Back

Applying the Taylor series

Practice Quiz • 15 min • 5 total points

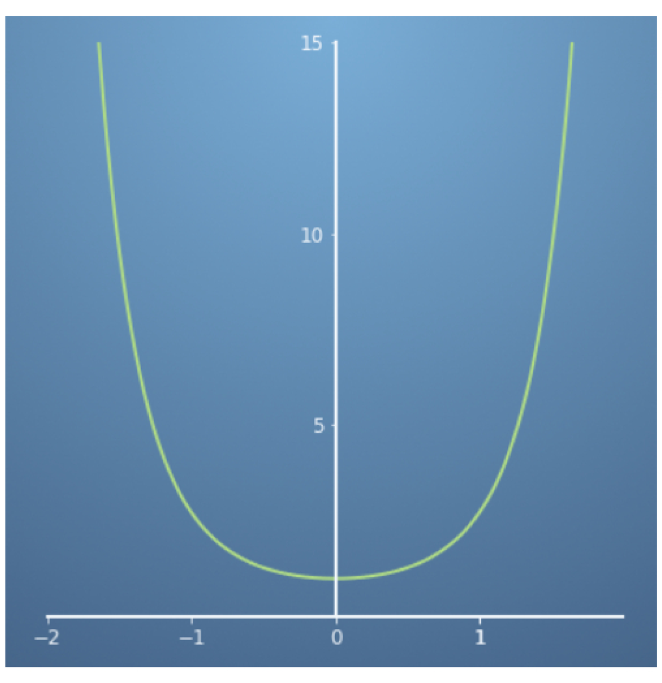
Congratulations! You passed!

Grade received 100% **To pass** 80% or higher

Go to next item

1. In the two previous videos, we have shown the short mathematical proofs for the Taylor series, and for special cases when the starting point is $x = 0$, the Maclaurin series. In these set of questions, we will begin to work on applying the Taylor and Maclaurin series formula to obtain approximations of functions.

1 / 1 point



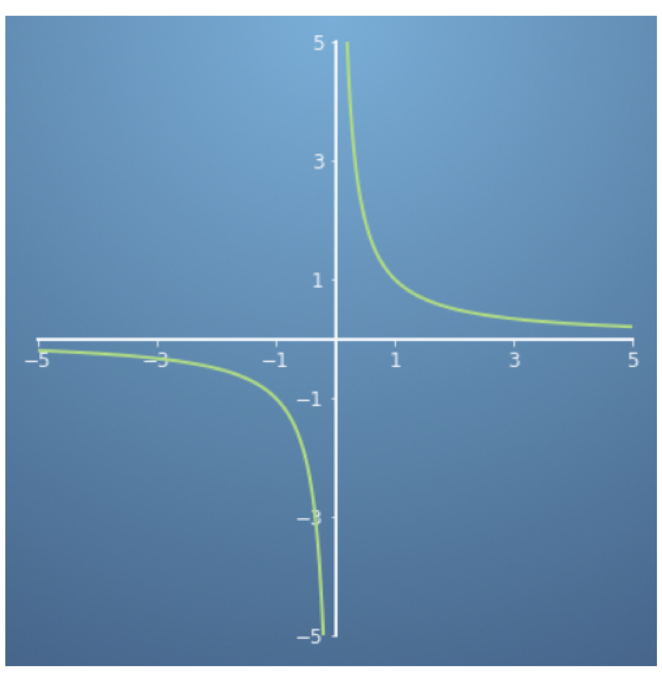
For the function $f(x) = e^{x^2}$ about $x = 0$, using the the Maclaurin series formula, obtain an approximation up to the first three non zero terms.

- ○ $f(x) = x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \dots$
- ○ $f(x) = 1 - x^2 - \frac{x^4}{2} \dots$
- ○ $f(x) = 1 + 2x + \frac{x^2}{2} + \dots$
- ◉ $f(x) = 1 + x^2 + \frac{x^4}{2} + \dots$

Correct
We find that only even powers of x appear in the Taylor approximation for this function.

2.

1 / 1 point



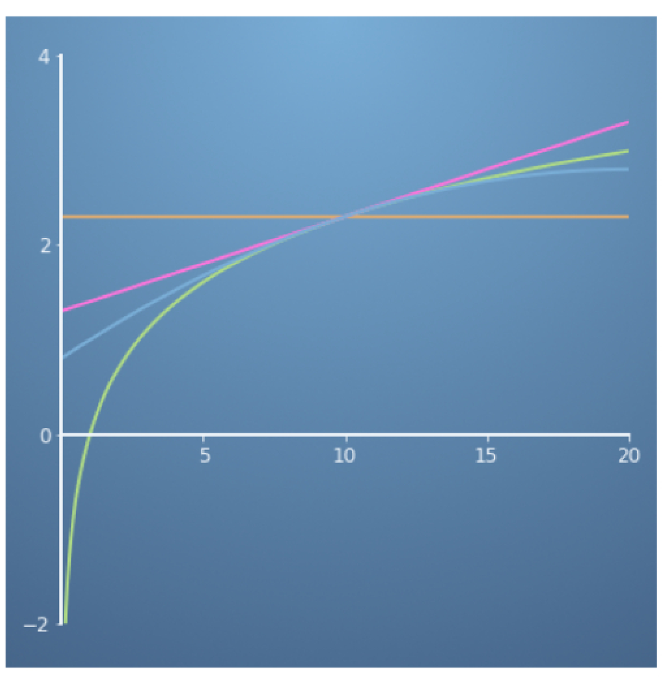
Use the Taylor series formula to approximate the first three terms of the function $f(x) = 1/x$, expanded around the point $p = 4$.

- ○ $f(x) = \frac{(x-4)}{16} + \frac{(x-4)^2}{64} - \frac{(x-4)^3}{256} \dots$
- ○ $f(x) = -\frac{1}{4} - \frac{(x+4)}{16} - \frac{(x+4)^2}{64} \dots$
- ◉ $f(x) = \frac{1}{4} - \frac{(x-4)}{16} + \frac{(x-4)^2}{64} + \dots$
- ○ $f(x) = \frac{1}{4} - \frac{(x+4)}{16} + \frac{(x+4)^2}{64} + \dots$

Correct
We find that only even powers of x appear in the Taylor approximation for this function.

3.

1 / 1 point



By finding the first three terms of the Taylor series shown above for the function $f(x) = \ln(x)$ (green line) about $x = 10$, determine the magnitude of the difference of using the second order taylor expansion against the first order Taylor expansion when approximating to find the value of $f(2)$.

- ◉ $\Delta f(2) = 0.32$
- ○ $\Delta f(2) = 1.0$
- ○ $\Delta f(2) = 0$
- ○ $\Delta f(2) = 0.5$

Correct
The second order Taylor approximation about the point $x = 10$ is
$f(x) = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200} \dots.$

So the first order approximation is

$g_1 = \ln(10) + \frac{(x-10)}{10}$

and the second order approximation is

$g_2 = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200}.$

So, the magnitude of the difference is

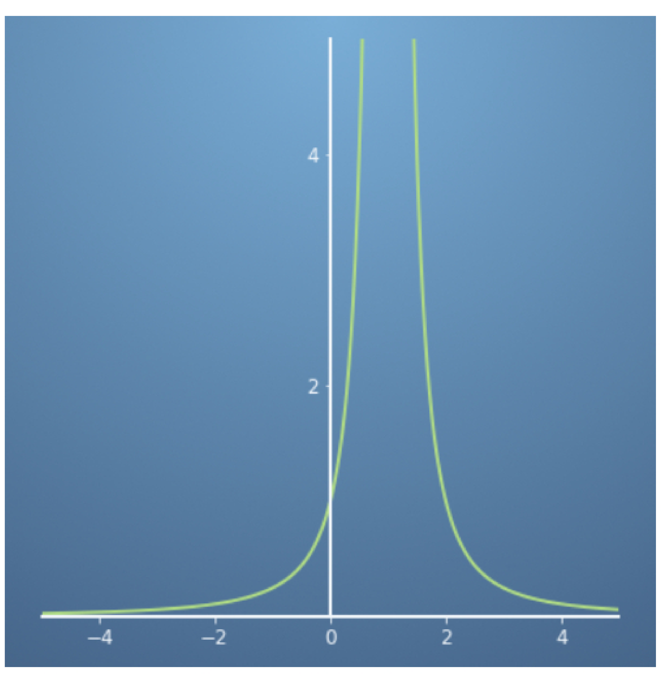
$|g_2(2) - g_1(2)| = |-\frac{(x-10)^2}{200}|$

and substituting in $x = 2$ gives us

$|g_2(2) - g_1(2)| = |-\frac{(2-10)^2}{200}| = 0.32$

4. In some cases, a Taylor series can be expressed in a general equation that allows us to find a particular n^{th} term of our series. For example the function $f(x) = e^x$ has the general equation $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Therefore if we want to find the 3^{rd} term in our Taylor series, subsitituting $n = 2$ into the general equation gives us the term $\frac{x^2}{2}$. We know the Taylor series of the function e^x is $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$. Now let us try a further working example of using general equations with Taylor series.

1 / 1 point



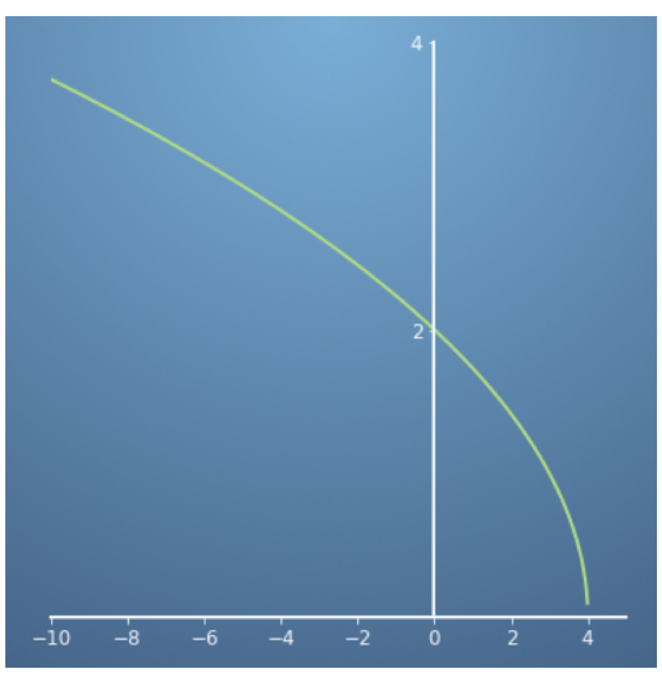
By evaluating the function $f(x) = \frac{1}{(1-x)^2}$ about the origin $x = 0$, determine which general equation for the n^{th} order term correctly represents $f(x)$.

- ○ $f(x) = \sum_{n=0}^{\infty}(2+n)(x)^n$
- ◉ $f(x) = \sum_{n=0}^{\infty}(1+n)x^n$
- ○ $f(x) = \sum_{n=0}^{\infty}(1+2n)(x)^n$
- ○ $f(x) = \sum_{n=0}^{\infty}(1+n)(-x)^n$

Correct
By doing a Maclaurin series approximation, we obtain $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots$, which satisfies the general equation shown.

5.

1 / 1 point



By evaluating the function $f(x) = \sqrt{4 - x}$ at $x = 0$, find the quadratic equation that approximates this function.

- ○ $f(x) = 2 + x + x^2 \dots$
- ◉ $f(x) = 2 - \frac{x}{4} - \frac{x^2}{64} \dots$
- ○ $f(x) = \frac{x}{4} - \frac{x^2}{64} \dots$
- ○ $f(x) = 2 - x - \frac{x^3}{64} \dots$

Correct
The quadratic equation shown is the second order approximation.