

# Zeeman effect

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## Introduction

The most important experimental technique triggering the development of the modern atomic theory is spectroscopy, the observation of the characteristic wavelengths of light that atoms can absorb and emit. A so-called wavelength dispersive element such as a waterdrop, a prism or a grating, spatially separates different wavelengths of light emitted from one source. If the light source emits white light, a band of all colours will be found after the light passes the wavelength dispersive element (*e.g.* rainbow). Observing the light from a lamp that contains only atoms of a certain element, one finds distinct lines instead of the complete band on the detector. The positioning of the lines on the detector is called a spectrum, and the corresponding wavelengths are hence called ‘spectral lines’ of the element. The additional information needed to be able to draw conclusions about the atomic structure from spectral lines was provided by the experimental observation of Rutherford that in an atom, electrons surround a heavy, positively charged nucleus in the center, and by Planck’s explanation of the blackbody spectrum by quanta of light, photons, whose energy  $E_{ph}$  is connected with the wavelength  $\lambda$  by

$$E_{ph} = h\nu = \frac{hc}{\lambda} , \tag{1}$$

where  $\nu$  is the frequency,  $h$  is the Planck constant and  $c$  is the speed of light. With this knowledge it became clear that electrons in an atom can occupy only certain energy levels  $E_i$ , and that the transition of an electron from a higher energy level  $E_2$  to a lower energy level  $E_1$  leads to the emission of a photon:  $E_2 - E_1 = E_{ph}$ . The energy needed to lift the electron to the higher, ‘excited’ energy level can be provided by absorption of a photon of the very same energy or by collisions. The increasing experimental precision reached with spectroscopy inspired scientists

to propose different atomic models, leading to the development of quantum mechanics. With refinements to the quantum mechanical description of the atom, relativistic quantum mechanics (necessary to explain the splitting of energy levels of same main quantum number  $n$ ) and quantum electrodynamics (QED, necessary to explain the splitting of levels with same quantum number  $j$  but different quantum number  $l$ ), theory is by now able to make highly accurate predictions about the atomic structure.

## Effects of an external magnetic field on the atomic structure

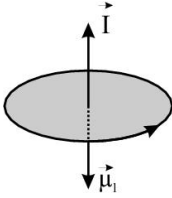


Figure 1: Classical model of an electron with angular momentum  $\mathbf{l}$  inducing an angular magnetic moment  $\vec{\mu}_l$  inducing an orbital magnetic moment

If the atom is placed in an external magnetic field, the atomic energy levels are split and the observed spectrum changes. This was first observed by the dutch physicist Pieter Zeeman and is hence called the Zeeman effect. In an intuitive approach to explain the Zeeman effect, consider an atom with an electron with charge  $e$  moving with the speed  $v$  around the nucleus on a Bohr orbit with radius  $r$ . This orbiting electron can be described by a current

$$I = -e \cdot \frac{v}{2\pi r} \quad (2)$$

$$\vec{\mu}_l = I \cdot \mathbf{A} = I \cdot \pi r^2 \mathbf{n} = \frac{evr}{2} \mathbf{n} , \quad (3)$$

where  $\mathbf{A} = \pi r^2 \mathbf{n}$  is the area vector perpendicular to the orbit area  $\pi r^2$ . The angular momentum of the electron with mass  $m_e$  is

$$\mathbf{l} = \mathbf{r} \times \mathbf{p} = m_e \cdot \mathbf{r} \cdot \mathbf{v} \cdot \mathbf{n} . \quad (4)$$

An external magnetic field  $\mathbf{B}$  interacts with the angular magnetic moment and, hence, changes the electron's potential energy:

$$\Delta E_{pot} = -\vec{\mu}_l \cdot \mathbf{B} = \frac{e}{2m_e} \cdot \mathbf{l} \cdot \mathbf{B} \quad (5)$$

Using the direction of the magnetic field vector  $\mathbf{B}$  as quantisation axis  $z$  and the quantised angular momentum of the electron

$$|\mathbf{l}| = \sqrt{l(l+1)}\hbar \quad \text{with} \quad l = 0, 1, \dots, n-1 \quad \text{and} \quad (6)$$

$$l_z = m_l \cdot \hbar \quad \text{with} \quad -l \leq m_l \leq l , \quad (7)$$

equation (5) can be simplified to

$$\Delta E_{pot} = \frac{e \cdot \hbar}{2m_e} \cdot m_l \cdot B = \mu_B \cdot m_l \cdot B , \quad (8)$$

where  $\mu_B$  is called the Bohr magneton. The energy shift of the electron's original energy level by  $\Delta E_{pot}$  due to the coupling of the external magnetic field to the electron's angular magnetic moment leads to a splitting of the original level into the previously degenerated  $2l + 1$  sublevels of same angular momentum  $l$  but different magnetic quantum number  $m_l$ .

The quantum mechanical way for deriving this energy splitting where also the electron spin is taken into account starts off by treating the external magnetic field as a perturbation  $\hat{H}_B$  of the unperturbed Hamilton Operator  $\hat{H}_0$ . In general, an atom with more than one electron has to be considered. In the case of an external magnetic field that is small as compared to the magnetic coupling energies between the  $i$  electrons' orbital and spin momenta  $\mathbf{l}_i$  and  $\mathbf{s}_i$ , respectively, one may use the so-called *LS*-coupling approximation. This approximation is appropriate when the coupling energy between the different electrons' orbital momenta is large as compared to the coupling of a single electron's orbital momentum with the same electron's spin momentum. Hence, all orbital and spin momenta are added separately to total orbital and total spin momenta  $\mathbf{L}$  and  $\mathbf{S}$  before determining the total angular momentum  $\mathbf{J}$ :

$$\mathbf{L} = \sum_i \mathbf{l}_i \quad \text{with} \quad |\mathbf{L}| = \sqrt{L(L+1)}\hbar , \quad (9)$$

$$\mathbf{S} = \sum_i \mathbf{s}_i \quad \text{with} \quad |\mathbf{S}| = \sqrt{S(S+1)}\hbar , \quad (10)$$

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \quad \text{with} \quad |\mathbf{J}| = \sqrt{J(J+1)}\hbar , \quad (11)$$

$$J_z = M_J \cdot \hbar \quad \text{with} \quad -J \leq M_J \leq J . \quad (12)$$

With these momenta introduced, the Hamiltonian of an atom in an external magnetic field is

$$\hat{H}_0 + \hat{H}_B = \hat{H}_0 - \frac{\mu_B}{\hbar} (\mathbf{J} + \mathbf{S}) \cdot \mathbf{B} , \quad (13)$$

and the solution for the shift of the level energy becomes

$$\Delta E_{pot} = \mu_B \cdot B \cdot M_J \cdot g_J \quad (14)$$

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} , \quad (15)$$

with the Landé factor  $g_J$ .

If all electron spins in the atom cancel each other, *i.e.* in the case of  $S = 0$ ,  $g_J = 1$  is

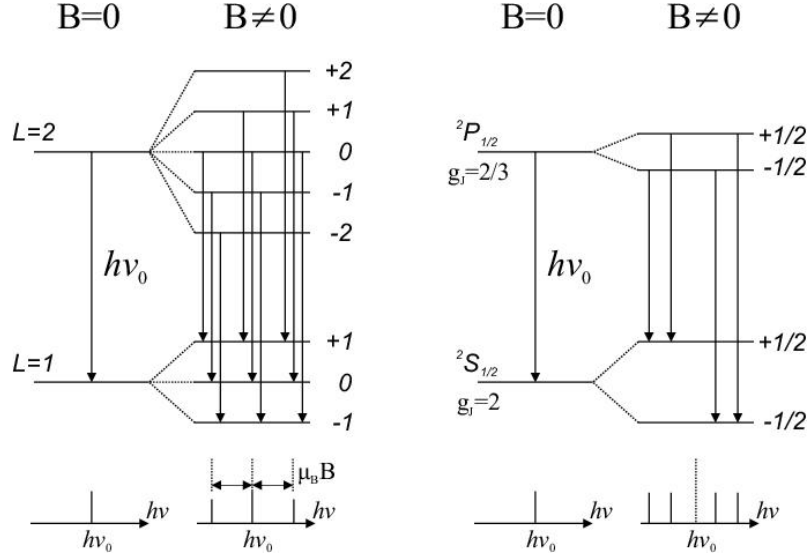


Figure 2: Left: Normal Zeeman effect for a  $d \rightarrow p$  transition. The field splits the degenerate  $M_J$  levels by  $\Delta E = \mu_B \cdot B \cdot M_J$ . Right: Anomalous Zeeman effect. Here, the levels are split by  $\Delta E = \mu_B \cdot B \cdot M_J \cdot g_J$ . In both cases, transitions can occur if  $\Delta m_J = 0, \pm 1$ .

obtained and equation (14) becomes identical to equation (5). For historical reasons, the splitting in absence of spin is called the *normal* Zeeman effect whereas, since the electron spin was not yet discovered and the observed features seen in the spectra were not understood, the more general case with  $S \neq 0$  was named the *anomalous* Zeeman effect. In the normal Zeeman effect, the size of the energy splitting from the unperturbed level energy depends only on the magnetic quantum number  $m_l$  and is, hence, of the same size for all levels in the atom. In the case of the anomalous Zeeman effect, the energy splitting depends on the quantum numbers  $J, L$  and  $S$ , which is reflected by more complicated structures observed in the corresponding spectra (see Fig. 2). In a strong external field, the  $LS$ -coupling approximation is no longer appropriate. The field-induced precessions are so rapid that one has to consider the total orbital momentum  $L$  and spin  $S$  as they individually precess about  $\mathbf{B}$ , that is, the effect of  $\mathbf{B}$  is effectively to decouple  $L$  from  $S$ , rendering  $J$  meaningless. This is called the Paschen-Back effect.

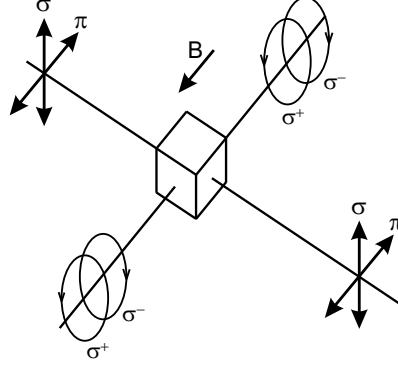


Figure 3: Isometric depiction of the polarisation of the different Zeeman components.

## Selection rules and polarisation of emitted light

In transitions from an excited level  $i$  to a lower level  $k$ , a photon with the difference energy  $E_{ph} = \Delta E = E_i - E_k$  is emitted. Still, as indicated in Fig. 2, not all energy differences between existing levels are actually found in the spectra. The reason is that not only the energy, but also momentum conservation, angular momentum conservation and symmetry rules play a role on whether or not a transition can occur. Only those transitions are allowed where the transition dipole matrix element

$$\mathbf{M}_{ik} = e \int \psi_i^* \mathbf{r} \psi_k dV \quad (16)$$

has at least one non-zero component

$$(M_{ik})_q = e \int \psi_i^* q \psi_k dq \quad , \text{ with } q = x, y, z . \quad (17)$$

Evaluating these three integrals yields that only those transitions have non-zero components where the change of the magnetic quantum number fulfills  $\Delta M_J = M_{J,i} - M_{J,k} = 0, \pm 1$ . Furthermore, electric dipole transitions require that the change of the total orbital momentum be  $\Delta L = L_i - L_k = \pm 1$  and that the spin does not change, *i.e.*  $\Delta S = 0$ .

If the quantisation axis (*i.e.* the direction of the magnetic field vector) is the  $z$ -axis, the matrix element component  $(M_{ik})_z$  is the only non-zero one for  $\Delta M_J = 0$  (so-called ‘ $\pi$ ’-) transitions. Considering a dipole and its characteristics of emission, this means the dipole is oscillating along the axis of the magnetic field vector and, hence, does not emit any radiation in that direction. Also, since the electric field vector is oscillating in only one direction, also the field vector of the emitted light wave oscillates in this direction, meaning that the light emitted by  $\Delta M_J = 0$  transitions

is linearly polarised (see Fig. 3).

In the case of  $\Delta M_J = \pm 1$  ( $\sigma'$ )- transitions,  $(M_{ik})_z$  is zero and both  $(M_{ik})_x$  and  $(M_{ik})_y$  are non-zero and equal in magnitude, but phase-shifted by  $\pi/2$ . This phase shift between the two superimposed dipoles leads to the observation of circularly polarised light when viewing in  $z$ -direction ('longitudinally'). Viewing the emitted light in  $x$ - or  $y$ -direction ('transversal'), only the projection of the superimposed dipoles is seen, and thus the light from  $\Delta M_J = \pm 1$  transitions appears linearly polarised when observed transversal to the magnetic field vector.

## The Experiment

The experiment consists of two parts. In the first part, the Zeeman effect in transverse and longitudinal directions should be observed, and the polarisation of the observed lines examined. The second part is dedicated to precision spectroscopy with a grating spectrometer. In both parts of the experiment the light emitted from a Cadmium lamp (Cd) will be used. The electronic configuration of Cadmium is  $[\text{Kr}]4d^{10}5s^2$ . In this configuration all spins of the shell electrons compensate for each other, such that  $S = 0$ . In visible light, the Cd spectrum consists of four lines, given in table 1. All measurements will be made on the red line,  $\lambda_0 = 643.8\text{nm}$  ( $\nu_0 = 465.7\text{THz}$ ), which arises from the transition between the two excited levels  $^1D_2 \rightarrow ^1P_1$  in the fifth shell of Cd.

Colour	violet	blue	green	red
Wavelength (nm)	467.9	480.1	508.7	643.8

Table 1: Spectral lines of Cadmium in the visible range.

## Part 1: Spectroscopy of the Zeeman Effect

As the expected relative shift in wavelength due to the external magnetic field is smaller than 0.1 nm, a very high resolution is necessary for the effect to be observable. In this experiment the so-called Lummer-Gehrcke Plate will be used (see figure 1). This is a quartz glass plate with extremely plane parallel surfaces. The light enters into the plate through the use of a prism, and is then reflected to and fro between the innersurfaces of the plate under an angle near to total internal reflection. In this way, a small part of the light escapes from the plate at every point of reflection, under an angle of  $\alpha \sim 90^\circ$  (to the normal of the plate). The escaping rays of light can be observed with a telescope fixed behind the plate, whose focal point is at infinity. As the thickness of the plate is a lot smaller than its length and additionally this has a higher coefficient of reflection, many light rays can interfere with one another. This leads to a very high resolution. As can be seen in figure 1, the optical path length difference between two light rays, one of which is delayed by the glass plate, is given by:

$$\Delta = 2n_2 \cdot \frac{d}{\cos \epsilon} - 2n_1 \cdot d \cdot \tan \epsilon \cdot \sin \alpha, \quad (1)$$

with the thickness of the plate  $d$ , the indices of refraction (braking indices)  $n_1$  and  $n_2$  outside and inside the place, and similarly with the angles of the light rays to the normal of the plate (outside and inside the plate)  $\alpha$  and  $\epsilon$ . With  $\sin \alpha = \frac{n_2}{n_1} \sin \epsilon$  and  $\sin^2 \epsilon + \cos^2 \epsilon = 1$  equation 1 becomes:

$$\Delta = 2n_2 \cdot d \cdot \cos \epsilon = 2 \cdot d \cdot \sqrt{n_2^2 - n_1^2 \sin^2 \alpha}. \quad (2)$$

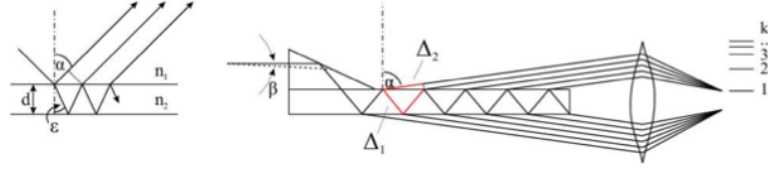


Figure 1: Schematic of the Lummer-Gehrcke-Plate

In the case of the Lummer-Gehrcke plate, the light is introduced into the plate through a prism, such that  $\alpha \sim 90^\circ$  at the limit of total internal reflection. For a plate with  $n_2 \sim n$  in air, ( $n_1 \sim 1$ ) equation 2 reduces to:

$$\Delta = \Delta_1 - \Delta_2 = 2d \cdot \sqrt{n^2 - 1} . \quad (3)$$

where the  $\Delta_i$  are marked in the right hand part of diagram 1. For constructive interference the path difference must satisfy  $\Delta = m \cdot \lambda$ , with  $k$  as a whole number and the wavelength  $\lambda$  of the interfering light. Whether constructive or destructive interference occurs when the Lummer-Gehrcke Plate is observed with the telescope, depends on the wavelength  $\lambda$ , the escape angle  $\alpha$  and hence also on the angle  $\beta$  under which the light enters the plate. For a given wavelength  $\lambda$  one recovers many orders  $k$  for different entering angles  $\beta$  at which the escaping light rays interfere constructively (compare diagram 2). The order  $k$  of wavelength  $\lambda + \Delta\lambda$  and order  $k + 1$  of wavelength  $\lambda$  overlap spatially when

$$k \cdot (\lambda + \Delta\lambda) = (k + 1) \cdot \lambda \rightarrow \Delta\lambda = \frac{\lambda}{k} . \quad (4)$$

With equation 3 the free spectral range of the Lummer-Gehrcke-Plate follows from

$$\Delta\lambda = \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}} . \quad (5)$$

To a first approximation, a small change in wavelength  $\delta\lambda \ll \Delta\lambda$  leads to a shift of

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \Delta\lambda , \quad (6)$$



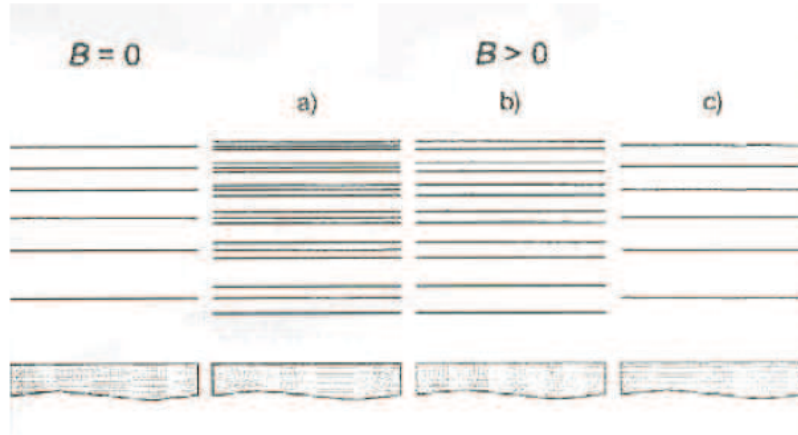


Figure 2: Interference pattern for the Zeeman effect, as would be observed in the transverse direction for the case a) without polarisation filter, b) with polarisation direction of the filter perpendicular to the magnetic field and c) with polarisation direction of the filter parallel to the magnetic field.

with the distance  $\delta a$  between the line with wavelength  $\lambda + \delta\lambda$  and the position of the line with wavelength  $\lambda$ , and the distance  $\Delta a$  between two orders  $k$  and  $k+1$  of wavelength  $\lambda$ . Therefore, for small shifts in wavelength as is the case for the Zeeman effect, one obtains the following:

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}}. \quad (7)$$

The experimental setup is sketched in figure 3. The Cd-lamp is placed between the pole pieces of the magnet. Light emitted from this lamp enters the Lummer-Gehrcke plate after passing through the red filter. This filter only lets the red part of the Cd spectrum through. The magnet can be turned through  $90^\circ$ , so that the lamp can be observed in transverse and longitudinal directions. The longitudinal direction may be observed through the hole in the pole piece. The interference pattern produced by the Lummer-Gehrcke plate can be observed through the telescope. In order to observe the different polarisations, a polarisation filter and a  $\lambda/4$  filter may be fixed between the plate and the telescope. With the height adjustment of the telescope, a crosshair in the eyepiece can be moved over the interference pattern.

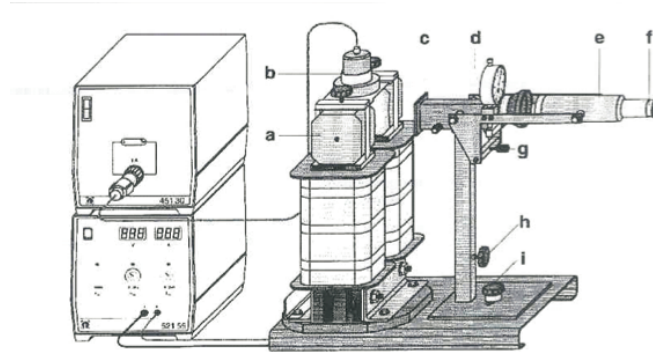


Figure 3: Drawing of the experimental set-up for investigating the normal Zeeman effect (Figure from Leybold, *Physics Leaflets*). a) Pole pieces of the magnets, b) Cd lamp, c) Red light filter, d) Lummer-Gehrcke Plate, e) Telescope, f) Eyepiece, g) Height adjustment for the telescope, h) Locking screw for the telescope holder, i) Locking screw for the base.

## Part 1: Execution (1-2 afternoons or a whole day)

- Use the teslameter (Hall probe) to determine the strength of the magnetic field at the position of the Cd-lamp. The magnetic field strength should be measured as a function of the current through the power supply of the magnets. Carry out the measurements once with increasing and once with decreasing current (Hysteresis effect). Take three measurements per current value and average.
- Carefully set the Cd lamp into the holder with the magnetic field turned off (without using force!) and observe the Zeeman effect in transverse and longitudinal directions with the telescope and with the camera. → Let turning the magnets be demonstrated by the supervisor first!
- Use the polarisation filter and the  $\lambda/4$  filter (how can the two be distinguished?) in order to verify the polarisations of the different Zeeman components with different observation directions (transverse and longitudinal).
- Using the **ScopePhoto** program, record an image of the  $\sigma$ - and  $\pi$ -lines for 10-12 orders of interference  $k$  with currents for the magnetic field of  $I = 10A$ ,  $I = 12A$  and  $I = 14A$ .
- **ScopePhoto program:**
  - Open program
  - TC (bring up previews)

- Settings (optimise gain, select time)
- Capture (record a spectrum)
- File → save as *file:C/Practical/your names*
- Open images from **ScopePhoto** in the program **ImageJ** to work on them there.
- **ImageJ program:**
  - Open image from ScopePhoto
  - Image → Colour → RGB split → (close blue and green)
  - Mark lines on the image (zoom and center the end points of the line)
  - PlugIns → Spectroscopy → Shift (produced from a projection of the image)
  - Save the image

## Security Advice:

- The Lummer-Gehrcke plate and its cover may not be taken away. The plate is very expensive!
- The current for the magnetic field should always be turned off when the Hall probe or the Cd lamp is being placed in or taken out of the holder.
- Never touch the Cd lamp with bare fingers. Moisture and grease on the glass can damage the lifetime of the lamp and the lamp will be very hot.

## Part 1: Evaluation

- Correct, if necessary (ask the supervisor), the values of the Hall probe with the correction factor on the board. Create a graph with the magnetic field strength as a function of the voltage. Assess the strength of the hysteresis effect. Use a linear fit to obtain the function  $B(U)$  and the magnetic field strength at  $I = 10V$ ,  $I = 12V$  and  $I = 14V$ .
- Note qualitatively what you see in observations (transverse and longitudinal) of the Zeeman effect. (For long evaluations: save images in the **ScopePhoto** program for the long protocol.)
- When using the polarisation filter and the  $\lambda/4$ -filter, note which lines one sees when, and why.

- In the transverse direction: Determine from the graphs, that you have produced with the program **ImageJ**, with the program **Origin** (Import: *Single ASCII*) the positions  $a$  of all  $\sigma$ - and  $\pi$ -lines for 10-12 orders of interference. This should be done for each of the three measurements with the currents  $I = 10V$ ,  $I = 12V$  and  $I = 14V$  for the magnetic field. Fit the line positions (always the fit main and two neighbouring maxima at once; zoom in, so that a triple peak fills the whole screen) with help from *Analysis; Fit multiple peaks*. Hence you obtain the line positions in the (pixel) units. What lineshape did you use for your fit? Justify your choice. Is the error, given by the fit, reasonable? If this is not the case, estimate a new error based on the region of the fitted curve.
- For each of the three magnetic fields, plot the order of interference  $k$  against the position  $a$  of the  $\pi$ -lines. Find a polynomial function  $k = f(a)$  (which order is sufficient?) to fit the trend of the points. The distance of the different orders on the y-axis is  $f(\Delta a) = f(a_2) - f(a_1) = 1$ . Determine the relation  $\delta a / \Delta a$  based on the fitted curve. Calculate  $\delta \lambda$  using equation 7 (parameters of the Lummer-Gehrcke plate:  $n = 1.4567$ ; thickness  $d = 4.04\text{mm}$ ). The error for  $\delta \lambda$  is determined from the standard deviation of the order. For  $\lambda$ , use the value that you determined in part II of the experiment.
- Calculate  $\mu_B$  for the three magnetic field strengths. Form an average and compare this with the literature value of  $\mu_B = 9.27400949 \cdot 10^{-24}\text{J/T}$ .
- Long evaluation: Plot  $\Delta E$  against  $B$ , fit a function on the plot and determine  $\mu_B$  based on this plot, in order to compare the value with the literature value.

## Part II: Precision Spectroscopy

The Lummer-Gehrcke plate, which served to determine the small differences between wavelengths in the first part of the experiment, can, however, only be used to determine the absolute wavelengths. The very precise determination of wavelengths should ensue with the help of a grating spectrometer in the second part.

### Czerny-Turner Spectrometer

The Czerny-Turner (CT) Spectrometer splits up light into discrete wavelengths through diffraction off a grating. The spectrum appears in the focal plane of the exit and is recorded there by a CCD camera. As shown in figure 4, a concave mirror (MC) is used to focus the light rays, which reach the spectrometer through the entrance slit. After the diffraction of the light off the grating, the light is focussed by another concave mirror onto the CCD camera.

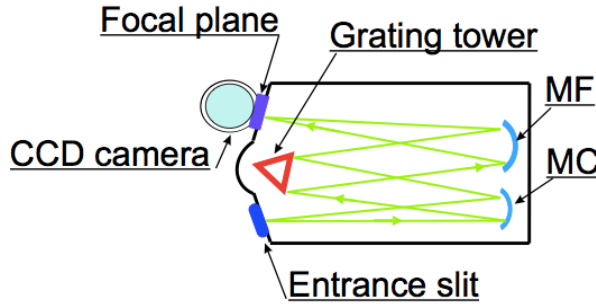


Figure 4: Schematic of the Czerny-Turner Spectrometer

### Grating Characteristics

At a grating, the angles of the incoming and outgoing light rays with wavelength  $\lambda$  satisfy the following relationship:

$$\sin \alpha + \sin \beta_k = kn\lambda , \quad (8)$$

with the angle of incidence  $\alpha$ , the diffraction angle  $\beta_k$  relative to the normal ( $N_G$ ) of the grating, the braking index  $k$  and the line density  $n$  (corresponding to the reciprocal of the grating constant  $1/d$ ). When  $k = 0$ , then equation 8 reduces to  $\alpha = \beta_0$ , the law of reflection. If the diffraction angle is assigned a fixed value, then the difference between  $\alpha$  and  $\beta_k$ , the so called deflection angle ( $D_V$ ) is constant (compare figure 5). The grating can concentrate a large portion of the diffracted radiation into one spectral order. Hence the intensity of the other orders decreases.

This redistribution of the intensity between the orders depends on the angle between the reflected elements and the grating surface, the so called *blaze angle*  $\omega_B$  (compare figure 5). In the visible spectrum the red region of the spectrum is diverted the least and the violet region the most.

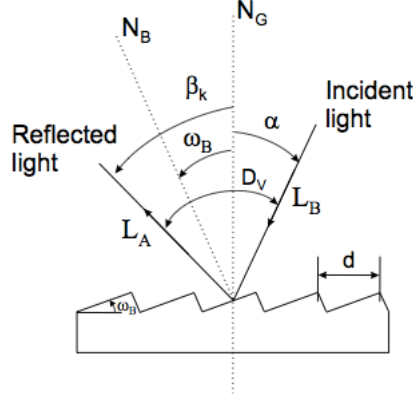


Figure 5: Schematic of the reflection off a grating.  $N_B$  is the *blaze normal* and  $\omega_B$  defines the *blaze angle*. The distance between consecutive lines is given by  $d$ .

### Dispersion

The derivation of the diffraction angle after the wavelength is also referred to as angular dispersion. It is a measure of the angular distance between light beams with adjacent wavelengths. Through the derivation of equation 8 for a fixed angle  $\alpha$  one obtains the following expression for the angular dispersion:

$$\frac{\partial \beta_k}{\partial \lambda} = \frac{kn}{\cos \beta_k}. \quad (9)$$

A high dispersion can be achieved if either a grating with a high line density ( $n$ ) or a coarse grating in high diffraction orders is used. The linear wavelength dispersion in the exit plane of a spectroscopic element is referred to as *reciprocal linear dispersion* and given in nm/mm. For measurement apparatus with focal length  $L_B$ , the reciprocal linear dispersion follows:

$$D(\lambda) = \frac{\partial \lambda}{\partial p} = \frac{kn \cos \beta_k}{L_B}, \quad (10)$$

with the distance  $p$  in mm. As the size of the measuring apparatus depends on the focal length of the optical system, it can be designed more compactly by using a grating with a higher line density.

As shown in equation 10, the relation between the pixel position  $p$  on the CCD camera and the real wavelength  $\lambda$  is given by the dispersion function  $D(\lambda)$ . This function depends on the wavelength. At least one reference point  $p_0$  in the spectrum (at a known wavelength  $\lambda_0$ ) is necessary in order to calibrate the wavelength

scale, if  $D(\lambda)$  is known. Then the value  $\lambda$  of every other line in the spectrum can be obtained through

$$\lambda = \lambda_0 + \int_{p_0}^p D(\lambda) dp . \quad (11)$$

This is often solvable numerically. Though if the dispersion function is independent of  $\lambda$ , then equation 11 can be expressed as follows

$$\lambda = \lambda_0 + D(p - p_0) , \quad (12)$$

and the desired wavelength  $\lambda$  can be determined. In this experiment reference lines should be used in order to find the dispersion function of the spectrometer. With the positions of the reference lines in the detector  $p_i$  and their wavelengths  $\lambda_i$ , the dispersion can be approximated by a polynomial function  $\lambda(p)$ .

$$\lambda(p) = A + B \cdot p + C \cdot p^2 + \dots , \quad (13)$$

with the free parameters A, B, C, etc.

### **Resolution**

The *resolution* or *chromatic resolution* of a grating describes its ability to distinguish neighbouring spectral lines. The resolution is in general defined as  $R = \frac{\lambda}{\Delta\lambda}$ , with the wavelength difference  $\Delta\lambda$  between two spectral lines of the same intensity, that can be distinguished. The limit of resolution of a grating is  $R = kN$ , with the number  $N$  of the illuminated lines of the grating. An appropriate expression for the resolution is obtained through the use of equation 8:

$$R = kN = \frac{Nd(\sin \alpha + \sin \beta_k)}{\lambda} . \quad (14)$$

The spectrometer is equipped with a grating with 1800 lines/mm.

## **CCD Camera**

A CCD camera is an arrangement of silicon solid state detectors, which are sensitive in the range 400 to 1100nm. They convert the incoming light into electrons through the internal photoelectric effect. These free electrons are saved in a rectangular arrangement of sensors, called pixels. The CCD camera provides information about the intensity and position at the same time. The size of the CCD-chips used is  $30 \times 12 \text{ mm}^2$  and consists of  $2048 \times 512$  pixels. Each pixel has an area of  $15 \times 15 \mu\text{m}^2$ . The camera is cooled with liquid Nitrogen in order to reduce the thermal noise and can integrate weak signals over hours without accumulating a disruptive background signal (less than 1 electron/pixel/hour). The typical readout noise is smaller than 3 electrons/pixel at  $-140^\circ \text{ C}$ .

<i>Species</i>	$\lambda$ (nm)	<i>Species</i>	$\lambda$ (nm)	<i>Species</i>	$\lambda$ (nm)
Ne I	585.24878	Ne I	621.72812	Ne I	667.82766
Ne I	588.18950	Ne I	626.64952	Ne I	671.70430
Ne I	594.48340	Ne I	630.47893	Ne I	692.94672
Ne I	597.55343	Ne I	633.44276	Ne I	702.40500
Ne I	602.99968	Ne I	638.29914	Ne I	703.24128
Ne I	607.43376	Ne I	640.22480	Ne I	717.39380
Ne I	609.61630	Ne I	650.65277	Ne I	724.51665
Ne I	614.30627	Ne I	653.28824	Ne I	743.88981
Ne I	616.35937	Ne I	659.89528		

Table 2: Neon reference lines for the calibration (source: <http://www.nist.gov/> )

## Calibration

### Calibration Lamp

In order to calibrate the spectrometer, for example to find the dispersion, reference lines will be necessary. For this reason, a neon lamp will be used for calibration. The necessary reference lines are listed in Table 2 (data from the NIST, *National Institute for Standards and Technology, spectral line database*, <http://www.nist.gov>) and are displayed in the form of an overview spectrum of the experiment.

## Part II: Execution (1 afternoon)

The aim of this experiment is to measure the wavelength of the red spectral line of Cadmium (Cd). Furthermore, another very weak line will be emitted from the Cd lamp with a wavelength of  $\approx 652$  nm, that should be identified and an element assigned.

- Questions to be prepared: The band width which will be observed by the detector is  $\approx 20$  nm. Therefore the grating must be moved and various regions of the Neon- and Cd-spectrums mapped. The grating converts the wavelength/energy into a geometrical value, namely the dispersion of the CCD detector. Hence, you must proceed carefully with the alignment of the various components along the light path of the experimental set-up. Are there sources of error in the set-up? How can these be prevented? What line shape do the spectral lines have? Give reasons.



- Consider a strategy for measuring the Cd-lines that will minimise the systematic errors (integration time of the camera, calibration procedure etc.).
- Discuss this strategy with the supervisor before you start taking measurements.
- Using an optical fiber in the optics set-up, connect the light from the neon lamp to the spectrometer.
- Long evaluation: newly optimise the optical set-up in front of the spectrometer.
- Use the program **Instrument Control Center** to take a spectrum of the Neon-calibration lamp and compare the images to the overview spectrum, which is displayed on the experiment. WARNING: The spectra, which are taken by the spectrometer, are reflected in the y-axis. Now attempt to find the characteristic region of the Neon-spectrum (around the expected wavelength for the Cadmium line) and make a separate recording of this region (5 lines). In order to move the window along the spectrum, in dialogue window “Spectrometer Control Dialogue” select “Image” and systematically change the value “TRIAX”. Vary the integration time and the slit size so that you use the whole intensity region of the CCD-detector (the maximum is at around 65000) and the lines are as narrow as possible.
- Now take an image of the Cd-line with the value found for “TRIAX”. The integration time must be suited to the intensity of the Cd-lamp.
- Long evaluation: Create an image, in which light from the Neon- and the Cd- lamps enter the spectrometer at the same time.
- **Instrument Control Center**
  - Open the program
  - Click “Yes” in the first window
  - Click “Nein” in the second window (IMPORTANT!)
  - The possible settings are listed for reference as screen shots in section “Spectrometer Control Program”

## Part II: Evaluation

- Fit the position of the 5 neon lines with the help of Origin (Import: *Thermo Glacatic .spc*. Fit a calibration function, such that pixels can be converted into wavelengths.

- Determine the wavelength of the Cd-line.
- Identify the unknown element based on data of the NIST Atomic Spectra Database ([http://physics.nist.gov/PhysRefData/ASD/lines\\_form.html](http://physics.nist.gov/PhysRefData/ASD/lines_form.html))
- Long evaluation: Create an origin graph from an image in which both Neon lines and Cd lines can be seen. Consider how to present the graph, such that the strong and weak intensity lines are portrayed equally.

## Spectrometer Control Program

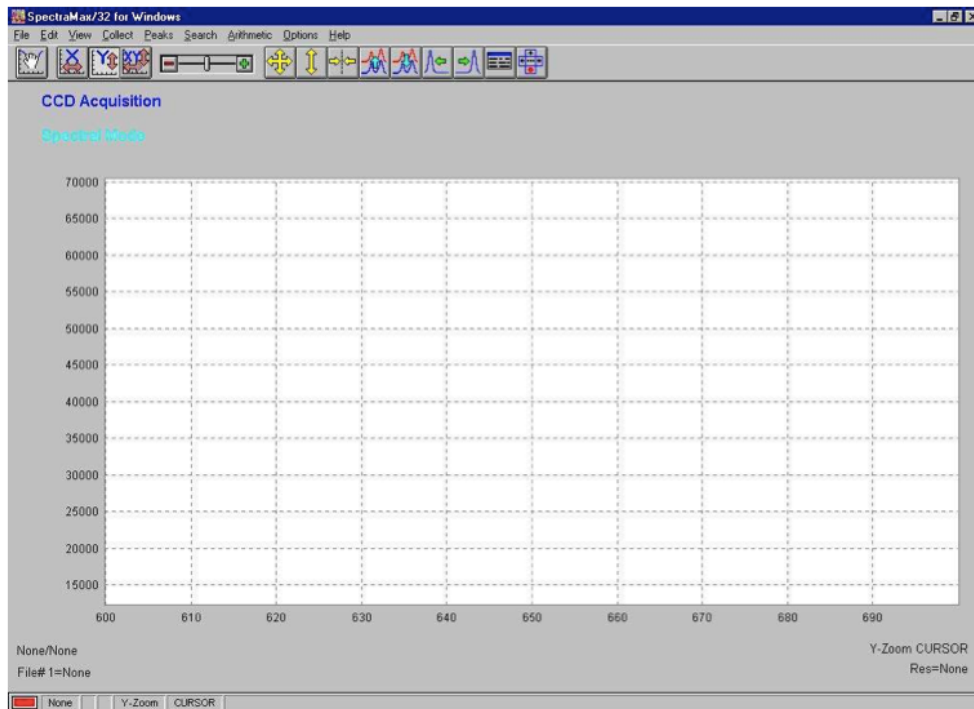


Figure 6: Main window of the spectrometer control program SPECTRAMAX. The dialogue box for the spectrometer is opened in a new window by clicking the button on which a list is depicted (second from the right). Figure 7

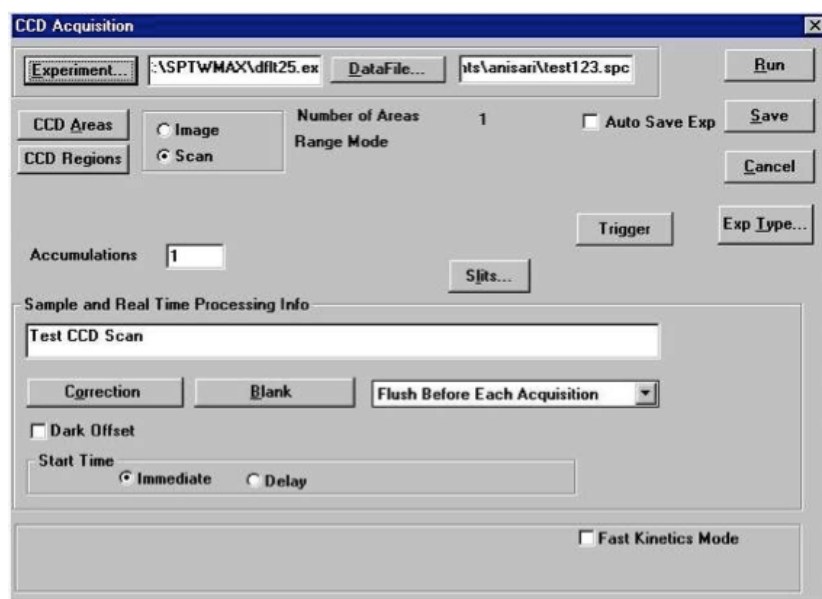


Figure 7: Spectrometer control dialogue box. Before taking data a name for the data must be chosen. The data taking starts after clicking “Run”. “CCD Areas” opens the dialogue box which is shown in figure 8. “Slits” opens the dialogue box, which is shown in figure 9. The other options are not necessary for this experiment.

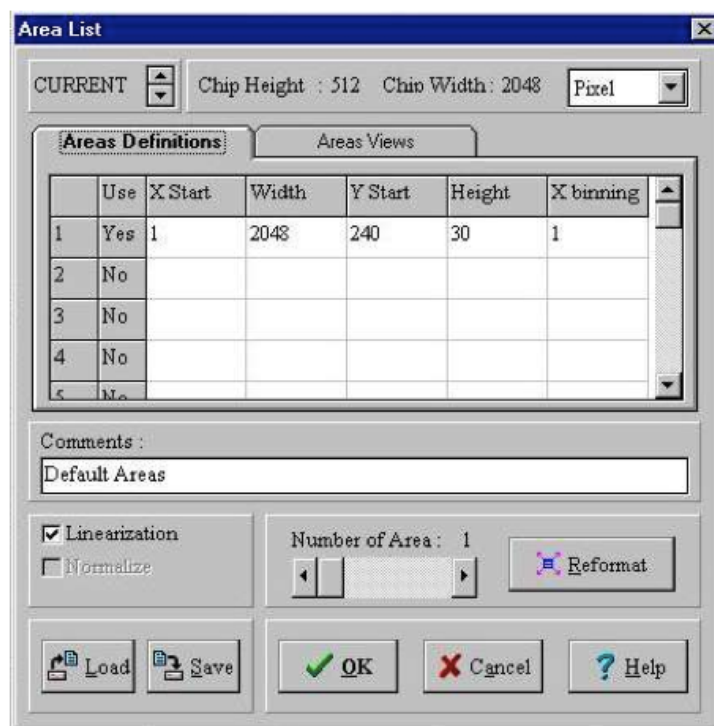


Figure 8: “CCD Areas” dialogue box. Here the pixels which should be readout can be defined. The cross in the control box at “Linearisation” should be removed. The field “Reformat” should be clicked once for safety, so that the complete chip of the CCD-camera will be used for the measurement.

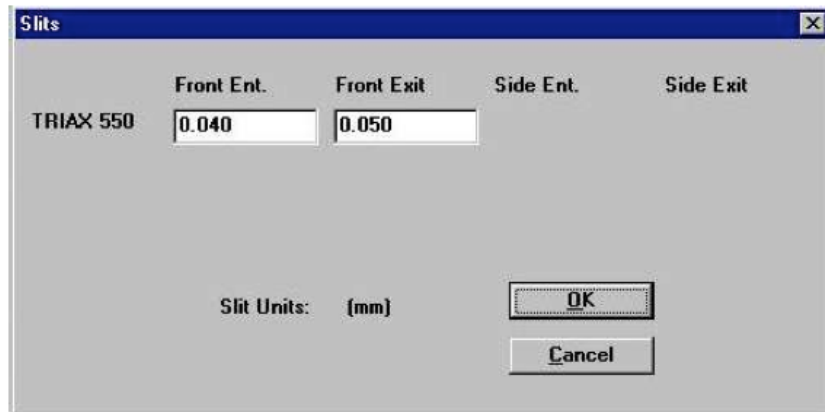


Figure 9: “Slits” dialogue box. The spectrometer has only one slit at the entrance. Here the width of the slit (in mm) can be set. Note that depending on the intensity profile, which illuminates the entrance slit, a change in the slit during the experiment can induce a systematic error.

## Preparation

Topics to be prepared are:

- Zeeman effect
- Hysteresis
- Hall effect
- Polarisation filter
- Interferometer and Lummer-Gehrcke Plate
- Grating spectrometer and Czerny-Turner Monochromator
- Charge Coupled Device (CCD) camera
- Internal photoelectric effect
- Line widths and line broadening