Quant Finance questions

Quantitative Proficiency Test (WorldQuant University)

Question 1

Not yet answered

Marked out of 1

Flag question

Let $x_0 \in (0, \infty)$ be the value of $x \in (0, \infty)$ that maximizes the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) := \ln x / x$ for every $x \in (0, \infty)$, and $y_0 := f(x_0)$ be the value of this maximum. Then the product $x_0 y_0$ is

Select one:

- $1/e$
- e
- 1
- 0

Question 2

Answer saved

Marked out of 1

Flag question

If $f : (0, \infty) \to \mathbb{R}$ is defined by $f(x) = \frac{x}{\ln(2x)}$ for every $x \in (0, \infty)$, then $f'(e/2)$ is

Select one:

- $1/e$
- $e/2$
- e (selected)
- 0

Question 3

Answer saved

Marked out of 1

Flag question

The value of the limit $\lim_{x \to \infty} x^2 e^{-x}$ is

Select one:

- 0 (selected)
- e^{-1}
- 1
- 2

Question 4

Answer saved

Marked out of 1

Flag question

The value of the integral
$\int_0^{\sqrt{\pi}/2} x\cos(x^2)\,dx$
is

Select one:

- 0
- $\frac{1}{2\sqrt{2}}$
- $\sqrt{\pi}$
- $\sqrt{\pi}\cos\pi$

Question 5

Answer saved

Marked out of 1

Flag question

Evaluate
$\int_0^{\infty} x^2 e^{-3x}\,dx$

Select one:

- ∞
- 0
- 2
- 2/27

Question 6

Not yet answered

Marked out of 1

Flag question

Let A, B and C be square invertible matrices of the same size. If C has no eigenvalue equal to -1, then $(AB + ACB)^{-1}$ is equal to

Select one:

- $A^{-1}B(I + C)^{-1}$
- $(I + C)^{-1}BA^{-1}$
- $(I + C)^{-1}B^{-1}A^{-1}$
- $B^{-1}(I + C)^{-1}A^{-1}$

Question 7

Answer saved

Marked out of 1

Flag question

Let

$$A := \begin{pmatrix} 0 & 9 \\ 3 & 13 \end{pmatrix}$$

and λ_1 and λ_2 be its (not necessarily distinct) eigenvalues. Then $det(A) + \lambda_1 + \lambda_2$ is

Select one:

- -14
- 10
- -12
- 0

Question 8

Not yet answered

Marked out of 1

Flag question

Let $f : (0, \infty) \to (0, \infty)$ and $g : (0, \infty) \to \mathbb{R}$ be defined by
$f(x) := \frac{1}{x}$ for every $x \in (0, \infty)$
and
$g(x) := \ln x$ for every $x \in (0, \infty)$.

Consider the following statements:
[i.] f is one-to-one
[ii.] f is onto
[iii.] g is one-to-one
[iv.] g is onto

Which of the statements are true?

Select one:

- i and iii
- None of the statements
- All of the statements
- i, ii. and iii.

Question 9

Answer saved

Marked out of 1

Flag question

The solution to the following differential equation
$y'' + 2y' + y = 0,\ y(0) = 2, y'(0) = 10$
is

Select one:

- $y(t) = e^{-t}$
- $y(t) = 2$
- $y(t) = 2e^{-t} + 12te^{-t}$
- $y(t) = 2e^{-t}$

Question 10

Not yet answered

Marked out of 1

Flag question

The complex number $z = 12 + 5i$ can also be written as $z = r(\cos\theta + i\sin\theta)$, where

Select one:

- $r = 12,\ \theta = 5.$
- $r = 12,\ \theta = \pi.$
- $r = 169,\ \theta = \arctan\frac{5}{12}$
- $r = 13,\ \theta = \arctan\frac{5}{12}$

Question 11

Not yet answered

Marked out of 1

Flag question

The following series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Select one:

- converges absolutely
- diverges to ∞
- converges conditionally
- diverges to $-\infty$

Question 12

Answer saved

Marked out of 1

Flag question

The value of $\lim_{n\to\infty} 1 - \frac{n}{2^n}$ is

Select one:

- 0
- 1
- $\ln 2.$
- ∞

Question 13

Answer saved

Marked out of 1

Flag question

An island consists of four kinds of people: Tetas, Jekas, Frekas and Hekas. The following information is known:

Every Heka is either a Teta or a Jeka, but not both
All Frekas are Jekas
No Frekas are Tetas

Consider the following statements:

[i.] No Tetas are Jekas
[ii.] Some Hekas are Frekas

Which of these statements are necessarily true based only on the information above?

Select one:

- Only i
- Only ii.
- All the statements
- None of the statements

Question 14

Not yet answered

Marked out of 1

Flag question

Consider the following statements concerning a positive integer n:

[i.] if n is a multiple of 9, then n^2 is a multiple of 3
[ii.] if n^2 is a multiple of 7, then n is a multiple of 7
[iii.] if n^2 is a multiple of 14, then n is a multiple of 7

Which of the statements are true?

Select one:

- None of the statements
- i. and iii.
- All the statements
- i. and ii.

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Question 15

Not yet answered

Marked out of 1

Flag question

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$f((x, y)) := x^2 e^{-x-2y}$, for every $(x, y) \in \mathbb{R}^2$.

The value of $f_x((0,0)) + f_y((0,0)) + f_{yy}((0,0))$ is:

Select one:

- 0
- Undefined
- 2
- -2

Question 16

Not yet answered

Marked out of 1

Flag question

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$f((x, y)) := -x^2 - 2y^2$, for every $(x, y) \in \mathbb{R}^2$.

The point (x_0, y_0) where f reaches its global maximum value is

Select one:

- (0,1)
- (1,0)
- (0,0)
- None of the above

Question 17

Not yet answered

Marked out of 1

Flag question

Evaluate
$\int_0^\infty \int_y^\infty 2ye^{-x^3}\,dxdy$
by changing the order of the integral. The answer is

Select one:

- $\frac{1}{2}$
- 1
- e
- $\frac{1}{3}$

Question 18

Not yet answered

Marked out of 1

Flag question

Let
$I := \iint_D y\,dA,$
where
$D = \{(x, y) : y \leq x \leq 4, 0 \leq y \leq 1\}.$
The value of I is

Select one:

- 8/3
- 5/3
- 1
- 1/3

Question 19

Not yet answered

Marked out of 1

Flag question

For each $n = 1, 2, 3, \ldots$, define
$f_n(x) := \frac{n^2x^3}{1+2n^2x^2}$, for every $x \in \mathbb{R}$.

Then the function f defined by
$f(x) := \lim_{n\to\infty} f_n(x)$

exists for each $x \in \mathbb{R}$ and is equal to

Select one:

- $f(x) = x$
- $f(x) = 0$
- $f(x) = x^2$
- $f(x) = \frac{x}{2}$

Question 20

Not yet answered

Marked out of 1

Flag question

Consider the following partial differential equation (PDE):
$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

where $u = u(x, y)$ is the unknown function.
Define the following functions:
$u_1(x, y) := \cos(2xy)$, $u_2(x, y) = \sin(x^2y)$ and $u_3(x, y) = e^{-(x^2+y^2)}$.

Which of these functions are solutions to the above PDE?

Select one:

- u_1 and u_3
- None of the functions.
- Only u_3.
- All the functions.

Question 21

Answer saved

Marked out of 1

Flag question

A group of 10 students received the following marks for a test:
58, 89,65, 78, 55,26,93,46,43,59.
The standard deviation of their marks is (to two decimal places)

Select one:

- 3745.44
- 61.20
- 437.29
- 20.91

Question 22

Answer saved

Marked out of 1

Flag question

A class has 60 students of which 25 are taught by teacher A and 35 are taught by teacher B. The class average (mean) mark for all the students is $\bar{x}$ and the mean mark for the students taught by teacher A is $\bar{x}_A$. The mean mark for students taught by teacher B $\bar{x}_B$ is

Select one:

- $\frac{35\bar{x}-25\bar{x}_A}{60}$
- $\frac{25\bar{x}-35\bar{x}_A}{25}$
- $\frac{60\bar{x}-25\bar{x}_A}{35}$
- $\frac{35\bar{x}+25\bar{x}_A}{60}$

Question 23
Answer saved
Marked out of 1
Flag question

In a class of 30 students, there are 17 girls and 13 boys. Five are 'A' students, and three of these 'A' students are girls. If a student is chosen at random, what is the probability of choosing a girl or an 'A' student?

Select one:
- 22/30
- 19/30
- 5/30
- 17/30

Question 24
Not yet answered
Marked out of 1
Flag question

A jar contains 6 red balls and 5 blue balls. Two balls are drawn at random from the hat without replacement. What is the probability that both the balls drawn are blue?

Select one:
- 10/11
- $\frac{\binom{5}{2}}{\binom{11}{2}}$
- 5/11
- $\frac{\binom{5}{2}}{\binom{6}{2}}$

Question 25
Answer saved
Marked out of 1
Flag question

A magician has a collection of 52 cards, with 26 red and 26 black cards. Four of these cards are classified as 'special', and two of the special cards are red. If a card is chosen at random from the 52 cards, what is the probability that the card is special or red?

Select one:
- 2/52
- 28/52
- 26/52
- 4/52

Question 26
Answer saved
Marked out of 1
Flag question

A jar contains 4 red balls and 7 blue balls. Two balls are drawn at random from the jar without replacement. What is the probability that exactly one of the balls drawn is red?

Select one:
- 28/55
- 4/11
- 1/2
- 28/121

Question 27

Not yet answered

Marked out of 1

Flag question

The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} cx^2 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

where $c > 0$ is a constant. Find $\mathbb{P}(X < 0.5)$.

Select one:

- 1/4
- 1
- 1/8
- 1/2

Question 28

Not yet answered

Marked out of 1

Flag question

The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} \frac{c}{x^2} & 1 \leq x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

where $c > 0$ is a constant. Find $\mathbb{P}(X > 25)$.

Select one:

- 0
- $1/25^2$
- 1
- 1/25

Question 29

Answer saved

Marked out of 1

Flag question

A class has 30 girls and 22 boys. A teacher selects 15 students at random from the class to participate in a competition. What is the probability that out of the 15 selected students, 6 of them are boys?

Select one:

- $\frac{\binom{15}{6}\binom{30}{9}}{\binom{52}{15}}$
- $\frac{\binom{22}{6}\binom{30}{9}}{\binom{52}{15}}$
- $\frac{6}{15}$
- $\frac{\binom{22}{6}\binom{30}{9}}{\binom{52}{15}}$

Question 30

Not yet answered

Marked out of 1

Flag question

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2} & 0 < y < 1,\ 0 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

What is $\mathbb{P}(X > Y)$?

Select one:

- 3/4
- 1/4
- $1/\sqrt{2}$
- 1/2

Question 31

Not yet answered

Marked out of 1

Flag question

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the conditional distribution of X given Y and use it to calculate $\mathbb{P}(X > 0.5|Y = 0.7)$.

Select one:

- 2/3
- 1
- 2/7
- 0

Question 32

Not yet answered

Marked out of 1

Flag question

The random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} 2e^{-x-y} & 0 < x < y < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

Find the conditional distribution of Y given X and use it to calculate $\mathbb{P}(Y > 2|X = 1)$.

Select one:

- 0
- e^{-2}
- 1/3
- 1/2

Question 33

Not yet answered

Marked out of 1

Flag question

The random variable X has the following moment generating function:
$M_X(t) = e^{2(e^t-1)}$.

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

Select one:

- $\mathrm{E}(X) = e, \mathrm{Var}(X) = e^2$
- $\mathrm{E}(X) = 1, \mathrm{Var}(X) = 2$
- $\mathrm{E}(X) = 0, \mathrm{Var}(X) = 1$
- $\mathrm{E}(X) = 2, \mathrm{Var}(X) = 2$

Question 34

Not yet answered

Marked out of 1

Flag question

The random variable X has the following moment generating function:
$M_X(t) = (1-2t)^{-3}, \ t < 1$.

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

Select one:

- $\mathrm{E}(X) = 0, \mathrm{Var}(X) = 1$
- $\mathrm{E}(X) = 6, \mathrm{Var}(X) = 12$
- $\mathrm{E}(X) = 1, \mathrm{Var}(X) = 2$
- $\mathrm{E}(X) = 2/3, \mathrm{Var}(X) = 2$

Question 35

Not yet answered

Marked out of 1

Flag question

Let X_1, X_2, X_3 be a random sample from the distribution of the random variable X with a Poisson distribution with rate $\lambda > 0$.
Define the following estimators for λ:
$\hat{\lambda}_1 := \frac{X_1+X_2+X_3}{3}$, $\hat{\lambda}_2 := X_1 + X_2 + X_3$, and $\hat{\lambda}_3 := \frac{1}{7}X_1 + \frac{3}{7}X_2$

Rank the estimators from the one with the smallest variance to the one with the largest variance.

Select one:

- $\hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_1$
- $\hat{\lambda}_1, \hat{\lambda}_3, \hat{\lambda}_2$
- $\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3$
- $\hat{\lambda}_3, \hat{\lambda}_1, \hat{\lambda}_2$

Question 36

Not yet answered

Marked out of 1

Flag question

Let X_1, X_2, X_3 be a random sample from the distribution of the random variable X with a normal distribution with unknown mean parameter μ and variance 1.
Define the following estimators for μ:
$\hat{\mu}_1 := \frac{X_1+X_2+X_3}{3}$, $\hat{\mu}_2 := X_1 + X_2 + X_3$, and $\hat{\mu}_3 := \frac{2}{5}X_1 + \frac{3}{5}X_2$

Rank the estimators from the one with the smallest variance to the one with the largest variance.

Select one:

- $\hat{\mu}_2, \hat{\mu}_3, \hat{\mu}_1$
- $\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3$
- $\hat{\mu}_3, \hat{\mu}_1, \hat{\mu}_2$
- $\hat{\mu}_1, \hat{\mu}_3, \hat{\mu}_2$

Question 37

Not yet answered

Marked out of 1

Flag question

Let $X_1, \ldots, X_n (n \geq 2)$ be a random sample from the distribution of the random variable X with the following probability density function:

$$f_X(x) = \begin{cases} \lambda^2 x e^{-\lambda x} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

where $\lambda > 0$ is an unknown parameter. The maximum likelihood estimator of λ is

Select one:

- $\frac{2}{n}\sum_{i=1}^{n} X_i$
- $2\sum_{i=1}^{n} X_i$
- $\frac{2}{\sum_{i=1}^{n} X_i}$
- $\frac{2n}{\sum_{i=1}^{n} X_i}$

Question 38

Not yet answered

Marked out of 1

Flag question

Let $X_1, \ldots, X_{25}$ be a random sample from a normal distribution with unknown mean μ and known variance $\sigma^2 = 100$. A sample $x_1, \ldots, x_{25}$ is collected and the sample mean from this sample is calculated to be $\bar{x} = 4$. Based on this sample, a 95% confidence interval for μ is

Select one:

- (0.08,3.92)
- (4,10)
- $(0, \infty)$
- (0,4)

Question 39

Not yet answered

Marked out of 1

Flag question

Let $X_1, \ldots, X_{15}$ be a random sample from a normal distribution with unknown mean μ and known variance $\sigma^2 = 12$. Consider the following hypotheses:
$H_0 : \mu = 10,\ H_1 : \mu > 10.$

A sample $x_1, \ldots, x_{15}$ is collected and the sample mean from this sample is calculated to be $\bar{x} = 12$. The test statistic z and p-value p for testing these hypotheses (UMP) based on this sample are:

Select one:

- $z = 2.24, p = 1.25\%$
- $z = -2.24, p = 1.25\%$
- $z = 2.24, p = 2.50\%$
- $z = 2.33, p = 1\%$

Question 40

Not yet answered

Marked out of 1

Flag question

The random variables X and Y are believed to be linearly related by the equation
$Y = a + bX + \epsilon,$
where a and b are constants and ϵ is a mean zero, normally distributed error term. The following sample of pairs (x, y) is collected:

x	1	2	3	4	5	6	7	8
y	590	750	1215	1335	1830	1920	2265	2670

Using the method of least squares, calculate the values of a and b to 3 decimal places.

Select one:

- $a = 112.84, b = -249.33$
- $a = 246.25, b = 294.58$
- $a = 1, b = 0$
- $a = 98.38, b = -78.22$