

**B. Tech. Third Semester (Artificial Intelligence and Data Science) /
SOE_21-22_ADS_203.1 Examination**

Course Code : AIDS 2201 /
AIML 2201

Course Name : Discrete Maths and
Graph Theory

Time : 3 Hours]

[Max. Marks : 50

Instructions to Candidates :—

- (1) Do not write anything on question paper except your exam seat number. 295974
- (2) Write the accurate question number in left margin of answer book along with answers.
- (3) All questions are compulsory.
- (4) All questions carry marks as indicated.

1. (A) If A and B are any two non-empty sets then show that :

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

4(CO1)

(B) Test the validity of the following argument :

If I study then I will not fail in mathematics.

If I do not play basketball then I will study.

But I fail in mathematics.

\therefore I must have played basketball.

4(CO1)

2. (A) If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and R be a relation on A given by $R = \{(x, y) : x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation.

4(CO2)

(B) Let $f: R \rightarrow R$ and $f(x) = x^3 + 1$ for all $x \in R$. Then show that f is bijective.
4(CO2)

3. (A) Let $(Q, *)$ be an algebraic structure where $x * y = \frac{x+y}{1}$. Determine which of the following properties holds for this structure :

- (i) Closure,
- (ii) Commutative,
- (iii) Associative,
- (iv) Identity.

Is it a group ?

5(CO3)

(B) Prove that the set $\{1, 2, 3, 4, 5, 6\}$ is a finite abelian group under multiplication modulo 7 as composition.
4(CO3)

4. (A) Show that $\{I, \oplus, \square\}$ is a commutative ring with identity where the operations \oplus and \square are defined as : For any $a, b \in I$, $a \oplus b = a + b - 1$ and $a \square b = a + b - ab$ where I is the set of integers.
4(CO3)

(B) Show that $S = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ for the operation "+" "X" is an integral domain.
4(CO3)

5. (A) Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $a, b \in \mathbb{R}$ is a field with respect to matrix addition and multiplication.
4(CO3)

(B) Define the Hasse diagram and For any integer n , let D_n denote the set of all divisor of n . Find the Hasse diagram for $n=36$, $n=75$, $n=105$.
4(CO3)

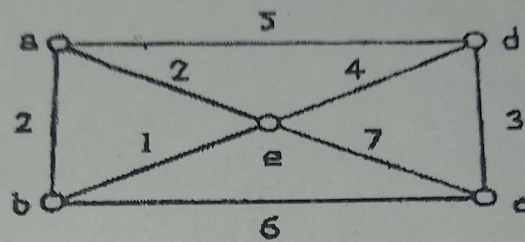
6. (A) Draw the digraph corresponding to adjacency matrix A , B , A^T and B^T where :

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Show that the digraph corresponding to A^T and B^T are isomorphic.

5(CO4)

- (B) Find the minimal spanning tree of the following graph using Prim's algorithm.



4(CO4)

