death Bernoulli's → 1705 (After 8 yr's Publish)

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Unit 3 : Special Probability Distribution

Syllabus: Binomial, Geometric, Poisson or Exponential, Normal distributions, Central Limit Thm statement

* **Binomial Distribution / Bernoulli Distribution.**
  Binomial distribution was discovered by James Bernoulli's in year 1700 & was 1st published in 1713

* Let a random experiment be performed repeatedly each repetition being called a trial.

* Event occurs → Success & Event not occurs → failure.

* Consider a random experiment with only two possible outcomes say "success" & "failure". Let 'p' denotes probability of success & 'q' $= 1-p$ denotes prob. of failure.

Suppose that experiment is independently repeated n-times & 'X' denotes no. of successes in n-trials. Hence X assumes values $0, 1, 2, \ldots, n$
Then;

success → x.
fail → n−x.

probability of exactly x-success in n-trials is given by prob. f^n.

$$f(x) = P(X=x) = {}^nC_x (p)^x \cdot q^{n-x}$$

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This prob. distribution is called Binomial or Bernoulli's distribution as it corresponds to successive terms in Binomial exp:

$$(p+q)^n = {}^nC_0\, p^0 q^n + {}^nC_1\, p^1 \cdot q^{n-1} + {}^nC_2 \cdot p^2 \cdot q^{n-2} + \dots + {}^nC_n \cdot p^n \cdot q^0$$

$$= \sum_{x=0}^{n} {}^nC_x\, p^x\, q^{n-x}$$

The discrete r.v. X having above prob. distributn $f(x)$ is said to be Binomial distributed.

* Properties of B.D: If random v. X is binomially distributed, then

(1) Mean of X is $\mu = np$

(2) Variance of X is $\sigma^2 = npq$

(3) Standard deviatn of X is $\sigma = \sqrt{npq}$

Ex Out of 800 families with 5 children each how many would you expect to have

(i) 3-boys (ii) 5 girls (iii) Either '2' or 3-boys.

Assume equal probabilities for boys & girls

Soln:

Let X be the discrete random variable denoting minors $n=5$, $p=0.5$ & $q=0.5$

$q = 1-0.5 = 0.5$

Now;

$$P(X=x) = {}^nC_x\, p^x\, (q)^{n-x}$$

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To find ~~1st~~ (i) $P(x=3)$

$$P(x=3) = {}^5C_3 \cdot (0.5)^3 \cdot (0.5)^{5-3} = 5/16.$$

$$\boxed{P(X=x=3) = 0.3125}$$

~~(ii)~~ $P(x=3) = P(3\text{-boys})$

$$= \frac{5!}{3!\,2!}\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2$$

$$= 5/16.$$

$\therefore$ The number of families having exactly 3 boys is $\frac{5}{16} \times 800 = \underline{\underline{250}}$

(ii) $P(X=5) = P(5\text{-girls}) = {}^5C_5\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{5}\right)^{5-5}$

$$= 1 \cdot \frac{1}{32} \cdot 1 = 1/32.$$

$\therefore$ The number of famililies having exactly 5-girls is $\frac{1}{32} \times 800 = \underline{\underline{25}}$.

(iii) $P(2\text{-boys or } 3\text{-boys}) = P(X=2) + P(X=3)$

$$= {}^5C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{5-2} + {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$$

$$= \frac{5!}{2!\,3!} \times \frac{1}{4} \times \left(\frac{1}{2}\right)^3 + \frac{5!}{3!\,2!} \times \frac{1}{8} \times \frac{1}{4}$$

$$= 2 \cdot 10 \times \left(\frac{1}{2}\right)^5 = 5/8.$$

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$\therefore$ The number of families having 2 or 3 boys is $\frac{5}{8} \times 800 = \underline{500}$

Q If on an average one ship in every 10 is wrecked (damage). Find prob. that out of 5 ships expected to arrive ~~4~~ atleast 4 will arrive safely.

→ Let X be discrete r.v. denoting ships

$n = 5$; $q = \frac{1}{10}$; $p = 1 - q = 1 - 0.1$

$\boxed{q = 0.1}$ $\boxed{p = 0.9}$

$P(X = x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

To find : $P(X \geq 4) = P(X=4) + P(X=5)$

$P(X \geq 4) = {}^{5}C_{4} \cdot \left(\frac{1}{9}\right)^{4} \cdot \left(\frac{1}{10}\right)^{5-4} + {}^{5}C_{5} \left(\frac{1}{9}\right)^{5} \cdot \left(\frac{1}{10}\right)^{0}$

$= \frac{5!}{4!\,1!} \left(\frac{1}{9}\right)^{4} \cdot \left(\frac{1}{10}\right)^{1} + 1 \cdot \left(\frac{1}{9}\right)^{5}$

$P(X \geq 4) = 0.9186$

Q An insurance salesman sells policies to 5 men, all of identical age & in good health. The prob. that a man of this particular age will be alive 30 yrs is $2/3$. Find prob. that in 30 yrs.

(i) All 5 men (ii) Atleast 3 men

(iii) Only 2 men (iv) at the most 1 man.

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(v) Atleast 1 man will be alive

→ We have no. of trials $n=5$, prob. of success is $p=2/3$ &
$q = 1-p = 1-2/3 = 1/3$.

(i) $P(X=5) = P(\text{all 5-men})$
$= {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{5-5} = \frac{32}{243}$

(ii) $P(\text{AtLeast 3-men}) = P(X=3) + P(X=4) + P(X=5)$
$= {}^5C_3 \cdot \left(\frac{2}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^{5-3} + {}^5C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{5-4} +$
${}^5C_5 \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^{5-5}$

$= \frac{5!}{3! \cdot 2!} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2 + \frac{5!}{4! \, 1!} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right) +$
$1 \cdot \left(\frac{2}{3}\right)^5 \cdot 1$

$= \frac{192}{243}$

(iii) $P(\text{only 2-men}) = P(X=2)$
$= {}^5C_2 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^{5-2} =$
$= \frac{5!}{2! \, 3!} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3 = \frac{40}{243}$.

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(iv) $P(\text{At Most one man}) = P(X=0)+P(X=1)$

$= {}^5C_0 \cdot \left(\frac{2}{3}\right)^0 \cdot \left(\frac{1}{3}\right)^{5-0} + {}^5C_1 \cdot \left(\frac{2}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^{5-1}$

$= \frac{5!}{0!\cdot 5!} \left(\frac{1}{3}\right)^5 + \frac{5!}{1!\,4!} \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4$

$= \frac{11}{243}$.

(v) $P(\text{Atleast 1 man}) = 1 - P(X=0) = P(X \geq 1)$

$= 1 - {}^5C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 = 1 - \frac{1}{243} = \frac{242}{243}$

H.W. A die is thrown 8-times & it is required to find prob. that 3- will show
(i) Exalctly 2- times (ii) Atleast 7 times
(iii) Atleast once

Solⁿ. We have no. of trials, $n = 8$
Prob. of success, $p = \frac{1}{6} = 0.1667$
Prob. and $q = 1-p = 1-\frac{1}{6} = \frac{5}{6} = 0.8333$

$P(X=x) = {}^nC_x \cdot p^x \cdot q^{n-x}$.

(i) $P(X=2) = {}^8C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^6 = 0.2605$

(ii) $P(X \geq 7) = P(X=7) + P(X=8) = {}^8C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right) + {}^8C_8 \left(\frac{1}{6}\right)^8 \left(\frac{5}{6}\right)^0$

(iii) $P(X \geq 1) = 1 - P(X=0) = 1 - {}^8C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^8$ $=0$

$= 0.7674$.

Binomial - discrete

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H.W. An underground mine has 5 pumps installed for pumping out storm water, the probability of any one pump failing during storm is $1/8$. What is prob. that

(i) Atleast 2 pumps will be working

(ii) All pumps will be working during particular storm

Sol^n $n = 5$ = No. of pumps.

$q = 1/8$; $p = 1 - 1/8 = 7/8$

(i) $P(X \geq 2) = 0.9989$

(ii) $P(X = 5) = {}^5C_5 \cdot (7/8)^5 \cdot (1/8)^0 = 0.5129$

* Poisson Distribution *

+ Poisson distribut^n is a limiting case of Binomial distribut^n.

* If the discrete r.v. X has probability f^n $f(x) = P(X = x) = \dfrac{\mu^x \cdot e^{-\mu}}{x!}$; $x = 0, 1, 2, \ldots$

then

X is said to be Poisson distributed & prob. distribut^n $f(x)$ is called Poisson distribution.

where;

mean $= \mu = np$; $\sigma^2 = \lambda = \mu$.

& $\sigma = \sqrt{np}$. $\sigma = \sqrt{\lambda}$

$\therefore$ mean $= \sqrt{\text{variance}}$

Q. If 3% of an electric bulbs manufactured by a Company are defective find the prob. that sample of 100 bulbs
(i) exactly 2 (ii) more than 5 (iii) b/w 1 & 3
(iv) Atmost 2 (v) Atleast 2 bulbs will be defective.

Solⁿ: Let X denotes no. of defective bulbs
Here; $n = 100$.
So Poisson's distributⁿ is gⁿ by

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-\mu}\cdot \mu^{x}}{x!}$$

where $\mu = n \cdot p$.
Now; we have p = defective bulb $= \frac{3}{100} = 0.03$

$\therefore \mu = (100)(0.03) = 3$.

(i) $P(X=2) = \frac{\mu^2 \cdot e^{-\mu}}{2!} = \frac{3^2 \cdot e^{-3}}{2!} = \frac{9}{2}(0.0497)$

$= 0.2241$

(ii) $P(X>5) = P(X=6) + P(X=7) + \dots + P(X=100)$

$= 1 - \{P(X=0) + P(X=1) + P(X=2) + P(X=3) + \dots + P(X=5)\}$

$$= 1 - \left\{\frac{3^0 \cdot e^{-3}}{0!} + \frac{3^1 \cdot e^{-3}}{1!} + \frac{3^2 \cdot e^{-3}}{2!} + \frac{3^3 \cdot e^{-3}}{3!} + \frac{3^4 \cdot e^{-3}}{4!} + \frac{3^5 \cdot e^{-3}}{5!}\right\}$$

$$= 1 - \left\{\left(1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}\right)(0.0479)\right\}$$

$= 0.08386$.

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(iii) $P(1 \le X \le 3) = P(X=1) + P(X=2) + P(X=3)$

$$= \frac{3^1 \cdot e^{-3}}{1!} + \frac{3^2 \cdot e^{-3}}{2!} + \frac{3^3 \cdot e^{-3}}{3!}$$

$$= \left[3 + \frac{9}{2} + \frac{27}{6}\right](0.0497) = \underline{\underline{0.5975}}$$

(iv) P(At the most 2) = $P(X \le 2)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{3^0 \cdot e^{-3}}{0!} + \frac{3^1 \cdot e^{-3}}{1!} + \frac{3^2 \cdot e^{-3}}{2!}$$

$$= (1 + 3 + 4.5)(0.0497) = \underline{\underline{0.4232}}$$

(V) P(At a least 2) = $P(X \ge 2) = 1 - \{P(X=0) + P(X=1)\}$

$$= 1 - \left\{\frac{3^0 \cdot e^{-3}}{0!} + \frac{3^1 \cdot e^{-3}}{1!}\right\}$$

$$= \underline{0.8008}$$

Q B/w hours 2.pm 4 4.p.m the average no. of phone calls per minute coming into switch board of company is 2.35. Find prob. that during one particular minute there will be at most 2 phone calls.

Sol^n If X denotes no. of phone calls per min, then it follows Poisson distribu^n with $\mu = 2.35$

$$P(X=x) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{(2.35)^x \cdot e^{-2.35}}{x!} \; ; x = 0, 1, 2, 3, \ldots$$

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$\therefore$ P(At most 2- phone call) $= P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \dfrac{(2.35)^0 \cdot e^{-2.35}}{0!} + \dfrac{(2.35)^1 \cdot e^{-2.35}}{1!} + \dfrac{(2.35)^2 \, e^{-2.35}}{2!}$

$= (1 + 2.35 + 2.7613)(0.0954) = \underline{0.583}$

Q The prob. of coal miner being killed in a mine accident during a year is 0.0004. Calculate prob. that in a mine employing 200 miners, there will be atleast one fatal accident in a year.

Sol.
Let X be a discrete random variable denoting minors.

$n = 200$, $p = 0.0004$ & $q = 1-p = 0.9996$

$\therefore$ Mean ; $\mu = n \cdot p = 200 \cdot (0.0004) = 0.08$

To find ; $P(x \geq 1)$

We have ; $P(X = x) = \dfrac{e^{-\mu} \cdot \mu^x}{x!}$

$P(X \geq 1) = P(X=1) + P(X=2) + \dots + P(X=200)$

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

$= 1 - \left\{ \dfrac{e^{-0.08}}{0!} (0.08)^0 \right\} = 1 - 0.92$

$= \underline{0.08}$

$4 \times 10^{-4} \times 2 \times 10^2$

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~~Q~~ In a certain factory producing cycle's tyres, there is a small chances of 1 in 500 tyres to be defective. The cycles are supplied in lots of 10. Using poisson's distributⁿ
Calculate appbox. no. of lot of 10.
Calculate approx. no. of lots containing
(i) No. defective (ii) one defective
(iii) Two defective tyres resp.

Q The number of monthly breakdowns of a computer is a random variable having a poisson distribution with mean equal to 1·8. Find prob. that this computer will function for a month.
(i) without breakdown
(ii) with atleast one breakdown.

Solⁿ:
Let X be a discrete r.v. having a poisson distributⁿ functⁿ denotes number of monthly breakdowns of computer.

Given ; mean $\mu = 1\cdot 8$.

Poisson distributⁿ ; $P(X=x) = \dfrac{(\mu)^x \cdot e^{-\mu}}{x!}$

(i) P(without breakdown) $= P(X=0) = \dfrac{(1\cdot 8)^0 \cdot e^{-1\cdot 8}}{0!}$

$= \underline{\underline{0\cdot 1653}}$

(ii) P(With atleast one breakdown) = $P(X \geq 1)$

$= 1 - P(X<1) = 1 - P(X=0)$

$= 1 - 0.1653 = \underline{0.8347}$

Q In a certain factory turning razor blades, there is a probability of $1/500$ for any blade to be defective. The blades are supplied in packet of 10. Using poisson distribution to calculate approx. -imate no. of packets containing
(i) No defective (ii) Two defective blades in consignment of 10,000 packets.

Sol^n:

Let X be a discrete random variable

Let n be blades are supplied in packet of 10

ie $n = 10$.

Let p be prob. for any blade to be defective. ie $p = 1/500$.

Now; Poisson distribut^n is g^n by

$$P(X=x) = \frac{\mu^x e^{-\mu}}{x!}$$

Here; $\mu = np = 10 \times \frac{1}{500} = \frac{1}{50} = \underline{0.02}$

(i) P(No defective) $= P(X=0) = \frac{(0.02)^0 \cdot e^{-0.02}}{0!}$

$= 0.9802$

But for Consigment of 10,000 packets

$P(X=0) = 0.9802 \times 10000 = \underline{9802}$

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(ii) $P(\text{Two defective}) = P(X=2) = \dfrac{(0.02)^2 \cdot e^{-0.02}}{2!}$

$= \dfrac{0.0004}{2} \cdot e^{-0.02}$

$= 0.00019604 = 1.96 \times 10^{-4}$

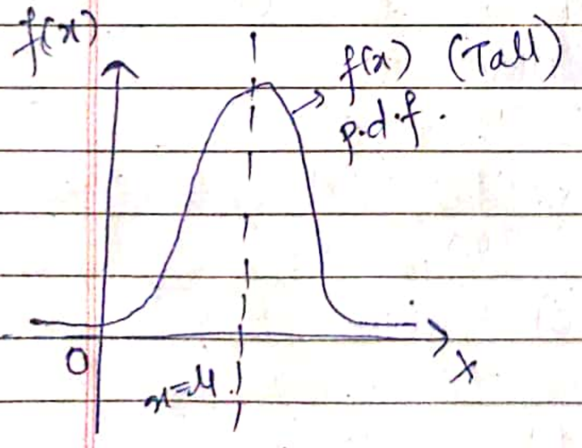
But consignment of 10,000 packets.

$P(X=2) = 1.96 \times 10^{-4} \times 10000 = 1.96 \simeq 2$

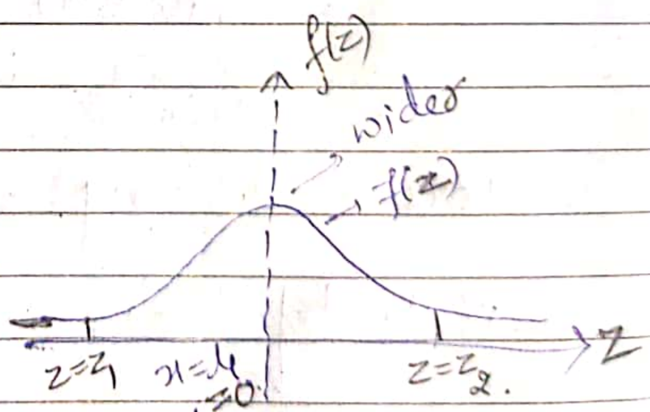
M.Imp : * **Normal distribution** * / Gaussian distribution
(use to find area of curve under the). (use for large population)

Through Graph: Normal distribution curve is bell shaped and symmetrical about mean line $x = \mu$. and x-axis is an asymptote to the curve.

Mean, mode & median are coincide in this Normal distribution.



Normal prob. curve
Tall ⇒ freq. dist. is more
Variance → ↓ है



Std. normal. prob. curve
Wider ⇒ More Std. deviation
ज्यादा data फैला हुआ है
Variation ज्यादा है।

Continuous

A random variable 'X' is said to follow normal distribution with parameter μ (mean) & σ^2 (Variance) if its prob. density f^n $f(x)$ is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad \because -\infty < x < \infty$$

$-\infty < \mu < \infty$

& $\sigma > 0$

The distribution f^n $F(x)$ of r.v. X is given by

$$F(x) = P(X \leq x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(v-\mu)^2/2\sigma^2} \, dv.$$

* If $X \sim N(\mu, \sigma^2)$ then $\frac{X-\mu}{\sigma} = z$ is the std. normal variable.

$E(z)=0$ } The mean of z is '0' & std. deviat^n is 1 $\sigma_z = 1$.

$\therefore$ The "density f^n" of std. random v. z is g^n by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad ; -\infty < z < \infty$$

& Corresponding distribut^n f^n is

$$f(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-v^2/2} \, dv = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_{0}^{z} e^{-v^2/2} \, dv$$

$z_1 = \frac{60-60}{5} = 0$

Note $P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2} \cdot dx.$

Ex 1) H.W Students of a class were given an aptitude test. There marks were found to be normally distributed with mean 60 & std deviation 5. what % of student scored more than equal to 60 marks?

Sol^n.

Let X be cts r.v. denoting marks

$\mu = \text{mean} = 60$, $\sigma = 5$ & $z = \frac{X-\mu}{\sigma}$

$z = \frac{X-60}{5}$.

$P(60 \le X < \infty)$
$P(a \le x < \infty)$

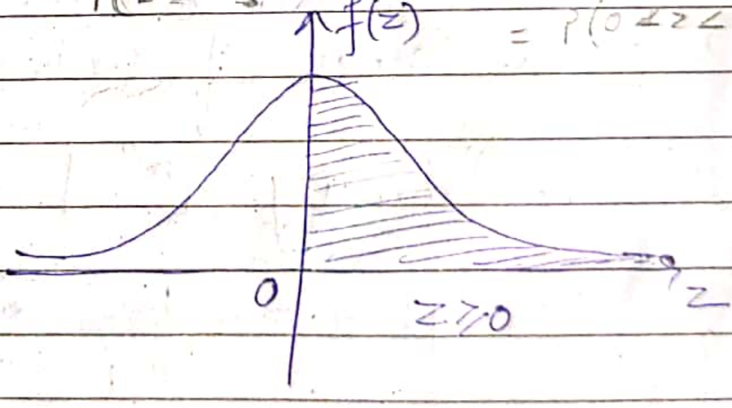
(i) To find $P(X \ge 60) = P\left(z \ge \frac{X-60}{5}\right)$

$\Rightarrow P(X \ge 60) = P\left(z \ge \frac{60-60}{5}\right) = P(z \ge 0)$

$= P(0 \le z \le \infty)$

$= P(0 < z < \infty)$

$\Rightarrow P(X \ge 60) = \underline{0.5}$



$\therefore$ % of students getting more that 60 marks

$P = 0.5 \times 100\% = 50\%$.

Note Properties (i) $F(0) = 0$; $P(z=0)$

(ii) $F(\infty) = 0.5 \Rightarrow P(0 < z < \infty)$

(iii) $F(-z) = -F(z) \Rightarrow P(z<0) = -P(z>0)$

Note: $P(a \le X \le b) = P(z_1 \le Z \le z_2)$; $= F(z_2) - F(z_1)$

where $z_1 = \frac{a-\mu}{\sigma}$ & $z_2 = \frac{b-\mu}{\sigma}$.

Q An aptitude test for selecting engineers in an industry is conducted on 100 candidate. The average score is 42 & std. deviatn of score is 24. Assuming normal distribun for score. Find

(i) No. of candidates whose score is more than 60

(ii) No. of candidates whose score lie b/w 30 & 60. (inclusive)

Soln

Let $n = 100$ = Total no. of candidates appearing for aptitude test

mean; $\mu = 42$ & std. deviatn; $\sigma = 24$.

(i) To find prob. of candidate whose score is more than 60

Let 'X' be r.v. denote score of candidate

∴ Score is more than 60 (ie $60 < X < \infty$)

∴ $P(X > 60) = P(z_1 < Z < z_2)$ ⟶ (A)

The standard r.v. corr. to $X = 60$; $z_1 = \frac{60-42}{24}$

But $z_1 = \frac{a-\mu}{\sigma} = \frac{60-42}{24} = 0.75$ $\quad \frac{X-\mu}{\sigma}$

$z_2 = \frac{b-\mu}{\sigma} = \frac{\infty - 42}{24} = \infty$.

ie standard normal variable corr. to score 60 is gn by

$z_1 = 0.75$ & $z_2 = \infty$

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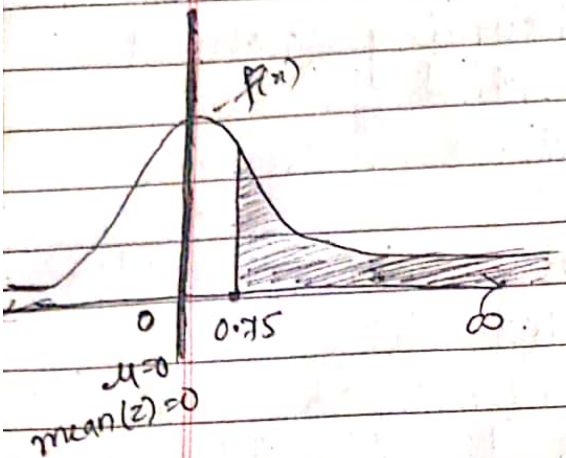
$\therefore P(X>60) = P(z_1 < z < z_2)$

$= P(0.75 < z < \infty)$

$= F(\infty) - F(0.75)$

$= 0.5 - 0.2734$

$= \underline{\underline{0.2266}}$



Hence no. of candidates scoring more than 60

$= 100 \times 0.2266$

$\simeq 23$.

(ii) Score lies b/w 30 to 60 (inclusive) i.e $P(30 \le X \le 60)$ $P(a \le X \le b)$

$\therefore$ Standard normal variable corr. to score 30 ($X=30$)

i.e $z_1 = \frac{a-\mu}{\sigma} = \frac{30-42}{24} = -0.5$ $\quad = \frac{X-\mu}{6}$

& Std. normal variable corr. to score 60 ($X=$)

i.e $z_2 = \frac{b-\mu}{6} = \frac{60-42}{24} = 0.75$

$\therefore P(30 \le X \le 60) = P(z_1 \le Z \le z_2)$

$= P(-0.5 < Z < 0.75)$

$= F(0.75) - F(-0.5)$

$= F(0.75) + F(0.5)$

$= 0.2734 + 0.1915$

$= \underline{\underline{0.4649}}$

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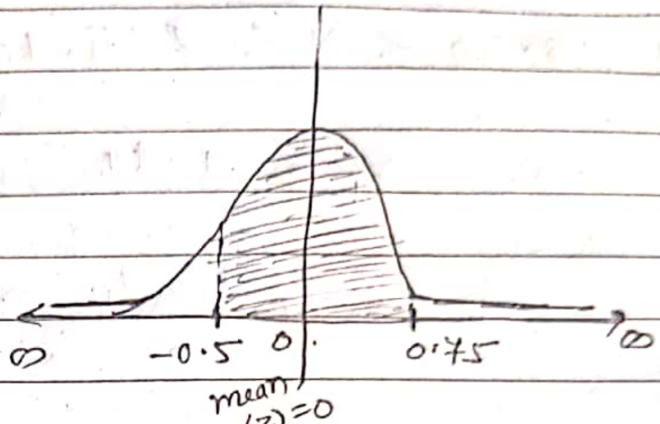
∴ Hence no. of candidate scoring b/w 30 to 60
$= 100 \times 0.4649 \cong 46$



Q If diameter of ball bearing are normally distributed with mean 15.60 mm & std deviation 0.06 mm. Determine % of ball bearing with diameters
(a) b/w 15.50 & 15.70 mm inclusive
(b) greater than 15.70 mm
(c) less than 15.40 mm
(d) Equal to 15.60 mm
Assume measurement to be recorded to nearest 0.01 mm

Solⁿ.

Let X be r.v. denote diameter of ball bearing which is normally distributed with mean, $\mu = 15.60$ mm &
S.D; $\sigma = 0.06$ mm

Let; z be standarized normal variable gⁿ by

$$z = \frac{X-\mu}{\sigma} = \frac{X-15.60}{0.06} \rightarrow (1)$$

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Now to find

(i) Prob. of ball bearing lies b/w 15.50 to 15.70 (inclusive)

$P(15.50 \le X \le 15.70) \Rightarrow P(15.49 < X < 15.71)$ (exclusive)

Now; $z_1 = \frac{a-\mu}{\sigma} = \frac{15.49 - 15.6}{0.06} = -1.833$

& $z_2 = \frac{b-\mu}{\sigma} = \frac{15.71 - 15.6}{0.06} = 1.833$

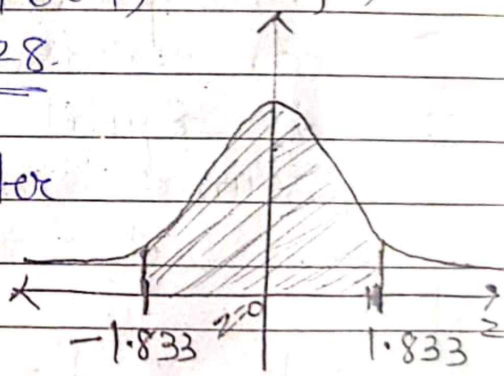
$\therefore P(15.50 \le X \le 15.70) = P(z_1 < X < z_2)$

$= P(-1.833 < X < 1.833)$

$= 2 \cdot$ (Area b/w $z=0$ to $z=1.8$

$= 2(0.4664)$

$= \underline{0.9328}$



% of ball bearing with diameter b/w 15.50 m/m & 15.70mm is $0.9328 \times 100\%$
$= 93.28\%$.

(2) Prob. of ball bearing greater than 15.70 mm ie $P(X > 15.70)$

ie $P(15.70 < X < \infty)$ $X = 15.70 + 0.01$

$\therefore$ Std. normal variable corr.

$z_1 = \frac{a-\mu}{\sigma} = \frac{15.71 - 15.6}{0.06}$

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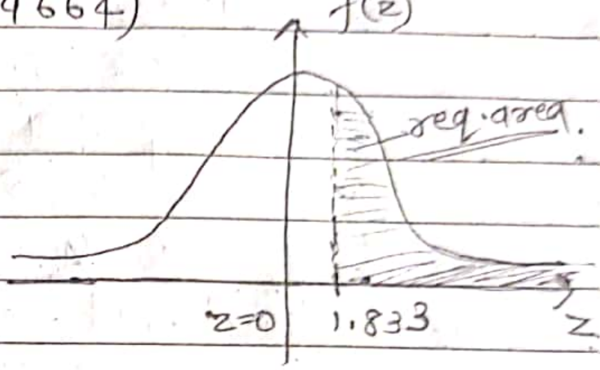
$z_1 = 1.833$ & $z_2 = \frac{\infty - 15.6}{0.06} = \infty$

$\therefore P(x > 15.70) = P(z_1 < z < z_2)$
$= P(1.833 < z < \infty)$
$= F(\infty) - F(1.833)$
$= 0.5 - (0.4664)$
$= 0.0336$



$\therefore$ % of ball bearing with diametter greater than 15.70 is 3.36%

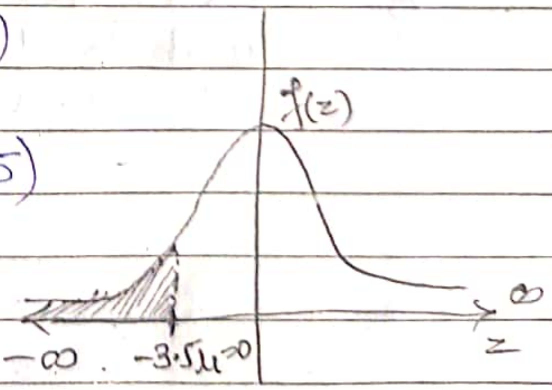
(3) To find: Prob of ball bearing less than 15.40mm ie $P(x < 15.40)$
ie $P(-\infty < x < 15.40)$

$\therefore$ The std. normal variable corr. to $x = 15.39$

$z_1 = \frac{a-\mu}{\sigma} = \frac{-\infty - 15.6}{0.06} = -\infty$

& $z_2 = \frac{b-\mu}{\sigma} = \frac{15.39 - 15.60}{0.06} = -3.5$

$\therefore P(x < 15.40) = P(-\infty < x < 15.40)$
$= P(z_1 < z < z_2)$
$= P(-\infty < z < -3.5)$
$= F(-3.5) - F(-\infty)$
$= F(\infty) - F(3.5)$
$= 0.5 - (0.4998)$



$P(x < 15.40) = 0.0002$

$\therefore$ % of ball bearing with diameter less than 15.40 mm is 0.02%

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(4) To find Prob. of ball bearing having diameter equal to 15.60 mm

i.e. $P(X = 15.60)$

i.e. in b/w $x = 15.59$ to 15.61

i.e. $P(15.59 < X < 15.61)$

∴ The standard normal variable Corr. to

$x = 15.59$

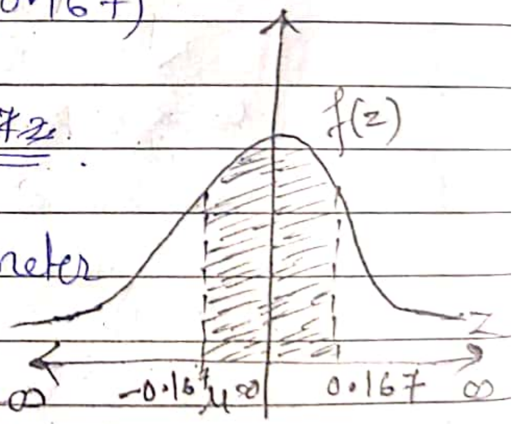
i.e. $z_1 = \frac{a-\mu}{\sigma} = \frac{15.59 - 15.60}{0.06} = -0.167$

~~& $z_2 = \frac{b-\mu}{\sigma}$~~

The standard normal variable Corr. to $x = 15.6$

i.e. $z_2 = \frac{b-\mu}{\sigma} = \frac{15.61 - 15.60}{0.06} = 0.167$

$$\therefore P(X = 15.60) = P(\cancel{-0.}15.59 < X < 15.61)$$
$$= P(-0.167 < X < 0.167)$$
$$= F(0.167) - F(-0.167)$$
$$= 2.F(0.167)$$
$$= 2.(0.06\cancel{7}5) = \cancel{X.272}$$
$$= 0.135$$



∴ % of ball bearing with diameter equal to 15.6 mm is 13.5%

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Ex The mean inside diameter of sample of 200 washers produced by a machine is 0.502 cm & std deviatn is 0.005 cm. The purpose for which these washers are intended allows a maximum tolerance in diameter of 0.496 to 0.508 cm, otherwise washers are considered defective. Determine % of defective washers produce by machine assuming diameters are normally distributed.

→

Soln: Let x be a cts r.v denoting washers

μ = mean = 0.502 cm & σ = 0.005 cm

& standard ~~deviatn~~ normal distributn is given by $z = \frac{x-\mu}{\sigma}$.

To find : $P(0.496 \le X \le 0.508) = P(z_1 \le z \le z_2)$

$= P(a \le x \le b)$

Now; standard normal variable corr. to $X = 0.496$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{0.496 - 0.502}{0.005} = -1.2$$

& std. normal variable corr to $X = 0.508$

ie $$z_2 = \frac{b-\mu}{\sigma} = \frac{0.508-0.502}{0.005} = 1.2$$

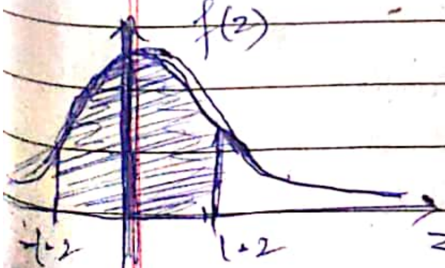
∴ $P(0.496 \le x \le 0.508) = P(z_1 \le z \le z_2)$

$= P(-1.2 \le z \le 1.2)$

$= 2.$ Area b/w $[z=0$ to $1.2]$

$= 2.F(1.2) = 2 \times 0.3849$

$= 0.7698$



$\therefore$ % of washers are attended intended allows maximum tolerance is $0.7698 \times 100\%$
$= 76.98\%$

$\therefore$ % of defective washers ; $= (100 - 76.98)\%$
$= \underline{\underline{23.02\%}}$

H.W The life of army shoes is normally distributed with mean 8 months & standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months (include 12^{th} month) ?

Solⁿ. Let x be continuous random variable denoting life of shoes.

Here: μ = mean = 8

std deviatⁿ; σ = ~~5000~~ 2 & $Z = \frac{X - \mu}{\sigma}$

(i) To find $P(X \geq 12) = P(12 \leq X < \infty)$
$= P(a \leq X < b)$

Now; Std normal variable Corr. to. X = 12

$\therefore z_1 = \frac{a - \mu}{\sigma} = \frac{12 - 8}{2} = \frac{4}{2} = \underline{\underline{2}}$

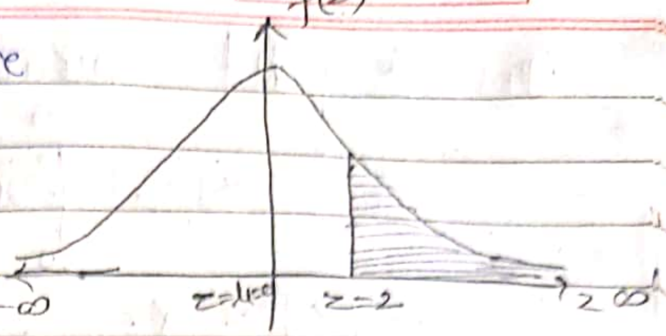
& Std. normal variable Corr. to $X = \infty$

$z_2 = \frac{b - \mu}{\sigma} = \frac{\infty - 8}{2} = \underline{\underline{\infty}}$

$\therefore P(X \geq 12) = P(z_1 \leq z \leq z_2) = P(2 \leq z < \infty)$
$= F(\infty) - F(2)$
$= 0.5 - 0.4772$
$= \underline{\underline{0.0228}}$

No. of pairs whose life is more than 12 months

$= 5000 \times 0.0228 = 114$



Replacement after 12 months

$= 5000 - 114 = 4886$ pair of shoes.

* Exponential Distribution *

Defn: A continuous random variable x is said to be exponentially distributed, if its density fn is

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & ; x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

* Properties of Exponential distribution.

① Mean ; $\mu = 1/\alpha$ ② Variance ; $\sigma^2 = 1/\alpha^2$

③ $\sigma = 1/\alpha$. i.e. $\mu = \sigma$

Ex Find mean, variance & std. deviation for exponential distributn gn by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & ; x \geq 0 \\ 0 & ; \text{otherwise.} \end{cases}$$

→

(i) To find mean; $\mu_1 = E[x]$

$$\Rightarrow E[x] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx = \int_{0}^{\infty} x \cdot \alpha \cdot e^{-\alpha x} \cdot dx$$

$$E[x] = \alpha \cdot \left\{ x \cdot \frac{e^{-\alpha x}}{-\alpha} - 1 \cdot \frac{e^{-\alpha x}}{\alpha^2} \right\}_0^{\infty}$$

$$= \alpha \left\{ 0 - \frac{0}{\cancel{\;}} - \left(-\frac{1}{\alpha^2}\right) \right\} = \frac{\alpha}{\alpha^2}$$

$$\boxed{E[x] = \frac{1}{\alpha}}$$

(ii) To find $Var(x)$ ie $Var(x) = E[x^2] - [E(x)]^2$

$$\therefore E[x^2] = \alpha \int_0^{\infty} x^2 \cdot e^{-\alpha x} \cdot dx$$

$$= \alpha \cdot \left\{ x^2 \cdot \frac{e^{-\alpha x}}{-\alpha} - \int_0^{\infty} 2x \cdot \frac{e^{-\alpha x}}{(-\alpha)} \cdot dx \right\}_0^{\infty}$$

$$= \alpha \left[\left\{ x^2 \cdot \frac{e^{-\alpha x}}{\alpha} \right\}_0^{\infty} + \frac{2}{\alpha} \cdot \int_0^{\infty} x \cdot e^{-\alpha x} \cdot dx \right]$$

$$= \alpha \left[\{0 - 0\} + \frac{2}{\alpha} \cdot \frac{\alpha}{\alpha^2} \right] = \frac{2}{\alpha^2}$$

$$E[x^2] = \frac{2}{\alpha^2}$$

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$\therefore Var(x) = \frac{2}{\alpha^2} - \left[\frac{1}{\alpha}\right]^2 = = \frac{1}{\alpha^2}$.

(iii) To find ; $\sigma = +\sqrt{var} = \frac{1}{\alpha}$.

Ex ② If a r.v. x has exponential distribution with mean $\mu = \frac{1}{\alpha} = \frac{1}{2}$. Calculate prob. that

(i) x ~~lie~~ will lie b/w 1 & 3

(ii) X is greater than 0.5

(iii) X is atmost 4.

Sol^n

Given ; mean ; $\mu = \frac{1}{\alpha} = \frac{1}{2}$ $\Rightarrow$ $\boxed{\alpha = 2}$

$\therefore$ Prob. density f^n for exponential distribution is given by

$$f(x) = \begin{cases} \alpha \cdot e^{-\alpha x} & ; x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

ie $f(x) = \begin{cases} 2 \cdot e^{-2x} & ; x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$

(i) To find $P(1 \leq X \leq 3)$

$$P(1 \leq x \leq 3) = \int_1^3 2 \cdot e^{-2x}\, dx = \int_{-\infty}^{\infty} f(x) \cdot dx .$$

$$= 2 \cdot \left[\frac{e^{-2x}}{(-2)}\right]_1^3$$

$$= -1\left[e^{-6} - e^{-2}\right] = e^{-2} - e^{-6}$$

$$= 0.1353 - 0.0024 = \underline{0.1329}$$

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(ii) To find $P(X \geq 0.5) = P(0.5 \leq X < \infty)$

$$\therefore P(0.5 \leq X < \infty) = \int_{0.5}^{\infty} 2 \cdot e^{-2x} \cdot dx$$

$$= (-1) \cdot \left[e^{-2x}\right]_{0.5}^{\infty} = e^{-1}$$

$$\Rightarrow P(0.5 \leq X < \infty) = \underline{\underline{0.3679}}$$

(iii) To find $P(X \leq 4)$

$$\Rightarrow P(0 \leq X \leq 4) = \int_{0}^{4} 2 e^{-2x} \cdot dx = (-1) \cdot \left[e^{-2x}\right]_{0}^{4}$$

$$= 1 - e^{-8} = \underline{\underline{0.9997}}$$

H.W Let Mileage of a particular tyre be r.v. X having probability density

$$f(x) = \begin{cases} \frac{1}{20} \cdot e^{-x/20} & \text{for } x \geq 0 \\ 0 & : \text{for } x < 0 \end{cases}$$

Find prob. that one of these tyres will last

(i) Atmost 10000 milies

(ii) Anywhere from 16000 to 24000 miles

(iii) At least 30000 miles.

Sol^n:

(i) To find $P(X \leq 10000)$

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$$P(X \le 1000) = P(0 \le X \le 10) = \int_0^{10} \frac{1}{20} \cdot e^{-x/20} \cdot dx$$

$$= \frac{1}{20} \cdot \frac{\left[e^{-x/20}\right]_0^{10}}{\left(-\frac{1}{20}\right)} = -\left[e^{-1/2} - 1\right]$$

$$= 1 - e^{-1/2} = \underline{\underline{0.3934}}$$

(ii) To find $P(1600 \le X \le 2400) = P(16 \le X \le 24)$

$$\rightarrow P(1600 \le X \le 2400) = \int_{16}^{24} \frac{1}{20} \cdot e^{-x/20} \cdot dx$$

$$= (-1) \cdot \left[e^{-x/20}\right]_{16}^{24} = e^{-4/5} - e^{-6/5}$$

$$= \underline{\underline{0.148}}$$

(iii) To find $P(X \ge 3000) = P(X \ge 30) = P(30 \le X < \infty)$

$$P(X \ge 30) = \int_{30}^{\infty} \frac{1}{20} \cdot e^{-x/20} \cdot dx = (-1) \cdot \left[e^{-x/20}\right]_{30}^{\infty}$$

$$= e^{-3/2}$$

$$P(30 \le X < \infty) = \underline{\underline{0.2231}}$$