Quantitative Proficiency Test 231 - Pdfcoffee

Quantitative Proficiency Test (WorldQuant University)

Quantitative Proficiency Test 2

Finance (WorldQuant University)

Started on	Monday, 11 November 2019, 1:44 AM
State	Finished
Completed on	Monday, 11 November 2019, 4:22 AM
Time taken	2 hours 38 mins
Marks	33/40
Grade	**83** out of 100
Feedback	**Congratulations!** You have successfully completed the *Quantitative Proficiency Test* with a passing grade and have been conditionally accepted into the MSc in Financial Engineering program. A confirmation with further instructions has been sent to your email address. **Please proceed with your application.**

Information

This section of the quiz will test your mathematical proficiency.

Question 1
Correct
Mark 1 out of 1

Let $x_0 \in \mathbb{R}$ be the value of $x \in \mathbb{R}$ that maximizes the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) := e^{-(x-1)^2}$ for every $x \in \mathbb{R}$,
and $y_0 := f(x_0)$ be the value of this maximum. Then $x_0 + y_0$ is

Select one:

- 1
- $2e^{-1}$
- 2 ✔
- e^{-1}

Question 2
Correct
Mark 1 out of 1

The value of the limit
$\lim_{x \to 0} \frac{\sin 2x}{x}$ is

Select one:

- ∞
- 1/2
- 2 ✔
- 1

Question 3
Correct
Mark 1 out of 1

Let $x_0 \in (0, \infty)$ be the value of $x \in (0, \infty)$ that maximizes the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) := \ln x / x$ for every $x \in (0, \infty)$,
and $y_0 := f(x_0)$ be the value of this maximum. Then the product $x_0 y_0$ is

Select one:

- e
- 1 ✔
- $1/e$
- 0

Question 4
Correct
Mark 1 out of 1

Evaluate
$\int_1^e \ln x \, dx$

Select one:

- $-e$
- 1 ✔
- e
- 0

Question 5
Correct
Mark 1 out of 1

Evaluate
$\int_0^{\pi/2} x \cos x \, dx.$

Select one:

- 1
- $\pi/2 - 1$ ✔
- π
- -1

Question 6
Incorrect
Mark 0 out of 1

Let A, B and C be square invertible matrices of the same size. If C has no eigenvalue equal to -1, then $(AB + ACB)^{-1}$ is equal to

Select one:

- $(I + C)^{-1}B^{-1}A^{-1}$ ✖
- $A^{-1}B(I + C)^{-1}$
- $B^{-1}(I + C)^{-1}A^{-1}$
- $(I + C)^{-1}BA^{-1}$

Question 7
Correct
Mark 1 out of 1

Let
$A := \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$
and λ_1 and λ_2 be its (not necessarily distinct) eigenvalues. Then $det(A) + \lambda_1 + \lambda_2$ is

Select one:
- 22
- 0
- $2 + i$
- 18 ✔

Question 8
Correct
Mark 1 out of 1

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to [0, \infty)$ be defined by
$f(x) = g(x) : x^2$ for every $x \in \mathbb{R}$.

Consider the following statements:
[i.] f is one-to-one
[ii.] f is onto
[iii.] g is one-to-one
[iv.] g is onto

Which of the statements are true?

Select one:
- None of the statements
- iv. ✔
- ii. and iv.
- All of the statements

Question 9
Correct
Mark 1 out of 1

The solution to the following differential equation
$y'' + 3y' - 4y = 0,\ y(0) = 5, y'(0) = 0$
is

Select one:

- $y(t) = 4e^t + e^{-4t}$ ✔
- $y(t) = e^{-4t}$
- $y(t) = 4e^t$
- $y(t) = 5$

Question 10
Correct
Mark 1 out of 1

The complex number $z = 3 + 4i$ can also be written as $z = re^{i\theta}$, where

Select one:

- $r = 5, \theta = \arctan \frac{4}{3}$ ✔
- $r = 5, \theta = \pi/4$
- $r = 25, \theta = \arctan \frac{4}{3}$
- $r = 3, \theta = 4$

Question 11
Incorrect
Mark 0 out of 1

The following series $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

Select one:

- converges to $e - 1$
- diverges to ∞
- converges to e^3 ✖
- converges to $e^3 - 1$

Question 12
Correct
Mark 1 out of 1

The value of $\lim_{n\to\infty} \frac{2^n}{n!}$ is

Select one:

- 0. ✔
- ∞.
- 1.
- 2.

Question 13
Incorrect
Mark 0 out of 1

An island consists of four kinds of people: Tetas, Jekas, Frekas and Hekas. The following information is known:

Every Heka is either a Teta or a Jeka, but not both
All Frekas are Jekas
No Frekas are Tetas

Consider the following statements:

[i.] No Tetas are Jekas
[ii.] Some Hekas are Frekas

Which of these statements are necessarily true based only on the information above?

Select one:

- Only ii. ✖
- All the statements
- None of the statements
- Only i

Question 14
Incorrect
Mark 0 out of 1

Consider the following statements concerning a positive integer n:

[i.] if n is a multiple of 9, then n^2 is a multiple of 3
[ii.] if n^2 is a multiple of 7, then n is a multiple of 7
[iii.] if n^2 is a multiple of 14, then n is a multiple of 7

Which of the statements are true?

Select one:
- All the statements
- i. and ii. ✖
- i. and iii.
- None of the statements

Question 15
Correct
Mark 1 out of 1

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$f((x, y)) := x^2 + 2y^2 - 3xy$, for every $x, y) \in \mathbb{R}^2$.

The value of $f_x((0,0)) + f_y((0,0)) + f_{xy}((0,0))$ is:

Select one:
- 0
- Undefined
- -3 ✔
- 3

Question 16
Incorrect
Mark 0 out of 1

Let
$I := \iint_D 2\,dA,$
where D is the interior of the region bounded by the curves $y = x^2\$$ and $y = x^3$ with $x \geq 0$ and $y \geq 0$.
The value of I is

Select one:
- 1/3
- 1/6
- 2/3
- 3/4 ✖

Question 17
Correct
Mark 1 out of 1

Evaluate
$\int_0^1 \int_x^1 \sin(y^2)\,dydx$
by changing the order of the integral. The answer is

Select one:
- $1 - \sin 1$
- cannot be determined
- $\frac{1}{2}(1 - \cos 1)$ ✔
- $\cos 1$

Question 18
Correct
Mark 1 out of 1

Evaluate
$\int_0^{\infty} \int_y^{\infty} 2ye^{-x^3}\, dxdy$
by changing the order of the integral. The answer is

Select one:

- 1
- $\frac{1}{3}$ ✔
- e
- $\frac{1}{2}$

Question 19
Correct
Mark 1 out of 1

For each $n = 1, 2, 3, \ldots$, define
$f_n(x) := x(1 - x^{2n})$, for every $x \in [-1, 1]$.

Then the function f defined by
$f(x) := \lim_{n \to \infty} f_n(x)$
exists for each $x \in [-1, 1]$ and is equal to

Select one:

- $f(x) = \begin{cases} x & |x| < 1 \\ 0 & |x| = 1 \end{cases}$ ✔
- $f(x) = x$
- $f(x) = \begin{cases} 0 & |x| < 1 \\ 1 & |x| = 1 \end{cases}$
- $f(x) = 0$

Question 20
Correct
Mark 1 out of 1

Consider the following partial differential equation (PDE):
$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

where $u = u(x, y)$ is the unknown function.
Define the following functions:
$u_1(x, y) := \cos(2xy)$, $u_2(x, y) = \sin(x^2 y)$ and $u_3(x, y) = e^{-(x^2+y^2)}$.

Which of these functions are solutions to the above PDE?

Select one:

- Only u_3.
- None of the functions. ✔
- u_1 and u_3
- All the functions.

Information

This section will test your statistical proficiency.

Question 21
Correct
Mark 1 out of 1

A bag contains 50 balls, of which 20 are red and 30 are green. The average (mean) mass for all the balls is $\bar{x}$ kg, and the mean mass for the red balls is $\bar{x}_R$ kg. The mean mass for the green balls $\bar{x}_G$ is

Select one:

- $\frac{30\bar{x}+20\bar{x}_R}{50}$
- $\frac{50\bar{x}-30\bar{x}_R}{20}$
- $\frac{30\bar{x}-20\bar{x}_R}{50}$
- $\frac{50\bar{x}-20\bar{x}_R}{30}$ ✔

Question 22
Correct
Mark 1 out of 1

A group of 10 students received the following marks for a test:
58, 89,65, 78, 55,26,93,46,43,59.
The standard deviation of their marks is (to two decimal places)

Select one:

- 3745.44
- 61.20
- 20.91 ✔
- 437.29

Question 23
Incorrect
Mark 0 out of 1

A magician has 20 coins in his pocket. Twelve of these coins are normal fair coins (with one head and one tail) and eight are defective coins with heads on both sides. The magician randomly draws a coin from his pocket and flips it. Given that the flipped coin shows a head, what is the probability that it is defective?

Select one:

- 1
- 1/2
- 8/20 ✖
- 4/7

Question 24
Correct
Mark 1 out of 1

A bag contains 20 balls: 8 red balls and 12 blue balls. Six of these balls are classified as "heavy", and three of the heavy balls are red. If a ball is chosen at random from the bag, what is the probability that it is red or heavy?

Select one:

- 14/30
- 11/20 ✔
- 8/30
- 6/30

Question 25
Correct
Mark 1 out of 1

A jar contains 5 red balls and 12 blue balls. Two balls are drawn at random from the hat without replacement. What is the probability that both balls drawn are red?

Select one:

- 5/17
- $\frac{\binom{5}{2}}{\binom{17}{2}}$ ✔
- $\frac{\binom{5}{2}}{\binom{12}{2}}$
- 10/17

Question 26
Correct
Mark 1 out of 1

A magician has a collection of 52 cards, with 26 red and 26 black cards. Four of these cards are classified as 'special', and two of the special cards are red. If a card is chosen at random from the 52 cards, what is the probability that the card is special or red?

Select one:

- 26/52
- 2/52
- 28/52 ✔
- 4/52

Question 27
Correct
Mark 1 out of 1

A class has 30 girls and 22 boys. A teacher selects 15 students at random from the class to participate in a competition. What is the probability that out of the 15 selected students, 6 of them are boys?

Select one:

- $\frac{6}{15}$
- $\frac{\binom{15}{6}\binom{30}{9}}{\binom{52}{15}}$
- $\frac{\binom{22}{6}\binom{30}{9}}{\binom{22}{15}}$
- $\frac{\binom{22}{6}\binom{30}{9}}{\binom{52}{15}}$ ✔

Question 28
Correct
Mark 1 out of 1

The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} \frac{c}{x^2} & 1 \leq x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

where $c > 0$ is a constant. Find $\mathbb{P}(X > 25)$.

Select one:

- 1
- 1/25 ✔
- $1/25^2$
- 0

Question 29
Correct
Mark 1 out of 1

The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} 6x(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $\mathbb{E}(X)$ and $\text{Var}(X)$.

Select one:

- $\mathbb{E}(X) = 0, \text{Var}(X) = 1$
- $\mathbb{E}(X) = 1/2, \text{Var}(X) = 1$
- $\mathbb{E}(X) = 1/2, \text{Var}(X) = 1/20$ ✔
- $\mathbb{E}(X) = 1/2, \text{Var}(X) = 3/10$

Question 30
Correct
Mark 1 out of 1

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} e^{-x-y} & 0 < x < \infty,\ 0 < y < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

What is $\mathbb{P}(X < Y)$?

Select one:

- 1/2 ✔
- e^{-1}
- 1
- 0

Question 31
Correct
Mark 1 out of 1

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the conditional distribution of X given Y and use it to calculate $\mathbb{P}(X > 0.5 | Y = 0.7)$.

Select one:

- 1
- 2/7 ✔
- 2/3
- 0

Question 32
Correct
Mark 1 out of 1

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} 4xy & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

What is $\text{Cov}(X,Y)$?

Select one:

- 0 ✔
- 2
- 1
- -1

Question 33
Incorrect
Mark 0 out of 1

The non-negative integer-valued discrete random variable X has the following probability generating function:
$G_X(s) = \frac{1}{2-s}$.

Find $\mathbb{P}(X < 2)$.

Select one:

- 1/8
- 1/4
- 1/2 ✖
- 3/4

Question 34
Correct
Mark 1 out of 1

The random variable X has the following moment generating function:
$M_X(t) = \frac{1}{1-t},\ t < 1.$

Find $\mathbb{E}(X)$ and $\text{Var}(X)$.

Select one:

- $\mathbb{E}(X) = \infty,\ \text{Var}(X) = \infty$
- $\mathbb{E}(X) = 1/2,\ \text{Var}(X) = 3/10$
- $\mathbb{E}(X) = 1,\ \text{Var}(X) = 1$ ✔
- $\mathbb{E}(X) = 0,\ \text{Var}(X) = 1$

Question 35

Correct

Mark 1 out of 1

Let X_1, X_2, X_3 be a random sample from the distribution of the random variable X with a normal distribution with unknown mean parameter μ and variance 1. Define the following estimators for μ:
$\hat{\mu}_1 := \frac{X_1+X_2+X_3}{3}$, $\hat{\mu}_2 := X_1 + X_2 + X_3$, and $\hat{\mu}_3 := \frac{2}{5}X_1 + \frac{3}{5}X_2$

Rank the estimators from the one with the smallest variance to the one with the largest variance.

Select one:

- $\hat{\mu}_1, \hat{\mu}_3, \hat{\mu}_2$ ✔
- $\hat{\mu}_2, \hat{\mu}_3, \hat{\mu}_1$
- $\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3$
- $\hat{\mu}_3, \hat{\mu}_1, \hat{\mu}_2$

Question 36
Correct
Mark 1 out of 1

Let $X_1, \ldots, X_n (n \geq 2)$ be a random sample from the distribution of the random variable X with the following probability density function:

$$f_X(x) = \begin{cases} \lambda^2 x e^{-\lambda x} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

where $\lambda > 0$ is an unknown parameter. The maximum likelihood estimator of λ is

Select one:

- $\frac{2}{\sum_{i=1}^{n} X_i}$
- $2\sum_{i=1}^{n} X_i$
- $\frac{2n}{\sum_{i=1}^{n} X_i}$ ✔
- $\frac{2}{n}\sum_{i=1}^{n} X_i$

Question 37
Correct
Mark 1 out of 1

Let $X_1, \ldots, X_n (n \geq 2)$ be a random sample from the distribution of the random variable X with a normal distribution with unknown mean parameter μ and variance 1. Define the following estimators for μ:
$\hat{\mu}_1 := \frac{1}{n}\sum_{i=1}^{n} X_i$, $\hat{\mu}_2 := \sum_{i=1}^{n} X_i$, and $\hat{\mu}_3 := \frac{2}{5}X_1 + \frac{3}{5}X_2$

Which of these estimators are unbiased?

Select one:

- a. All of them
- b. None of them
- c. $\hat{\mu}_1 and \hat{\mu}_3$ ✔
- d. Only $\hat{\mu}_1$

Question 38
Correct
Mark 1 out of 1

Let $X_1, \ldots, X_{16}$ be a random sample from a normal distribution with unknown mean μ and known variance $\sigma^2 = 81$. A sample $x_1, \ldots, x_{16}$ is collected and the sample mean from this sample is calculated to be $\bar{x} = 8$. Based on this sample, a 95% confidence interval for μ is

Select one:

- $(-\infty, 8)$
- (3.59,12.41) ✔
- (0,8)
- $(8, \infty)$

Question 39
Correct
Mark 1 out of 1

Let $X_1, \ldots, X_{36}$ be a random sample from a normal distribution with unknown mean μ and known variance $\sigma^2 = 121$. A sample $x_1, \ldots, x_{36}$ is collected and the sample mean from this sample is calculated to be $\bar{x} = 22$. Based on this sample, a 95% confidence interval for μ is

Select one:

- $(0, \infty)$
- (18.41,25.59) ✔
- $(22, \infty)$
- (0,22)

Question 40
Correct
Mark 1 out of 1

The random variables X and Y are believed to be linearly related by the equation $Y = a + bX + \epsilon$,

where a and b are constants and ϵ is a mean zero, normally distributed error term. The following sample of pairs (x, y) is collected:

x	10	20	30	40	50	60	70
y	5	10	23	34	40	54	75

Using the method of least squares, calculate the values of a and b to 3 decimal places.

Select one:

- $a = 10.571, b = -1.125$
- $a = 65.587, b = -87.214$
- $a = 0, b = 1$
- $a = -10.571, b = 1.125$ ✔

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