B. Tech. First Semester (Artificial Intelligence and Data Science) / B. Tech. – 21– 22– SOE – ADS– 203 Examination

Course Code : AIDS 2101/AIML 2101 /CSD 2101

Course Name : Calculus, Sequences and Series

Time : 2 Hours] [Max. Marks : 40

Instructions to Candidates :—
    (1)   All questions are compulsory.
    (2)   All questions carry marks as indicated.
    (3)   Use of Logarithmic tables, non – programmable calculator is permitted.

All the questions are compulsory.

1.   (A1) Show that the sequence $\{x_n\}$, where $x_n = \left\{ \dfrac{1}{(n+1)^2} + \dfrac{1}{(n+2)^2} + ... + \dfrac{1}{(n+n)^2} \right\}$,

        Converges to 0.   3(CO1)

  (A2) Define Absolute convergence of series. Also test the absolute convergence

      of the series $\Sigma \dfrac{(-1)^n}{5n+1}$.   3(CO1)

2.   (A1) If $y = \tan^{-1} x$, prove that
        $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$.   4(CO1)

  (A2) Expand log (cos x) in ascending powers of x upto and including the term x^4 and calculate $\log_{10} \cos\left(\frac{\pi}{12}\right)$ up to three places.   3(CO1)

3.   (A1) If $u = f(x^2 + 2yz, \ y^2 + 2xz)$ then find the value of

      $(y^2 - xz) \dfrac{\partial u}{\partial x} + (x^2 - yz) \dfrac{\partial u}{\partial y} + (z^2 - xy) \dfrac{\partial u}{\partial z}$ 4(CO2)

  (A2) If $y_1 = \dfrac{x_2 x_3}{x_1}$, $y_2 = \dfrac{x_1 x_3}{x_2}$, $y_3 = \dfrac{x_1 x_2}{x_3}$

      Find the value of : $\dfrac{\partial(x_1, \ x_2, \ x_3)}{\partial(y_1, \ y_2, \ y_3)}$ 3(CO2)

4.   (A1) Evaluate $\displaystyle\int_0^1 \dfrac{x^2 dx}{\sqrt{1-x^4}} \cdot \int_0^1 \dfrac{dx}{\sqrt{1-x^4}}$ 3(CO3)

(A2) Trace the curve $y^2 = x^2(1-x^2)$.

3(CO3)

5. (A1) Evaluate $\iint\limits_{R} ydxdy$, where R is the region bounded by parabolas $y^2 = 4x$ and $x^2 = 4y$.

4(CO3)

(A2) Evaluate $\int\limits_{0}^{a}\int\limits_{y}^{a} \dfrac{x^2}{(x^2 + y^2)^{3/2}} dydx$, by changing it to polar Coordinates.

3(CO3)

6. (A1) Solve $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = e^{-2x} + \sin x$

3(CO4)

(A2) Solve $(3x + 2)^2 \dfrac{d^2y}{dx^2} + 5(3x + 2)\dfrac{dy}{dx} - 3y = x^2 + x + 1$

4(CO4)

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