| Seat No. | |
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[5352]-561

S.E. (Computer) (First Semester) EXAMINATION, 2018

DISCRETE MATHEMATICS

(2015 PATTERN)

Time : Two Hours                          Maximum Marks : 50

*N.B.* :— (*i*)  Neat diagrams must be drawn wherever necessary.

(*ii*)  Figures to the right indicate full marks.

(*iii*)  Assume suitable data, if necessary.

1.    (*a*)   Prove :                                                              [4]

$$1^3 + 2^3 + 3^3 + .... + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

(*b*)   Prove that the set of rational numbers is countably infinite. [4]

(*c*)   Let A = {1, 2, 3} and $f_1$ and $f_2$ are functions from A to B given by :                                    [4]

$f_1 = \{(1, 2), (2, 3), (3, 1)\}$  and

$f_2 = \{(1, 2), (2, 1), (3, 3)\}$

Compute  $f_1 \circ f_2$  and  $f_2 \circ f_1$

P.T.O.

**2.** (*a*) Compute the transitive closure of given diagraph using Warshall's algorithm : [4]
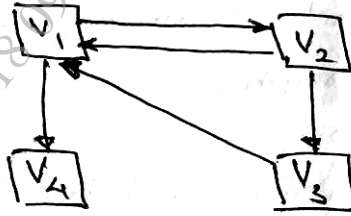


Fig. 2(*a*) Graph G(V, E)

(*b*) Show that the relation R is "Less than" from A to B where : [4]

A = {1, 2, 8} and

B = {1, 2, 3, 5}

Find :

(*i*) R in Roster form

(*ii*) Domain and Range of R.

(*c*) Explain with example, notation used and mathematical expression to describe the following terms : [4]

(*i*) Membership

(*ii*) Subset

(*iii*) Equality between sets

(*iv*) Union of sets.

**3.** (*a*) Write an algorithm for generating permutation of {1, 2, .....  $n$}. Apply it for $n$ = 3 case. [4]

(*b*) Solve the following :

(*i*) How many different car number plates are possible with 2 letters followed by 3 digits. [4]

(*ii*) How many of these number plates begin with 'MH'.

(*c*) Consider a graph G(V, E) where V = {$v_1$, $v_2$, $v_3$} & deg ($v_2$) = 4 : [4]

(*i*) Does such simples graph exists ? If not, why ?

(*ii*) Does such a multigraph exists ? If yes, give example.

*Or*

**4.** (*a*) Explain the following in brief : [4]

(*i*) Subgraphs and spanning subgraph

(*ii*) Isomorphic graph

(*iii*) Bipartite graph

(*iv*) Adjacency matrix and incidence matrix of undirected graph.

(*b*) Apply Dijkastra's Algorithm to find the shortest path from vertex $v_1$ to $v_5$ in the graph show below in Fig. 4.(*b*). [4]
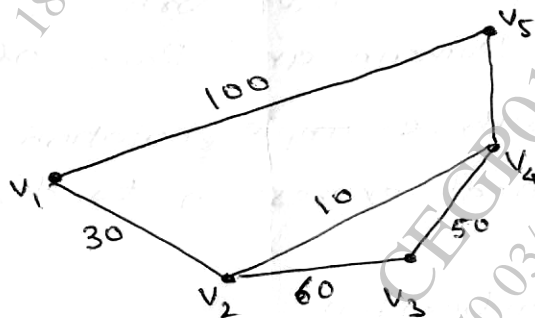


Fig. 4(*b*). Weighted Graph

(*c*) In how many ways can a cricket team of eleven players be chosen out of a batch of 14 players. How many of them will : [4]

(*i*) Include a particular player.

(*ii*) Exclude a particular player.

**5.** (*a*) Determine the maximum flow in the transport network shown in Fig. 5. (*a*) using Labelling procedure. Determine the corresponding min. cut. [7]
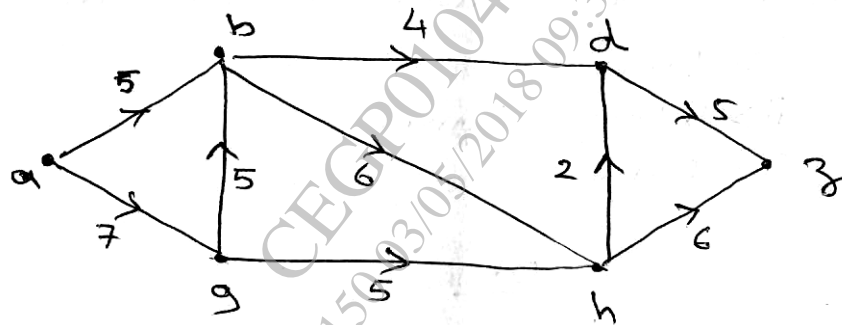


Fig. 5.(*a*) Graph Transport network.

(*b*) Explain the following terms : [6]

(*i*) Spanning trees

(*ii*) Properties of trees

(*iii*) M-ary tree.

**6.** (*a*) Give the stepwise construction of minimum spanning tree using Prim's Algorithm for the following graph shown in Fig. 6(*a*) Obtain the total cost of minimum spanning tree. [7]
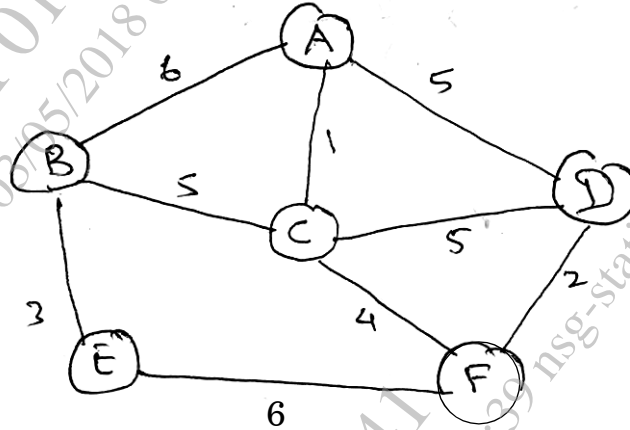


Fig. 6.(*a*) Graph G.

(*b*) Explain the following : [6]

(*i*) Game tree

(*ii*) Kruskal's Algorithm.

**7.** (*a*) Let R = {0, 60, 120, 180, 240, 300} and * = binary operation so that for *a* and *b* in R, *a* * *b* is overall angular rotation corresponding to successive rotation by *a* and by *b*. Show (R, *) is a group. [7]

(*b*) Explain the following terms with examples : [6]

(*i*) Ring

(*ii*) Integral Domain

(*iii*) Field.

**8.** (*a*) Show that $(I, \oplus, \odot)$ is a commutative ring with identity where $\oplus$ and $\odot$ are defined as : [7]

$$a \oplus b = a + b - 1 \quad \text{and}$$

$$a \odot b = a + b$$

(*b*) Explain the following terms : [6]

(*i*) Monoids

(*ii*) Sub-group

(*iii*) Group codes.