Q.1

(A)(A1) State Cauchy's Second theorem of Limits

(A2) Prove that the sequence $\{x_n\}$, where $x_n = \frac{2n-7}{3n+2}$ is monotonic increasing sequence. further show that it is bounded and tends to limit $\frac{2}{3}$

(B)(B1) Test the convergence of the series $2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots + \frac{(n+1)x^n}{n^3} + \ldots$

Q.2

(A) If $\frac{1}{y_m} + \frac{-1}{y_m} = 2x$, prove that

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$$

(B) Given $f(x) = x^3 + 8x^2 + 15x - 24$, find $f\left(\frac{11}{10}\right)$ by using Taylor's theorem.

Q.3

(A) If $u = \sin^{-1}\left[\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}}\right]$ Prove that.

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\tan u[\tan^2 u - 1]$$

(B) An open rectangular tank is to have a capacity 256 cu. Meter - find the dimension of the tank so that its surface area is minimum.

Sign.