UNIT 4 & UNIT-5 → CO3

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Unit 4 :- Sampling Theory

Syllabus: Population & sample, Statistical Inference, sampling with & without replacement, Population parameters, sample statistics, Sampling distribution of means, Sampling distribution of proportion.

Definitions:

(1) Population: Populatn is the set under study

Ex (1) Populatn selected about different-different blood group (set)

(2) Height of the students in university

(3) No. of student given exam in 2021 of HSC (via online).

Types of population

Finite Populatn: Finite / countable individual

Infinite Population: ∞ / uncountable individual present.

Ex ① No. of students in a class.

Note: 3121[?] populatn is small & we study each & every element in that populatn then it is called as Census

Ex ① No. of decimals b/w 1 or 2

② using No. of alphabet we can make ∞-words

③ A bag contains tea leaves so. no of particles are ∞!

(small) → Populatⁿ को हर एक CH. की study करना called census

* Sample: A finite subset of a population is known as sample.

(large populatⁿ की study करना impossible है, so we need sample selectⁿ)

Ex Blood Test sample. (To find/count RBC/WBC)

Ex Corona Test as per area wise. (Covid-19) Test (Red zone / green zone area)

* Size :- Size of the populatⁿ (sample) is the no. of element in the populatⁿ (sample)

It is denoted by 'N' (n)

Size → populatⁿ ⟶ N

Size → sample ⟶ n.

* ~~जिस~~ जिस method से हम populatⁿ में से sample को select ~~करेंगे~~ use sampling कहेंगे.

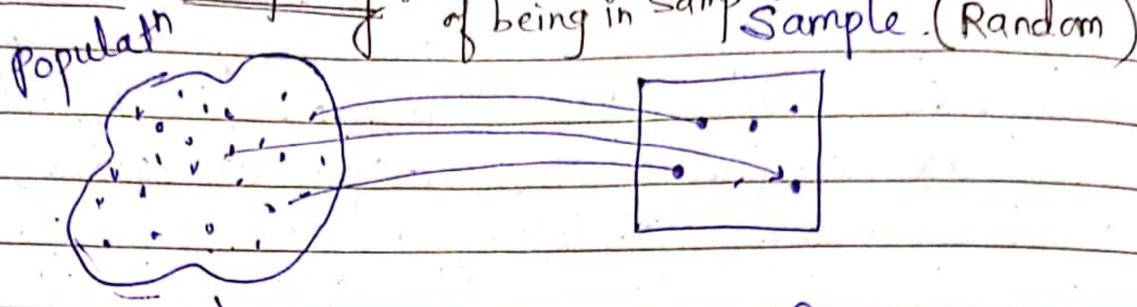
* Sampling: Sampling is the process of drawing samples from gⁿ population.

Objective of Sampling → कम से कम efforts में, कम से कम time में, कम से कम money खर्च करके populatⁿ के bare में ज्यादा से ज्यादा information निकाले.

* Large and Small Sampling: If $n \geq 30$ then sampling is said to be large sampling

& if $n < 30$ then sampling is said to be Small Sampling or Exact Sampling.

* **Random Sampling** :- Each member of population has same chance of being in sample.



Populatn के हर एक element के chance equal रहेंगे Sample में select होने के लिए, This sampling is called Random Sampling.

$\boxed{\text{Random Sampling}}$

Simple R.S. with Replacement

↓

Populatn (sample) में जितने भी elements हैं उन्हें "एक से ज्यादा बार" Select कर सकते हैं.

एक element sample में से select करके दूसरी बार वही ~~samp~~ elt sample में डाल देना

Q: (σ - populatn)

No. of Sample $= N^n$

(a) $\mu = \mu_{\bar{x}}$ (b) $SD = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Simple R.S. without replacement.

↓

(Sample) Populatn का जो element है वह एक ही बार Sample में आयेगा (उस elt को वापस populatn में नहीं डालेंगे)

(Finite populatn)

* No. of sample $= {}^{N}C_n$

(a) Mean $= \mu_{\bar{x}} = \mu$

(b) S.D $= \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

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S.R.S. with Replacement :- Sampling where each member of a populatⁿ may be chosen more than once is called Sampling with replacement. * Mean = $\mu_{\bar{x}} = \mu$ + S.D = $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

S.R.S. without Replacement :- Sampling where each member of populatⁿ can not be chosen more than once is called Sampling without replacement.

Ex : Group activities, (selectⁿ of students grp)

* **Statistical Inference** : S.I. deals with methods of arriving at valid or logical generalizations and predictions about population using information contained in sample alone.

* **Parameters** :- Statistical measures or constant obtained from population are known as Parameters ie mean, std. deviatⁿ, variance, prop.
It is denoted by Mean ; μ ; std deviatⁿ = σ, Populatⁿ proportion = p.

* **Statistics** :- Statistical measures or constant obtained from "sample" are known as Statistics. mean, std. deviation & variance
It is denoted by
Sample Mean = $\bar{X}$, sample std deviatⁿ = s, Sample proportion = P

(statistical constant sample से लें तो statistics होता)

	Populat^n	Sample
Mean →	μ	$\bar{X}$ $\mu_{\bar{X}}$
std-deviat^n →	σ	s
proportion →	p	P
size →	N	n
Variance →	σ^2	s^2

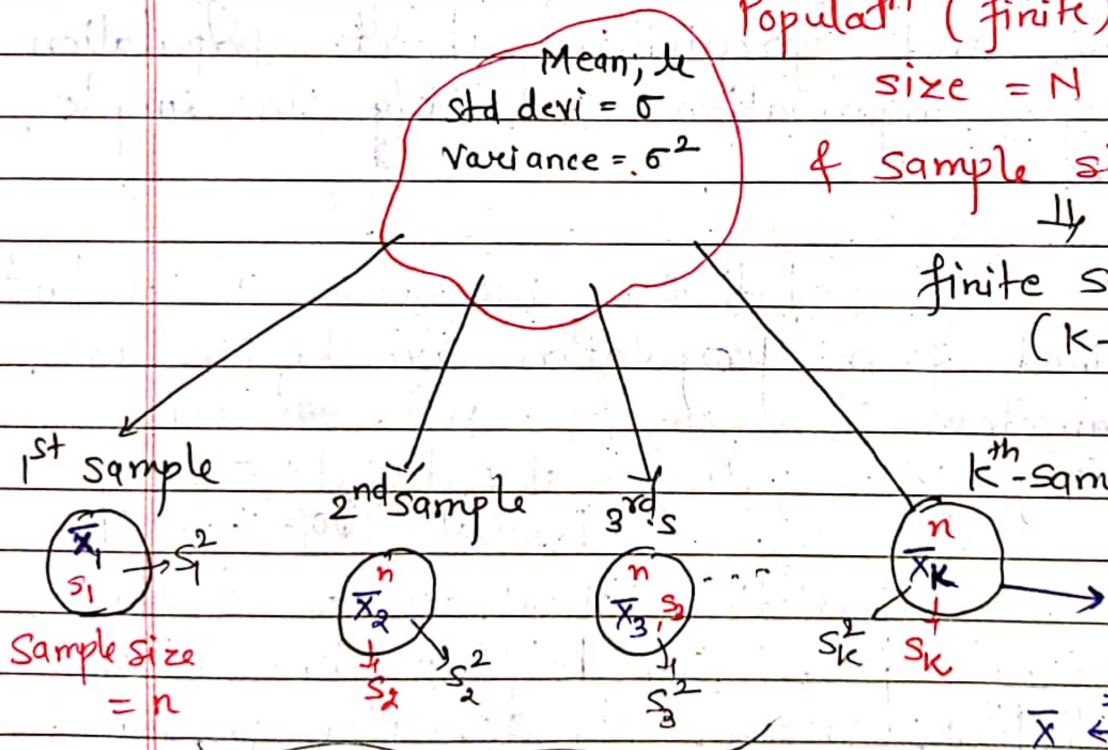
* Sample के statistics को populat^n के parameter को estimate करते हैं

$$Var(x) = E[(x-\mu)^2] \;;\; \sigma = +\sqrt{Var(x)}$$

$$\Rightarrow \sigma = \sqrt{E[x-\mu]^2} = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}$$

* Sampling distribut^n of Mean *

Populat^n (finite)
size = N
& sample size = n
⇓
finite sample selected
(K-sample selected)



→ To find (statistics)
i.e To find
$\bar{X}$ ← mean, variance, s_i^2
s_i ← std. devat^n

Theorem

(3) If populat^n is of size 'N', & sampling without replacement & if sample size "n ≤ N"
then;

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$$

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Sampling Distributⁿ of mean.

$$\begin{aligned} \bar{x}_1 &\to P(\bar{x}_1) \\ \bar{x}_2 &\to P(\bar{x}_2) \\ \bar{x}_3 &\to P(\bar{x}_3) \\ &\vdots \\ \bar{x}_k &\to P(\bar{x}_k) \end{aligned}$$

std deviatⁿ of sampling distributⁿ is called "std error"

S. Dist of std. deviatⁿ.

$$\begin{aligned} s_1 &\to P(s_1) \\ s_2 &\to P(s_2) \\ s_3 &\to P(s_3) \\ &\vdots \\ s_k &\to P(s_k) \end{aligned}$$

* **Definitⁿ of Sampling distributⁿ of means:-**

Let $f(x)$ be probability distributⁿ of some given population from which we can draw a sample of size 'n'.
Then it is natural to look for prob. distributⁿ of sample statistic $\bar{x}$, which is called sampling distributⁿ of means.
Or Sampling distributⁿ for Sample mean

Imp Theorems:

(1) Mean of sampling distributⁿ of means, denoted by $\mu_{\bar{x}}$ is given by
$$\mu_{\bar{x}} = \mu = E[\bar{x}]$$
i.e. Expected value of sample mean is the population mean

2) If populatⁿ is '∞' or populatⁿ is finite a sampling with replacement, then Variance of Sampling distributⁿ of means, denoted by $\sigma_{\bar{x}}^2$, given by
$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = E\left[(\bar{x}-\mu)^2\right]$$
where, σ^2 is variance of populatⁿ

* Sampling Distribut^n of Means

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Ex A population consist of four numbers $P = \{2, 3, 4, 5\}$. Consider all possible distinct samples of size two with replacement (infinite). Find

(a) Population mean (μ)
(b) populat^n std. deviat^n (σ)
(c) The sampling dist. of means ~~$(\bar{x})$~~ ~~$(\mu_{\bar{x}})$~~ $(\bar{x})$.
(d) Mean of S.D. of means $(\sigma_{\bar{x}})$
(e) Mean of Sampling Distribution of means $(\mu_{\bar{x}})$.

Verify (c) & (e) directly from (a) & (b) by use of suitable formulae

Sol^n.

(a) To find population mean (μ)

$$\mu = \frac{2+3+4+5}{4} = 3.5$$

(b) To find Populat^n std. deviat^n (σ)

$$\sigma_x = \sqrt{Var(x)} = \sqrt{E[(x-\mu)^2]} = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}$$

$$\sigma = \sqrt{\frac{(2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2}{4}}$$

$$\sigma = \sqrt{\frac{(1.5)^2 + (0.5)^2 + (0.5)^2 + (1.5)^2}{4}} = \sqrt{\frac{5}{4}}$$

$$\sigma = \sqrt{1.25} = 1.118033$$

(c) Sampling distribut^n of mean $\bar{x}$

1st to find mean of each sample:

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Total No. of samples (∞) with replacement $= N^n = 4^2 = 16$

(a, b)	2	3	4	5
2	(2,2)	(2,3)	(2,4)	(2,5)
3	(3,2)	(3,3)	(3,4)	(3,5)
4	(4,2)	(4,3)	(4,4)	(4,5)
5	(5,2)	(5,3)	(5,4)	(5,5)

Mean of each sample.

Samp! distn of mean.

$\bar{X} = \frac{a+b}{2}$

	2	3	4	5
2	2	2.5	3.0	3.5
3	2.5	3	3.5	4.0
4	3	3.5	4.0	4.5
5	3.5	4.0	4.5	5.0

mean of samp. dist. of means.

(c) ⇒ Mean, $\mu_{\bar{x}} = \frac{2 + 2.5 + 3.0 + \cdots + 5}{16} = 3.5$

(d) Mean of S.D. of means $(\sigma_{\bar{x}}) = \sqrt{\frac{\Sigma(x-3.5)^2}{16}}$

$$\sigma_{\bar{x}} = \sqrt{\frac{(2-3.5)^2 + (2.5-3.5)^2 + \cdots + (5-3.5)^2}{16}}$$

$\sigma_{\bar{x}} = 0.6455$

Note Replacement / without replacement. S.R.S. $\Rightarrow \mu_{\bar{x}} = \mu$
ie Mean of sampling dist. of mean = mean of populatn.

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Verificatn; mean = $\mu_{\bar{x}} = \mu = 3.5$

$n\uparrow, \sigma_{\bar{x}}\downarrow$ $\left\{ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cancel{\sqrt{\frac{N-n}{N-1}}} = \frac{1.11803}{\sqrt{2}} = \frac{1.11803}{1.4142} \right.$

$n\uparrow, \sigma_{\bar{x}}\downarrow$ $\sigma_{\bar{x}} = \cancel{0.5590}$ $\underline{0.79180}$.

Ex A populatn consist of 4 number.. solve above example 1 without replacement.
To find: (1) (μ) (2) σ (3) $\bar{x}$ (4) $\sigma_{\bar{x}}$
(5) $\mu_{\bar{x}}$

$\Rightarrow$

(a) To find: populatn mean;

$$\mu = \frac{2+3+4+5}{4} = 3.5$$

(b) To find population std. deviatn (σ)

$$\sigma = \sqrt{\frac{(2-3.5)^2+(3-3.5)^2+(4-3.5)^2+(5-3.5)^2}{4}}$$

$$\sigma = 1.11803$$

(c) Total no. of sample (finite) without replacement $= {}^{N}C_{n} = {}^{4}C_{2} = \frac{4!}{2!\,2!}$

$$= \frac{4 \times 3 \times \cancel{2!}}{\cancel{2!} \cdot 2!} = \underline{\underline{6}}$$

$= \{ (2,3), (2,4), (2,5), (3,4), (3,5), (4,5) \}$

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Mean of each sample : $(\bar{x})$

$\bar{x} = \frac{(a+b)}{2} = \{2.5, 3, 3.5, 3.5, 4, 4.5\}$ = Sampling distribution of mean.

(e) Mean of Sampling dist. of means ; $\mu_{\bar{x}}$

$\cancel{\mu_{\bar{x}} = E[(\bar{x}-\mu)^2]}$

$$\mu_{\bar{x}} = \frac{2.5+3+3.5+3.5+4+4.5}{6} = \frac{21}{6}$$

$$\boxed{\mu_{\bar{x}} = 3.5}$$

(d) Mean of S.D. of mean ; $\sigma_{\bar{x}}$ = (std. Error of mean)

$$\sigma_{\bar{x}} = \sqrt{Var(\bar{x})} = \sqrt{E(\bar{x}-\mu)^2}$$

$$= \sqrt{\frac{(2.5-3.5)^2+(3-3.5)^2+(3.5-3.5)^2+(0)^2+(4-3.5)^2+(4.5-3.5)^2}{6}}$$

$$= \sqrt{15/6}$$

$$\boxed{\sigma_{\bar{x}} = 0.6455}$$ or

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{N-n}{N-1}\right)} = \frac{1.11803}{\sqrt{2}}\sqrt{\left(\frac{4-2}{4-1}\right)}$$

$= \cancel{0.52704}$ 0.6455

solve it for mean → sampling distributⁿ for mean.

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Ex. Find mean & std. deviation of Sampling distribution of ~~Variance~~ mean (S.D. of $\bar{x}$) for populatⁿ $P = \{2, 3, 4, 5\}$ by drawing samples of size two. (a) With replacement
(b) without replacement.

Solⁿ: (a) With replacement

(i) To find

Q A population consists of 5 numbers 2, 3, 6, 8, 11. Consider all possible samples of size two which can be drawn with replacement from this populatⁿ. Find
(a) mean of population
(b) std. deviation of populatⁿ
(c) Mean of sampling distributⁿ of means,
(d) Std. deviation of sampling distributⁿ of means.

(Also solve IT FOR WITHOUT REPLACEMENT).

Imp ***

Solⁿ:

(a) Mean of populatⁿ; $\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5}$

$N=5$

$$\boxed{\mu = 6.0}$$

(b) Std. deviatⁿ of populatⁿ;

$$\sigma = \sqrt{\frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}}$$

* $\sigma = 3.29$

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(Replacement)

(C) Mean of sampling dist. of means ; $\mu_{\bar{x}}$;

1^{st} to find sampling dist. of means ; $(\bar{X})$

The total no. of samples with replacement is g^n by $N^n = 5^2 = \underline{25}$

(a, b)	2	3	6	8	11
2	(2,2)	(2,3)	(2,6)	(2,8)	(2,11)
3	(3,2)	(3,3)	(3,6)	(3,8)	(3,11)
6	(6,2)	(6,3)	(6,6)	(6,8)	(6,11)
8	(8,2)	(8,3)	(8,6)	(8,8)	(8,11)
11	(11,2)	(11,3)	(11,6)	(11,8)	(11,11)

Mean of each sample ; $\bar{X}$

$\bar{X} = \frac{a+b}{2}$	2	3	6	8	11
2	2.0	2.5	4	5	6.5
3	2.5	3.0	4.5	5.5	7.0
6	4.0	4.5	6.0	7.0	8.5
8	5.0	5.5	7.0	8.0	9.5
11	6.5	7.0	8.5	9.5	11.0

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Now, mean; $\mu_{\bar{x}} = \frac{2 + 2.5 + 4.0 + \dots + 9.5 + 11.0}{25}$

$\mu_{\bar{x}} = \frac{150}{25} = 6.0$

ie $\boxed{\mu = \mu_{\bar{x}}}$

(d) std. deviatn of sampling dist. of means; $\sigma_{\bar{x}}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.29}{\sqrt{2}} = \underline{\underline{2.3263}}$

WITHOUT REPLACEMENT

(a) $\mu = 6.0$ (b) $\sigma = 3.29$ (same as for replace)

(c) Mean of sampling dist. of means; $\mu_{\bar{x}}$

1st to find sampling of mean; $\bar{x} = ?$

The total no of samples without replacement is gn by $NC_n = {}^5C_2 = \frac{5!}{2!\,3!} = \frac{5 \times 4}{2} = \underline{\underline{10}}$

${}^5C_2 = \{(2,3), (2,6), (2,8), (2,11), (3,6), (3,8), (3,11), (6,8), (6,11), (8,11)\}$

Now; mean of each sample

$\bar{x} = \frac{a+b}{2}$ $\{2.5, 4, 5, 6.5, 4.5, 5.5, 7.0, 7.0, 8.5, 9.5\}$

mean of sampling distributⁿ of means is

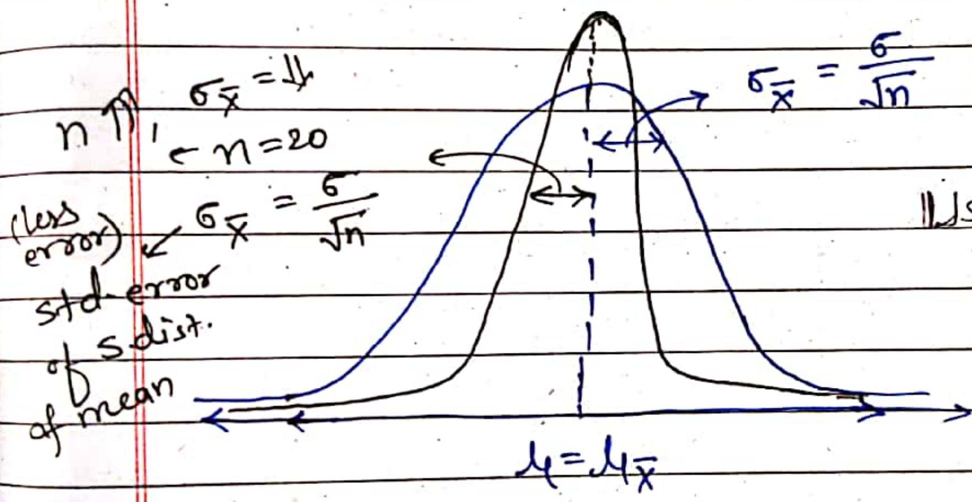
$$\mu_{\bar{x}} = \frac{2.5 + 4.0 + 5 + 6.5 + 4.5 + 5.5 + 7.0 + 7 + 8.5 + 9.5}{10}$$

$\boxed{\mu_{\bar{x}} = 6.0}$ i.e. $\boxed{\mu_{\bar{x}} = \mu}$

(d) std. deviatⁿ of sampling of means;

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{(N-n)}{(N-1)}} = \frac{3.29}{\sqrt{2}}\sqrt{\left(\frac{5-2}{5-1}\right)} = 2.01$$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$; $n = 10$ → (more) Error.



Use of std error of mean $\sigma_{\bar{x}}$

→ (1) Confidence interval को set करने में

(2) Test of significance
i.e. populatⁿ parameter का study करते हैं तथा hypothesis को test करते हैं uss case में std. error of mean का use करते हैं.

→ Do it for sampling distribution for mean.

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Q. Find mean and S.D. of sampling distribution of mean ~~Variance (S.D.V)~~ for the population 2, 3, 4, 5 by drawing samples of size two
(a) with replacement
(b) without replacement.

Solⁿ: **WITH Replacement**

To find sampling distributⁿ Variance (S.D.V)

The total no. of samples with replacement is gⁿ by $N^n = 4^2 = \underline{\underline{16}}$.

(a, b)	2	3	4	5
2	(2,2)	(2,3)	(2,4)	(2,5)
3	(3,2)	(3,3)	(3,4)	(3,5)
4	(4,2)	(4,3)	(4,4)	(4,5)
5	(5,2)	(5,3)	(5,4)	(5,5)

Mean of each sample.

$\bar{x} = \left(\frac{a+b}{2}\right)$	2	3	4	5
2	2.0	2.5	3	3.5
3	2.5	3	3.5	4
4	3	3.5	4	4.5
5	3.5	4	4.5	5

H.W. Ex

Find mean & s.d. of S.D.M. (Sampling distribution of mean) for populatⁿ 1, 2, 3 with $n=2$ & sampling (a) with replacement (b) without replacement

Solⁿ $\mu = 2$, $\sigma^2 = 2/3$

(a) $\mu_{\bar{x}} = \frac{18}{9}$, $\sigma_{\bar{x}}^2 = 1/3 = \frac{\sigma^2}{n}$

(b) $\mu_{\bar{x}} = 2$, $\sigma_{\bar{x}}^2 = 1/6 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = 1/6$.

H.W. Ex

Determine mean & s.d. of S.D of ~~Variance~~ mean (Sampling distributⁿ) for populatⁿ 3, 7, 11, 15 with $n=2$ with sampling
(a) with replacement (b) without replacement

⇒ $N^n = 4^2 = 16$

Mean :	3	5	7	9	11	13	15
freq :	1	2	3	4	3	2	1
Variance :	0	4	16	36			

(a) $\mu_{s^2} = 10$ (b) $\sigma_{s^2}^2 = 11.489$

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* Sampling distribution of means (σ KNOWN)

Ex ① Determine the mean & s.d. of sampling distributⁿ of means of 300 random samples each of size $n = 36$ are drawn from population of $N = 1500$ which is normally distributed with mean $\mu = 22.4$ & s.d 'σ' $= 0.048$, if sampling is done
(a) with replacement
(b) without replacement

Solⁿ: Let $N = 1500$; Mean ; $\mu = 22.4$
s.d ; $\sigma = 0.048$ & $n = 36$

(i) With replacement
(a) Mean ; $\mu_{\bar{x}} = \mu = 22.4$
(b) S.d. of sampling distributⁿ of means

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = 0.008 = 8 \times 10^{-3}$$

(ii) Without replacement
(a) Mean; $\mu_{\bar{x}} = \mu = 22.4$

(b) s.d. of sampling distributⁿ of means,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = 0.008 \sqrt{\frac{1500-36}{1500-1}}$$

$$= 7.90 \times 10^{-3} = 0.00790 .$$

Q ② Assume that hts of 3000 male students at university are normally distributed with mean 68.0 inches & std. deviation 3.0 inches. If 80 samples consisting of 25 students each are obtained, what would be mean & std. deviation of resulting sample of mean

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if sampling were done
(a) With replacement (b) Without replacement

Sol^n:

G^n: Population; $N = 3000$ male students
Mean of populat^n; $\mu = 68.0$ inches
Std. deviation of populat^n; $\sigma = 3.0$ inches

Sample size; $n = 25$ (80 samples आहेत)

To find; Mean of sample of means i.e. $\mu_{\bar{x}} = ?$
Std. deviat^n of sample of means; i.e. $\sigma_{\bar{x}} = ?$

(i) **For With replacement :-**

(a) Mean; $\mu_{\bar{x}} = \mu = 68.0$
(b) std. deviat^n; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = 0.6$ inches

(ii) **Without replacement :-**

(a) Mean; $\mu_{\bar{x}} = \mu = 68.0$ inches
(b) std. deviation; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

$$\sigma_{\bar{x}} = 0.6 \sqrt{\frac{3000-25}{3000-1}} = 0.5975 \text{ inches}$$

Q3 The sampling distribut^n of means of 300 random samples of size 36 is drawn from a populat^n of 1500 which is normally distributed with mean 22.4 & S.D. of 0.048. Determine the expected

OR $z = \dfrac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$

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Number of random samples having their means

(a) Between 22.39 & 22.41

(b) Greater than 22.42 (c) Less than 22.37

(d) Less than 22.38 or more than 22.41.

Solⁿ :

Let size of populatⁿ ; $N = 1500$
Mean of populatⁿ ; $\mu = 22.4$
S.D. of populatⁿ ; $\sigma = 0.048$.

$z = \dfrac{X - \mu}{\sigma/\sqrt{n}}$

Size of sample ; $n = 36$
Mean of sample mean ; $\mu_{\bar{x}} = \mu = 22.4$
S.D. of sample of mean ; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} \simeq 0.048$.

(i) To find $P(22.39 \le X \le 22.41)$

Now; $P(a \le X \le b) = P\left(\dfrac{a-\mu}{\sigma/\sqrt{n}} \le \dfrac{X-\mu}{\sigma/\sqrt{n}} \le \dfrac{b-\mu}{\sigma/\sqrt{n}}\right)$

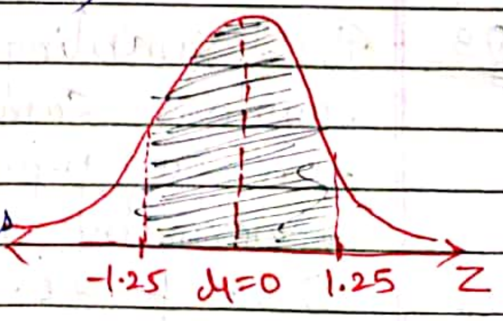
$\Rightarrow P(22.39 \le X \le 22.41) = P\left(\dfrac{22.39 - 22.4}{\frac{0.048}{\sqrt{36}}} \le Z \le \dfrac{22.41 - 22.4}{\frac{0.048}{\sqrt{36}}}\right)$

$\Rightarrow P(22.39 \le X \le 22.41) = P(-1.25 \le Z \le 1.25)$

$= 2 \cdot$ Area b/w $(z = 0$ to $z = 1.25)$

$= 2 \cdot (0.3944)$

$= 0.7888$



∴ Expected No. samples

N = Probability × No. of Samples

$= 0.7888 \times 300$

$N = 236.64 \simeq \underline{237}$

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(ii) To find $P(X \geq 22.42) = P(22.42 \leq X < \infty)$

$$\Rightarrow P(X \geq 22.42) = P\left(\frac{22.42-22.4}{\frac{0.048}{\sqrt{36}}} \leq z < \infty\right)$$

$$= P(2.50 \leq z < \infty)$$

$$= F(\infty) - F(2.50) = 0.5 - 0.4938$$

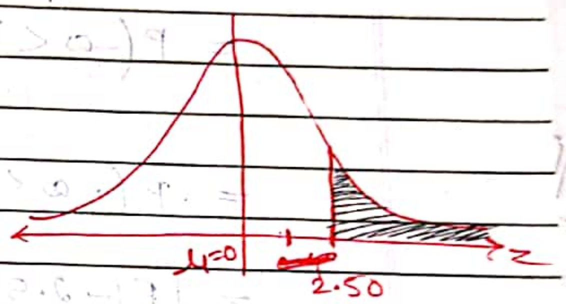
$$= \underline{0.0062}$$

∴ Expected no. of sample;

N = Prob of X No. of samples

$N = 0.0062 \times 300$

$\underline{\underline{N = 2}}$



(iii) To find $P(X \leq 22.37) = P(-\infty \leq X \leq 22.37)$

$$\Rightarrow P(-\infty < X \leq 22.37) = P\left(-\infty < z \leq \frac{22.37-22.48}{\frac{0.048}{\sqrt{36}}}\right)$$

$$= P(-\infty < z \leq -3.75)$$

$$= F(-3.75) - F(-\infty) = F(\infty) - F(3.75)$$

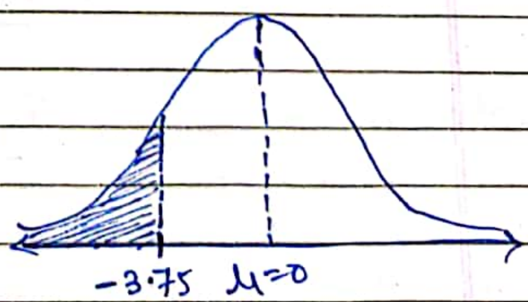
$$= 0.5 - 0.4999 = \underline{\underline{0.0001}}$$

∴ Expected no. of sample;

N = Prob of X No. of Samples

$= 0.0001 \times 300$

$N = 0.03 \cong \underline{\underline{0}}$



(iv) To find $P(x \le 22.38$ or $x \ge 22.41)$

$$\therefore P(x \le 22.38) + P(x \ge 22.41) = P(-\infty < X \le 22.38) + P(22.41 \le X < \infty)$$

$$= P\left(-\infty < Z < \frac{b-\mu}{\sigma/\sqrt{n}}\right) + P\left(\frac{a-\mu}{\sigma/\sqrt{n}} < Z < \infty\right)$$

$$= P\left(-\infty < Z \le \frac{22.38 - 22.4}{0.048/\sqrt{36}}\right) + P\left(\frac{22.41 - 22.4}{0.048/\sqrt{36}} \le Z < \infty\right)$$

$$= P(-\infty < Z \le -2.53) + P(1.26 \le Z < \infty)$$

$$= [F(-2.53) - F(-\infty)] + [F(\infty) - F(1.26)]$$

$= [0.5 -$

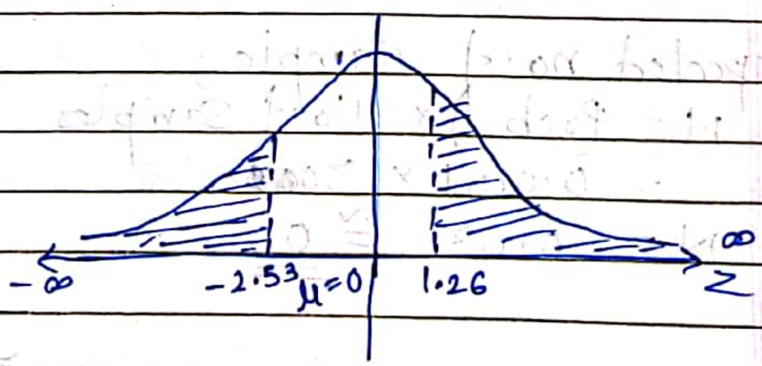
$= 0.0057 + 0.1038$

$= 0.1095$

$\therefore$ Expected No. samples; N = Prob × No. of samples

$N = 0.1095 \times 300 = 32.85 \cong 33$

$\Rightarrow$ $N = 33$.



H.W

Q Determine prob. that sample mean area covered by sample of 40 of 1 litre paint boxes will be b/w 510 to 520 sq. feet given that a 1 litre of such paint boxes covers on average 513.3 sq. feet with s.d. of 31.5 sq. ft.

Soln

Let μ = mean = 513.3 ; $\sigma = 31.5$; $n = 40$

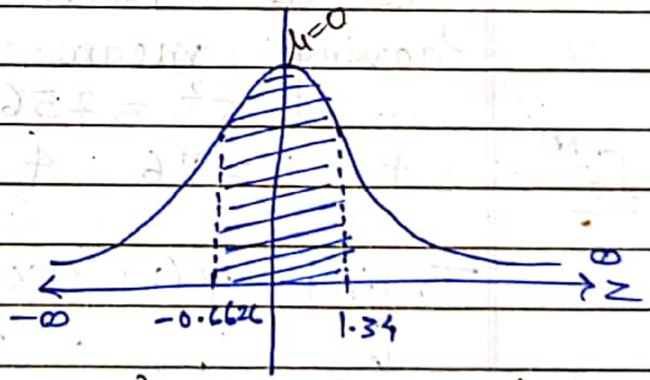
& $z = \frac{x-\mu}{\sigma/\sqrt{n}}$

To find $P(510 \le x \le 520) = P\left(\frac{a-\mu}{\sigma/\sqrt{n}} \le z \le \frac{b-\mu}{\sigma/\sqrt{n}}\right)$

$$= P\left(\frac{510-513.1}{31.5/\sqrt{40}} \le z \le \frac{520-513.1}{31.5/\sqrt{40}}\right)$$

$\Rightarrow P(510 \le x \le 520) = P(-0.6626 \le z \le 1.3452)$

$$= F(1.3452) + F(0.6626)$$
$$= F(1.35) + F(0.66)$$
$$= 0.4115 + 0.2454 = \underline{\underline{0.6569}}$$



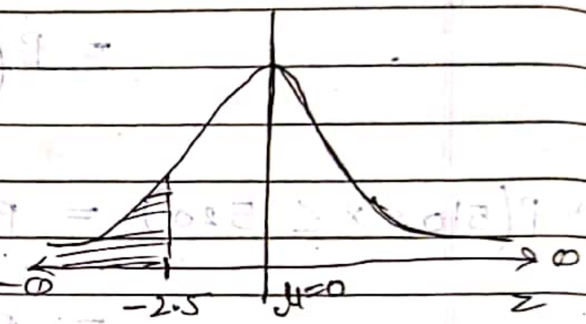
H.W Calc. Prob that random sample of 16 computers will have an average life of less than 775 hours assuming that length of life of computers is approx. normally distributed with mean 800 hours & std. devi. 40 hours.

Solⁿ. Let $\mu = 800$; $\sigma = 40$; $n = 16$; $z = \frac{X-\mu}{\sigma/\sqrt{n}}$

To find : $P(X \le 775)$

$$P(X \le 775) = P[-\infty < X \le 775)$$
$$= P\left(-\infty < Z \le \frac{775-800}{40/\sqrt{16}}\right)$$
$$= P[-\infty < Z \le (-2.5)]$$
$$= F(-2.5) - F(-\infty)$$
$$= F(\infty) - F(2.5)$$
$$= 0.5 - F(2.5) = 0.5 - 0.4938$$

$$P(X \le 775) = \underline{0.0062}$$



① Determine prob. that $\bar{X}$ will be between 75 & 78 if a random sample of size 100 is taken from an infinite populatⁿ having mean $\mu = 76$ & variance $\sigma^2 = 256$.

Solⁿ. Let $\mu = 76$ & $\sigma = \sqrt{256} = 16$ & $z = \frac{X-\mu}{\sigma/\sqrt{n}}$

To find $P(75 \le X \le 78)$

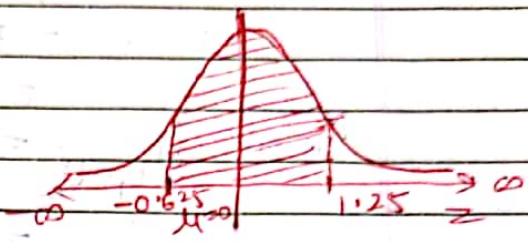
$$= P\left[\frac{75-76}{16/\sqrt{100}} \le Z \le \frac{78-76}{16/\sqrt{100}}\right]$$

$$\Rightarrow \quad = P[-0.625 \le Z \le 1.25]$$

$\rightarrow$ $P(75 \le X \le 78) = F(1.25) - F(-0.625)$

$= F(1.25) + F(0.625)$

$= 0.2340 + 0.3944 = \underline{0.6284}$



* Sampling distribution of differences and sum (More than 1 mean)

Note: (i) $\mu_{S_1 - S_2} = \mu_{S_1} - \mu_{S_2}$

(ii) $\mu_{S_1 + S_2} = \mu_{S_1} + \mu_{S_2}$

(iii) $\sigma_{S_1 - S_2} = \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2} = \sigma_{S_1 + S_2}$

Q Let U_1 be a variable that stands for any of an elements of population 3, 7, 8 & U_2 a variable that stands for any of the elements of population 2, 4. Compute

(a) μ_{U_1} (b) μ_{U_2} (c) $\mu_{U_1 - U_2}$ (d) σ_{U_1}

(e) σ_{U_2} (f) $\sigma_{U_1 - U_2}$ H.W. (g) $\mu_{U_1 + U_2}$ (h) $\sigma_{U_1 + U_2}$

↳ Ans: 9 ↳ Ans: 2.3803

Sol^n.

(a) μ_{U_1} = mean of populat^n $U_1 = \dfrac{3+7+8}{3} = 6$

(b) μ_{U_2} = mean of populat^n $U_2 = \dfrac{2+4}{2} = 3$

$U_1 = \{3, 7, 8\} \to 6 \to \mu_{U_1}$

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(c) The populatn consisting of differences of any member of U_1 & any member of U_2 is

$3-2 = 1$ $\quad 7-2 = 5$ $\quad 8-2 = 6$ $\quad$ OR $\quad 3-4 = -1$ $\quad 7-4 = 3$ $\quad 8-4 = 4$

Then $\mu_{U_1-U_2}$ = mean of $(U_1 - U_2)$

$$= \frac{1+5+6+(-1)+3+4}{6} = 3$$

This shows

$$\boxed{\mu_{U_1-U_2} = \mu_{U_1} - \mu_{U_2}}$$

(d) $\sigma^2_{U_1}$ = Variance of populatn U_1 $= E[(\bar{x} - \mu_{\bar{x}})^2]$

$$= E[(\bar{x} - \mu_{U_1})^2] = \frac{\Sigma(\bar{x} - \mu_{U_1})^2}{n}$$

$$= \frac{(3-6)^2 + (7-6)^2 + (8-6)^2}{3} = \frac{14}{3}$$

$\sigma^2_{U_1} = \frac{14}{3}$ $\quad$ ie $\quad \sigma_{U_1} = \sqrt{\frac{14}{3}}$

(e) $\sigma^2_{U_2}$ = Variance of populatn U_2 $= E[(\bar{x} - \mu_{U_2})^2]$

$$= \frac{(2-3)^2 + (4-3)^2}{2} = 1$$

$\Rightarrow \boxed{\sigma_{U_2} = 1}$

(f) $\sigma^2_{U_1-U_2}$ = Variance of populatn (U_1-U_2)

$$= \frac{(1-3)^2+(5-3)^2+(6-3)^2+(-1-3)^2+(5-3)^2+(4-3)^2}{6}$$

$\sigma^2_{U_1-U_2} = \frac{17}{3}$ ie $\sigma_{U_1-U_2} = \sqrt{\frac{17}{3}}$

This show;

ie $\sigma_{U_1-U_2} = \sqrt{\frac{17}{3}} = \sqrt{\sigma^2_{U_1}+\sigma^2_{U_2}} = \sqrt{\frac{14}{3}+1}$

ie $\sigma^2_{U_1-U_2} = \sigma^2_{U_1}+\sigma^2_{U_2}$

* A certain type of electric light bulb has a mean lifetime ~~bulbs~~ of 1500 hrs & a std. deviation of 150 hrs. Three bulbs are connected so that when one burns out, another will go on. Assuming that lifetimes are normally distributed, what is prob. that lighting will take place for (a) Atleast 5000 hrs

(b) Atmost 4200 hrs.

Solⁿ

Let mean life time of bulbs 1, 2, 3 be L_1, L_2, L_3 then

$\mu_{L_1}+\mu_{L_2}+\mu_{L_3} = \mu_{L_1+L_2+L_3}$ = 1500 + 1500 + 1500 = 4500 hrs.

& $\sigma_{L_1+L_2+L_3} = \sqrt{\sigma^2_{L_1}+\sigma^2_{L_2}+\sigma^2_{L_3}} = \sqrt{3(150)^2} = 260$ hrs.

Let X be combined lifetime

(a) $P(5000 \le X) = P(5000 \le X < \infty)$

$= P(z_1 \le z \le z_2)$

$z_1 = \frac{a-\mu}{\sigma}$

combined lifetime

combine lifetime mean

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$\rightarrow$ $P(5000 \le X < \infty) = P\left(\frac{5000-4500}{260} \le z \le \infty\right)$

$= P(1.92 \le z < \infty)$

$= F(\infty) - F(1.92)$

$= \underline{0.0274}$

(b) $P(X \le 4200) = P(-\infty < X \le 4200)$

$= P(z_1 \le z \le z_2)$

$= P\left(-\infty < z \le \frac{4200-4500}{260}\right)$

$= P(-\infty < z \le -1.15)$

$= F(-1.15) - F(-\infty)$

$= F(\infty) - F(1.15)$

$P(X \le 4200) = \underline{0.1251}$

H.W The Electric light bulbs of manufacturer 'A' have a mean lifetime of 1400 hrs with standard deviation of 200 hrs; While those of manufacturer 'B' have a mean lifetime of 1200 hrs with a Standard deviation of 100 hrs. If random Samples of 125 bulbs of each brand are tested, what is prob. that the brand 'A' bulbs will have a mean lifetime that is atleast (a) 160 hrs
(b) 250 hrs more than the brand B bulbs?

Sol.

Let $\overline{X}_A$ & $\overline{X}_B$ denote mean lifetimes of samples A & B resply.

Then $\mu_{\bar{X}_A - \bar{X}_B} = \mu_{\bar{X}_A} - \mu_{\bar{X}_B} = 1400 - 1200 = 200\text{ hrs.}$

& $\sigma_{\bar{X}_A - \bar{X}_B} = \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}} = \sqrt{\frac{(200)^2}{125} + \frac{(100)^2}{125}}$

$\sigma_{\bar{X}_A - \bar{X}_B} = 20\text{ hrs}$

The standardized variable for differences in means that

$$z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_{\bar{X}_A} - \mu_{\bar{X}_B})}{\sigma_{\bar{X}_A} - \sigma_{\bar{X}_B}}$$

$$z = \frac{(\bar{X}_A - \bar{X}_B) - 200}{20}$$ is normally distributed.

(a) $P(160 \le X < \infty) = P(z_1 \le z \le z_2)$

$= P\left(\frac{160 - 200}{20} \le z < \infty\right)$

$= P(-2 \le z < \infty) = F(\infty) - F(-2)$

$= F(\infty) + F(2)$

$= 0.5 + 0.4772 = 0.9772$

Q The mean voltage of a battery is 15 & std devi. is 0.2. Find the prob. that four such batteries connected in series will have combined voltage of 60.8 or more volts.

Sol^n.

To find ; $P(X \ge 60.8) = ?$

Let mean of single battery $= 15$

& std. deviation $= 0.2$.

Let four batteries are A, B, C & D.

$$\therefore \mu_{A+B+C+D} = \mu_A + \mu_B + \mu_C + \mu_D$$
$$= 15 + 15 + 15 + 15 = \underline{\underline{60}}.$$

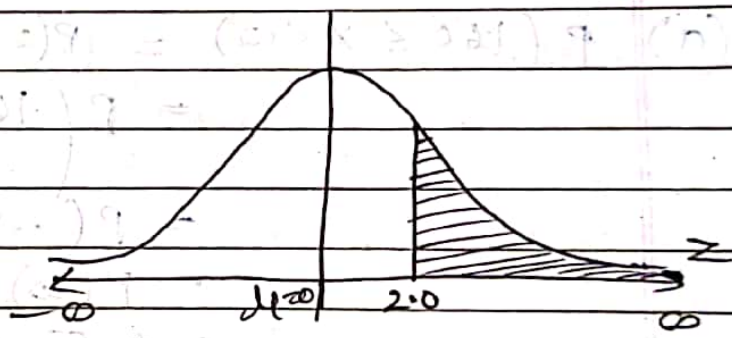
$$\& \quad \sigma_{A+B+C+D} = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2} = \sqrt{4(0.2)^2}$$

$$\sigma_{A+B+C+D} = 0.4.$$

Now; $P(X \geq 60.8) = P(60.8 \leq X < \infty) = P(z_1 \leq Z \leq z_2)$

$$= P\left(\frac{60.8 - 60}{0.4} \leq Z < \infty\right)$$
$$= P(2.00 \leq Z < \infty)$$
$$= F(\infty) - F(2.0)$$

$$\therefore P(X \geq 60.8) = 0.5 - 0.4773 = \underline{\underline{0.0227}}$$



* Sampling distribution of Proportion

(a) <u>With replacement</u> :- (a) Mean ; $\mu_p = p$

(b) std. deviatⁿ ; $\sigma_p = \sqrt{\frac{pq}{n}}$

(b) <u>Without replacement</u> :-

(a) Mean; $\mu_p = p$.

(b) std. deviation; σ_p =

$$\sigma_{\bar{x}} = \sigma_{\bar{x}} = \sqrt{\frac{pq}{n}\left[\frac{N-n}{N-1}\right]}$$

<u>Ex</u> Find probability that in 120 tosses of a fair coin
(a) b/w 40% & 60% will be heads
(b) 5/8 or more will be heads

<u>Solⁿ.</u>

In this populatⁿ; probability of head is $p = \frac{1}{2}$

& prob. of tail is $q = 1-p = \frac{1}{2}$

Now; Mean ; $\mu_p = p = \frac{1}{2}$

& standard deviation; $\sigma_p = \sqrt{\frac{pq}{n}}$

$$\Rightarrow \sigma_p = \sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{120}} = 0.0456$$

{ ∵ 120 tosses of coin as a sample from ∞-population of all possible tosses of coin }.

<u>To find</u> : P (40% ≤ X ≤ 60%) will be heads
ie $P(0.40 \le X \le 0.60)$

$$z_1 = \frac{a-\mu_p}{\sigma_p} \quad \& \quad z_2 = \frac{b-\mu_p}{\sigma_p}$$

$$\therefore P(0.4 \le X \le 0.6) = P(z_1 \le z \le z_2)$$

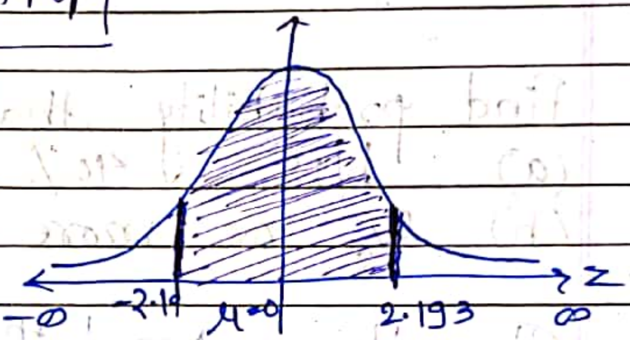
$$= P\left[\frac{0.4-0.5}{0.0456} \le z \le \frac{0.6-0.5}{0.0456}\right]$$

$$= P[-2.193 \le z \le 2.193]$$

$$= F(2.193) - F(-2.193)$$

$$= 2 \cdot F(2.193) = 2 \times 0.4887$$

$$\boxed{P(0.4 \le X \le 0.6) = 0.9774}$$



(b) $P(\frac{5}{8}$ or more will be heads$) = P(X \ge \frac{5}{8})$

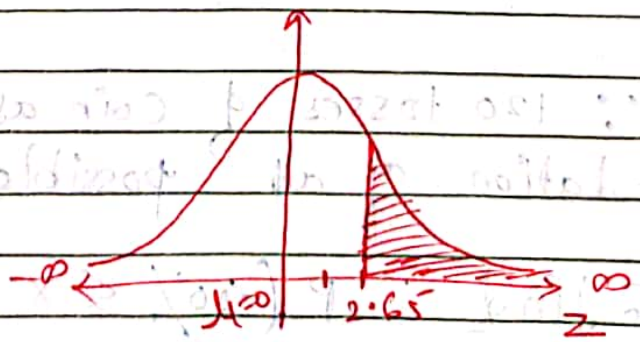
$$P(X \ge 0.6250) = P(0.6250 \le X < \infty)$$

$$= P(z_1 \le z \le z_2)$$

$$= P\left(\frac{0.6250-0.5}{0.0456} \le z < \infty\right) = P(2.65 \le z < \infty)$$

$$= F(\infty) - F(2.65)$$

$$= 0.5 - 0.4960 = \underline{\underline{0.0040}}$$



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Ex The election returns showed that a certain candidates received 46% of the votes. Determine prob. that a poll of (a) 200 (b) 1000 people selected at random from the voting population would have shown a majority of votes in favour of candidates (prob. of win).

Solⁿ.

Here prob. of certain candidates received 46% of votes ie $p = \frac{46}{100} = 0.46$

$\therefore$ Mean; $\mu_p = p = 0.46$ & $q = 1 - p = 0.54$

$\cdot$ std. deviatⁿ; $\sigma_p = \sqrt{\frac{p \cdot q}{n}}$

(a) Here; $n = 200$. $\therefore \sigma_p = \sqrt{\frac{(0.46)(0.54)}{200}}$

$\Rightarrow \sigma_p = \underline{0.0352}$

(**Note**: This proportⁿ can also be obtained by realizing that 101 or more indicates a majority, but this is a cts variable is 100.5 & proportⁿ is $\frac{100.5}{200} = 0.5025$)

Or: Since $\frac{1}{2n} = \frac{1}{2(200)} = \frac{1}{400} = 0.0025$ is the correction factor for discrete variable, so majority is indicate in sample, if proportⁿ in favor of candidate is $0.50 + 0.0025$
$= 0.5025$ or more.

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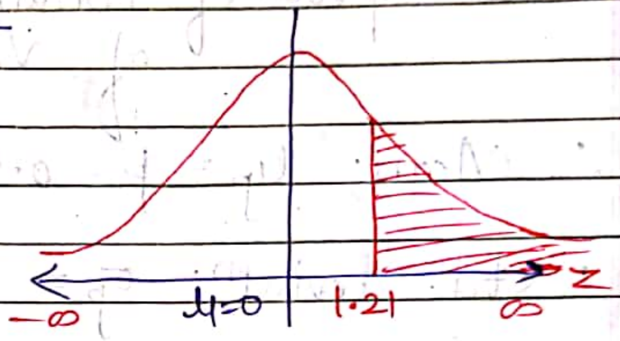
To find $P(x \geq 0.5025) = P(0.5025 \leq X < \infty)$
$= P(z_1 \leq z < z_2)$
$= P\left(\frac{a-\mu_p}{\sigma_p} \leq z < \frac{b-\mu_p}{\sigma_p}\right)$
$= P\left(\frac{0.5025 - 0.46}{0.0352} \leq z < \infty\right)$
$= P(1.21 \leq z < \infty)$
$= F(\infty) - F(1.21) = 0.50 - 0.3869$

$P(x \geq 0.5025) = \underline{\underline{0.1131}}$



(b) When $n = 1000$.
Here; $\mu_p = p = 0.46$

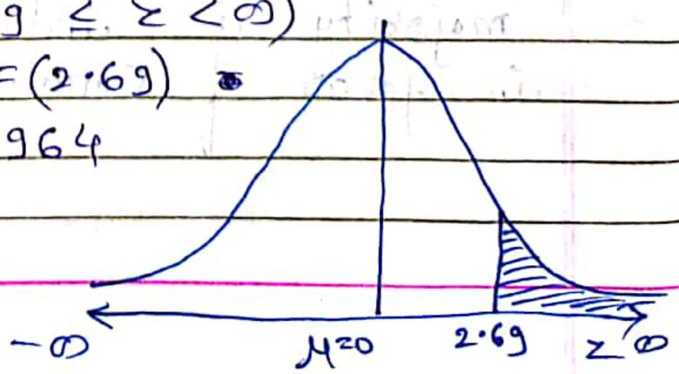
& std. deviation; $\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.46)(0.54)}{1000}}$

$\boxed{\sigma_p = 0.0158}$

Correctn factor, $\frac{1}{2n} = \frac{1}{2000}$
$= 0.0005$

To find: $P(x \geq 0.5025) = P(0.5025 \leq X < \infty)$
$= P(z_1 \leq z \leq z_2)$
$= P\left(\frac{0.5025 - 0.46}{0.0158} \leq z < \infty\right)$
$= P(2.69 \leq z < \infty)$
$= F(\infty) - F(2.69)$
$= 0.5 - 0.4964$

$P(x \geq 0.5025) = \underline{\underline{0.0036}}$



Q. It has been found that 2% of the tools produced by a certain machine are defective. What is prob. that in shipment of 400 such tools (a) 3% or more (b) 2% or less will prove defective?

Solⁿ.

Here; probability of tools produced by certain machine are defective is 2% = $\underline{0.02}$

i.e. $p = 0.02$

Here; $n = 400$

∴ Mean; $\mu_p = p = 0.02$ & $q = 1 - p = 0.98$

& std. deviatⁿ; $\sigma_p = \sqrt{\dfrac{pq}{n}} = \sqrt{\dfrac{(0.02)(0.98)}{400}}$

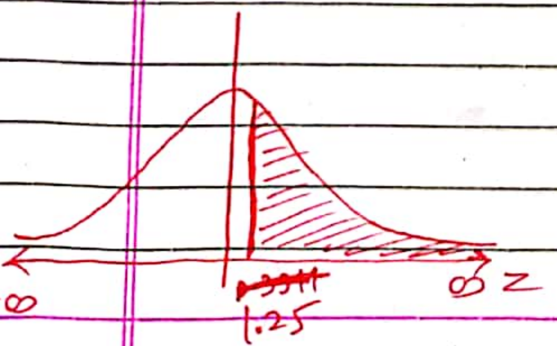
$\Rightarrow \boxed{\sigma_p = 0.007}$

* Using Correction for discrete variables,

$\dfrac{1}{2n} = \dfrac{1}{2(400)} = 0.00125$

(a) 3% or more i.e. 0.03 or more

$$\begin{aligned}
\therefore P(X \geq 0.03 - 0.00125) &= P(0.03 - 0.00125 \leq X < \infty) \\
&= P(0.02875 \leq X < \infty) \\
&= P(z_1 \leq Z \leq z_2) \\
&= P\left(\frac{0.02875 - 0.02}{0.007} \leq Z < \infty\right) \\
&= P(1.25 \leq Z < \infty) \\
&= F(\infty) - F(1.25) \\
&= 0.50 - 0.3944 = \underline{0.1056}
\end{aligned}$$



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(b) 2% or less ie ~~p~~ 0.02 or less.

$$P(X \le 0.02 + 0.00125) = P(-\infty < X \le 0.02125)$$

$$= P\left(-\infty < Z \le \frac{0.02125 - 0.02}{0.007}\right)$$

$$= P(-\infty < Z \le 0.18)$$

$$= F(0.18) + F(\infty) = 0.0714 + 0.5$$

$$= \underline{\underline{0.5714}}$$