CVT: Thoughts and a Proposed Design

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Abstract

This document is a collection of my initial thoughts and ideas on CVT systems, their uses, what we want from them, and my idea for one. My idea is a dual-differential CVT system, which is a single-input and single-output CVT system that uses two differentials: one to split the input power into a high and low speed reducers (creating a power differential) and one to combine them again. It may have been done before (and I have found evidence for this), but I believe those explorations lack depth.

1 Why are CVTs Useful?

1.1 Mechanical Power

The mechanical power of some rotating thing is given by the following equation:

$$P_m = \tau \cdot \omega \tag{1}$$

You can think of τ (torque) and ω (speed) as the length and width of a rectangle, the rectangle would have an area of P_m.
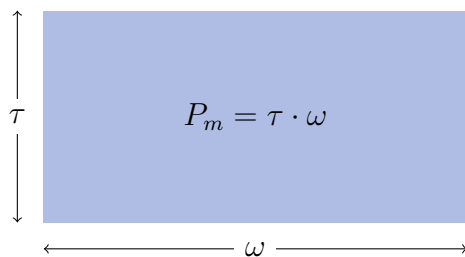

Figure 1: Rectangular representation of mechanical power.

If we fixed P_m to be some constant, there would be an infinite number of rectangles with that area, and thus there are an infinite number of $\tau : \omega$ ratios for any P_m. Each of these rectangles is representative of one single gear ratio.
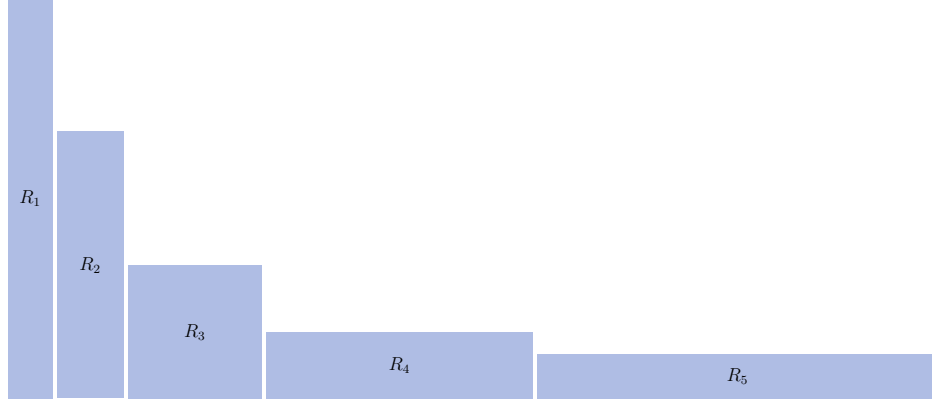
Figure 2: Various examples of rectangles with the same area P_m.

Layering these rectangles gives us a crude approximation of a curve, where each ω (x-axis) has a corresponding τ (y-axis):
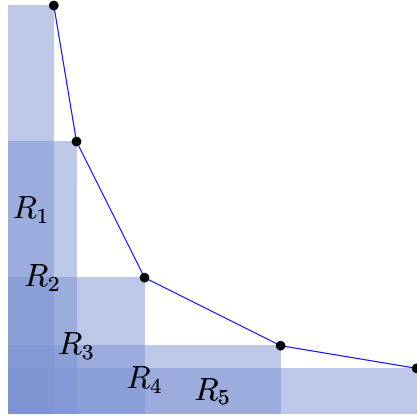

Figure 3: Crude approximation of optimal power graph with 5 rectangles.

Depending on the torque requirements, you could choose between discrete reduction ratios. This is how a conventional bicycle works with its chain and sprocket transmission and manual gear shifter.

If we had a bike transmission with an infinite amount of these sprockets, we wouldn't have to choose between a handfull of discrete, sub-optimal reduction ratios. Instead, we could just choose the exact ratio for our requirements. This is the idea for a CVT.

Specifically extending the bicycle example, we would end up with something like the V-Belt CVTs we currently use in some cars.

1.2 Optimal Torque-Speed Curve w/ Fixed P_m

We can bring this CVT idea back to the torque-speed graph by adding more and more rectangles (approaching infinity), giving us a continuous curve. This type of curve is aptly named: it is a rectangular hyperbola.
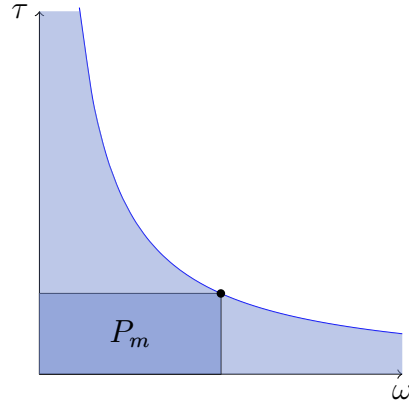
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Figure 4: Optimal torque-speed curve with a constant P_m

This curve, the rectangular hyperbola (aka: $y = \frac{1}{x}$), is the ideal torque-speed curve in terms of power, we are always at peak performance. The function for this curve is

$$\tau = \frac{P_m}{\omega} \tag{2}$$

Remember that P_m is a constant.

1.3 Torque-Speed Curve of a Motor

1.3.1 Basic Torque-Speed Graph

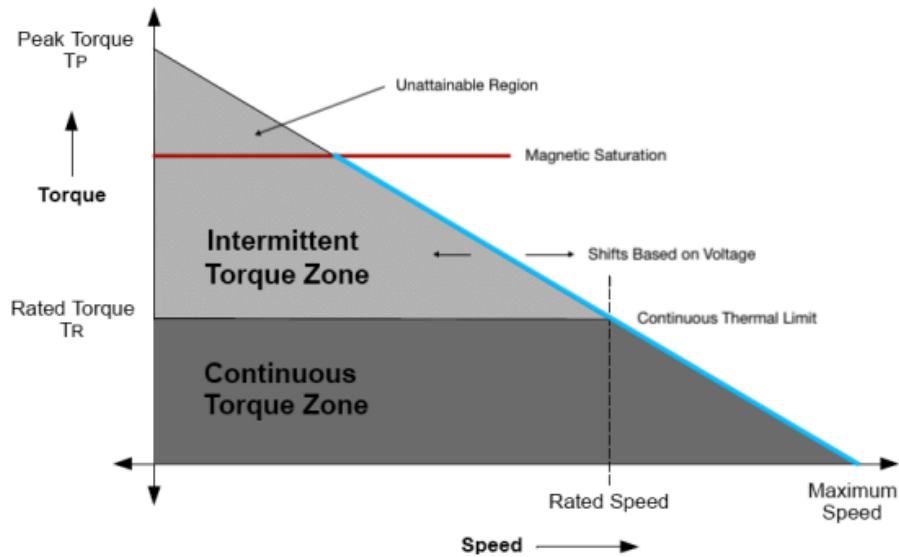One thing that can help our intution for this curve is looking at the torque-speed curve of a motor.

Figure 5: Torque-speed curve of a motor from Adi Mehrotra's Dynos for Dummies

This curve is a simple linear function, connecting the y-intercept (peak torque τ_p or $\tau_{\max}$) to the x-intercept (maximum speed $\omega_{\max}$).
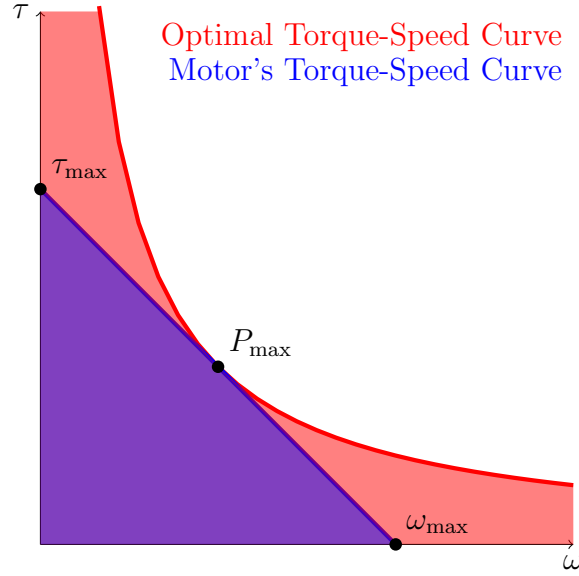

Figure 6: Rectangular hyperbola and motor torque-speed curve.

If you compare this to the rectangular hyperbola, we see that there is one point where they intersect, which is also the point of maximum power output for a motor.

1.3.2 Motor Efficiency Graphs and CVT Use-Cases

For the obvious use-case for CVT, where we want to use the maximum amount of power while the torque demands are varying (different terrains, driving uphill vs. downhill, etc.), we would always run our input at the speed at the intersection point, and then the transmission would vary the output speed depending on the exact torque demand.

However, what if we are driving on these different terrains, and there is a speed limit? Or a traffic light? In this case, we are not trying to maximize speed or power, we are trying to conserve energy, and almost never get to use $P_{\max}$ in the first place. It is an energy-efficiency optimization.

Energy efficiency, μ_m, is defined as the ratio of mechanical power output to electrical power input. This means that we want to maximize μ_m:

$$\mu_m = \frac{P_m}{P_e} \tag{3}$$

Usually, energy efficiency for a motor drops off significantly at lower speeds, while it increases at higher speeds.
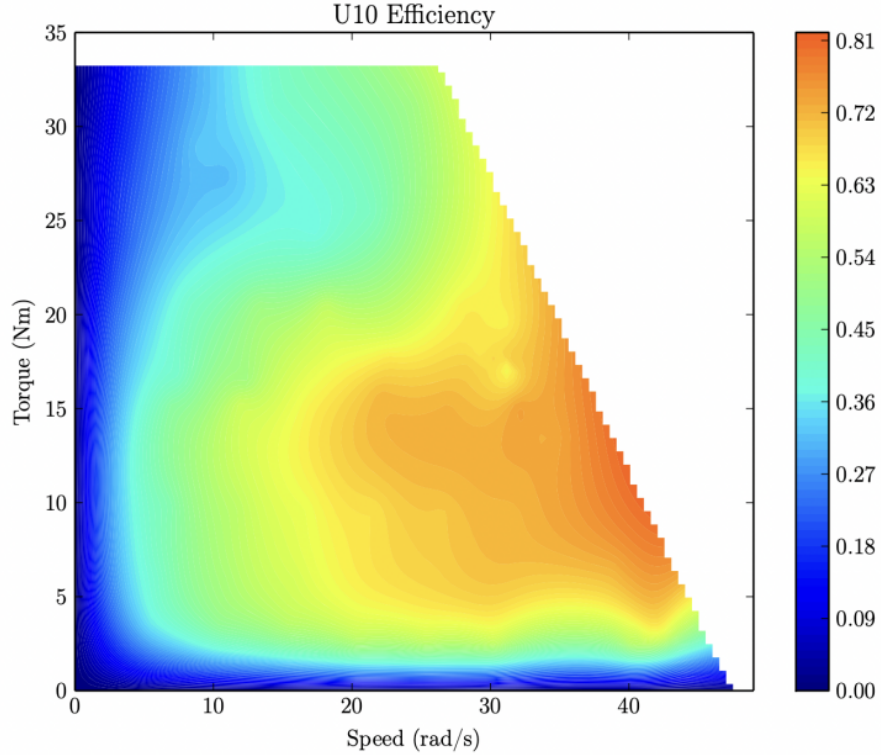
Figure 7: The torque/speed/efficiency map for the U10 from Elijah Stanger-Jones' masters thesis. (Found in Adi Mehrotra's Dynos for Dummies)

Thus, in the case of a vehicle forced to drive at low speeds, it would be beneficial to run the motor at a higher speed, and choose a reduction ratio that significantly decreases speed. Because we are already running our motor at a high speed, we have to be using $< P_{\max}$, and thus we would be using less energy.

There is an unfortunate issue related to this: it only goes one way. We can set our motor to run at a high speed, and choose to gear it down, maintaining high energy efficiency and low power at low speeds. But, if we set our motor at a low speed and geared it up, we could not expect high energy efficiency at high speeds, so we would benefit by simply running our motor at a higher speed and gearing closer to a 1:1 reduction. Thankfully, running our motor at those high speeds is already more efficient, so we are not losing much.

2   CVT Design

2.1   Constraints

Going back to the bike chain and sprocket example, I earlier explained that we could "have an infinite number of sprockets", and that would yield a CVT. That is actually not true. What I really meant was an infinite amount of sprockets **between** two discrete sizes, the

largest and a smallest.

We want the curve to be the continous on a closed interval $[\omega_1, \omega_2]$, where ω_1 is the minimum expected operating speed, and ω_2 is the maximum expected operating speed.
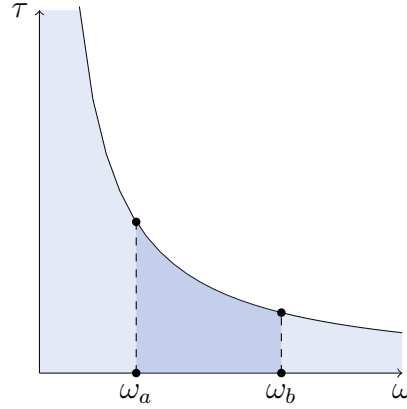

Figure 8: Important section of the rectangular hyperbola.

Of course, we can, and likely will, operate at speeds both lower and higher than within this interval, but we cannot use this continous curve. Instead, we will multiply the points of peak speed and peak torque by the highest and lowest reduction ratios, respectively.

Specifically:

$$\omega_a = \omega_1 \cdot R_{\max} \tag{4}$$