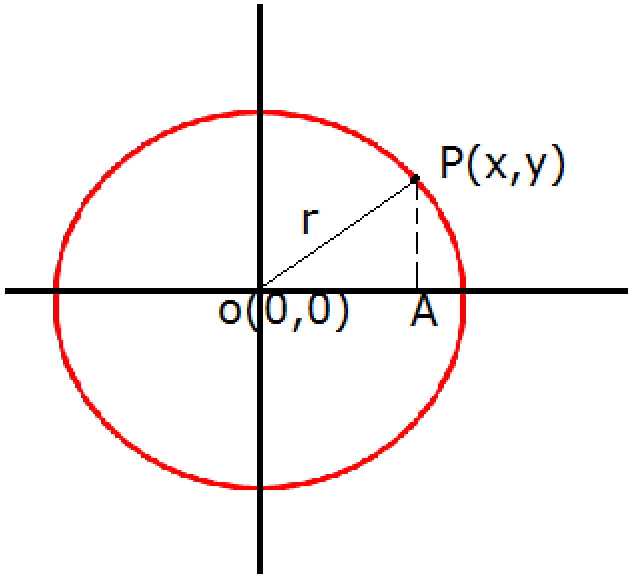
Circle Drawing

- 1. Polynomial Method
- 2. Trigonometric Method

Polynomial Method



OP = r

OA = OP cosΘ = r. cos 45 = r/1.414

r^2 = x^2 +y^2

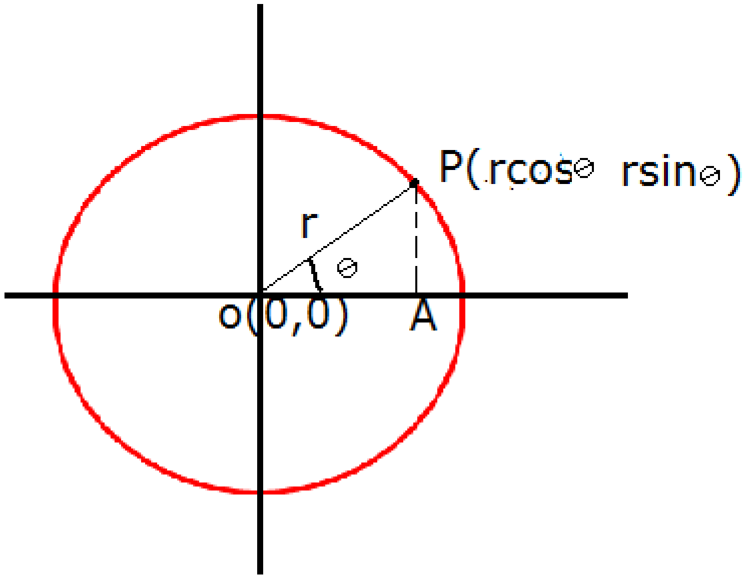
Y = Sqrt (r^2 – x^2)

Algorithm

1. Read (h , k)
2. X=0, y=r, end = r/1.414
3. While(x<=end)

```
        // plot 8 points, each point of 1 octant
    y = Sqrt (r^2 – x^2)
    plot(x+h,y+k)
    plot(-x+h, y+k)
    .........
    x= x+1
End while
```

Trigonometric Method



Algorithm:

1. Read r & angle of increment (Φ)
2. Plot (0,r) so Θ = 90 degree
3. X= r cos Θ , y = r sin Θ
4. Reflect point in 8 ways
5. Θ = Θ +Φ
6. If Θ < 45 degree, then go to step 3
7. stop

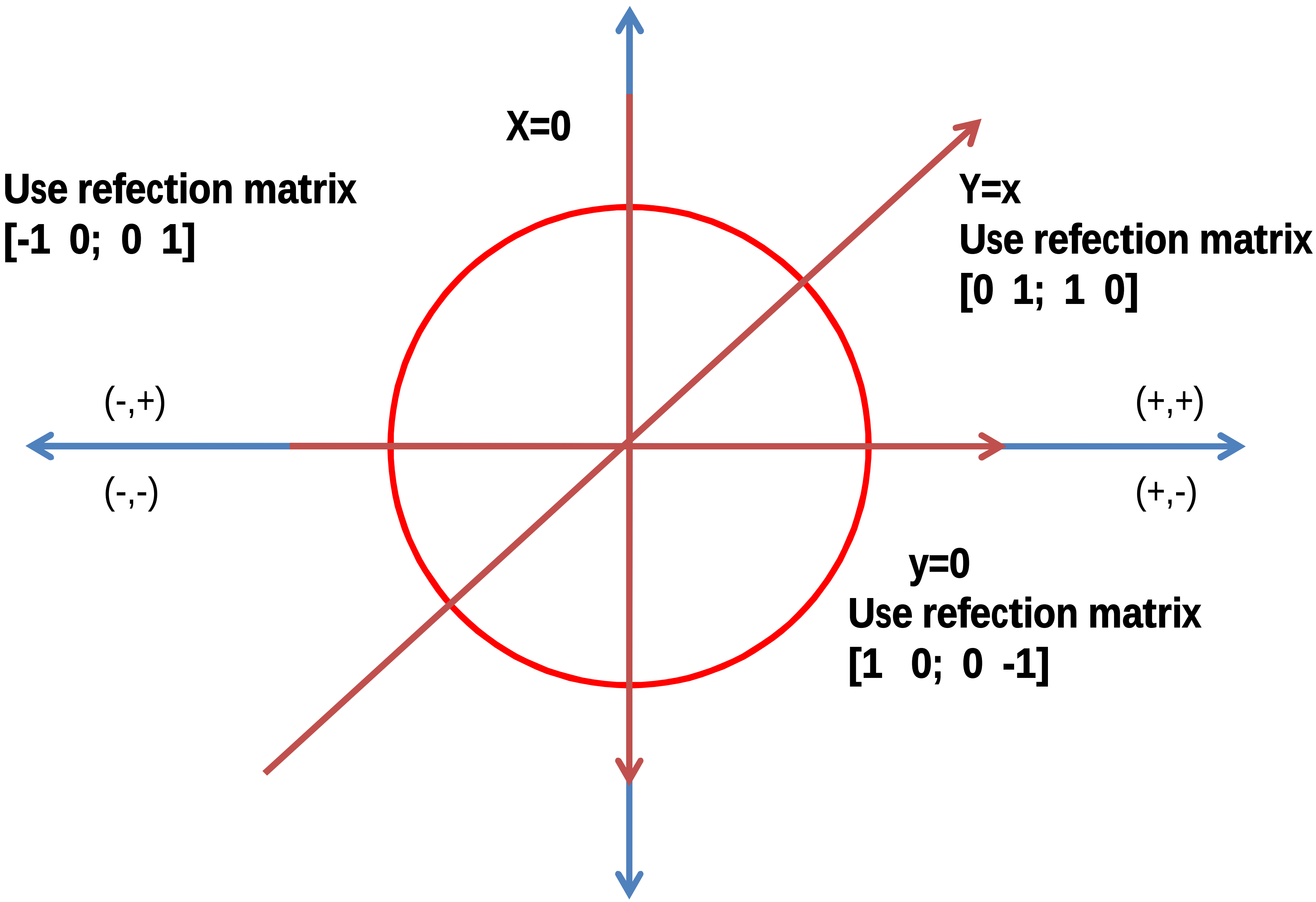
DDA for Circle Drawing

- Can plot Circle by 2 methods
  - 1. trigonometric method(polar eq / parametric)
  - 2. geometric method(polynomial eq. / Direct)

DDA

- 1. Read r, calculate ϵ
- 2.Initialize : st-x=r, st-y=0
- 3. x1=st-x, y1=st-y
- 4. do{
    - X2=x1+ ϵy1
    - Y2=y1- ϵx2
    - Plot(x2,y2)
    - X1=x2, y1=y2}
    - While((y1-st-y)<ϵ || (st-x-x1)> ϵ)

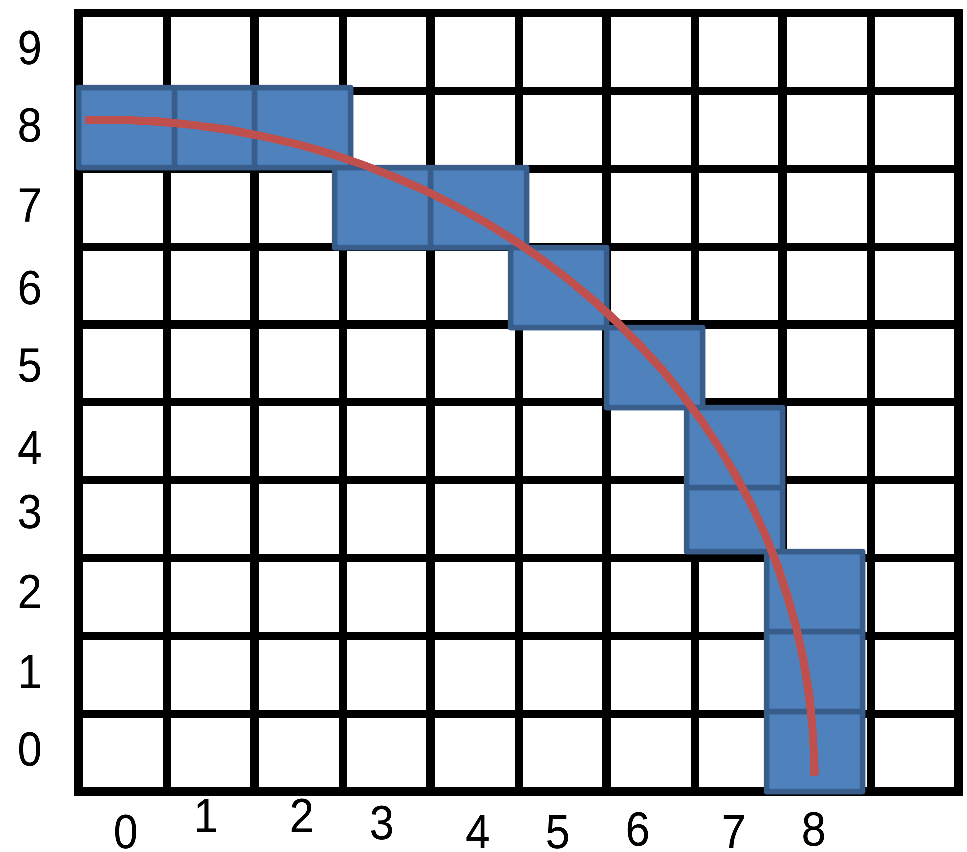
Bresenham's circle generation



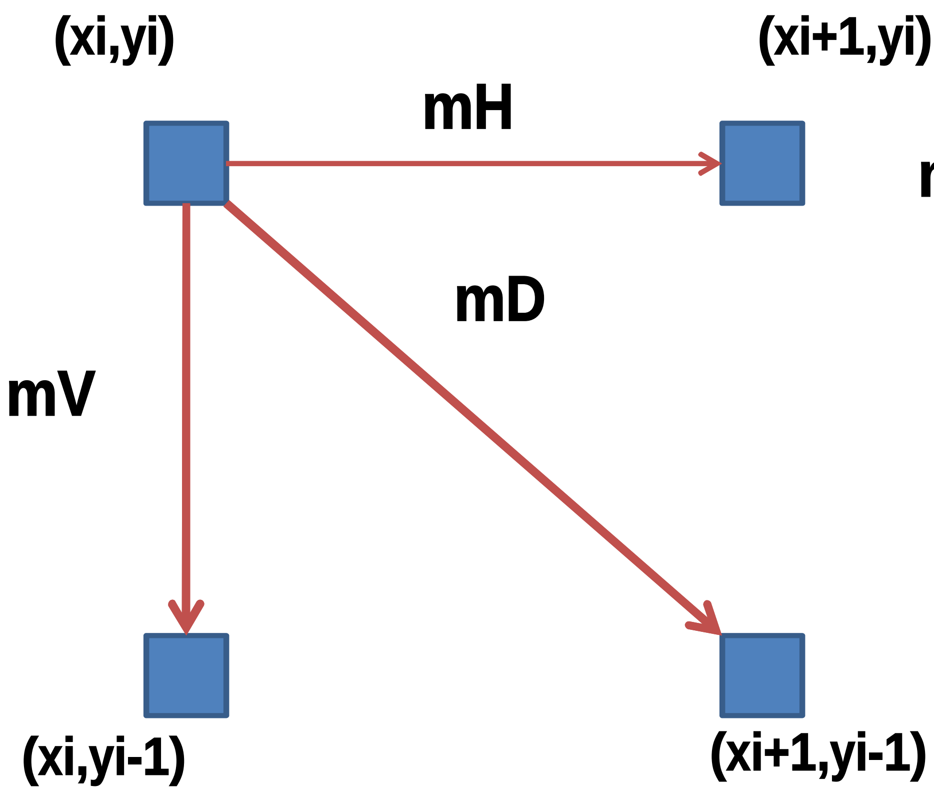
Circle in first quadrant

- Radius →8
- Start pt → (0,8)
- End pt. → (8,0)

- X is incrementing
- 0→….→8
- Y is decreasing
- 8→….→0



Circle in first quadrant



$$mH = | (xi+1)^2 + (yi)^2 - (R)^2 |$$

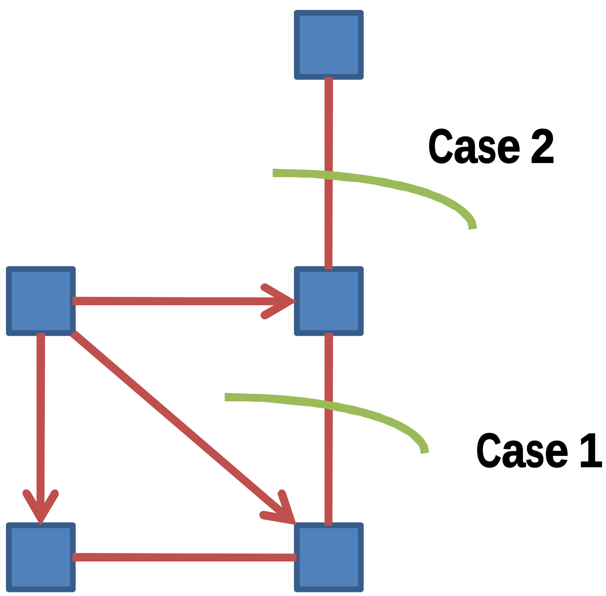
$$mD = | (xi+1)^2 + (yi-1)^2 - (R)^2 |$$

$$mV = | (xi)^2 + (yi-1)^2 - (R)^2 |$$

Algorithm selects pixel which minimizes the sq of distance between one of the 3 pixel and true circle.

Cases depends upon position of diagonal point

- **Δi=Diff Bet the Sq of the dist from center of circle to diagonal pixel & dist to a point on circle R^2**
- **$\Delta i = | (x_i+1)^2 + (y_i-1)^2 - (R)^2 |$**
- **If $\Delta i < 0$ Then diagonal pt is inside the circle**



$\delta = mH - mD$ [case 1]

$= | (x_i+1)^2 + y_i^2 - R^2 | - | (x_i+1)^2 + (y_i-1)^2 - R^2 |$

If ($\delta <= 0$)

choose pt (x_i+1, y_i) // horizontal pt

Else

choose pt (x_i+1, y_i-1) // diagonal pt

Cont…

mD is always inside the circle & mH is outside. Thus (mH-mD)

$$\delta = ((x_i+1)^2+y_i^2-R^2) + ((x_i+1)^2+(y_i-1)^2-R^2)$$

$$= 2(x_i+1)^2 + y_i^2 + (y_i-1)^2 - 2R^2$$

$$= [2(x_i+1)2 + (y_i2-2y_i+1) + (y_i-1)2 - 2R2] + 2y_i-1$$

$$= [2(x_i+1)2 + (y_i-1)2 + (y_i-1)2 - 2R2] + 2y_i-1$$

$$= [2(x_i+1)2 + 2(y_i-1)2 - 2R2] + 2y_i-1$$

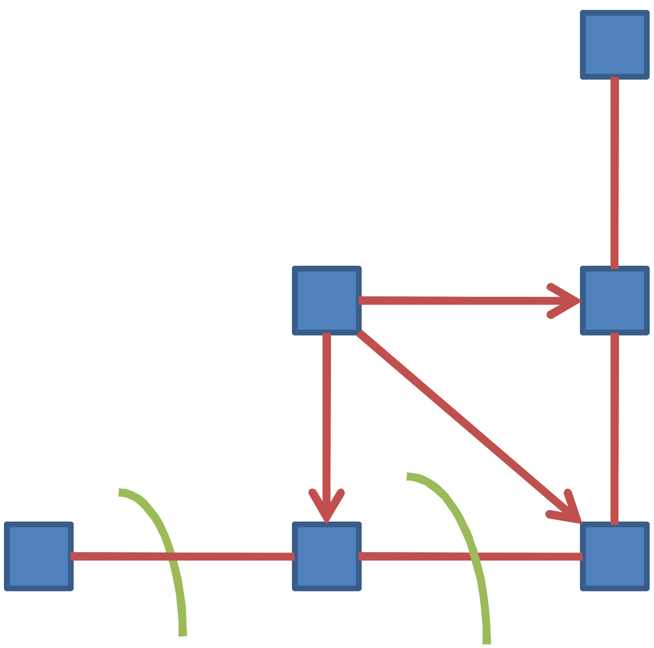
$$= 2 * [(x_i+1)2 + (y_i-1)2 - R2] + 2y_i-1$$

$$= 2 * \Delta_i + 2y_i-1$$

Δ

Cases depends upon position of diagonal point [case 3]

- **$\Delta i = | (xi+1)^2 + (yi-1)^2 - (R)^2 |$**
- **If $\Delta i > 0$ Then diagonal pt is outside the circle**



δ=mD-mV
$= |(xi+1)^2 + (yi-1)^2 - R^2| - |(xi)^2 + (yi-1)^2 - R^2|$

If (δ<=0)
choose pt (xi+1,y1-1) // diagonal pt
Else
choose pt (xi,yi-1)// vertical pt

Cont..

As mD is outside the Circle while mV is inside for case 3, this allows to rewrite (mD-mV)

$$\Delta = ((xi+1)^2+(yi-1)^2-R^2) + ((xi)^2+(yi-1)^2-R^2)$$

to complete the term of (Xi)^2, +& - (2Xi+1)

= (xi+1)2 + xi2 + 2(yi-1)2 -2R2

= [(xi+1)2 + (xi2+2xi+1)+ 2(yi-1)2 -2R2] -2xi-1

= [(xi+1)2 + (xi+1)2+ 2(yi-1)2 -2R2] – 2xi-1

= [2(xi+1)2 + 2(yi-1)2 -2R2] – 2xi-1

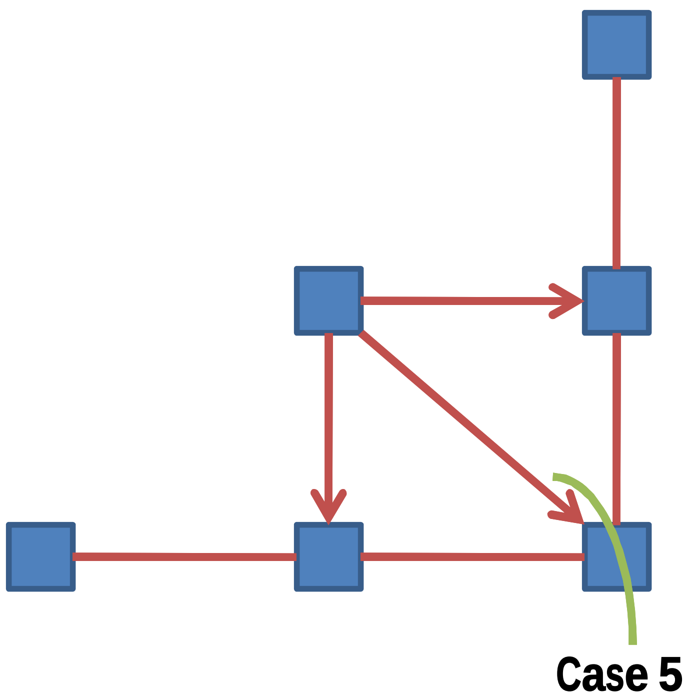
=2 * [(xi+1)2 + (yi-1)2 -R2] – 2xi-1

Δ

= 2 * Δi - 2xi-1

Cases depends upon position of diagonal point [case 5]

- **Δi=| (xi+1)^2 + (yi-1)^2 –(R)^2 |**
- **If Δi = 0 Then diagonal pt is on the circle**



Summary

If $\Delta i < 0$

$\delta = mH - mD$

If ($\delta <= 0$)

choose pt (xi+1,yi) // horizontal pt

Else

choose pt (xi+1,yi-1) // diagonal pt

Else If $\Delta i > 0$

$\delta = mD - mV$

If ($\delta <= 0$)

choose pt (xi+1,yi-1) // diagonal pt

Else

choose pt (xi,yi-1)// vertical pt

Else // $\Delta i = 0$

Choose diagonal pt

Mh

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i$

$$\Delta_{i+1} = (x_{i+1} + 1)^2 + (y_{i+1} - 1)^2 - R^2$$

$$= (x_{i+1})^2 + 1 + 2x_{i+1} + (y_{i+1})^2 + 1 - 2y_{i+1} - R^2$$

$$= (x_i + 1)^2 + 1 + 2x_{i+1} + (y_i)^2 + 1 - 2y_i - R^2$$

$$= \Delta_i + 2x_{i+1} + 1$$

Md

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i - 1$

$$\Delta_{i+1} = (x_{i+1} + 1)^2 + (y_{i+1} - 1)^2 - R^2$$

$$= (x_{i+1})^2 + 1 + 2x_{i+1} + (y_{i+1})^2 + 1 - 2y_{i+1} - R^2$$

$$= (x_i + 1)^2 + 1 + 2x_{i+1} + (y_i - 1)^2 + 1 - 2y_{i+1} - R^2$$

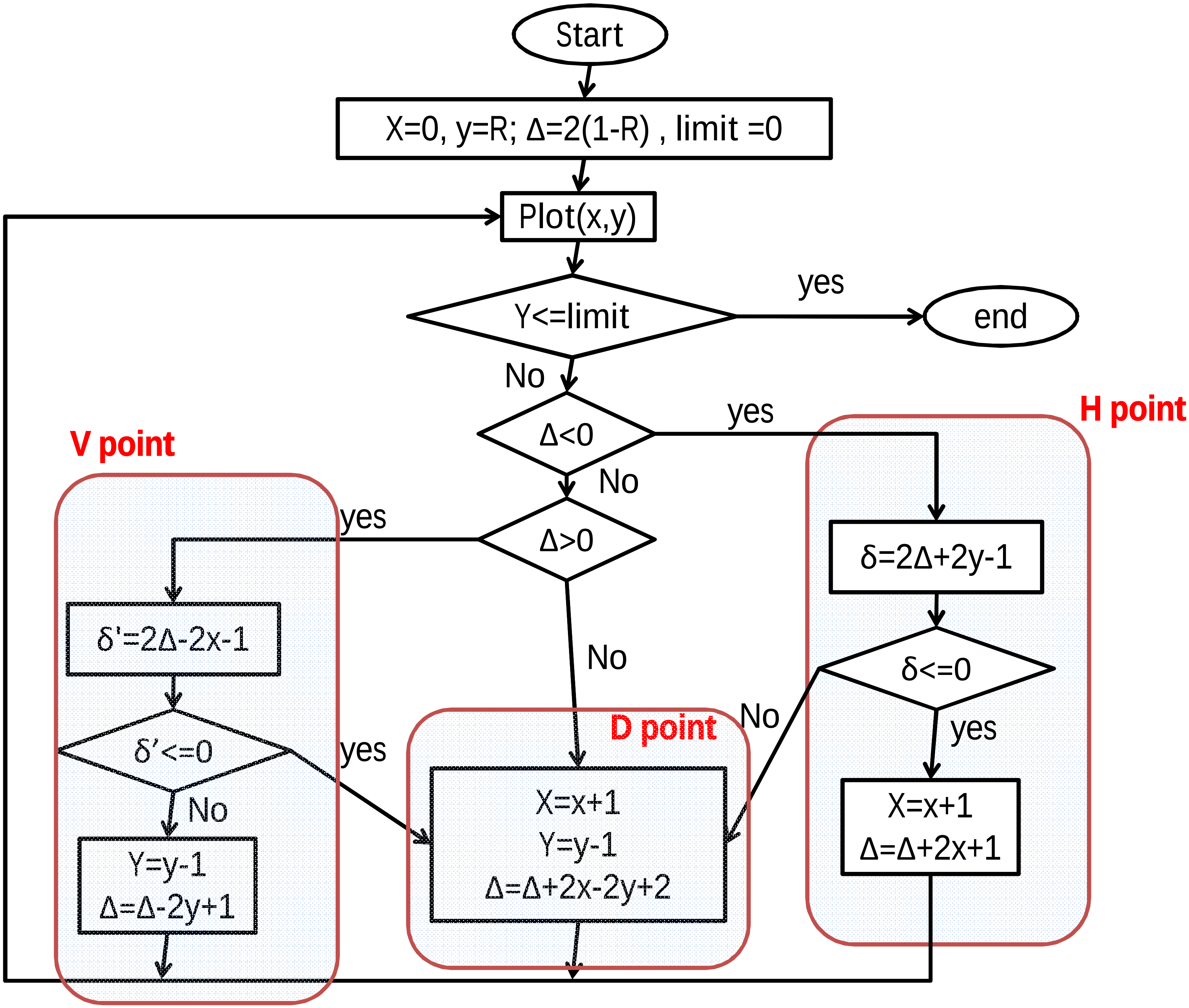
$$= \Delta_i + 2x_{i+1} - 2y_{i+1} + 2$$

Mv

$x_{i+1} = x_i$

$y_{i+1} = y_i - 1$

$$\Delta_{i+1} = (x_{i+1} + 1)^2 + (y_{i+1} - 1)^2 - R^2$$
$$= (x_{i+1})^2 + 1 + 2x_{i+1} + (y_{i+1})^2 + 1 - 2y_{i+1} - R^2$$
$$= (x_i)^2 + 1 + 2x_i + (y_i - 1)^2 + 1 - 2y_{i+1} - R^2$$
$$= \Delta_i - 2y_{i+1} + 1$$



Start
X=0, y=R; Δ=2(1-R) , limit =0
Plot(x,y)
Y<=limit
yes
end
No
Δ<0
yes
H point
No
Δ>0
yes
V point
δ=2Δ+2y-1
δ<=0
δ'=2Δ-2x-1
δ'<=0
No
yes
D point
No
yes
No
X=x+1
Y=y-1
Δ=Δ+2x-2y+2
Y=y-1
Δ=Δ-2y+1
X=x+1
Δ=Δ+2x+1

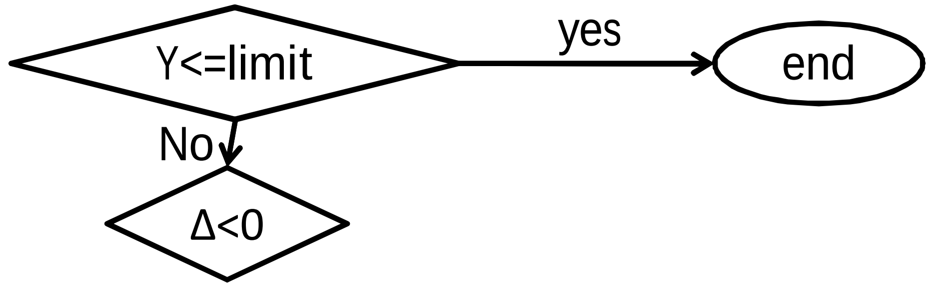
Draw circle for radius =8

$X=0,\ y=R;\ \Delta=2(1-R)\ ,\ limit=0$

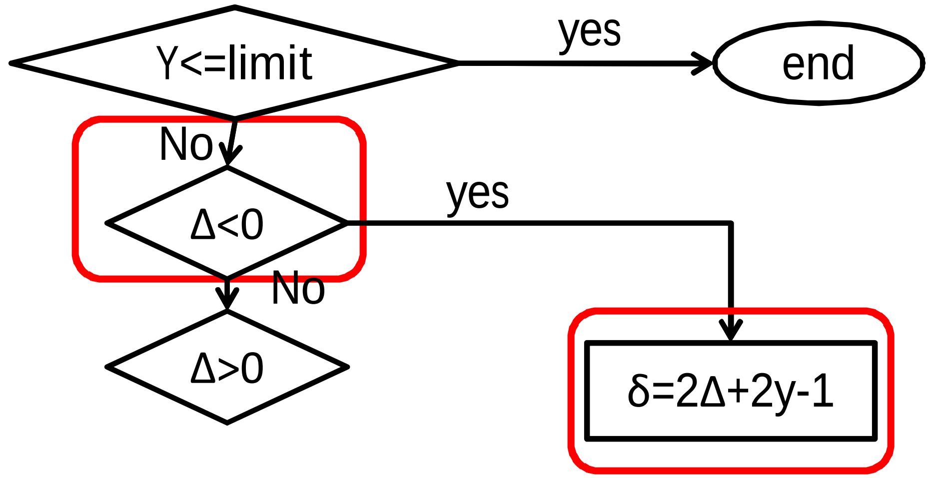
- X=0
- Y=R=8
- Δ=2(1-8)=2*(-7)=-14
- Limit=0

$Plot(x,y)$

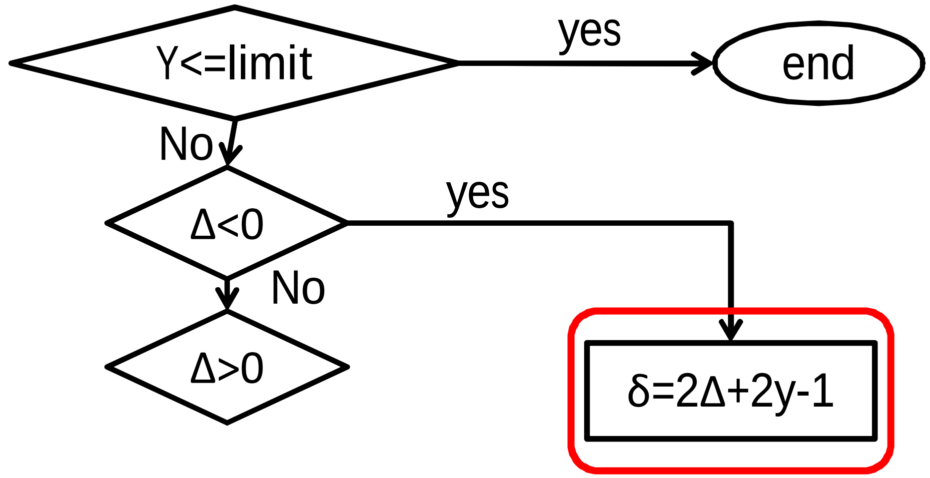
- Plot (0,8)



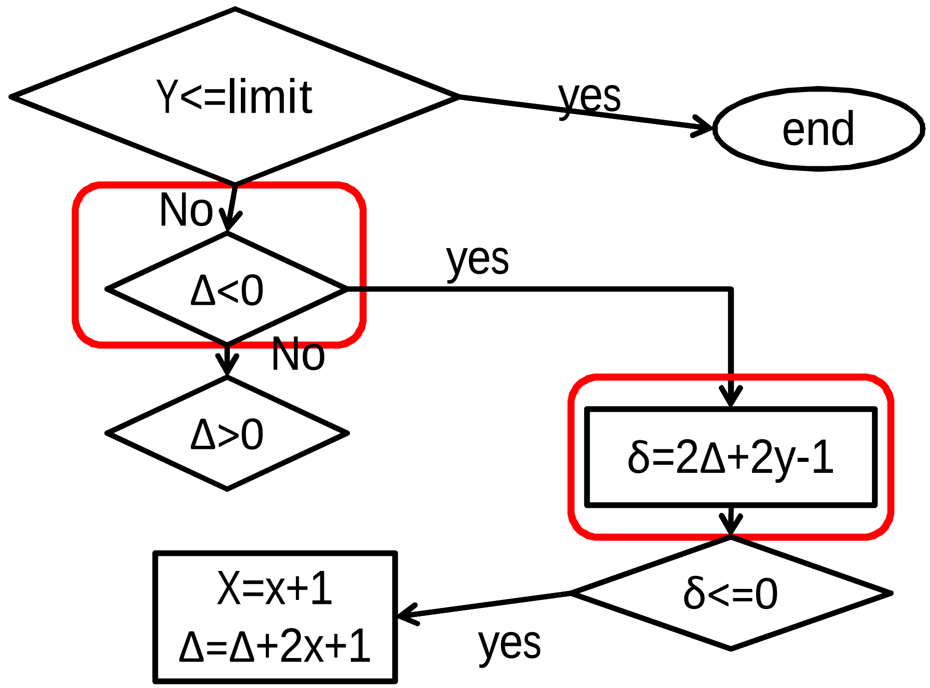
Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8



Sr.no.	Setpixel	Δ	Δ	δ'	x	y
1	(0,8)	-14			0	8



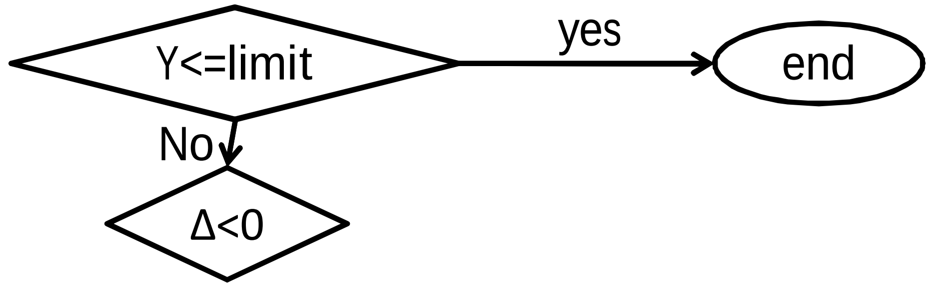
Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
			=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13			



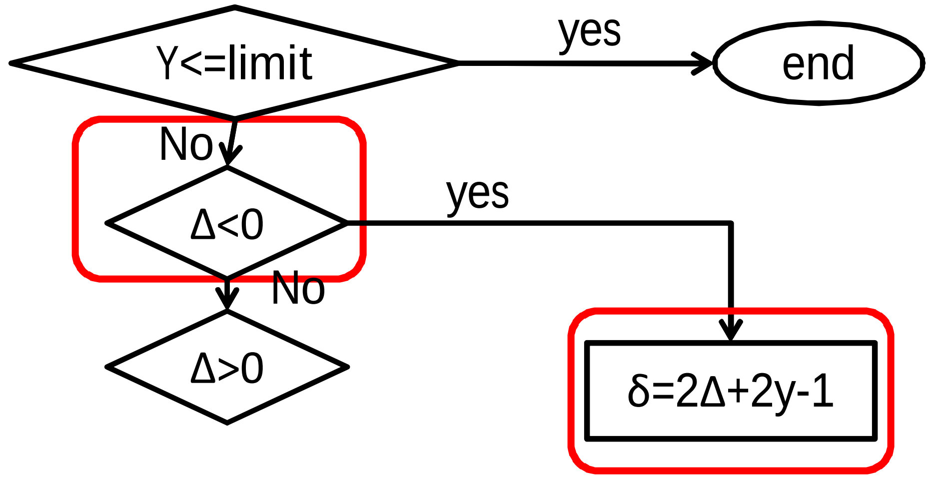
Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
			=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13			
					=0+1=1	
		=-14+(2*1)+1 =-11				

Plot(x,y)

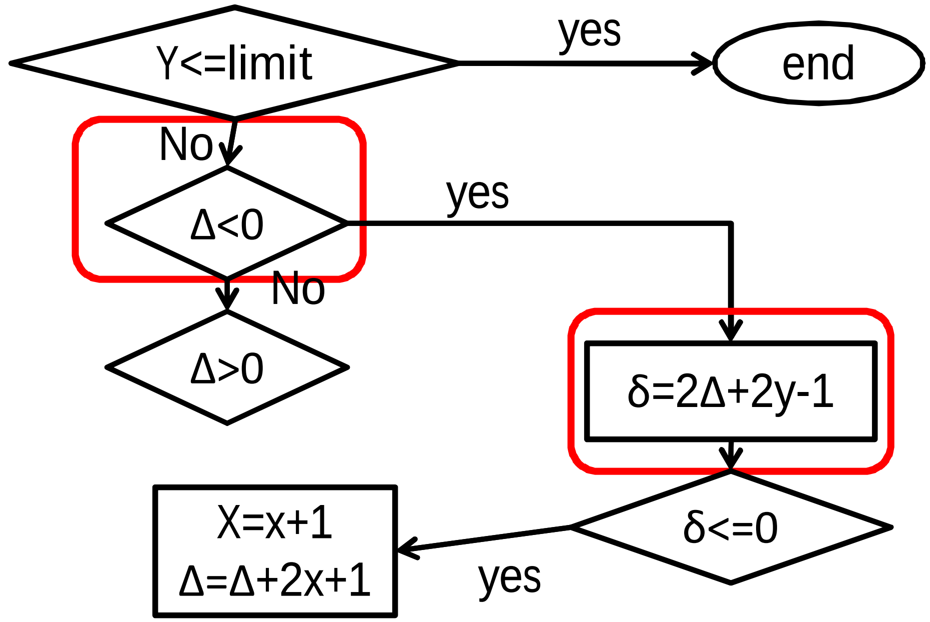
Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
		=-14+(2*1)+1 =-11	=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13		=0+1=1	
2	(1,8)					



Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
		=-14+(2*1)+1 =-14+3=-11	=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13		=0+1=1	
2	(1,8)					



Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
		=-14+(2*1)+1 =-11	=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13		=0+1=1	
2	(1,8)					
			=-22+16-1=-7			



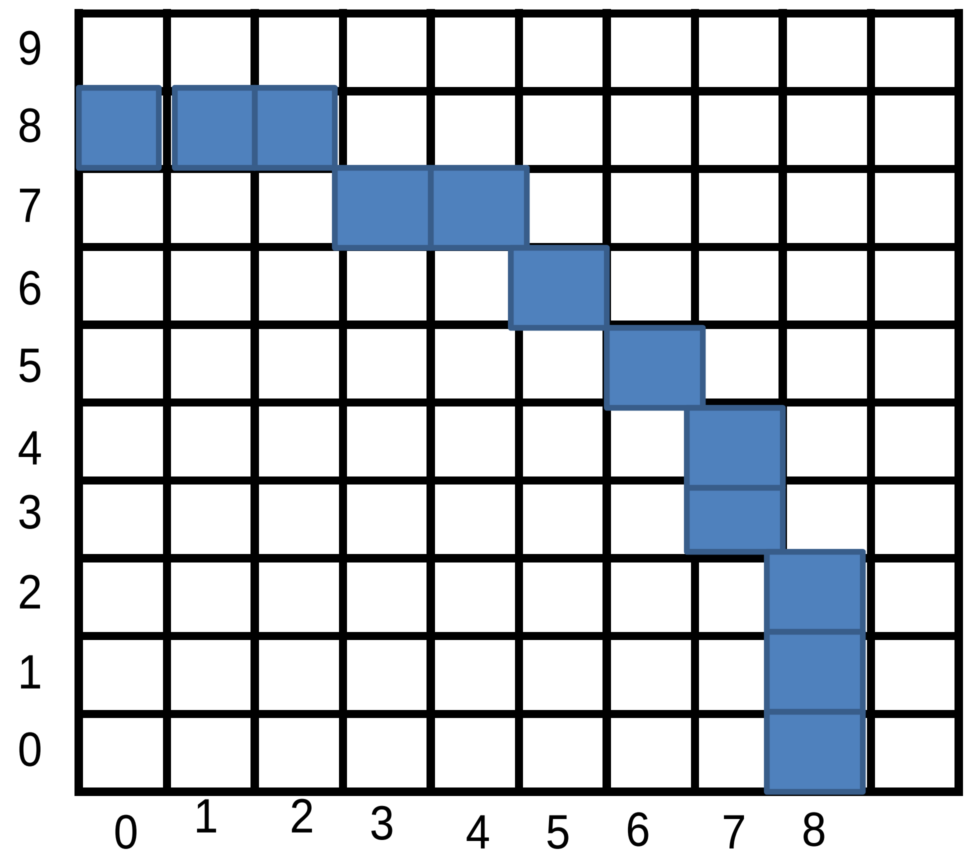
Sr.no.	Setpixel	Δ	δ	δ'	X	y
1	(0,8)	-14			0	8
			=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13		=0+1=1	
		=-14+(2*1)+1 =-11				
2	(1,8)		=-22+16-1=-7			
					=1+1=2	
		=-11+(2*2)+1 =-6				

Plot(x,y)

Sr.no.	Setpixel	Δ	δ	δ'	x	y
1	(0,8)	-14			0	8
		=-14+(2*1)+1 =-11	=2*(-14)+2*8-1 =-28+16-1 =-12-1=-13		=0+1=1	
2	(1,8)					
			=-26+16-1=-9			
					=1+1=2	
		=-11+(2*2)+1 =-6				
3	(2,8)					

Selected pixel

1) 0 8
2) 1 8
3) 2 8
4) 3 7
5) 4 7
6) 5 6
7) 6 5
8) 7 4
9) 7 3
10) 8 2
11) 8 1
12) 8 0



Complete circle generation

- To generate the next part, multiply the coordinate matrix of selected pixel by transformation matrix for reflection through y axis and then by transformation matrix used for reflection through x-axis

Example

- Plot the circle using bresenham’s algo with R=3 & center is (0,0).