## term-2-all

$$[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{+}] = 0$$

$$[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{-}] = 0$$

$$\begin{split} [\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}] = & + X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0} \\ & + \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} + X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0} \\ & - \eta(\sigma)X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} - X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma} \\ & - \eta(\sigma)X_{iN}^{o\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} - X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma} \\ & + \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma} - X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{d\leftarrow-\sigma} \\ & + \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}] = & + X^{\sigma\leftarrow0}_{jN} X^{0\leftarrow0}_{iN} X^{0\leftarrow\sigma}_{iR} \\ & + \eta(\sigma) X^{\sigma\leftarrow0}_{jN} X^{0\leftarrow0}_{iN} X^{-\sigma\leftarrow d}_{iR} + X^{-\sigma\leftarrow d}_{jN} X^{d\leftarrow0}_{iN} X^{0\leftarrow-\sigma}_{iR} \\ & - \eta(\sigma) X^{-\sigma\leftarrow d}_{jN} X^{d\leftarrow0}_{iN} X^{\sigma\leftarrow d}_{iR} - X^{\sigma\leftarrow\sigma}_{iN} X^{0\leftarrow\sigma}_{iR} X^{\sigma\leftarrow0}_{jN} \\ & - \eta(\sigma) X^{\sigma\leftarrow\sigma}_{iN} X^{-\sigma\leftarrow d}_{iN} X^{\sigma\leftarrow0}_{jN} - X^{-\sigma\leftarrow\sigma}_{iN} X^{0\leftarrow-\sigma}_{iR} X^{\sigma\leftarrow0}_{jN} \\ & + \eta(\sigma) X^{-\sigma\leftarrow\sigma}_{iN} X^{\sigma\leftarrow d}_{iR} X^{\sigma\leftarrow0}_{jN} \end{split}$$

$$\begin{split} [\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}] = & + X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow d} \\ & + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow \sigma} + X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow d} \\ & - \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow -\sigma} - X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma} \\ & - \eta(\sigma) X_{iN}^{d \leftarrow d} X_{iR}^{0 \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} - X_{iN}^{d \leftarrow 0} X_{iR}^{\sigma \leftarrow d} X_{jN}^{0 \leftarrow \sigma} \\ & + \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{iR}^{0 \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} \\ \end{split}$$

$$\begin{split} [\hat{H}_{t,Ni}^{0},\hat{H}_{t,cross}^{-\oplus}] = & + X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow\sigma} \\ & - \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow d} X_{iR}^{-\sigma\leftarrow0} + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow d} X_{iR}^{d\leftarrow-\sigma} \\ & + \eta(\sigma) X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow d} X_{iR}^{\sigma\leftarrow0} - X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{d\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} \\ & - \eta(\sigma) X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} - X_{iN}^{\sigma\leftarrow-\sigma} X_{iR}^{d\leftarrow\sigma} X_{jN}^{-\sigma\leftarrow d} \\ & + \eta(\sigma) X_{iN}^{\sigma\leftarrow-\sigma} X_{iR}^{-\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$[\hat{H}_{t,Ni}^{0},\hat{H}_{t,R}]=0$$

$$\begin{split} [\hat{H}^+_{t,Ni},\hat{H}^-_{t,Ni}] = & + X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} \\ & - X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} + X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} \\ & - X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} \\ & - X_{jN}^{\sigma \leftarrow \sigma} X_{iN}^{-\sigma \leftarrow -\sigma} + X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow -\sigma} \\ & + X_{jN}^{-\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow -\sigma} \end{split}$$

$$[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{0\oplus}] = 0$$

$$\begin{split} [\hat{H}^{+}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}] = & + \eta(\sigma)X^{d\leftarrow 0}_{iN}X^{0\leftarrow \sigma}_{jN}X^{0\leftarrow -\sigma}_{iR} \\ & - X^{d\leftarrow 0}_{iN}X^{0\leftarrow \sigma}_{jN}X^{\sigma\leftarrow d}_{iR} + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iN}X^{0\leftarrow \sigma}_{iR} \\ & + X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow 0}_{iN}X^{-\sigma\leftarrow d}_{iR} - \eta(\sigma)X^{\sigma\leftarrow \sigma}_{iN}X^{0\leftarrow \sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \\ & - X^{\sigma\leftarrow \sigma}_{iN}X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow -\sigma}_{jN} - \eta(\sigma)X^{-\sigma\leftarrow \sigma}_{iN}X^{0\leftarrow -\sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \\ & + X^{-\sigma\leftarrow \sigma}_{iN}X^{\sigma\leftarrow d}_{iR}X^{d\leftarrow -\sigma}_{jN} - \eta(\sigma)X^{-\sigma\leftarrow \sigma}_{iN}X^{0\leftarrow -\sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \end{split}$$

$$[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{+\ominus}] = 0$$

$$\begin{split} [\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{-\oplus}] = & + \eta(\sigma)X_{iN}^{d \leftarrow d}X_{jN}^{0 \leftarrow \sigma}X_{iR}^{d \leftarrow -\sigma} \\ & + X_{iN}^{d \leftarrow d}X_{jN}^{0 \leftarrow \sigma}X_{iR}^{\sigma \leftarrow 0} + \eta(\sigma)X_{jN}^{d \leftarrow -\sigma}X_{iN}^{0 \leftarrow d}X_{iR}^{d \leftarrow \sigma} \\ & - X_{jN}^{d \leftarrow -\sigma}X_{iN}^{0 \leftarrow d}X_{iR}^{-\sigma \leftarrow 0} - \eta(\sigma)X_{iN}^{-\sigma \leftarrow -\sigma}X_{iR}^{d \leftarrow -\sigma}X_{jN}^{0 \leftarrow \sigma} \\ & - X_{iN}^{-\sigma \leftarrow -\sigma}X_{iR}^{\sigma \leftarrow 0}X_{jN}^{0 \leftarrow \sigma} - \eta(\sigma)X_{iN}^{-\sigma \leftarrow -\sigma}X_{iR}^{d \leftarrow -\sigma}X_{jN}^{0 \leftarrow \sigma} \\ & + X_{iN}^{\sigma \leftarrow -\sigma}X_{iR}^{-\sigma \leftarrow 0}X_{jN}^{0 \leftarrow \sigma} \end{split}$$

$$[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,R}] = 0$$

$$\begin{split} [\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}] &= + \, \eta(\sigma) X_{iN}^{\sigma\leftarrow\sigma} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow0} \\ &\quad + X_{iN}^{\sigma\leftarrow\sigma} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow-\sigma} + \eta(\sigma) X_{iN}^{\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{-\sigma\leftarrow0} \\ &\quad - X_{iN}^{\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow\sigma} - \eta(\sigma) X_{iN}^{0\leftarrow0} X_{iR}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} \\ &\quad - X_{iN}^{0\leftarrow0} X_{iR}^{d\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{0\leftarrow d} X_{iR}^{-\sigma\leftarrow0} X_{jN}^{\sigma\leftarrow0} \\ &\quad + X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$\begin{split} [\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}] &= + \, \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{-\sigma\leftarrow-d} \\ &\quad + X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{0\leftarrow\sigma} + \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow-d} \\ &\quad - X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{0\leftarrow-\sigma} - \eta(\sigma) X_{iN}^{d\leftarrow d} X_{iR}^{-\sigma\leftarrow d} X_{jN}^{\sigma\leftarrow0} \\ &\quad - X_{iN}^{d\leftarrow d} X_{iR}^{0\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} - \eta(\sigma) X_{iN}^{d\leftarrow0} X_{iR}^{\sigma\leftarrow d} X_{jN}^{-\sigma\leftarrow d} \\ &\quad + X_{iN}^{d\leftarrow0} X_{iR}^{0\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$[\hat{H}_{t,Ni}^-, \hat{H}_{t,cross}^{-\oplus}] = 0$$

$$[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,R}] = 0$$

$$\begin{split} [\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}] = & + X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma} \\ & + X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow d} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow0} \\ & - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow-\sigma} + X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} \end{split}$$

$$[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}]=0$$

$$[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}] = -X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow 0}$$

$$\begin{split} [\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}] &= + X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma} + X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{d\leftarrow\sigma} + X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow}X_{jR}^{\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow}X_{jR}^{d\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow}X_{jR}^{\sigma\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma} \\ &- X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow-d} - X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{0\leftarrow\sigma} \\ &- X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} - \eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow}X_{iN}^{0\leftarrow\sigma} + X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow\sigma} \end{split}$$

$$[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{+\ominus}] = + \, X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow d}$$

$$[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}] = 0$$

$$\begin{split} [\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,R}] &= + X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow0} X_{jR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow0} X_{jR}^{-\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow d} X_{jR}^{-\sigma\leftarrow0} \\ &+ X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow0} X_{jR}^{d\leftarrow\sigma} + X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{0\leftarrow\sigma} \\ &+ X_{iN}^{\sigma\leftarrow0} X_{iR}^{o\leftarrow0} X_{jR}^{d\leftarrow\sigma} + X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{-\sigma\leftarrow d} - X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{\sigma\leftarrow d} - X_{iR}^{\sigma\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow0} \\ &- \eta(\sigma) X_{iR}^{\sigma\leftarrow\sigma} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{\sigma\leftarrow0} - X_{iR}^{d\leftarrow d} X_{jR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow0} \\ &- \eta(\sigma) X_{iR}^{d\leftarrow d} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{\sigma\leftarrow0} - X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{o\leftarrow0} \\ &+ \eta(\sigma) X_{iR}^{-\sigma\leftarrow\sigma} X_{jR}^{\sigma\leftarrow d} X_{iN}^{\sigma\leftarrow0} - \eta(\sigma) X_{jR}^{-\sigma\leftarrow0} X_{iR}^{o\leftarrow0} \\ &+ X_{jR}^{d\leftarrow\sigma} X_{iR}^{o\leftarrow d} X_{iN}^{o\leftarrow0} \end{split}$$

$$\begin{split} [\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\oplus}] = & + X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} \\ & + X_{iN}^{d \leftarrow d} X_{iR}^{0 \leftarrow 0} - X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow d} \\ & - X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} + X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} \end{split}$$

$$\begin{split} [\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}] &= + \, \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jR}^{0\leftarrow \sigma} \\ &\quad + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow -\sigma} X_{iN}^{0\leftarrow -\sigma} \\ &\quad + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow -\sigma} X_{iN}^{-\sigma\leftarrow -\sigma} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow -\sigma} \\ &\quad + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jR}^{\sigma\leftarrow -\sigma} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow -\sigma} \\ &\quad + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow 0} X_{jR}^{-\sigma\leftarrow -\sigma} - X_{iN}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow -\sigma} \\ &\quad + \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow d} X_{jR}^{d\leftarrow -\sigma} - \eta(\sigma) X_{iR}^{\sigma\leftarrow -\sigma} X_{jR}^{0\leftarrow -\sigma} X_{iN}^{d\leftarrow -\sigma} \\ &\quad - X_{iR}^{\sigma\leftarrow \sigma} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} - \eta(\sigma) X_{iR}^{-\sigma\leftarrow \sigma} X_{jR}^{0\leftarrow -\sigma} X_{iN}^{d\leftarrow -\sigma} \\ &\quad - X_{iR}^{d\leftarrow d} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} - \eta(\sigma) X_{iR}^{-\sigma\leftarrow \sigma} X_{jR}^{0\leftarrow -\sigma} X_{iN}^{d\leftarrow -\sigma} \\ &\quad + X_{iR}^{-\sigma\leftarrow \sigma} X_{jR}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} - X_{jR}^{-\sigma\leftarrow 0} X_{iR}^{d\leftarrow -\sigma} \\ &\quad + \eta(\sigma) X_{iR}^{d\leftarrow \sigma} X_{iR}^{0\leftarrow d} X_{iN}^{d\leftarrow -\sigma} \end{split}$$

$$\begin{split} [\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}] &= +\eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow d}X_{jR}^{\sigma\leftarrow 0}\\ &+ X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow d}X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}\\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow \sigma}X_{jR}^{\sigma\leftarrow 0}\\ &+ X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0}\\ &+ X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow \sigma}X_{jR}^{d\leftarrow \sigma} - \eta(\sigma)X_{jR}^{\sigma\leftarrow 0}X_{iR}^{o\leftarrow 0}X_{iN}^{-\sigma\leftarrow d}\\ &- X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d} - X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow 0}X_{iN}^{-\sigma\leftarrow d}\\ &- \eta(\sigma)X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d} + X_{iR}^{d\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d}\\ &- X_{jR}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d} + \chi_{iR}^{d\leftarrow 0}X_{jR}^{o\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d}\\ &- \eta(\sigma)X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow d} + \eta(\sigma)X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d}\\ &- X_{iR}^{d\leftarrow \sigma}X_{iR}^{\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d}\\ &- X_{iR}^{d\leftarrow \sigma}X_{iR}^{\sigma\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow d} \end{pmatrix}$$