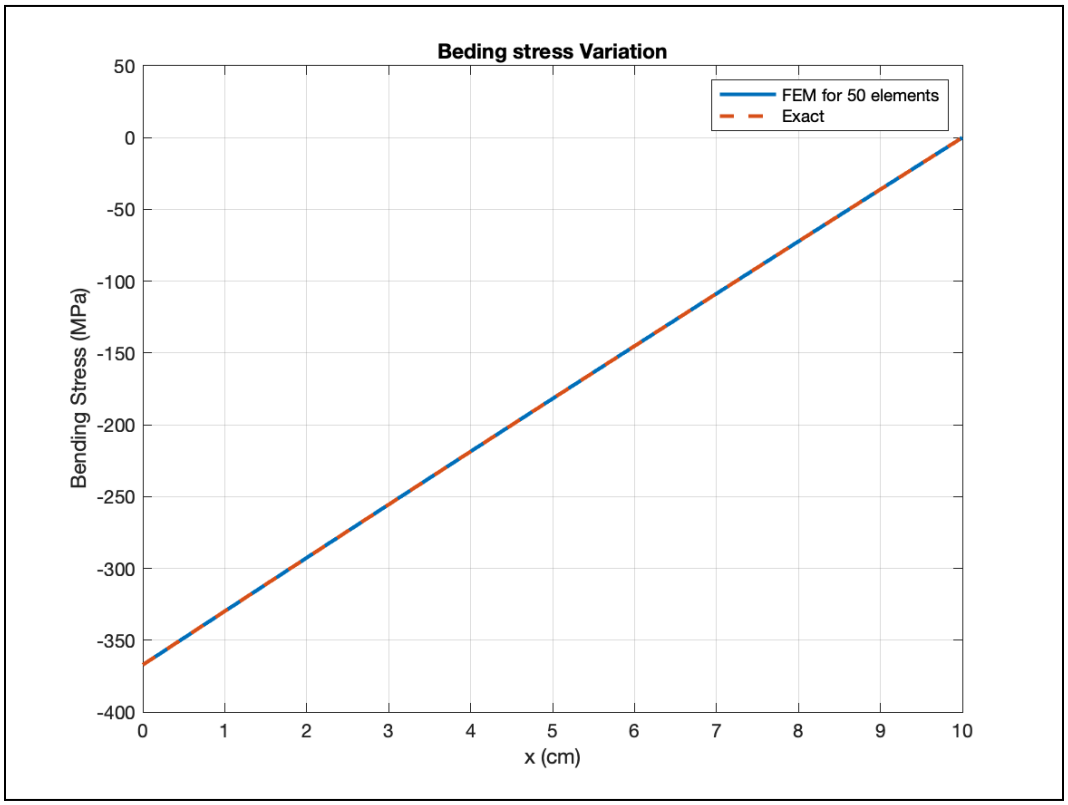
Bending stress on the topmost line of the beam along its entire length

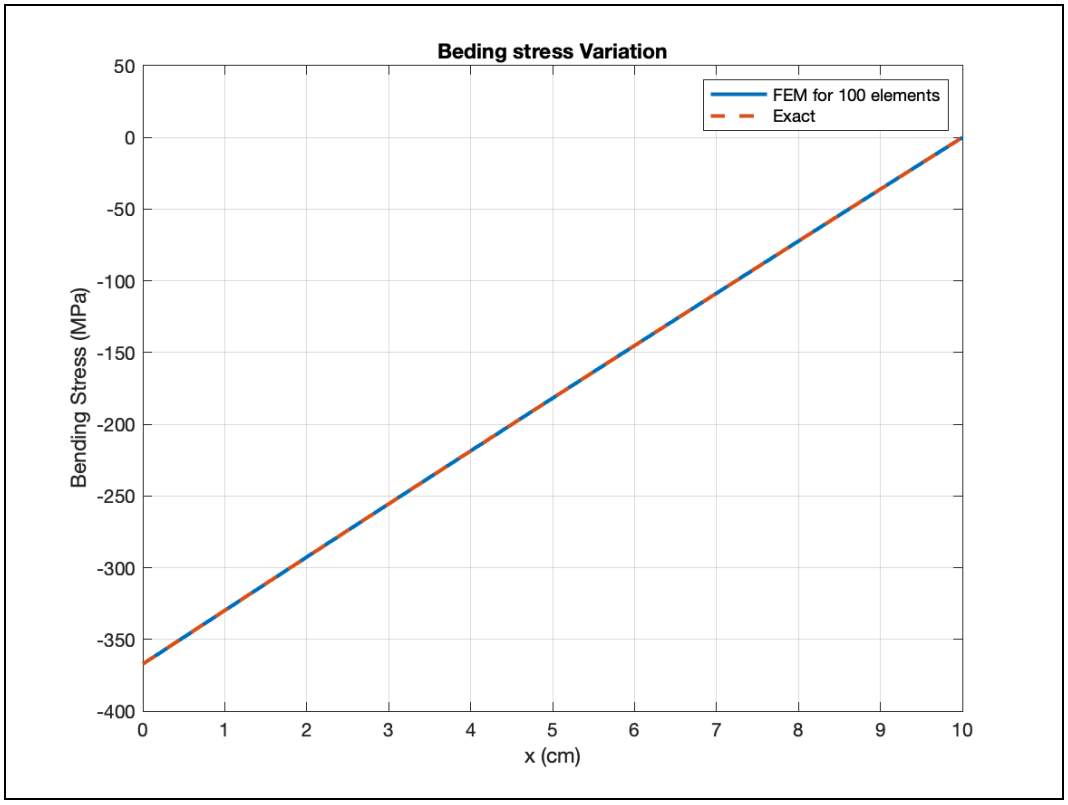
Plots for Bending stress variation along the length of the beam.






Beding stress Variation
FEM for 10 elements
Exact
Bending Stress (MPa)
x (cm)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
0
1
2
3
4
5
6
7
8
9
10


Beding stress Variation
FEM for 50 elements
Exact
Bending Stress (MPa)
x (cm)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
0
1
2
3
4
5
6
7
8
9
10


Beding stress Variation
FEM for 100 elements
Exact
Bending Stress (MPa)
x (cm)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
0
1
2
3
4
5
6
7
8
9
10

Conclusion

1. For Plots of Continuous Variation of Transverse displacement for 1, 4, 10, 50, and 100 elements.

For the given shear force and moment values of boundary conditions values for the fem solution and exact solutions are matching exactly.

At x=0 cm the value of transverse displacement, u = 0
At x=10 cm the value of transverse displacement, u = 1.218 mm
Values for fem solutions and exact solutions match at all points with maximum value at x=10 cm

2. For Plots of Continuous Variation of Slope of Transverse displacement for 1, 4, 10, 50, and 100 elements.

For the given shear force and moment values of boundary conditions values for the fem solution and exact solutions are matching exactly.

At x=0 cm the value of transverse displacement slope, du/dx = 0
At x=10 cm the value of transverse displacement slope, du/dx = 0.01824
Values for fem solutions and exact solutions match at all points

3. For Plots of Continuous Variation of Shear Force for 1, 4, 10, 50, and 100 elements.

For the given shear force and moment values of boundary conditions values for the fem solution and exact solutions are matching exactly.

At x=0 cm the value of Shear force, v = 624 N
At x=10 cm the value of Shear force, v = 600 N

Values of shear force remain the same for each local element for the FEM solution and this value varies from 624 N to 600 N

FEM solution and exact solution values match at nodal points.

4. For Plots of Continuous Variation of Moment for 1, 4, 10, 50, and 100 elements.

For the given shear force and moment values of boundary conditions values for the fem solution and exact solutions are matching exactly.

At x=0 cm the value of Moment, M = 61.2 N-m
At x=10 cm the value of Moment, M = 0 N-m

Values for fem solutions and exact solutions match at all points and linearly vary from 61.2 to 0 N-m.

5. For Plots of Continuous Variation of Bending stress on the topmost line along its length for 1, 4, 10, 50, and 100 elements.

For the given shear force and moment values of boundary conditions values for the fem solution and exact solutions are matching exactly.

At x=0 cm the value of Bending stress, **σ** = **-**367.2 MPa
At x=10 cm the value of Bending stress, **σ** = 0 MPa

Values for fem solutions and exact solutions match at all points and linearly vary from **-**367.2 to zero MPa. and is maximum at x = 0.