# MATH 446/OR 481
# Project 6

November 29, 2022

**Project Conclusion**

In this project, we fit various models of varying complexity to the electrical power usage data over three 6-year periods - 1992 - 1997, 2002 - 2007, and 2012 - 2017.

The linear model is a poor fit to the data, and is evident visually and by the large root mean squared errors (RMSE) obtained. To model the oscillating data, we add sinusoids to our model. On evaluation, we observe that the model fit improves visually with addition of sinusoids, and the RMSE also decreases.

In summary, the following four models have been considered -

$$y = c_1 + c_2 t \tag{1}$$
$$y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t \tag{2}$$
$$y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t + c_5 \cos 4\pi t + c_6 \sin 4\pi t \tag{3}$$
$$y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t + c_5 \cos 4\pi t + c_6 \sin 4\pi t + c_7 \cos 6\pi t + c_8 \sin 6\pi t \tag{4}$$

Model 3 and 4 fare similarly in terms of RMSE, and Model 3 can be considered to be the best balance of accuracy and computational expense.

The plots, model details, and result discussion follow for all datasets and models. The procedure for computing the coefficients is outlined in the Appendix.

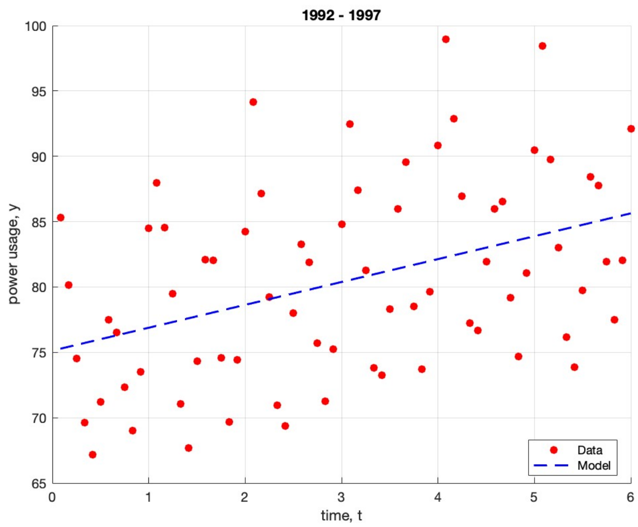
**Model 1** - $y = c_1 + c_2 t$

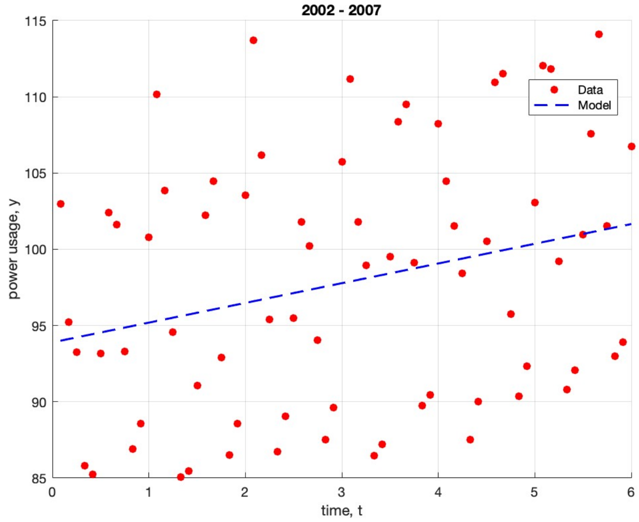
The command window output is shown below, which summarizes the model details - coefficients, and RMSE.

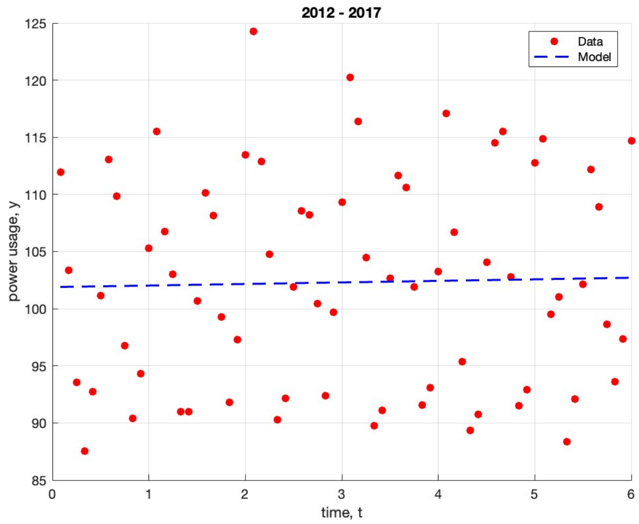
```
Command Window
Model Equation - y = c_1 + c_2 * t

Model for data from 1992 - 1997
    coeffs    values
    ------    ------

    "c_1"     75.132
    "c_2"     1.7502

    RMS Error = 6.79707084

Model for data from 2002 - 2007
    coeffs    values
    ------    ------

    "c_1"     93.899
    "c_2"     1.2929

    RMS Error = 7.99100986

Model for data from 2012 - 2017
    coeffs    values
    ------    -------

    "c_1"      101.89
    "c_2"     0.13604

    RMS Error = 9.17746541

fx >>
```

The coefficient $c_2$ represents the rate of increase of power consumption over time. The period with fastest rising energy consumption is hence 1992 - 1997, because of highest value of $c_2$.

The raw data is plotted with the linear model for all three datasets.




2002 - 2007
power usage, y
time, t
Data
Model
115
110
105
100
95
90
85
0
1
2
3
4
5
6


2012 - 2017
power usage, y
time, t
Data
Model
125
120
115
110
105
100
95
90
85
0
1
2
3
4
5
6

**Model 2** - $y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t$

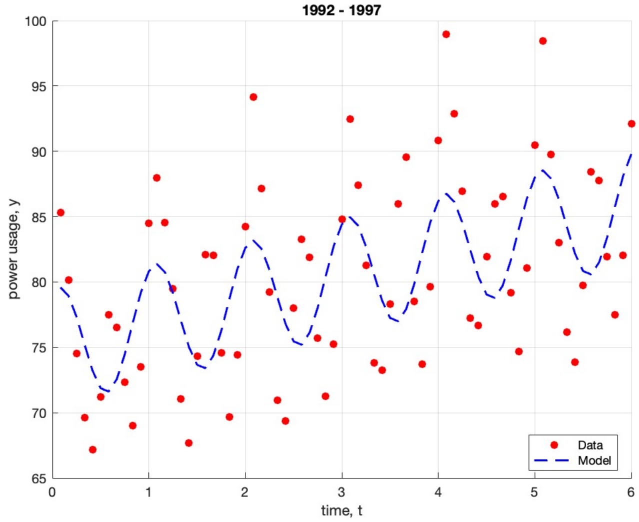
The command window output is shown below, which summarizes the model details - coefficients, and RMSE.

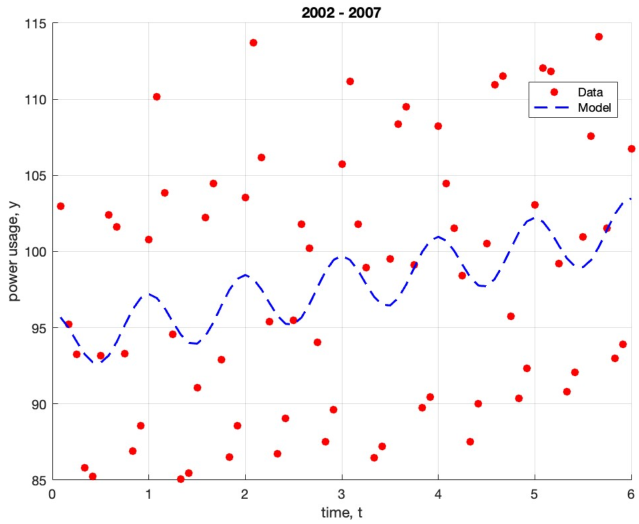
```
Command Window
Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + c_4 * sin(2 * pi * t)

Model for data from 1992 - 1997
    coeffs    values
    ------    ------

    "c_1"     75.004
    "c_2"     1.7924
    "c_3"     4.0262
    "c_4"     1.8917

    RMS Error = 6.02586502

Model for data from 2002 - 2007
    coeffs     values
    ------    --------

    "c_1"       94.015
    "c_2"       1.2548
    "c_3"       1.9408
    "c_4"     -0.21598

    RMS Error = 7.87108646

Model for data from 2012 - 2017
    coeffs     values
    ------    --------

    "c_1"       102.09
    "c_2"     0.071678
    "c_3"       2.6804
    "c_4"     -0.52323

    RMS Error = 8.97269472

fx >>
```

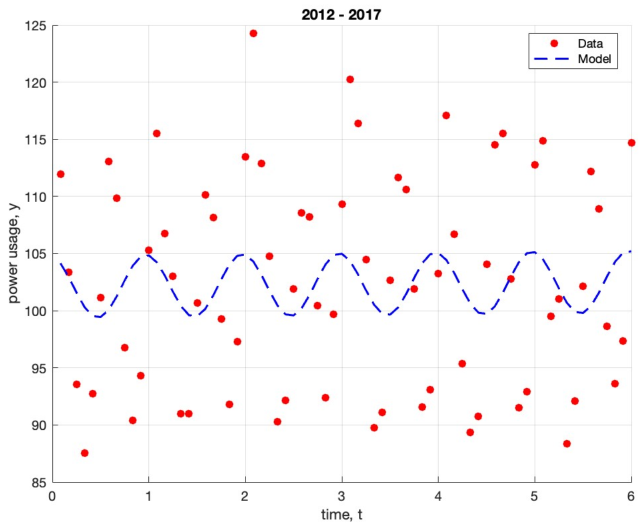
We see slight changes in the coefficient $c_2$ and RMSE, compared to Model 1.

The RMSE has decreased, but not very significantly. The model type has changed, and we can no longer strictly consider $c_2$ as the rate of increase in load consumption. $c_2$ would now be a less accurate estimate of the rate of increase in load consumption as compared to Model 1.

The raw data is plotted with the linear model for all three datasets.


1992 - 1997
power usage, y
time, t
Data
Model
100
95
90
85
80
75
70
65
0
1
2
3
4
5
6


2002 - 2007
power usage, y
time, t
Data
Model
115
110
105
100
95
90
85
0
1
2
3
4
5
6


2012 - 2017
power usage, y
time, t
Data
Model
125
120
115
110
105
100
95
90
85
0
1
2
3
4
5
6

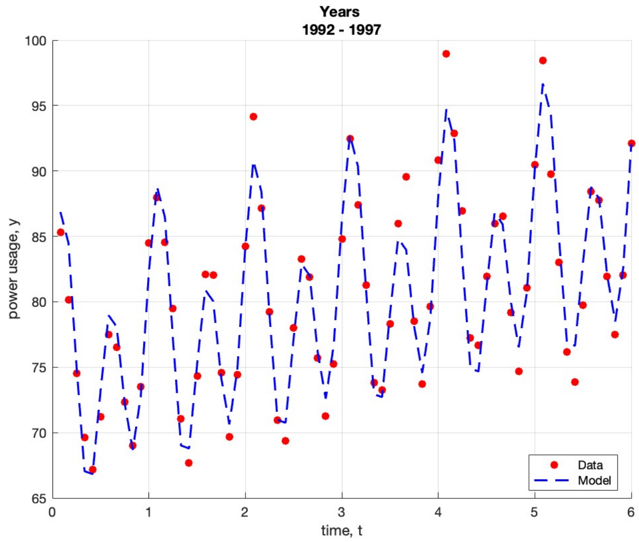
**Model 3** - $y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t + c_5 \cos 4\pi t + c_6 \sin 4\pi t$

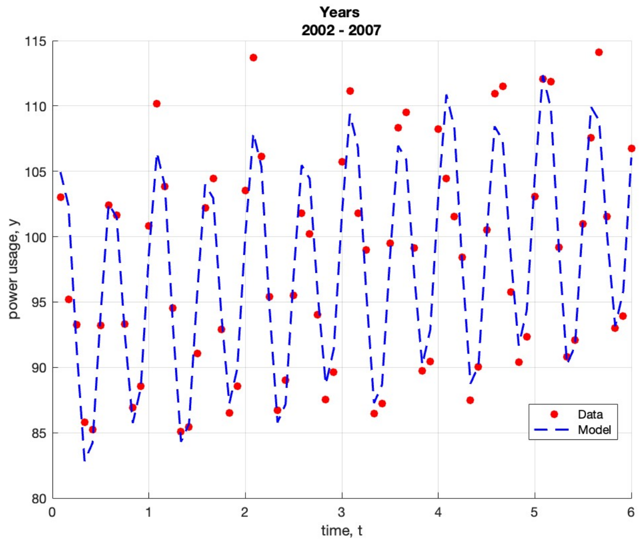
The command window output is shown below, which summarizes the model details - coefficients, and RMSE.

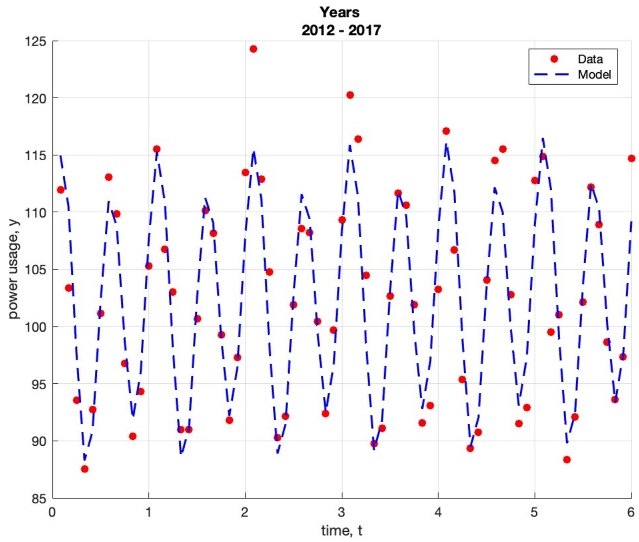
```
Command Window
Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + c_4 * sin(2 * pi * t)
                                 + c_5 * cos(4 * pi * t) + c_6 * sin(4 * pi * t)

Model for data from 1992 - 1997
    coeffs    values
    ------    ------

    "c_1"      74.49
    "c_2"     1.9613
    "c_3"     4.0121
    "c_4"     1.9442
    "c_5"     1.7945
    "c_6"     7.9365

    RMS Error = 1.81406481

Model for data from 2002 - 2007
    coeffs     values
    ------    --------

    "c_1"       93.328
    "c_2"       1.4806
    "c_3"       1.9219
    "c_4"     -0.14574
    "c_5"       1.9198
    "c_6"       10.333

    RMS Error = 2.62254835

Model for data from 2012 - 2017
    coeffs     values
    ------    --------

    "c_1"       101.39
    "c_2"      0.30157
    "c_3"       2.6612
    "c_4"     -0.45173
    "c_5"       3.3484
    "c_6"       11.323

    RMS Error = 3.31025953

fx >>
```

We see that the RMSE has dropped significantly when compared to Model 1. $c_2$ has remained similar to Model 2 for the first 2 datasets, but changes to 4 times the magnitude for the third dataset, when compared to Model 2.

The raw data is plotted with the linear model for all three datasets.


Years
1992 - 1997
power usage, y
time, t
Data
Model


Years
2002 - 2007
power usage, y
time, t
Data
Model


Years
2012 - 2017
power usage, y
time, t
Data
Model

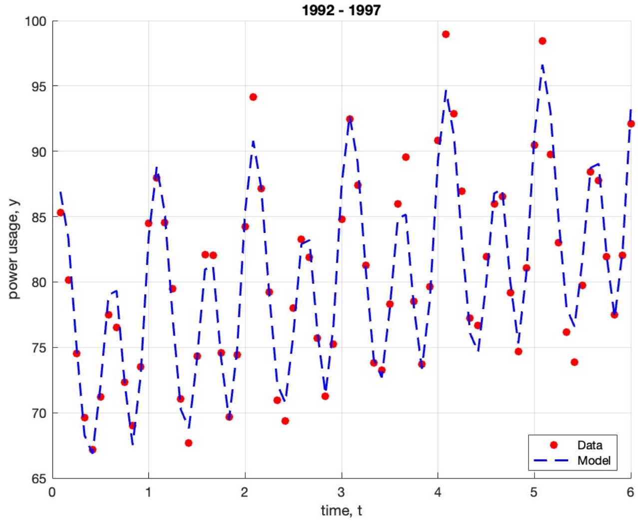
**Model 4** - $y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t + c_5 \cos 4\pi t + c_6 \sin 4\pi t + c_7 \cos 6\pi t + c_8 \sin 6\pi t$

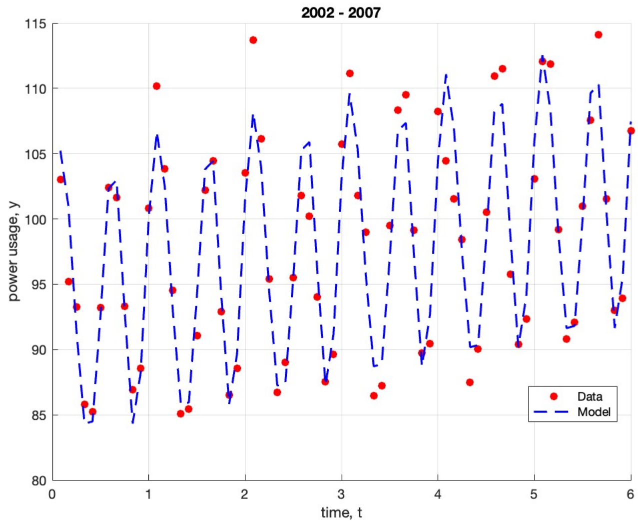
The command window output is shown below, which summarizes the model details - coefficients, and RMSE.

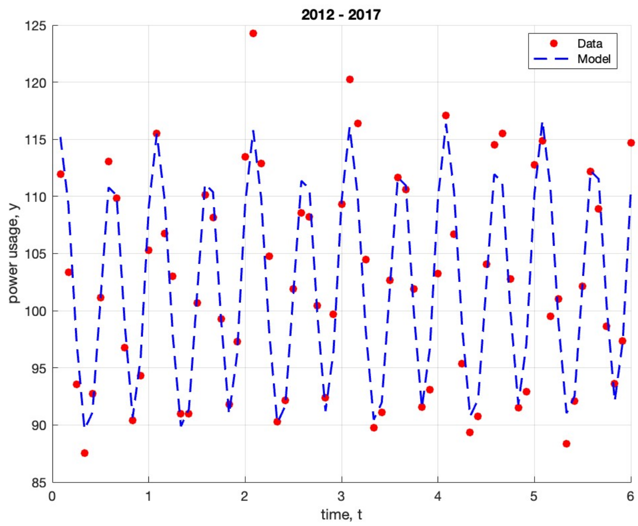
```
Command Window
Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + c_4 * sin(2 * pi *t)
                                   + c_5 * cos(4 * pi *t) + c_6 * sin(4 * pi *t)
                                   + c_7 * cos(6 * pi * t) + c_8 * sin(6 * pi *t)

Model for data from 1992 - 1997
    coeffs      values
    ------    ----------

    "c_1"         74.542
    "c_2"         1.9442
    "c_3"         4.0136
    "c_4"         1.9389
    "c_5"          1.796
    "c_6"          7.934
    "c_7"         1.1952
    "c_8"     -0.0081727

    RMS Error = 1.60543530

Model for data from 2002 - 2007
    coeffs     values
    ------    --------

    "c_1"       93.381
    "c_2"       1.4632
    "c_3"       1.9234
    "c_4"     -0.15117
    "c_5"       1.9213
    "c_6"        10.33
    "c_7"       1.4537
    "c_8"      0.22615

    RMS Error = 2.40758319

Model for data from 2012 - 2017
    coeffs     values
    ------    --------

    "c_1"       101.43
    "c_2"      0.28628
    "c_3"       2.6625
    "c_4"     -0.45648
    "c_5"       3.3497
    "c_6"        11.32
    "c_7"       1.2825
    "c_8"      0.20585

    RMS Error = 3.18039942

fx >>
```

We see that the RMSE is similar to that obtained with Model 4, and we might be overfitting the data if we include more sinusoids. $c_2$ is similar to what was obbtained in Model 3.

The raw data is plotted with the linear model for all three datasets.


1992 - 1997
power usage, y
time, t
Data
Model


2002 - 2007
power usage, y
time, t
Data
Model


2012 - 2017
power usage, y
time, t
Data
Model

# Appendix - Fitting a model to given data

Say we want to fit Model 2 - $y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t$ to a given dataset $(t_i, w_i)$ where $i = 1, 2, ..., N$ (the procedure is identical for other models). Then, we want the coefficients $c_1, c_2, c_3, c_4$ such that the root mean squared error (RMSE) defined as

$$RMSE = E = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - w_i)^2}$$

is a minimum. Here, $y_i = y(t_i)$.

Using differential calculus, the RMSE is minimum when the partial derivatives of the error w.r.t each coefficient are zero. Thus, we have the condition

$$\frac{\partial E}{\partial c_j} = 0$$

where $j = 1, 2, ..., 4$.

This can be simplified as

$$\sum_{i=1}^{N}(y_i - w_i)\frac{\partial y_i}{\partial c_j} = 0$$

for $j = 1, 2, ..., 4$.

Further simplifying,

$$\sum_{i=1}^{N} y_i \frac{\partial y_i}{\partial c_j} = \sum_{i=1}^{N} w_i \frac{\partial y_i}{\partial c_j}$$

for $j = 1, 2, ..., 4$.

Now, $y_i = y(t_i) = c_1 + c_2 t_i + c_3 \cos 2\pi t_i + c_4 \sin 2\pi t_i$ and

$$\frac{\partial y_i}{\partial c_1} = 1$$
$$\frac{\partial y_i}{\partial c_2} = t_i$$
$$\frac{\partial y_i}{\partial c_3} = \cos 2\pi t_i$$
$$\frac{\partial y_i}{\partial c_4} = \sin 2\pi t_i$$

Combining, we now have a set of four equations in four variables $c_1, c_2, ..., c_4$ to be solved for the values of the coefficients, which can be accomplished by matrix methods.

$$\begin{bmatrix} \sum 1 & \sum t_i & \sum \cos 2\pi t_i & \sum \sin 2\pi t_i \\ \sum t_i & \sum t_i^2 & \sum t_i \cos 2\pi t_i & \sum t_i \sin 2\pi t_i \\ \sum \cos 2\pi t_i & \sum t_i \cos 2\pi t_i & \sum \cos 2\pi t_i \cos 2\pi t_i & \sum \cos 2\pi t_i \sin 2\pi t_i \\ \sum \sin 2\pi t_i & \sum t_i \sin 2\pi t_i & \sum \sin 2\pi t_i \cos 2\pi t_i & \sum \sin 2\pi t_i \sin 2\pi t_i \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \sum w_i \\ \sum t_i w_i \\ \sum w_i \cos 2\pi t_i \\ \sum w_i \sin 2\pi t_i \end{bmatrix}$$

We have the matrix equation $Ac = b$ and the solution in MATLAB is $c = A \backslash b$.

# MATLAB Code - p6_1.m

```
clc; clear; close all

fprintf('Model Equation - y = c_1 + c_2 * t\n')
fprintf('\n')

% time vector
t = ((1:72)/12)';

% load data
load e02.txt
load e12.txt
load e92.txt

y = [e92 e02 e12];

years_names = ["1992 - 1997", "2002 - 2007", "2012 - 2017"];
coeffs_names = ["c_1"; "c_2"];

% compute and evaluate model
for i = 1:size(y, 2)

y_data = y(:, i);
A = zeros(2, 2);
b = zeros(2, 1);

% first equation
A(1, 1) = length(y_data);
A(1, 2) = sum(t);
b(1) = sum(y_data);

% second equation
A(2, 1) = A(1, 2);
A(2, 2) = sum(t.*t);
b(2) = sum(t.*y_data);

% solve for coefficients
coeffs = A\b;

% print model info and slope
```

```
fprintf('Model for data from %s\n', years_names(i))
tbl = table;
tbl.coeffs = coeffs_names;
tbl.values = coeffs;
disp(tbl)

% predicted values
y_predict = coeffs(1) + coeffs(2)*t;

% root mean squared error
rmse = sqrt(mean((y_data - y_predict).^2));
fprintf('    RMS Error = %.8f\n', rmse)
fprintf('\n')

% plot data with model
figure()
scatter(t, y_data, 'ro', 'filled')
hold on
plot(t, y_predict, 'b--', 'LineWidth', 1.5)
hold off
xlabel('time, t')
ylabel('power usage, y')
titlestring = years_names(i);
title(titlestring)
legend('Data', 'Model', 'Location', 'best')
grid

end
```

# MATLAB Code - p6_2.m

```
clc; clear; close all

fprintf(['Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + ' ...
'c_4 * sin(2 * pi * t)\n'])
fprintf('\n')

% time vector
t = ((1:72)/12)';

% load data
load e02.txt
load e12.txt
load e92.txt

y = [e92 e02 e12];

years_names = ["1992 - 1997", "2002 - 2007", "2012 - 2017"];
coeffs_names = ["c_1"; "c_2"; "c_3"; "c_4"];

% compute and evaluate model
for i = 1:size(y, 2)

y_data = y(:, i);
A = zeros(4, 4);
b = zeros(4, 1);

% first equation
A(1, 1) = length(y_data);
A(1, 2) = sum(t);
A(1, 3) = sum(cos(2*pi*t));
A(1, 4) = sum(sin(2*pi*t));
b(1) = sum(y_data);

% second equation
A(2, 1) = A(1, 2);
A(2, 2) = sum(t.*t);
A(2, 3) = sum(t.*cos(2*pi*t));
A(2, 4) = sum(t.*sin(2*pi*t));
b(2) = sum(t.*y_data);
```

```
% third equation
A(3, 1) = A(1, 3);
A(3, 2) = A(2, 3);
A(3, 3) = sum(cos(2*pi*t).*cos(2*pi*t));
A(3, 4) = sum(cos(2*pi*t).*sin(2*pi*t));
b(3) = sum(cos(2*pi*t).*y_data);

% fourth equation
A(4, 1) = A(1, 4);
A(4, 2) = A(2, 4);
A(4, 3) = A(3, 4);
A(4, 4) = sum(sin(2*pi*t).*sin(2*pi*t));
b(4) = sum(sin(2*pi*t).*y_data);

% solve for coefficients
coeffs = A\b;

% print model info and slope
fprintf('Model for data from %s\n', years_names(i))
tbl = table;
tbl.coeffs = coeffs_names;
tbl.values = coeffs;
disp(tbl)

% predicted values
y_predict = coeffs(1) + coeffs(2)*t + coeffs(3)*cos(2*pi*t) + ...
coeffs(4)*sin(2*pi*t);

% root mean squared error
rmse = sqrt(mean((y_data - y_predict).^2));
fprintf('    RMS Error = %.8f\n', rmse)
fprintf('\n')

% plot data with model
figure()
scatter(t, y_data, 'ro', 'filled')
hold on
plot(t, y_predict, 'b--', 'LineWidth', 1.5)
hold off
xlabel('time, t')
```

```
ylabel('power usage, y')
titlestring = years_names(i);
title(titlestring)
legend('Data', 'Model', 'Location', 'best')
grid

end
```

# MATLAB Code - p6_3.m

```
clc; clear; close all

fprintf(['Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + ' ...
'c_4 * sin(2 * pi * t)\n                                   + ' ...
'c_5 * cos(4 * pi * t) + c_6 * sin(4 * pi * t)\n'])
fprintf('\n')

% time vector
t = ((1:72)/12)';

% load data
load e92.txt
load e02.txt
load e12.txt

y = [e92 e02 e12];

years_names = ["1992 - 1997", "2002 - 2007", "2012 - 2017"];
coeffs_names = ["c_1"; "c_2"; "c_3"; "c_4"; "c_5"; "c_6"];

% compute and evaluate model
for i = 1:size(y, 2)

y_data = y(:, i);
A = zeros(6, 6);
b = zeros(6, 1);

% first equation
A(1, 1) = length(y_data);
A(1, 2) = sum(t);
A(1, 3) = sum(cos(2*pi*t));
A(1, 4) = sum(sin(2*pi*t));
A(1, 5) = sum(cos(4*pi*t));
A(1, 6) = sum(sin(4*pi*t));
b(1) = sum(y_data);

% second equation
A(2, 1) = A(1, 2);
A(2, 2) = sum(t.*t);
```

```
A(2, 3) = sum(t.*cos(2*pi*t));
A(2, 4) = sum(t.*sin(2*pi*t));
A(2, 5) = sum(t.*cos(4*pi*t));
A(2, 6) = sum(t.*sin(4*pi*t));
b(2) = sum(t.*y_data);

% third equation
A(3, 1) = A(1, 3);
A(3, 2) = A(2, 3);
A(3, 3) = sum(cos(2*pi*t).*cos(2*pi*t));
A(3, 4) = sum(cos(2*pi*t).*sin(2*pi*t));
A(3, 5) = sum(cos(2*pi*t).*cos(4*pi*t));
A(3, 6) = sum(cos(2*pi*t).*sin(4*pi*t));
b(3) = sum(cos(2*pi*t).*y_data);

% fourth equation
A(4, 1) = A(1, 4);
A(4, 2) = A(2, 4);
A(4, 3) = A(3, 4);
A(4, 4) = sum(sin(2*pi*t).*sin(2*pi*t));
A(4, 5) = sum(sin(2*pi*t).*cos(4*pi*t));
A(4, 6) = sum(sin(2*pi*t).*sin(4*pi*t));
b(4) = sum(sin(2*pi*t).*y_data);

% fifth equation
A(5, 1) = A(1, 5);
A(5, 2) = A(2, 5);
A(5, 3) = A(3, 5);
A(5, 4) = A(4, 5);
A(5, 5) = sum(cos(4*pi*t).*cos(4*pi*t));
A(5, 6) = sum(cos(4*pi*t).*sin(4*pi*t));
b(5) = sum(cos(4*pi*t).*y_data);

% sixth equation
A(6, 1) = A(1, 6);
A(6, 2) = A(2, 6);
A(6, 3) = A(3, 6);
A(6, 4) = A(4, 6);
A(6, 5) = A(5, 6);
A(6, 6) = sum(sin(4*pi*t).*sin(4*pi*t));
b(6) = sum(sin(4*pi*t).*y_data);
```

```
% solve for coefficients
coeffs = A\b;

% print model info and slope
fprintf('Model for data from %s\n', years_names(i))
tbl = table;
tbl.coeffs = coeffs_names;
tbl.values = coeffs;
disp(tbl)

% predicted values
y_predict = coeffs(1) + coeffs(2)*t + coeffs(3)*cos(2*pi*t) + ...
coeffs(4)*sin(2*pi*t) + coeffs(5)*cos(4*pi*t) + coeffs(6)*sin(4*pi*t);

% root mean squared error
rmse = sqrt(mean((y_data - y_predict).^2));
fprintf('    RMS Error = %.8f\n', rmse)
fprintf('\n')

% plot data with model
figure()
scatter(t, y_data, 'ro', 'filled')
hold on
plot(t, y_predict, 'b--', 'LineWidth', 1.5)
hold off
xlabel('time, t')
ylabel('power usage, y')
titlestring = ['Years ', years_names(i)];
title(titlestring)
legend('Data', 'Model', 'Location', 'best')
grid

end
```

# MATLAB Code - p6_4.m

```
clc; clear; close all

fprintf(['Model Equation - y = c_1 + c_2 * t + c_3 * cos(2 * pi * t) + ' ...
'c_4 * sin(2 * pi * t)\n                                   + ' ...
'c_5 * cos(4 * pi * t) + c_6 * sin(4 * pi * t)\n' ...
'                                    ' ...
'+ c_7 * cos(6 * pi * t) + c_8 * sin(6 * pi *t)\n'])
fprintf('\n')

% time vector
t = ((1:72)/12)';

% load data
load e92.txt
load e02.txt
load e12.txt

y = [e92 e02 e12];

years_names = ["1992 - 1997", "2002 - 2007", "2012 - 2017"];
coeffs_names = ["c_1"; "c_2"; "c_3"; "c_4"; "c_5"; "c_6"; "c_7"; "c_8"];

% compute and evaluate model
for i = 1:size(y, 2)

y_data = y(:, i);
A = zeros(8, 8);
b = zeros(8, 1);

% first equation
A(1, 1) = length(y_data);
A(1, 2) = sum(t);
A(1, 3) = sum(cos(2*pi*t));
A(1, 4) = sum(sin(2*pi*t));
A(1, 5) = sum(cos(4*pi*t));
A(1, 6) = sum(sin(4*pi*t));
A(1, 7) = sum(cos(6*pi*t));
A(1, 8) = sum(sin(6*pi*t));
b(1) = sum(y_data);
```

```
% second equation
A(2, 1) = A(1, 2);
A(2, 2) = sum(t.*t);
A(2, 3) = sum(t.*cos(2*pi*t));
A(2, 4) = sum(t.*sin(2*pi*t));
A(2, 5) = sum(t.*cos(4*pi*t));
A(2, 6) = sum(t.*sin(4*pi*t));
A(2, 7) = sum(t.*cos(6*pi*t));
A(2, 8) = sum(t.*sin(6*pi*t));
b(2) = sum(t.*y_data);

% third equation
A(3, 1) = A(1, 3);
A(3, 2) = A(2, 3);
A(3, 3) = sum(cos(2*pi*t).*cos(2*pi*t));
A(3, 4) = sum(cos(2*pi*t).*sin(2*pi*t));
A(3, 5) = sum(cos(2*pi*t).*cos(4*pi*t));
A(3, 6) = sum(cos(2*pi*t).*sin(4*pi*t));
A(3, 7) = sum(cos(2*pi*t).*cos(6*pi*t));
A(3, 8) = sum(cos(2*pi*t).*sin(6*pi*t));
b(3) = sum(cos(2*pi*t).*y_data);

% fourth equation
A(4, 1) = A(1, 4);
A(4, 2) = A(2, 4);
A(4, 3) = A(3, 4);
A(4, 4) = sum(sin(2*pi*t).*sin(2*pi*t));
A(4, 5) = sum(sin(2*pi*t).*cos(4*pi*t));
A(4, 6) = sum(sin(2*pi*t).*sin(4*pi*t));
A(4, 7) = sum(sin(2*pi*t).*cos(6*pi*t));
A(4, 8) = sum(sin(2*pi*t).*sin(6*pi*t));
b(4) = sum(sin(2*pi*t).*y_data);

% fifth equation
A(5, 1) = A(1, 5);
A(5, 2) = A(2, 5);
A(5, 3) = A(3, 5);
A(5, 4) = A(4, 5);
A(5, 5) = sum(cos(4*pi*t).*cos(4*pi*t));
A(5, 6) = sum(cos(4*pi*t).*sin(4*pi*t));
```

```
A(5, 7) = sum(cos(4*pi*t).*cos(6*pi*t));
A(5, 7) = sum(cos(4*pi*t).*sin(6*pi*t));
b(5) = sum(cos(4*pi*t).*y_data);

% sixth equation
A(6, 1) = A(1, 6);
A(6, 2) = A(2, 6);
A(6, 3) = A(3, 6);
A(6, 4) = A(4, 6);
A(6, 5) = A(5, 6);
A(6, 6) = sum(sin(4*pi*t).*sin(4*pi*t));
A(6, 7) = sum(sin(4*pi*t).*cos(6*pi*t));
A(6, 8) = sum(sin(4*pi*t).*sin(6*pi*t));
b(6) = sum(sin(4*pi*t).*y_data);

% seventh equation
A(7, 1) = A(1, 7);
A(7, 2) = A(2, 7);
A(7, 3) = A(3, 7);
A(7, 4) = A(4, 7);
A(7, 5) = A(5, 7);
A(7, 6) = A(6, 7);
A(7, 7) = sum(cos(6*pi*t).*cos(6*pi*t));
A(7, 8) = sum(cos(6*pi*t).*sin(6*pi*t));
b(7) = sum(cos(6*pi*t).*y_data);

% eighth equation
A(8, 1) = A(1, 8);
A(8, 2) = A(2, 8);
A(8, 3) = A(3, 8);
A(8, 4) = A(4, 8);
A(8, 5) = A(5, 8);
A(8, 6) = A(6, 8);
A(8, 7) = A(7, 8);
A(8, 8) = sum(sin(6*pi*t).*sin(6*pi*t));
b(8) = sum(sin(6*pi*t).*y_data);

% solve for coefficients
coeffs = A\b;

% print model info and slope
```

```
fprintf('Model for data from %s\n', years_names(i))
tbl = table;
tbl.coeffs = coeffs_names;
tbl.values = coeffs;
disp(tbl)

% predicted values
y_predict = coeffs(1) + coeffs(2)*t + coeffs(3)*cos(2*pi*t) + ...
coeffs(4)*sin(2*pi*t) + coeffs(5)*cos(4*pi*t) + ...
coeffs(6)*sin(4*pi*t) + coeffs(7)*cos(6*pi*t) + ...
coeffs(8)*sin(6*pi*t);

% root mean squared error
rmse = sqrt(mean((y_data - y_predict).^2));
fprintf('    RMS Error = %.8f\n', rmse)
fprintf('\n')

% plot data with model
figure()
scatter(t, y_data, 'ro', 'filled')
hold on
plot(t, y_predict, 'b--', 'LineWidth', 1.5)
hold off
xlabel('time, t')
ylabel('power usage, y')
titlestring = years_names(i);
title(titlestring)
legend('Data', 'Model', 'Location', 'best')
grid

end
```