World Quant University Proficiency Test

Quantitative Proficiency Test (WorldQuant University)

If $f:(0,\infty)\to\mathbb{R}$ is defined by $f(x)=\frac{x}{\ln(2x)}$ for every $x\in(0,\infty)$, then $f'(e/2)$ is

Select one:

- $1/e$
- 0
- $e/2$
- e

Let $x_0\in(0,\infty)$ be the value of $x\in(0,\infty)$ that maximizes the function $f:(0,\infty)\to\mathbb{R}$ defined by $f(x):=\ln x/x$ for every $x\in(0,\infty)$, and $y_0:=f(x_0)$ be the value of this maximum. Then the product x_0y_0 is

Select one:

- e
- $1/e$
- 0
- 1

If $f:\mathbb{R}\Rightarrow\mathbb{R}$ is defined by $f(x)=e^{-\cos x}\sin(x)$ for every $x\in\mathbb{R}$ then $f'(0)$ is

Select one:

- 1
- e^{-1}
- e
- 0

Question 4

Not yet answered

Marked out of 1

Flag question

The value of the integral
$\int_0^{\pi/2} \cos(x) e^{-\sin x}\, dx$
is

Select one:

- $\cos 1$
- $e^{-\sin 1}$
- $1 - e^{-1}$
- e^{-1}

Question 5

Not yet answered

Marked out of 1

Flag question

Evaluate
$\int_0^{\pi/2} x \cos x\, dx.$

Select one:

- 1
- -1
- $\pi/2 - 1$
- π

Question 6

Not yet answered

Marked out of 1

Flag question

Let *A*, *B* and *C* be square invertible matrices of the same size. If $B^TB = I = BB^T$ (X^T is the *transpose* of the matrix X) and *C* has no eigenvalue equal to -1, then $(AB^T + AB^TC)^{-1}$

is equal to

Select one:

- $AB(I + C)$
- $A^{-1}B^T(I + C)^{-1}$
- $(I + C)^{-1}BA^{-1}$
- $A^{-1}B(I + C)^{-1}$

Question 7

Not yet answered

Marked out of 1

Flag question

Let *A*, *B* and *C* be square invertible matrices of the same size. If $B^TB = I = BB^T$ (X^T is the *transpose* of the matrix X), then $(AB^TC)^{-1}$

is equal to

Select one:

- $A^{-1}BC^{-1}$
- CBA^{-1}
- $A^{-1}B^TC^{-1}$
- $C^{-1}BA^{-1}$

Question 8

Not yet answered

Marked out of 1

Flag question

Let $f : \mathbb{R} \to \mathbb{R}$and $g : \mathbb{R} \to \mathbb{R}$ be defined by
$f(x) := \sin x$ for every $x \in \mathbb{R}$
and
$g(x) := e^x$ for every $x \in \mathbb{R}$

Consider the following statements:

[i.] f is one-to-one
[ii.] f is onto
[iii.] g is one-to-one
[iv.] g is onto

Which of the statements are true?

Select one:

- None of the statements
- iii
- All of the statements
- ii. and iii.

Question 9

Not yet answered

Marked out of 1

Flag question

The solution to the following differential equation
$y'' + 2y' + y = 0,\ y(0) = 2, y'(0) = 10$
is

Select one:

- $y(t) = 2e^{-t} + 12te^{-t}$
- $y(t) = 2e^{-t}$
- $y(t) = e^{-t}$
- $y(t) = 2$

Question 10

Not yet answered

Marked out of 1

Flag question

The complex number $z = 2e^{i\frac{\pi}{4}}$ can also be written as

Select one:

- $z = \sqrt{2}i.$
- $z = \sqrt{2} + \sqrt{2}i.$
- $z = 2 + 2i.$
- $z = \sqrt{2}.$

Question 11

Not yet answered

Marked out of 1

Flag question

The following series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Select one:

- converges absolutely
- diverges to $-\infty$
- converges conditionally
- diverges to ∞

Question 12

Not yet answered

Marked out of 1

Flag question

The value of $\lim_{n\to\infty} \frac{2^n}{n!}$ is

Select one:

- 1.
- 2.
- ∞.
- 0.

Question 13

Not yet answered

Marked out of 1

Flag question

Consider the following statements concerning a positive integer n:

[i.] if n is a multiple of 9, then n^2 is a multiple of 3
[ii.] if n^2 is a multiple of 7, then n is a multiple of 7
[iii.] if n^2 is a multiple of 14, then n is a multiple of 7

Which of the statements are true?

Select one:

- i. and ii.
- i. and iii.
- All the statements
- None of the statements

Question 14

Not yet answered

Marked out of 1

Flag question

An island consists of four kinds of people: Tetas, Jekas, Frekas and Hekas. The following information is known:

All Frekas are both Jekas and Tetas
No Hekas are Jekas
No Hekas are Tetas

Consider the following statements:

[i.] Some Hekas are Frekas
[ii.] No Frekas are Hekas
[iii.] All Tetas are Frekas
Which of these statements are necessarily true based only on the information above?

Select one:

- None of the statements
- All the statements
- ii. and iii.
- Only ii.

Question 15

Not yet answered

Marked out of 1

Flag question

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$f((x, y)) := \sin(xy)$, for every $(x, y) \in \mathbb{R}^2$.

The value of $f_x((1, 0)) + f_y((1, 0)) + f_{xx}((1, 0))$ is:

Select one:

- Undefined
- 1
- 0
- -1

Question 16

Not yet answered

Marked out of 1

Flag question

Let
$I := \iint_D 2\, dA$,
where D is the interior of the region bounded by the curves $y = x^2\$$ and $y = x^3$ with $x \geq 0$ and $y \geq 0$.
The value of I is

Select one:

- 2/3
- 3/4
- 1/3
- 1/6

Question 17

Not yet answered

Marked out of 1

Flag question

Evaluate
$\int_0^1 \int_x^1 \sin(y^2)\, dydx$
by changing the order of the integral. The answer is

Select one:

- cannot be determined
- $1 - \sin 1$
- $\cos 1$
- $\frac{1}{2}(1 - \cos 1)$

Question 18

Not yet answered

Marked out of 1

Flag question

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$f((x, y)) := x^2 + 2y^2 - 3xy$, for every $x, y) \in \mathbb{R}^2$.

The value of $f_x((0, 0)) + f_y((0, 0)) + f_{xy}((0, 0))$ is:

Select one:

- -3
- 0
- Undefined
- 3

Question 19

Answer saved

Marked out of 1

Flag question

For each $n = 1, 2, 3, \ldots$, define
$f_n(x) := \frac{n^2x^3}{1+2n^2x^2}$, for every $x \in \mathbb{R}$.

Then the function f defined by
$f(x) := \lim_{n\to\infty} f_n(x)$

exists for each $x \in \mathbb{R}$ and is equal to

Select one:

- (●) $f(x) = \frac{x}{2}$
- ( ) $f(x) = x^2$
- ( ) $f(x) = x$
- ( ) $f(x) = 0$

Question 20

Answer saved

Marked out of 1

Flag question

Consider the following partial differential equation (PDE):
$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

where $u = u(x, y)$ is the unknown function.
Define the following functions:
$u_1(x, y) := \cos(2xy)$, $u_2(x, y) = \sin(x^2y)$ and $u_3(x, y) = e^{-(x^2+y^2)}$.

Which of these functions are solutions to the above PDE?

Select one:

- ( ) Only u_3.
- ( ) u_1 and u_3
- ( ) All the functions.
- (●) None of the functions.

Information

Flag question

This section will test your statistical proficiency.

Question 21

Answer saved

Marked out of 1

Flag question

A class has 25 students of which 10 are boys (you may assume that the other 15 are girls). The class average (mean) height is $\bar{x}$ metres and the mean height for boys is $\bar{x}_B$ metres. The mean height for girls $\bar{x}_G$ is

Select one:

- $\frac{25\bar{x}-10\bar{x}_B}{15}$
- $\frac{15\bar{x}-10\bar{x}_B}{25}$
- $\frac{25\bar{x}+10\bar{x}_B}{15}$
- $\frac{15\bar{x}-10\bar{x}_B}{10}$

Question 22

Not yet answered

Marked out of 1

Flag question

A group of 10 students received the following marks for a test:
58, 89,65, 78, 55,26,93,46,43,59.
The standard deviation of their marks is (to two decimal places)

Select one:

- 20.91
- 437.29
- 61.20
- 3745.44

Question 23

Not yet answered

Marked out of 1

Flag question

A jar contains 6 red balls and 5 blue balls. Two balls are drawn at random from the hat without replacement. What is the probability that both the balls drawn are blue?

Select one:

- $\frac{\binom{5}{2}}{\binom{6}{2}}$
- $\frac{\binom{5}{2}}{\binom{11}{2}}$
- 10/11
- 5/11

Question 24

Not yet answered

Marked out of 1

Flag question

Let A, B and C be the following sets:
$A :=$ {prime numbers}, $B :=$ {positive odd numbers}
and
$C :=$ {positive integers less than or equal to 10}.

The set $D := (A \cap B) \cap C$ is

Select one:

- {3,5,7}
- {1,3,4,5,6,7,8,9,10}
- {2,3,4,5,6,7,8,10}
- {2,3,4,5,6,7,8,9,10}

Question 25

Not yet answered

Marked out of 1

Flag question

Let A, B and C be the following sets:
A :={composite numbers}={2,3,4,5,6,7,8, ...}\{prime numbers}={4,6,8,9,10, ...}
B :={positive odd numbers}
and
C :={positive integers less than or equal to 10}.

The set $D := (A \cup B) \cap C$ is

Select one:

- {2,3,4,5,6,7,8,9,10}
- {1,3,4,5,6,7,8,9,10}
- {2,3,4,5,6,7,8,10}
- {1,3,5,7,9, ...}

Question 26

Not yet answered

Marked out of 1

Flag question

A magician has a collection of 52 cards, with 26 red and 26 black cards. Four of these cards are classified as 'special', and two of the special cards are red. If a card is chosen at random from the 52 cards, what is the probability that the card is special or red?

Select one:

- 26/52
- 28/52
- 2/52
- 4/52