Math 3660 - Spring 2022
Mathematical Models in Economics

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1/26/22

Profit maximizing firms

Assignment 5

Instructions

Save a copy of this notebook, complete the exercises, save and submit on Brightspace your final version by start of class Tues. Feb. 22.

Nikhil Jayswal

Edit the line above that reads Enter your name here, inserting instead your name. You may also want to save to a file name that includes your name. By adding your name into the file and file name, you reduce the chance I will mix-up your submission with someone else's.

Exercises

Remember to clear your variables.

In[1]:=
```
Clear[pr, pw, qr, qw, mcw, mcr]
```

1. Follow through the supply chain calculation for a linear demand form. Suppose consumer demand is 10000 at $50 and 8000 at $55, and define a retail demand function.

We assume a linear form of the retail demand function, and use the two given demand-price values to compute the coefficients.

In[2]:=
```
qr[pr_] := c1 * pr + c2
```

In[3]:=
```
calibrationequations = {qr[50] == 10000, qr[55] == 8000}
coeffs = Solve[calibrationequations, {c1, c2}]
```

Out[3]= $\{50\,c1 + c2 == 10000,\ 55\,c1 + c2 == 8000\}$

Out[4]= $\{\{c1 \to -400,\ c2 \to 30000\}\}$

In[5]:=
```
qr[pr_] = qr[pr] /. coeffs[[1]]
```

Out[5]= $30000 - 400\,pr$

2. Suppose retailer's cost is $5 per unit, so his total marginal cost is the wholesale price plus the $5. Define the retail profit as a function of wholesale and retail price. Compute and define the profit maximizing retail price as a function of wholesale price. (What is the pass through rate for a linear demand?)

In[6]:=
```
retailprofit[pw_, pr_] = pr * qr[pr] - 5 * qr[pr] - pw * qr[pr]
```

Out[6]= $-5\,(30000 - 400\,pr) + (30000 - 400\,pr)\,pr - (30000 - 400\,pr)\,pw$

In[7]:=
```
maxretailprofitcondition = D[retailprofit[pw, pr], pr] == 0
```

Out[7]= $32000 - 800\,pr + 400\,pw == 0$

In[8]:=
```
maxretailprofitprice = Solve[maxretailprofitcondition, {pr}]
```

Out[8]= $\left\{\left\{pr \to \frac{80 + pw}{2}\right\}\right\}$

In[9]:=
```
maxretailprofitprice = maxretailprofitprice[[1]]
```

Out[9]= $\left\{pr \to \frac{80 + pw}{2}\right\}$

The above formula defines retail price as a function of the wholesale price.

In[10]:=
```
pr[pw_] = pr /. maxretailprofitprice
```

Out[10]=
$$\frac{80 + pw}{2}$$

3. Define the effective wholesale demand as a function of wholesale price. Suppose the wholesale marginal cost is $20. Define the wholesale profit, compute the profit maximizing wholesale price.

In[11]:=
```
qw[pw_] = qr[pr[pw]]
```

Out[11]=
$$30000 - 200\,(80 + pw)$$

In[12]:=
```
wholesaleprofit[pw_] = pw * qw[pw] - 20 * qw[pw]
```
Out[12]=
$$-20\,(30\,000 - 200\,(80 + pw)) + pw\,(30\,000 - 200\,(80 + pw))$$

In[13]:=
```
maxwholesaleprofitcondition = D[wholesaleprofit[pw], pw] == 0
```
Out[13]=
$$34\,000 - 200\,pw - 200\,(80 + pw) == 0$$

In[14]:=
```
maxwholesaleprofitprice = Solve[maxwholesaleprofitcondition, pw]
```
Out[14]=
$$\{\{pw \to 45\}\}$$

In[15]:=
```
maxwholesaleprofitprice = maxwholesaleprofitprice[[1]]
```
Out[15]=
$$\{pw \to 45\}$$

Thus, the profit maximizing wholesale price is 45$.

4. With this optimal wholesale price, what is the resulting optimal retail price? What is the demand? What are the profits of manufacturer and retailer?

In[16]:=
```
optimalretailprice = pr[pw] /. maxwholesaleprofitprice
```
Out[16]=
$$\frac{125}{2}$$

Thus the optimal retail price is 62.5$.

In[17]:=
```
demandatoptimalprice = qw[pw] /. maxwholesaleprofitprice
```
Out[17]=
```
5000
```

In[18]:=
```
demandatoptimalprice = qr[pr] /. {pr → optimalretailprice}
```
Out[18]=
```
5000
```

The demand at the optimal price is 5000. We verify that the wholesale and retail demand are the same.

In[19]:=
```
manufacturerprofit = wholesaleprofit[pw] /. maxwholesaleprofitprice
```
Out[19]=
```
125 000
```

In[20]:=
```
retailerprofit =
 retailprofit[pw, pr] /. maxwholesaleprofitprice /. {pr → optimalretailprice}
```
Out[20]=
```
62 500
```

The manufacturer makes a profit of 12500$ and the retailer makes a profit of 62500$.

5. Compute the profit maximizing price for the vertically integrated firm with

retail price set to maximize total manufacturing and retail profits.

In[21]:=
```
mergedprofit[pr_] = pr * qr[pr] - (20 + 5) * qr[pr]
```

Out[21]=
$$-25\,(30\,000 - 400\,\text{pr}) + (30\,000 - 400\,\text{pr})\,\text{pr}$$

In[22]:=
```
maxprofitequation = D[mergedprofit[pr], pr] == 0
```

Out[22]=
$$40\,000 - 800\,\text{pr} == 0$$

In[23]:=
```
maxprofitsolution = Solve[maxprofitequation, pr]
```

Out[23]=
$$\{\{\text{pr} \to 50\}\}$$

In[24]:=
```
maxprofitprice = pr /. maxprofitsolution
```

Out[24]=
$$\{50\}$$

The profit maximizing price for the integrated firm us $50.

In[25]:=
```
profit = mergedprofit[maxprofitprice]
```

Out[25]=
$$\{250\,000\}$$

The integrated firm makes a maximum profit of 250000$ which is more than the sum of profits the retailer and manufacturer were making separately.