# Emaxx Algorithm and Data Structures

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Published with GitBook

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# Book from e-maxx.ru

When I was training for ACM, I have learned a lot from the blog http://e-maxx.ru/algo/. This is a great site which explains a lot of basic and advanced algorithms and data structures.

Unfortunately, this book is in Russian. I think this great resource should be available to more people. I hope translating this blog to English will help more people to access easily to this resource. It's also a way for me to contribute to competitive programming community.

*By the way, English is my secondary language. So there will be definitely some spelling error in this book.*

# Euler's Totient function

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## Definition

Euler's Totient function $$\phi (n)$$ (sometimes denoted $$\varphi(n)$$ or $${\it phi}(n)$$) is the number of numbers from 1 to $$n$$, coprime with $$n$$. In other words, this is the number of numbers in the interval $$[1; n]$$, the greatest common divisor of which with $$n$$ equals one.

The first few values of the function:

$$\phi(1) = 1$$

$$\phi(2) = 1$$

$$\phi(3) = 2$$

$$\phi(4) = 2$$

$$\phi(5) = 4$$

## Properties

The following three simple properties of the Euler function are enought to learn how to calculate it for any number:

- If $$p$$ is a prime number, then $$\phi(p) = p - 1$$

  (This is obvious, because any number except itself $$p$$, is coprime to it)

- If $p$ is a positive integer, $a$ is a natural number, then $\phi (p^a)=p^a-p^{a-1}$

  (Since the number of $p^a$ are not only relatively prime number of the form $pk(k \in \mathcal{N})$ which $p^a/p = p^{a - 1}$ numbers)

- If $a$ and $b$ are coprime, then $\phi(ab) = \phi(a)\phi(b)$ (**multiplicativity** in Euler's function) (This fact follows from **the theorem of China**. Consider an arbitrary number $z \le ab$. Let $x$ and $y$ be the remainders of the division $z$ into $a$ and $b$ respectively. Then $z$ is coprime to $ab$ if and only if $z$ coprime to $a$ and $b$ individually, in other words, $x$ coprime to $a$ and $y$ coprime to $b$