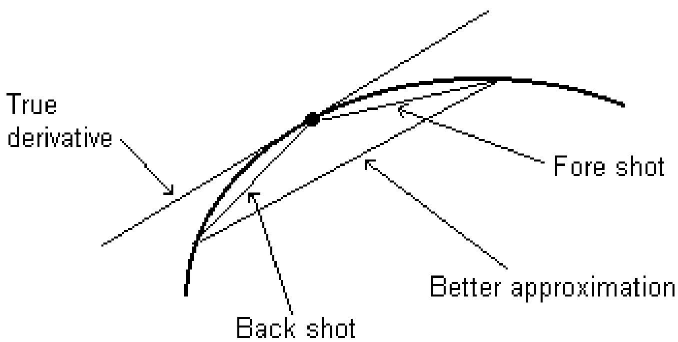
Centered Difference Approximations for Sensitivities

The following will demonstrate how to increase the accuracy of the sensitivity calculations for possible situations where sharp peaks or notches could cause sensitivity calculation errors to become unacceptable. The technique is known as Centered Difference Approximation. The reference is given in the M-file.



Since the normalized sensitivity is a scaled derivative, a better approximation of the slope of a curve at a point can be obtained by averaging two perturbations, one forward (fore shot) and one backward (back shot).

This is what is done in the M-file centdiff.m below using the dc diff amp as an example. The errors are a function of the perturbation size (dpf in the M-file) and the function being differentiated. Although the errors using the single perturbation method could be considered negligible, in tolerance analysis problems described above they will be excessive.

```
% Centered difference approximation for sensitivities
% Ref:  Numerical Methods for Engineers,
% S.C. Chapra & R.P. Canale, McGraw-Hill, 3rd ed, 1998, p.93
% File:  centdiff.m; updated 11/03/06.  Eliminates {Nc Nc} arrays Q and B.
%
clear;clc
E1=1;E2=-1;R1=10;R2=100;R3=10;R4=100;
RR=[R1 R2 R3 R4 E1 E2];
Vo=DA2(RR) % Differential amplifier
Nc=length(RR);dpf=0.0001;Q=1+dpf;B=1-dpf;
Qx=ones(1,Nc);Bx=ones(1,Nc);
for p=1:Nc
   Qx(p)=Q;Bx(p)=B;
   if p > 1;Qx(p-1)=1;Bx(p-1)=1;end; % Reset previous
   RRx=RR.*Qx;Vr=DA2(RRx); %  fore shot
   RRx=RR.*Bx;Vb=DA2(RRx); %  back shot
   Sen1(p)=(Vr/Vo-1)/dpf; % fore shot only
   Sen2(p)=(Vr-Vb)/(2*Vo*dpf); % fore and back shot; centered difference
   D(p)=1e6*(Sen1(p)/Sen2(p)-1); % (Meas-True)/True = Meas/True-1
end
format short g
Sen1
Sen2
disp(' ')
disp('Error (in ppm) of Sen1 compared to the more accurate Sen2:')
disp(D)
%
function y = DA2(X);
% function for dc diffamp
% X=[R1 R2 R3 R4 E1 E2]
R1=X(1);R2=X(2);R3=X(3);R4=X(4);E1=X(5);E2=X(6);
y=E1*(1+R2/R1)/(1+R3/R4)-E2*R2/R1;
```