Math 3660 - Spring 2022
Mathematical Models in Economics

Steven Tschantz

1/17/22

Profit maximizing firms

Assignment 4

Instructions

Save a copy of this notebook, complete the exercises, save and submit on Brightspace your final version by start of class Tues. Feb. 15.

Nikhil Jayswal

Edit the line above that reads Enter your name here, inserting instead your name. You may also want to save to a file name that includes your name. By adding your name into the file and file name, you reduce the chance I will mix-up your submission with someone else's.

Exercises

Your task in this assignment is to implement the logit demand model for three products. Calibrate the demand model to initial prices and quantities and specified aggregate elasticity and an own price elasticity for one product. Then determine the marginal costs that would make the initial prices and quantities of the products in Nash equilibrium for separate firms owning the three products. Compute the Nash equilibrium if two of the firms merge and price their two products jointly so as to maximize their total profit in competition with the third firm. Compare the initial prices and quantities to the prices and quantities predicted for the post merger scenario.

1. Let eta1, eta2, eta3, lambda, and M be parameters for a logit demand model

with three products. Write definitions for the demand functions q1[p1,p2,p3], q2[p1,p2,p3], and q3[p1,p2,p3] of the three products and for q0[p1,p2,p3] the demand for the outside good.

Clear the parameters eta1, eta2, eta3, lambda, and M, and the variables p1, p2, p3, q1, q2, q3, and q0.

```
In[1]:= Clear[eta1, eta2, eta3, lambda, M]
        Clear[p1, p2, p3, q1, q2, q3, q0]
```

Define q1[p1_, p2_, p3_] =.. writing out the logit demand formula for product 1. Do the same for q2, q3, and q0.

```
In[3]:= q0[p1_, p2_, p3_] :=
         M / (1 + Exp[(eta1 - p1) / lambda] + Exp[(eta2 - p2) / lambda] + Exp[(eta3 - p3) / lambda])

In[4]:= q1[p1_, p2_, p3_] := M * Exp[(eta1 - p1) / lambda] /
          (1 + Exp[(eta1 - p1) / lambda] + Exp[(eta2 - p2) / lambda] + Exp[(eta3 - p3) / lambda])

In[5]:= q2[p1_, p2_, p3_] := M * Exp[(eta2 - p2) / lambda] /
          (1 + Exp[(eta1 - p1) / lambda] + Exp[(eta2 - p2) / lambda] + Exp[(eta3 - p3) / lambda])

In[6]:= q3[p1_, p2_, p3_] := M * Exp[(eta3 - p3) / lambda] /
          (1 + Exp[(eta1 - p1) / lambda] + Exp[(eta2 - p2) / lambda] + Exp[(eta3 - p3) / lambda])
```

As a check, verify that q1[p1,p2,p3]+q2[p1,p2,p3]+q3[p1,p2,p3]+q0[p1,p2,p3] simplifies to M.

```
In[7]:= Simplify[q0[p1, p2, p3] + q1[p1, p2, p3] + q2[p1, p2, p3] + q3[p1, p2, p3]]

Out[7]= M
```

2. Write definitions for the aggregate elasticity ae[p1,p2,p3] and elasticity matrix elasticitymatrix[p1,p2,p3].

Define ae[p1_, p2_, p3_] =.. writing out the formula for the aggregate elasticity of the inside goods in a logit demand model.

```
In[8]:= AveragePrice[p1_, p2_, p3_] :=
         (p1 * q1[p1, p2, p3] + p2 * q2[p1, p2, p3] + p3 * q3[p1, p2, p3]) /
          (q1[p1, p2, p3] + q2[p1, p2, p3] + q3[p1, p2, p3])

In[9]:= ae[p1_, p2_, p3_] := - (AveragePrice[p1, p2, p3] / lambda) *
          (1 - (q1[p1, p2, p3] + q2[p1, p2, p3] + q3[p1, p2, p3]) / M)
```

Define elasticitymatrix[p1_, p2_, p3_] =.. using the definition of own and cross price elasticities.

```
In[10]:= elasticitymatrix[p1_, p2_, p3_] := {
           {D[q1[p1, p2, p3], p1] * p1 / q1[p1, p2, p3],
            D[q1[p1, p2, p3], p2] * p2 / q1[p1, p2, p3], D[q1[p1, p2, p3], p3] * p3 / q1[p1, p2, p3]},
           {D[q2[p1, p2, p3], p1] * p1 / q2[p1, p2, p3], D[q2[p1, p2, p3], p2] * p2 / q2[p1, p2, p3],
            D[q2[p1, p2, p3], p3] * p3 / q2[p1, p2, p3]}, {D[q3[p1, p2, p3], p1] * p1 / q3[p1, p2, p3],
            D[q3[p1, p2, p3], p2] * p2 / q3[p1, p2, p3], D[q3[p1, p2, p3], p3] * p3 / q3[p1, p2, p3]}
          }
```

3. Suppose initial prices and quantities for the three products are given as follows. Solve for the eta1, eta2, and eta3 parameters in terms of lambda and M.

In[11]:=
```
Clear[p1i, p2i, p3i, q1i, q2i, q3i];
```

In[12]:=
```
p1i = 10.;
p2i = 12.;
p3i = 14.;
q1i = 100.;
q2i = 110.;
q3i = 90.;
TableForm[{{p1i, q1i}, {p2i, q2i}, {p3i, q3i}},
 TableHeadings → {{"product 1", "product 2", "product 3"}, {"price", "quantity"}}]
```

Out[18]//TableForm=

	price	quantity
product 1	10.	100.
product 2	12.	110.
product 3	14.	90.

In[19]:=
```
calibrationequations =
 {q1[p1i, p2i, p3i] == q1i, q2[p1i, p2i, p3i] == q2i, q3[p1i, p2i, p3i] == q3i}
```

Out[19]=

$$\left\{\frac{e^{\frac{-10.+eta1}{lambda}} M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}} == 100.,\ \frac{e^{\frac{-12.+eta2}{lambda}} M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}} == 110.,\ \frac{e^{\frac{-14.+eta3}{lambda}} M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}} == 90.\right\}$$

In[20]:=
```
calibrationsolutions = Solve[calibrationequations, {eta1, eta2, eta3}][[1]]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[20]=

$$\left\{eta1 \to 1.\ lambda\ Log\left[-\frac{100.\ e^{10./lambda}}{300.-1.\ M}\right],\ eta2 \to 1.\ lambda\ Log\left[-\frac{110.\ e^{12./lambda}}{300.-1.\ M}\right],\ eta3 \to 1.\ lambda\ Log\left[-\frac{90.\ e^{14./lambda}}{300.-1.\ M}\right]\right\}$$

4. Suppose the aggregate elasticity is -2 and the own price elasticity of product 1 is -4 at the initial prices above. Solve for the lambda and M that give these elasticities. Assign the numerical values of eta1, eta2, eta3, lambda, and M that you have found here and from 3 above. Evaluate the aggregate elasticity and elasticity matrix functions at the initial prices for the calibrated demand

functions.

In[21]:= **AggregateElasticityEquation = ae[p1i, p2i, p3i] == -2**

Out[21]=

$$-\left(\left(\left(\frac{10.\,e^{\frac{-10.+eta1}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{12.\,e^{\frac{-12.+eta2}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{14.\,e^{\frac{-14.+eta3}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}\right)\left(1-\frac{\frac{e^{\frac{-10.+eta1}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{e^{\frac{-12.+eta2}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{e^{\frac{-14.+eta3}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}}{M}\right)\right)\Big/\left(lambda\left(\frac{e^{\frac{-10.+eta1}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{e^{\frac{-12.+eta2}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}+\frac{e^{\frac{-14.+eta3}{lambda}}M}{1+e^{\frac{-10.+eta1}{lambda}}+e^{\frac{-12.+eta2}{lambda}}+e^{\frac{-14.+eta3}{lambda}}}\right)\right)\right) == -2$$

In[22]:= **AggregateElasticityEquation = AggregateElasticityEquation /. calibrationsolutions**

Out[22]=

$$-\left(\left(\left(\frac{10.\,e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{12.\,e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{14.\,e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}\right)\left(1-\frac{1}{M}\left(\frac{e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}\right)\right)\right)\Big/\left(\text{lambda}\left(\frac{e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}+\frac{e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}M}{1+e^{\frac{-10.+1.\,\text{lambda}\,\text{Log}\left[-\frac{100.\,e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda}\,\text{Log}\left[-\frac{110.\,e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda}\,\text{Log}\left[-\frac{90.\,e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}}\right)\right)==-2$$

In[23]:=
```
Product1ElasticityEquation =
  {D[q1[p1, p2, p3], p1] * p1 / q1[p1, p2, p3] /. {p1 → p1i, p2 → p2i, p3 → p3i}} == -4
```

Out[23]=

$$\left\{\frac{1}{M} 10.\, e^{-\frac{-10.+\text{eta1}}{\text{lambda}}} \left(1+e^{\frac{-10.+\text{eta1}}{\text{lambda}}}+e^{\frac{-12.+\text{eta2}}{\text{lambda}}}+e^{\frac{-14.+\text{eta3}}{\text{lambda}}}\right) \left(\frac{e^{\frac{2(-10.+\text{eta1})}{\text{lambda}}} M}{\left(1+e^{\frac{-10.+\text{eta1}}{\text{lambda}}}+e^{\frac{-12.+\text{eta2}}{\text{lambda}}}+e^{\frac{-14.+\text{eta3}}{\text{lambda}}}\right)^2 \text{lambda}}-\frac{e^{\frac{-10.+\text{eta1}}{\text{lambda}}} M}{\left(1+e^{\frac{-10.+\text{eta1}}{\text{lambda}}}+e^{\frac{-12.+\text{eta2}}{\text{lambda}}}+e^{\frac{-14.+\text{eta3}}{\text{lambda}}}\right) \text{lambda}}\right)\right\} == -4$$

In[24]:=
```
Product1ElasticityEquation = Product1ElasticityEquation /. calibrationsolutions
```

Out[24]=

$$\left\{\frac{1}{M} 10.\, e^{-\frac{-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}} \left(1+e^{\frac{-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda Log}\left[-\frac{110.\, e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda Log}\left[-\frac{90.\, e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}\right)\right.$$

$$\left(\frac{e^{\frac{2\left(-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]\right)}{\text{lambda}}} M}{\left(1+e^{\frac{-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda Log}\left[-\frac{110.\, e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda Log}\left[-\frac{90.\, e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}\right)^2 \text{lambda}}-\right.$$

$$\left.\left.\frac{e^{\frac{-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}} M}{\left(1+e^{\frac{-10.+1.\,\text{lambda Log}\left[-\frac{100.\, e^{10./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-12.+1.\,\text{lambda Log}\left[-\frac{110.\, e^{12./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}+e^{\frac{-14.+1.\,\text{lambda Log}\left[-\frac{90.\, e^{14./\text{lambda}}}{300.-1.\,M}\right]}{\text{lambda}}}\right) \text{lambda}}\right)\right\} == -4$$

In[25]:=
```
parametersolutions =
  Solve[{AggregateElasticityEquation, Product1ElasticityEquation}, {lambda, M}][[1]]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[25]=
```
{lambda → 1.93723, M → 444.231}
```

In[26]:=
```
lambda = lambda /. parametersolutions
```

Out[26]=
```
1.93723
```

In[27]:=
```
M = M /. parametersolutions
```

Out[27]=
```
444.231
```

Now that lambda and M have been obtained, we can evaluate the numerical values of eta1, eta2, and eta3.

In[28]:=
```
etaValues = calibrationsolutions /. parametersolutions
```
Out[28]=
```
{eta1 → 9.2905, eta2 → 11.4751, eta3 → 13.0864}
```

In[29]:=
```
eta1 = eta1 /. etaValues
```
Out[29]=
```
9.2905
```

In[30]:=
```
eta2 = eta2 /. etaValues
```
Out[30]=
```
11.4751
```

In[31]:=
```
eta3 = eta3 /. etaValues
```
Out[31]=
```
13.0864
```

In[32]:=
```
AggregateElasticity = ae[p1i, p2i, p3i]
```
Out[32]=
```
-2.
```

In[33]:=
```
ElasticityMatrix = elasticitymatrix[p1, p2, p3] /. {p1 → p1i, p2 → p2i, p3 → p3i}
```
Out[33]=
```
{{-4., 1.53385, 1.46413}, {1.16201, -4.66056, 1.46413}, {1.16201, 1.53385, -5.76268}}
```

In[34]:=
```
MatrixForm[ElasticityMatrix]
```
Out[34]//MatrixForm=

$$\begin{pmatrix} -4. & 1.53385 & 1.46413 \\ 1.16201 & -4.66056 & 1.46413 \\ 1.16201 & 1.53385 & -5.76268 \end{pmatrix}$$

5. Suppose mc1, mc2, and mc3 are the marginal costs of the three products. Write definitions for the profits on products 1, 2 and 3, profit1[p1,p2,p3], profit2[p1,p2,p3], and profit3[p1,p2,p3].

Clear parameters mc1, mc2, and mc3, and variables profit1, profit2, and profit3.

In[35]:=
```
Clear[mc1, mc2, mc3, profit1, profit2, profit3]
```

Define profit1[p1_, p2_, p3_] =.. as usual, ignoring fixed costs.

In[36]:=
```
profit1[p1_, p2_, p3_] := p1 * q1[p1, p2, p3] - mc1 * q1[p1, p2, p3]
```

In[37]:=
```
profit2[p1_, p2_, p3_] := p2 * q2[p1, p2, p3] - mc2 * q2[p1, p2, p3]
```

In[38]:=
```
profit3[p1_, p2_, p3_] := p3 * q3[p1, p2, p3] - mc3 * q3[p1, p2, p3]
```

6. Write a definition for the first order conditions defining a Nash equilibrium at prices p1, p2, and p3 between three profit maximizing firms each owning one product, foconditions[p1,p2,p3].

Define foconditions[p1_, p2_, p3_] =.. to be a list of three equations.

```
In[39]:= foconditions[p1_, p2_, p3_] := {D[profit1[p1, p2, p3], p1] == 0,
           D[profit2[p1, p2, p3], p2] == 0, D[profit3[p1, p2, p3], p3] == 0}
```

7. Solve for the marginal costs such that the initial prices given above would be Nash equilibrium prices for the three firms. Assign these numerical values to the marginal costs variables.

In the past, we have been given marginal costs and solved the first order conditions for the equilibrium prices. Here we want to assume the prices are in equilibrium and solve instead for the marginal costs these conditions would imply.

```
In[40]:= equations = foconditions[p1, p2, p3] /. {p1 → p1i, p2 → p2i, p3 → p3i}
Out[40]=
         {-300. + 40. mc1 == 0, -402.661 + 42.7218 mc2 == 0, -428.641 + 37.0458 mc3 == 0}

In[41]:= marginalcosts = Solve[equations, {mc1, mc2, mc3}][[1]]
Out[41]=
         {mc1 → 7.5, mc2 → 9.4252, mc3 → 11.5706}

In[42]:= mc1 = mc1 /. marginalcosts
Out[42]=
         7.5

In[43]:= mc2 = mc2 /. marginalcosts
Out[43]=
         9.4252

In[44]:= mc3 = mc3 /. marginalcosts
Out[44]=
         11.5706
```

8. Write a definition for the total profit on products 1 and 2, the profit that would be maximized when the firms owning these products merge, profit12[p1,p2,p3].

Define profit12[p1_, p2_, p3_] =.. now given calibrated marginal costs for the products.

```
In[45]:= profit12[p1_, p2_, p3_] :=
          p1 * q1[p1, p2, p3] - mc1 * q1[p1, p2, p3] + p2 * q2[p1, p2, p3] - mc2 * q2[p1, p2, p3]
```

9. Write a definition for the first order conditions defining a Nash equilibrium in the post merger scenario where the merged firm sets prices on products 1 and 2 in competition with the firm owning product 3.

In[46]:=
```
NashEquilibirumEquations[p1_, p2_, p3_] := {D[profit12[p1, p2, p3], p1] == 0,
   D[profit12[p1, p2, p3], p2] == 0, D[profit3[p1, p2, p3], p3] == 0}
```

10. Solve the first order conditions for a post merger Nash equilibrium. Compute the post merger equilibrium prices and quantities, and compare to the initial prices and quantities. Compare the profits of the firms before and after the merger.

In[47]:=
```
PostMergerEquilibriumPrices =
  FindRoot[NashEquilibirumEquations[p1, p2, p3], {{p1, p1i}, {p2, p2i}, {p3, p3i}}]
```

Out[47]=
```
{p1 → 10.6947, p2 → 12.6199, p3 → 14.0776}
```

In[48]:=
```
PostMergerEquilibriumQuantities =
  {q1[p1, p2, p3], q2[p1, p2, p3], q3[p1, p2, p3]} /. PostMergerEquilibriumPrices
```

Out[48]=
```
{81.5807, 93.2714, 100.964}
```

The profits before the merger were as follows.

In[49]:=
```
{profit1[p1i, p2i, p3i], profit2[p1i, p2i, p3i], profit3[p1i, p2i, p3i]}
```

Out[49]=
```
{250., 283.228, 218.648}
```

After the merger, the profits are as follows.

In[50]:=
```
{profit12[p1, p2, p3], profit3[p1, p2, p3]} /. PostMergerEquilibriumPrices
```

Out[50]=
```
{558.595, 253.12}
```

After the merger, the profits of all parties have increased.