Values of terms used to calculate the commutators

$$\hat{H}^{0}_{t,Ni} = -t \sum_{\langle i,j\rangle\sigma} (X^{\sigma\leftarrow 0}_{iN} X^{0\leftarrow\sigma}_{jN} + X^{-\sigma\leftarrow d}_{iN} X^{d\leftarrow -\sigma}_{jN} + h.c) \tag{1}$$

$$\hat{H}^{+}_{t,Ni} = -t \sum_{\langle i,j\rangle\sigma} \eta(\sigma)(X^{d\leftarrow -\sigma}_{iN} X^{0\leftarrow\sigma}_{jN} + X^{d\leftarrow -\sigma}_{jN} X^{0\leftarrow\sigma}_{iN}) \tag{2}$$

$$\hat{H}^{-}_{t,Ni} = -t \sum_{\langle i,j\rangle\sigma} \eta(\sigma)(X^{\sigma\leftarrow 0}_{iN} X^{-\sigma\leftarrow d}_{jN} + X^{\sigma\leftarrow 0}_{jN} X^{-\sigma\leftarrow d}_{iN}) \tag{3}$$

$$\hat{H}^{0\oplus}_{t,cross} = -t_{\perp} \sum_{i\sigma} X^{0\leftarrow\sigma}_{iN} (X^{\sigma\leftarrow 0}_{iR} + \eta(\sigma) X^{d\leftarrow -\sigma}_{iR}) \tag{4}$$

$$\hat{H}^{0\ominus}_{t,cross} = -t_{\perp} \sum_{i\sigma} X^{\sigma\leftarrow 0}_{iN} (X^{0\leftarrow\sigma}_{iR} + \eta(\sigma) X^{-\sigma\leftarrow d}_{iR}) \tag{5}$$

$$\hat{H}^{+\ominus}_{t,cross} = -t_{\perp} \sum_{i\sigma} X^{d\leftarrow -\sigma}_{iN} (X^{-\sigma\leftarrow d}_{iR} + \eta(\sigma) X^{0\leftarrow\sigma}_{iR}) \tag{6}$$

$$\hat{H}^{-\oplus}_{t,cross} = -t_{\perp} \sum_{i\sigma} X^{-\sigma\leftarrow d}_{iN} (X^{d\leftarrow -\sigma}_{iR} + \eta(\sigma) X^{\sigma\leftarrow 0}_{iR}) \tag{7}$$

$$\hat{H}_{t,R} = -t \sum_{\langle i,j\rangle\sigma} (X^{\sigma\leftarrow 0}_{iR} + \eta(\sigma) X^{d\leftarrow -\sigma}_{iR})(X^{0\leftarrow\sigma}_{jR} + \eta(\sigma) X^{-\sigma\leftarrow d}_{jR}) + h.c \tag{8}$$

where

$$\eta(\sigma) = \begin{cases} 0 & \text{if} \quad \sigma = \uparrow \\ 1 & \text{if} \quad \sigma = \downarrow \end{cases}$$