Computing Project 3

Properties of Area

1. Introduction

This project describes a MATLAB program that computes various properties of a cross-section. Three different cross-sections are implemented – rectangle, hollow rectangle and an L shape.

2. Program Description

The execution flow of the program is as follows:

(1) Ask the user to input the type of cross-section. The program allows for 3 types of cross-sections, and each load is represented by a number that the user has to input.

```
1 = Rectangle, 2 = Hollow Rectangular Tube, 3 = Equal-Angle L shape
```

(2) Depending upon the type of cross-section, the program calculates the area, centroid, the moment of inertia tensor, the principal values of the moment of inertia tensor and the polar moment of inertia.

(3) The basic algorithm is as follows:

1. Store coordinates of vertices in array $\mathbf{x}$

$$\mathbf{x} = \left\{ \begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_N & y_N \\ x_1 & y_1 \end{matrix} \right\}$$

2. Initialize properties $A = 0,\ \mathbf{p} = \mathbf{0},\ \mathbf{R} = \mathbf{0}$
3. Loop over the sides, $i = 1:N$
   a. Compute the unit vectors $\mathbf{n}_i$ and $\mathbf{m}_i$ from $\mathbf{x}$
   b. Compute $\sin\varphi_i = (\mathbf{n}_i \times \mathbf{m}_i)\cdot \mathbf{e}_1$
   c. Compute the triangle contributions
   $$A_i = \int_{A_i} dA,\ \mathbf{p}_i = \int_{A_i} \mathbf{r}\, dA, \text{ and } \mathbf{R}_i = \int_{A_i} \mathbf{r} \otimes \mathbf{r}\, dA$$
   d. Add the contributions to the whole
   $$A \leftarrow A + A_i,\ \mathbf{p} \leftarrow \mathbf{p} + \mathbf{p}_i,\ \mathbf{R} \leftarrow \mathbf{R} + \mathbf{R}_i$$
4. Finish the computation of $\mathbf{c}$ and $\mathbf{J}$
$$\mathbf{c} = \mathbf{p}/A,\quad \mathbf{J} = \mathbf{R} - A[\mathbf{c} \otimes \mathbf{c}]$$
5. Compute principal values of $\mathbf{J}$
6. Print and plot results

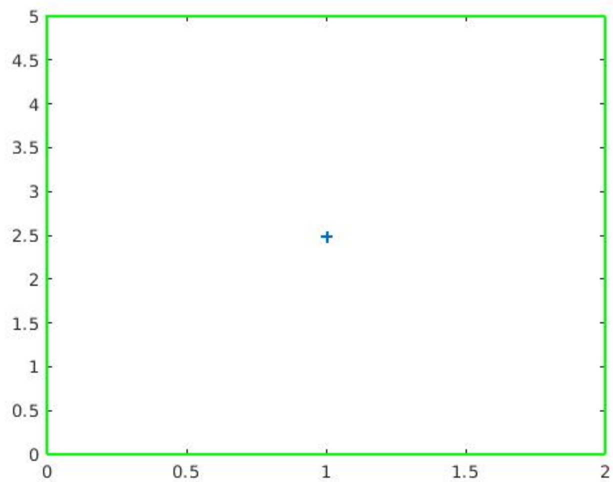
3. File Structure
The program consists of 2 files:

- CP3_Input
- AreaProp.m
- Simpson.m

The file "AreaProp.m" is the core script that runs and interacts with the other file that contains a helper routine that generates coordinates for the cross-section.

4. Results

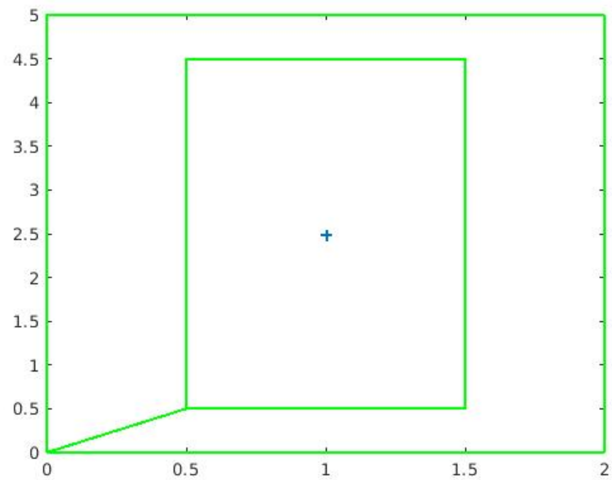
Case 1 : Rectangle of width 2 and height 5



The '+' represents the centroid.

```
Polygon Type (Rectangle = 1, Hollow Rectangular Tube = 2, Equal-Angle L shape = 3) = 1

Area = 10.000
Centroid = (1.000000, 2.500000)
Moment of Inertia Tensor is:
J =

    3.3333    0.0000
    0.0000   20.8333

Principal values of moment of inertia are:
Jmax =

   20.8333


Jmin =

    3.3333

Polar Moment of Inertia = 24.167
```

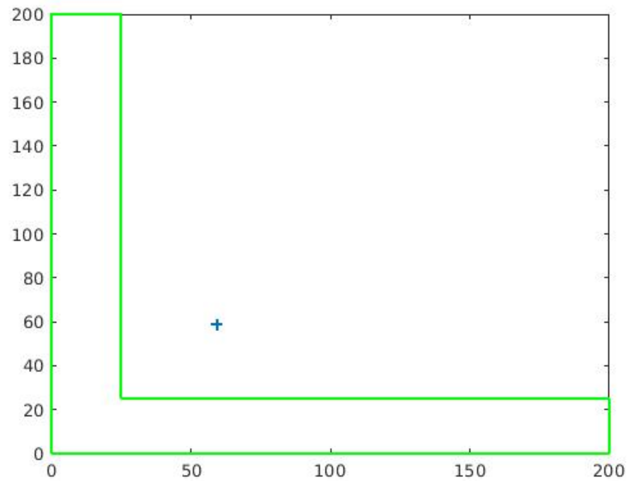
Case 2 : Hollow Rectangular Tube of width 2 and height 5 and wall thickness 0.5



The '+' represents the centroid.

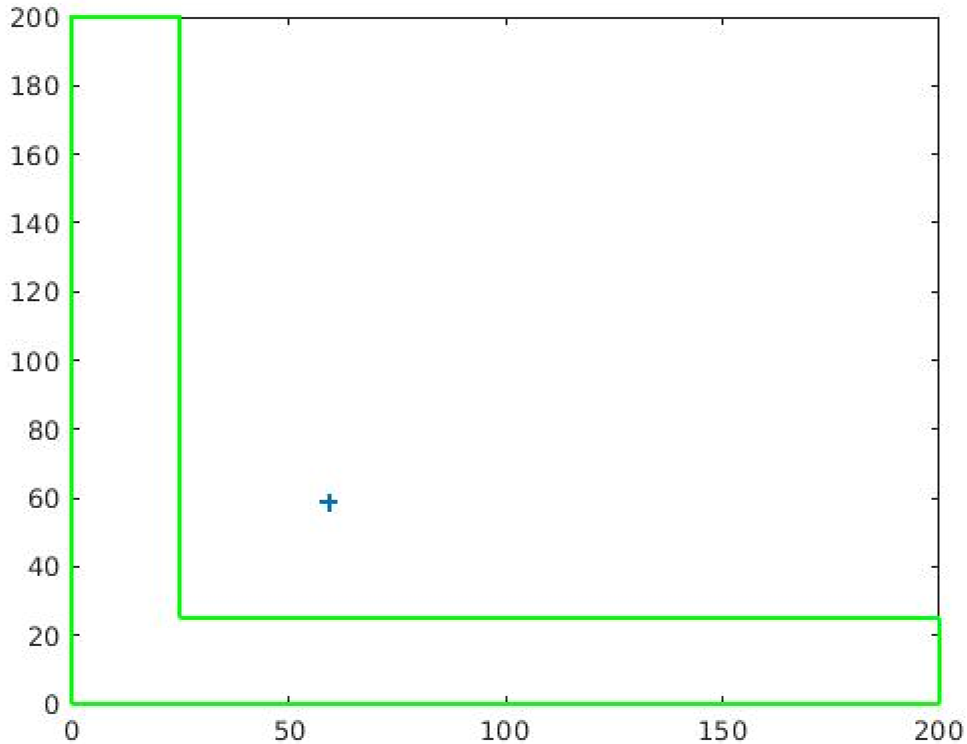
```
Polygon Type (Rectangle = 1, Hollow Rectangular Tube = 2, Equal-Angle L shape = 3) = 2

Area = 6.000
Centroid = (1.000000, 2.500000)
Moment of Inertia Tensor is:
J =

    3.0000    0.0000
    0.0000   15.5000

Principal values of moment of inertia are:
Jmax =

   15.5000


Jmin =

     3

Polar Moment of Inertia = 18.500
```

Case 3 : Equal Angle L-shape with side length 200 and width 25




```
Polygon Type (Rectangle = 1, Hollow Rectangular Tube = 2, Equal-Angle L shape = 3) = 3

Area = 9375.000
Centroid = (59.166667, 59.166667)
Moment of Inertia Tensor is:
J =

   1.0e+07 *

    3.4759   -2.0417
   -2.0417    3.4759

Principal values of moment of inertia are:
Jmax =

   5.5176e+07


Jmin =

   1.4342e+07

Polar Moment of Inertia = 69518229.167
```

5. Results for benchmark case




```
Area = 9375
Centroid Location = (59.167, 59.167)
J(yy)    =  3.4759e+07
J(xx)    =  3.4759e+07
J(yz)    = -2.0417e+07
J(max)   =  5.5176e+07
J(min)   =  1.4342e+07
J(polar) =  6.9518e+07
```