term-3-all

$$[[\hat{H}^0_{t,Ni}, \hat{H}^+_{t,Ni}], \hat{H}^0_{t,Ni}] = 0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}^+_{t,Ni}],\hat{H}^+_{t,Ni}] = 0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}^+_{t,Ni}],\hat{H}^-_{t,Ni}]=0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}^{+}_{t,Ni}],\hat{H}^{0\oplus}_{t,cross}]=0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}^{+}_{t,Ni}],\hat{H}^{0\ominus}_{t,cross}]=0$$

$$[[\hat{H}^{0}_{t,Ni}, \hat{H}^{+}_{t,Ni}], \hat{H}^{+\ominus}_{t,cross}] = 0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{+}], \hat{H}_{t,cross}^{-\oplus}] = 0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}^+_{t,Ni}],\hat{H}_{t,R}]=0$$

$$[[\hat{H}^0_{t,Ni}, \hat{H}^-_{t,Ni}], \hat{H}^0_{t,Ni}] = 0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}^-_{t,Ni}],\hat{H}^+_{t,Ni}]=0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}^-_{t,Ni}],\hat{H}^-_{t,Ni}]=0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}^{-}_{t,Ni}],\hat{H}^{0\oplus}_{t,cross}]=0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{-}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}^{-}_{t,Ni}],\hat{H}^{+\ominus}_{t,cross}]=0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{-}], \hat{H}_{t,cross}^{-\oplus}] = 0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{-}], \hat{H}_{t,R}] = 0$$

$$\begin{split} [[\hat{H}^0_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^0_{t,Ni}] = & -X^{0\leftarrow\sigma}_{iN}X^{\sigma\leftarrow0}_{iR}X^{0\leftarrow0}_{jN} \\ & -\eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR}X^{0\leftarrow0}_{jN} - X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR}X^{d\leftarrow d}_{jN} \\ & +\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-d}_{jN} - X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{\sigma\leftarrow0}_{iR} \\ & -\eta(\sigma)X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR} - X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR} \\ & +\eta(\sigma)X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR} - X^{d\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} \end{split}$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,cross}^{0\oplus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{-}_{t,Ni}] = & + \eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow 0}_{jN}X^{-\sigma\leftarrow 0}_{iR} \\ & - \eta(\sigma)X^{\sigma\leftarrow 0}_{iN}X^{0\leftarrow d}_{jN}X^{-\sigma\leftarrow 0}_{iR} - X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow 0}_{jN}X^{d\leftarrow \sigma}_{iR} \\ & + X^{\sigma\leftarrow 0}_{iN}X^{0\leftarrow d}_{jN}X^{d\leftarrow \sigma}_{iR} + \eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{-\sigma\leftarrow \sigma}_{jN}X^{-\sigma\leftarrow 0}_{iR} \\ & - X^{\sigma\leftarrow d}_{iN}X^{-\sigma\leftarrow \sigma}_{jN}X^{d\leftarrow \sigma}_{iR} - \eta(\sigma)X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow \sigma}_{jN}X^{-\sigma\leftarrow 0}_{iR} \\ & + X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow -\sigma}_{jN}X^{d\leftarrow \sigma}_{iR} - \eta(\sigma)X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow \sigma}_{jN}X^{-\sigma\leftarrow 0}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{0\oplus}_{t,cross}] = & -\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{0\leftarrow-\sigma}_{jN}\\ & -\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{0\leftarrow-\sigma}_{jN}\end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & + X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} \\ & + X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} + X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow-\sigma} \\ & + X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} + X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow0} X_{jN}^{0\leftarrow\sigma} \\ & + X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{0\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow-d} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{d\leftarrow-\sigma} \\ & + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{0\leftarrow0} X_{jN}^{d\leftarrow-\sigma} - X_{iN}^{-\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow-\sigma} X_{jN}^{0\leftarrow\sigma} \\ & + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow0} X_{jN}^{d\leftarrow-\sigma} - X_{iN}^{-\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow-\sigma} X_{jN}^{0\leftarrow\sigma} \\ & + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & -\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow-\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & - \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} + \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ & + \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow0}_{iR}X^{0\leftarrow\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{-\oplus}_{t,cross}] = & -X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow\sigma}_{jN}X^{d\leftarrow0}_{iR} \\ & -X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow\sigma}_{jN}X^{d\leftarrow0}_{iR} \end{split}$$

```
[[\hat{H}^0_{t.Ni},\hat{H}^{0\oplus}_{t.cross}],\hat{H}_{t.R}] = + X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0\leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow \sigma}
                                                                                     +~\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma}+X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow0}
                                                                                      - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} + X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}
                                                                                      + \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}
                                                                                      -X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow d}+X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0}
                                                                                      + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} + X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}
                                                                                      - \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}
                                                                                      - X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{-\sigma \leftarrow d} + X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{-\sigma \leftarrow 0}
                                                                                     - \, \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow \sigma} - X_{iN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow 0}
                                                                                      - \eta(\sigma) X_{iN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} - X_{iN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{-\sigma \leftarrow 0}
                                                                                     +~\eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma}-X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow d}X_{jN}^{0\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0}
                                                                                      - \eta(\sigma) X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}
                                                                                      + \ X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} - X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow 0}
                                                                                     - \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow -\sigma} - X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}
                                                                                     + \ \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} + \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}
                                                                                      + X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} - X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}
                                                                                     + \ \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow \sigma} X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}
                                                                                     +~\eta(\sigma)X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}-X_{iR}^{d\leftarrow0}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                     + \ X_{iR}^{d\leftarrow 0} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{0\leftarrow d} X_{jN}^{d\leftarrow -\sigma} - X_{jR}^{\sigma\leftarrow 0} X_{iR}^{0\leftarrow 0} X_{iN}^{\sigma\leftarrow\sigma} X_{jN}^{0\leftarrow\sigma}
                                                                                     +~X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}-X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                     +\,X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}+X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}
                                                                                      -X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}-\eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                     +~\eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}-\eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                     +~\eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}+\eta(\sigma)X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}
                                                                                      - \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} + \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma}
                                                                                      - \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} - X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma}
                                                                                     +~X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow d}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}-X_{jR}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                     + \ X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} + X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma}
                                                                                      -X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma}-X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}
                                                                                     +\,X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}+\eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow0}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}
                                                                                      - \, \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} - \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma}
                                                                                     + \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} + \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}
                                                                                      -\eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{iN}^{d\leftarrow-\sigma}
```

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = & -X^{\sigma\leftarrow0}_{iN}X^{0\leftarrow\sigma}_{iR}X^{\sigma\leftarrow\sigma}_{jN} \\ & -\eta(\sigma)X^{\sigma\leftarrow0}_{iN}X^{\sigma\leftarrow0}_{iR}X^{\sigma\leftarrow\sigma}_{jN} - X^{\sigma\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR}X^{\sigma\leftarrow\sigma}_{jN} \\ & +\eta(\sigma)X^{-\sigma\leftarrow0}_{iN}X^{\sigma\leftarrow0}_{iR}X^{\sigma\leftarrow\sigma}_{jN} - X^{\sigma\leftarrow0}_{iN}X^{0\leftarrow0}_{jN}X^{0\leftarrow\sigma}_{iR} \\ & -\eta(\sigma)X^{\sigma\leftarrow0}_{iN}X^{0\leftarrow0}_{jN}X^{-\sigma\leftarrow\sigma}_{iR} - X^{-\sigma\leftarrow0}_{iN}X^{d\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iR} \\ & +\eta(\sigma)X^{-\sigma\leftarrow0}_{iN}X^{d\leftarrow0}_{jN}X^{\sigma\leftarrow\sigma}_{iR} - X^{\sigma\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{jN}X^{o\leftarrow\sigma}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{+}_{t,Ni}] = & + \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} \\ & - \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} - X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} \\ & + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{d\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma} \\ & - X_{iN}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow d} - \eta(\sigma)X_{jN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} \\ & + X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d} \end{split}$$

$$[[\hat{H}_{t,Ni}^{0},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{-}]=0$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & + X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iR}^{0\leftarrow0} \\ & + X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow-\sigma} X_{iR}^{-\sigma\leftarrow\sigma} \\ & + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow-\sigma} X_{iR}^{0\leftarrow0} - X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow\sigma} X_{iR}^{\sigma\leftarrow-\sigma} \\ & + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow-\sigma} X_{iR}^{\sigma\leftarrow\sigma} + X_{iN}^{0\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ & + X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow-d} X_{jN}^{\sigma\leftarrow0} + X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow-\sigma} X_{jN}^{\sigma\leftarrow0} \\ & + X_{iN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow-d} X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & -\eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d}X_{jN}^{\sigma\leftarrow0} \\ & -\eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d}X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & -X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow d}_{iR}X^{\sigma\leftarrow 0}_{jN}\\ & -X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow d}_{iR}X^{\sigma\leftarrow 0}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{-\oplus}_{t,cross}] = & -\eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow 0} \\ & + \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{0\leftarrow 0}X_{jN}^{\sigma\leftarrow 0} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow 0} \\ & -\eta(\sigma)X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{\sigma\leftarrow 0}X_{iR}^{d\leftarrow d}X_{jN}^{-\sigma\leftarrow d} \\ & + \eta(\sigma)X_{iN}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{1}, \hat{H}_{t,cross}^{1}], \hat{H}_{t,R}] &= + X_{jN}^{ro-2} X_{ii}^{10} X_{ji}^{10} X_{ii}^{10} X_{ji}^{10} - X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} - X_{ji}^{10} - X_{ji}^{10} X_{ji}^{10} - X_{ji}^$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = &-X^{d\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow d}_{iR}X^{d\leftarrow d}_{jN} \\ &-\eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow\sigma}_{iR}X^{d\leftarrow d}_{jN} - X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow d}_{iR}X^{0\leftarrow 0}_{jN} \\ &+\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR}X^{0\leftarrow 0}_{jN} - X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow d}_{iR} \\ &-\eta(\sigma)X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow\sigma}_{iR} - X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow d}_{iR} \\ &+\eta(\sigma)X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR} \\ \end{split}$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{-}_{t,Ni}] = & + \eta(\sigma)X^{-\sigma\leftarrow0}_{iN}X^{d\leftarrow d}_{jN}X^{\sigma\leftarrow d}_{iR} \\ & - \eta(\sigma)X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{jN}X^{\sigma\leftarrow d}_{iR} - X^{-\sigma\leftarrow 0}_{iN}X^{d\leftarrow d}_{jN}X^{0\leftarrow -\sigma}_{iR} \\ & + X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{jN}X^{0\leftarrow -\sigma}_{iR} + \eta(\sigma)X^{\sigma\leftarrow \sigma}_{jN}X^{-\sigma\leftarrow 0}_{iN}X^{\sigma\leftarrow d}_{iR} \\ & - X^{\sigma\leftarrow \sigma}_{jN}X^{-\sigma\leftarrow 0}_{iN}X^{0\leftarrow -\sigma}_{iR} - \eta(\sigma)X^{-\sigma\leftarrow \sigma}_{jN}X^{\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow d}_{iR} \\ & + X^{-\sigma\leftarrow \sigma}_{jN}X^{\sigma\leftarrow 0}_{iN}X^{0\leftarrow -\sigma}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{0\oplus}_{t,cross}] = & -\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ & + \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} + \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ & - \eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow-\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = &-X^{-\sigma\leftarrow0}_{iN}X^{d\leftarrow-\sigma}_{jN}X^{0\leftarrow d}_{iR}\\ &-X^{-\sigma\leftarrow0}_{iN}X^{d\leftarrow-\sigma}_{jN}X^{0\leftarrow d}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & -\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & -\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{-\ominus}_{t,cross}] = &+ X_{iN}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} \\ &+ X_{iN}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} + X_{iN}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} \\ &+ X_{iN}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} + X_{iN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma} \\ &+ X_{iN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} - X_{iN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} \\ &+ X_{iN}^{\sigma \leftarrow 0} X_{iR}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} - X_{iN}^{\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} \\ &+ X_{iN}^{\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} - X_{iN}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} \end{split}$$

$$\begin{split} [|\hat{H}_{t,Ni}^{h},\hat{H}_{t,cross}^{h}],\hat{H}_{t,R}] &= + \eta(\sigma)X_{iN}^{-\sigma} - \sigma X_{jN}^{-\sigma} - \sigma X_{jR}^{-\sigma} - \sigma$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = &-X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow -\sigma}_{iR}X^{-\sigma\leftarrow -\sigma}_{jN} \\ &-\eta(\sigma)X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow 0}_{iR}X^{-\sigma\leftarrow -\sigma}_{jN} - X^{\sigma\leftarrow d}_{iN}X^{d\leftarrow \sigma}_{iR}X^{-\sigma\leftarrow -\sigma}_{jN} \\ &+\eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{-\sigma\leftarrow 0}_{iN}X^{-\sigma\leftarrow -\sigma}_{jN} - X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow 0}_{jN}X^{d\leftarrow \sigma}_{iR} \\ &+\eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow 0}_{jN}X^{-\sigma\leftarrow 0}_{iR} - X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow d}_{jN}X^{d\leftarrow -\sigma}_{iR} \\ &-\eta(\sigma)X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow d}_{jN}X^{\sigma\leftarrow 0}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{+}_{t,Ni}] = & + \eta(\sigma)X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} \\ & - \eta(\sigma)X^{\sigma\leftarrow-\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} - X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR} \\ & + X^{\sigma\leftarrow-\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR} + \eta(\sigma)X^{d\leftarrow}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} \\ & - X^{d\leftarrow}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR} - \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iR} \\ & + X^{d\leftarrow-\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow0}_{iR} \end{split}$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{-}_{t,Ni}] = 0$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni}, \hat{H}^{-\oplus}_{t,cross}], \hat{H}^{0\oplus}_{t,cross}] = & -X^{0\leftarrow -\sigma}_{iN} X^{d\leftarrow 0}_{iR} X^{-\sigma\leftarrow d}_{jN} \\ & -X^{0\leftarrow -\sigma}_{iN} X^{d\leftarrow 0}_{iR} X^{-\sigma\leftarrow d}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = &-\eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &+\eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{d\leftarrow d}X_{jN}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &-\eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{0\leftarrow0}X_{jN}^{\sigma\leftarrow0} \\ &+\eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & + X^{\sigma\leftarrow0}_{jN}X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow d}_{iR} \\ & - X^{\sigma\leftarrow0}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR} + X^{\sigma\leftarrow0}_{jN}X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow-\sigma}_{iR} \\ & + X^{-\sigma\leftarrow d}_{jN}X^{d\leftarrow-\sigma}_{iN}X^{d\leftarrow d}_{iR} + X^{-\sigma\leftarrow d}_{jN}X^{d\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR} \\ & - X^{-\sigma\leftarrow d}_{jN}X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow-\sigma}_{iR} + X^{d\leftarrow-\sigma}_{jN}X^{-\sigma\leftarrow-\sigma}_{iR}X^{-\sigma\leftarrow d}_{jN} \\ & + X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow0}_{iR}X^{-\sigma\leftarrow d}_{jN} + X^{d\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{-\sigma\leftarrow d}_{jN} \\ & + X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow0}_{iR}X^{-\sigma\leftarrow d}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{-\oplus}_{t,cross}] = & -\eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{iR}X^{-\sigma\leftarrow d}_{jN} \\ & -\eta(\sigma)X^{\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{iR}X^{-\sigma\leftarrow d}_{jN} \end{split}$$

 $[[\hat{H}_{t,Ni}^{0},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,R}] = + X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow\sigma}$ $+~\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow0}X_{jR}^{-\sigma\leftarrow d}-\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow d}X_{jR}^{-\sigma\leftarrow0}$ $+~X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow d}X_{jR}^{d\leftarrow\sigma}-\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0}$ $-~X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma}-\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow0}$ $+ \ X_{jN}^{\sigma \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} + \eta(\sigma) X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $+ \ X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} + X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $- \ \eta(\sigma) X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} + \eta(\sigma) X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow 0}$ $+ \ X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} + \eta(\sigma) X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}$ $- \ X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $- \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{d \leftarrow -\sigma} - X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow 0} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow 0} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $- \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{-\sigma \leftarrow 0}$ $+ \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{d \leftarrow \sigma} - X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow 0} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{0 \leftarrow \sigma}$ $- \ \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow 0} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{-\sigma \leftarrow 0}$ $-X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow d}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow\sigma}+\eta(\sigma)X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow0}$ $+ \ X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{d \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{-\sigma \leftarrow 0}$ $-~X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow\sigma}+X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $-~X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}+\eta(\sigma)X_{iR}^{d\leftarrow0}X_{jR}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $-\eta(\sigma)X_{iR}^{d\leftarrow 0}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow d}-\eta(\sigma)X_{jR}^{\sigma\leftarrow 0}X_{iR}^{0\leftarrow 0}X_{jN}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d}$ $+ \ \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} - \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow 0} X_{iN}^{0 \leftarrow d}$ $- \ \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} + \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d}$ $+~\eta(\sigma)X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}-X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d}$ $+~X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{iN}^{-\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}-X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $-\,X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d} + X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}$ $+ \ X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} + X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d}$ $- \ X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} - \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d}$ $+~\eta(\sigma)X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}+\eta(\sigma)X_{jR}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $-\eta(\sigma)X_{iR}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow d}+\eta(\sigma)X_{iR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $+ \ \eta(\sigma) X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow d} - \eta(\sigma) X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d}$ $-~\eta(\sigma)X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d}-X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow0}X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d}$ $+ \ X_{jR}^{d\leftarrow\sigma} X_{iR}^{0\leftarrow0} X_{iN}^{\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} - X_{jR}^{d\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow d}$ $-\,X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\,\sigma}X_{jN}^{-\sigma\leftarrow\,d}X_{iN}^{d\leftarrow\,d} + X_{jR}^{d\leftarrow\,\sigma}X_{iR}^{\sigma\leftarrow\,\sigma}X_{iN}^{-\sigma\leftarrow\,\sigma}X_{jN}^{-\sigma\leftarrow\,d}$ $+X_{iR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow d}$

$$[[\hat{H}^0_{t,Ni},\hat{H}_{t,R}],\hat{H}^0_{t,Ni}]=0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,R}], \hat{H}_{t,Ni}^{+}] = 0$$

$$[[\hat{H}_{t,Ni}^{0},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{-}]=0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,R}], \hat{H}_{t,cross}^{0\oplus}] = 0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}_{t,R}],\hat{H}^{0\ominus}_{t,cross}]=0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}_{t,R}],\hat{H}^{+\ominus}_{t,cross}]=0$$

$$[[\hat{H}^{0}_{t,Ni},\hat{H}_{t,R}],\hat{H}^{-\oplus}_{t,cross}] = 0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}_{t,R}],\hat{H}_{t,R}] = 0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,Ni}^{0}]=0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,Ni}^{+}] = &+ \eta(\sigma) X_{iN}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} \\ &- \eta(\sigma) X_{iN}^{d \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{d \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{iN}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow -\sigma} \\ &- \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{iN}^{d \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow \sigma} \\ &- \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{d \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{iN}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow -\sigma} \\ &+ \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow -\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,Ni}^{-}] &= -\eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0} - \eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow0} + \eta(\sigma)X_{jN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{jN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0} - \eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow0} + \eta(\sigma)X_{jN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow0} - \eta(\sigma)X_{jN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-}_{t,Ni}],\hat{H}^{0\oplus}_{t,cross}] &= -X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow d}_{jN}X^{\sigma\leftarrow 0}_{iR} \\ &- X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow d}_{jN}X^{-\sigma\leftarrow 0}_{iR} + X^{d\leftarrow d}_{jN}X^{0\leftarrow\sigma}_{iN}X^{\sigma\leftarrow 0}_{iR} \\ &+ \eta(\sigma)X^{d\leftarrow d}_{jN}X^{0\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{iR} + X^{d\leftarrow d}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow 0}_{iR} \\ &- \eta(\sigma)X^{d\leftarrow d}_{jN}X^{0\leftarrow-\sigma}_{iR}X^{d\leftarrow\sigma}_{iR} + X^{0\leftarrow\sigma}_{iN}X^{\sigma\leftarrow 0}_{iR}X^{-\sigma\leftarrow \sigma}_{jN} \\ &- X^{0\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow 0}_{iR}X^{-\sigma\leftarrow \sigma}_{jN} + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR}X^{-\sigma\leftarrow \sigma}_{jN} \\ &- \eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR}X^{-\sigma\leftarrow \sigma}_{jN} + X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow 0}_{iR}X^{\sigma\leftarrow \sigma}_{jN} \\ &- X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow 0}_{iR}X^{\sigma\leftarrow \sigma}_{jN} - \eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{\sigma\leftarrow \sigma}_{jN} \\ &+ \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{\sigma\leftarrow -\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,cross}^{0\ominus}] &= -X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma} - X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma} + X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow0} - \chi_{iN}^{\sigma\leftarrow0}X_{iR}^{o\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} \\ &+ X_{iN}^{\sigma\leftarrow}X_{iR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow0}X_{jN}^{d\leftarrow\sigma} \\ &- X_{iN}^{-\sigma\leftarrow0}X_{iR}^{o\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} + X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{jN}^{d\leftarrow\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,cross}^{+\ominus}] &= + X_{iN}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{-\sigma\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{0\leftarrow \sigma} + X_{iN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{\sigma\leftarrow d} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{0\leftarrow -\sigma} - X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d} \\ &- X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow \sigma}X_{iR}^{\sigma\leftarrow d} + X_{iN}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow \sigma} \\ &- X_{iN}^{d\leftarrow \sigma}X_{iR}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow -\sigma} + \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma}X_{jN}^{\sigma\leftarrow \sigma} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow \sigma}X_{jN}^{\sigma\leftarrow -\sigma} + X_{iN}^{d\leftarrow \sigma}X_{iR}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma} \\ &- X_{iN}^{d\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,cross}^{-\oplus}] &= -X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0} - X_{jN}^{\sigma\leftarrow \sigma}X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma} \\ &- \eta(\sigma)X_{jN}^{\sigma\leftarrow \sigma}X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow 0} + X_{iN}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0} + X_{jN}^{-\sigma\leftarrow \sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow -\sigma} \\ &+ \eta(\sigma)X_{jN}^{-\sigma\leftarrow \sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow 0} - X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow 0} \\ &+ X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow d} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow 0}X_{jN}^{0\leftarrow 0} \\ &- X_{iN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow 0} + X_{iN}^{\sigma\leftarrow 0}X_{jN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jN}^{0\leftarrow 0} - \chi_{iN}^{\sigma\leftarrow 0}X_{jN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jN}^{0\leftarrow 0} \end{split}$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,R}]=0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{0\oplus}], \hat{H}_{t,Ni}^{0}] = 0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,Ni}^{+}]=0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{0\oplus}], \hat{H}_{t,Ni}^{-}] = 0$$

$$[[\hat{H}_{t,Ni}^+,\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{0\oplus}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$[[\hat{H}_{t,Ni}^+,\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,cross}^{-\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,R}] = 0$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = & + \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow0}_{jN}X^{0\leftarrow-\sigma}_{iR} \\ & - X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow0}_{jN}X^{\sigma\leftarrow d}_{iR} + \eta(\sigma)X^{d\leftarrow0}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR} \\ & + X^{d\leftarrow0}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow d}_{iR} + \eta(\sigma)X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR} \\ & - X^{-\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow d}_{iR} + \eta(\sigma)X^{\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow\sigma}_{iR} \\ & + X^{\sigma\leftarrow-\sigma}_{jN}X^{d\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow d}_{iR} \end{split}$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{-}] &= -X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow d} \\ &+ X_{iN}^{\sigma\leftarrowd}X_{iR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow d}X_{jN}^{d\leftarrow d} \\ &- X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow-\sigma}X_{jN}^{d\leftarrow d} + X_{iN}^{-\sigma\leftarrow d}X_{iR}^{0\leftarrow-\sigma}X_{jN}^{d\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow d}X_{jN}^{d\leftarrow d} - X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d} - X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow d} + X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{o\leftarrow-\sigma} \\ &- \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow-d} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow0}X_{iR}^{o\leftarrow-\sigma} \\ &- \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow-d} + X_{iN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{0\leftarrow 0}_{iR} \\ & - \eta(\sigma)X^{d\leftarrow \sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{\sigma\leftarrow -\sigma}_{iR} + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{\sigma\leftarrow \sigma}_{iR} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iN}X^{0\leftarrow 0}_{iR} + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iN}X^{\sigma\leftarrow -\sigma}_{iR} \\ & - \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow -\sigma}_{iN}X^{-\sigma\leftarrow \sigma}_{iR} + \eta(\sigma)X^{0\leftarrow \sigma}_{iN}X^{\sigma\leftarrow \sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \\ & + \eta(\sigma)X^{0\leftarrow \sigma}_{iN}X^{d\leftarrow d}_{iR}X^{d\leftarrow -\sigma}_{jN} + \eta(\sigma)X^{0\leftarrow \sigma}_{iN}X^{-\sigma\leftarrow -\sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \\ & + \eta(\sigma)X^{0\leftarrow \sigma}_{iN}X^{d\leftarrow d}_{iR}X^{d\leftarrow -\sigma}_{jN} + \eta(\sigma)X^{0\leftarrow \sigma}_{iN}X^{-\sigma\leftarrow -\sigma}_{iR}X^{d\leftarrow -\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni}, \hat{H}^{0\ominus}_{t,cross}], \hat{H}^{0\ominus}_{t,cross}] = &-X^{-\sigma \leftarrow 0}_{iN} X^{0\leftarrow d}_{iR} X^{d\leftarrow -\sigma}_{jN} \\ &-X^{-\sigma \leftarrow 0}_{iN} X^{0\leftarrow d}_{iR} X^{d\leftarrow -\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & -\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & -\eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{d\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{-\oplus}_{t,cross}] = & -X^{\sigma\leftarrow d}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & +X^{-\sigma\leftarrow d}_{iN}X^{0\leftarrow 0}_{iR}X^{d\leftarrow-\sigma}_{jN} + X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & -X^{-\sigma\leftarrow 0}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} + X^{\sigma\leftarrow 0}_{iN}X^{d\leftarrow d}_{iR}X^{0\leftarrow\sigma}_{jN} \\ & +X^{\sigma\leftarrow 0}_{iN}X^{-\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \end{split}$$

 $[[\hat{H}_{t,Ni}^+,\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,R}] = + X_{iN}^{d\leftarrow 0} X_{jN}^{0\leftarrow \sigma} X_{iR}^{0\leftarrow d} X_{jR}^{\sigma\leftarrow 0}$ $+~\eta(\sigma)X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jR}^{d\leftarrow-\sigma}+\eta(\sigma)X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow0}X_{jR}^{0\leftarrow-\sigma}$ $-X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow \sigma}X_{iR}^{0\leftarrow 0}X_{jR}^{\sigma\leftarrow d}-\eta(\sigma)X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow \sigma}X_{iR}^{\sigma\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma}$ $- \ X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}$ $- \ X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} + \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{jR}^{0 \leftarrow 0}$ $+ X_{iN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} - X_{iN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow 0}$ $+~\eta(\sigma)X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}X_{iR}^{0\leftarrow d}X_{jR}^{d\leftarrow\sigma}+\eta(\sigma)X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{jR}^{0\leftarrow\sigma}$ $+ \ X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} - \eta(\sigma) X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}$ $- X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}$ $- \ \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}$ $- X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow -\sigma}$ $-X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}X_{jR}^{\sigma\leftarrow d}-X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}X_{jR}^{\sigma\leftarrow}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow d} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}$ $+ \ X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $-\eta(\sigma)X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}+\eta(\sigma)X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}X_{j$ $+ \ \eta(\sigma) X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} - X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $-X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow d}X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}+X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}$ $+ \ X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iR}^{d \leftarrow d} X_{jR}^{0 \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0}$ $+~\eta(\sigma)X_{iR}^{d\leftarrow d}X_{jR}^{0\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}-X_{iR}^{d\leftarrow d}X_{jR}^{-\sigma\leftarrow d}X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}$ $+~X_{iR}^{d\leftarrow d}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}+X_{jR}^{\sigma\leftarrow 0}X_{iR}^{0\leftarrow d}X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow\sigma}$ $- \ X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow d} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} + \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $-\eta(\sigma)X_{jR}^{d\longleftarrow\sigma}X_{iR}^{0\longleftarrow d}X_{iN}^{-\sigma\longleftarrow\sigma}X_{jN}^{d\longleftarrow\sigma}-\eta(\sigma)X_{iR}^{-\sigma\longleftarrow\sigma}X_{jR}^{0\longleftarrow\sigma}X_{iN}^{d\longleftarrow0}X_{jN}^{0\longleftarrow\sigma}$ $- \eta(\sigma) X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} + \eta(\sigma) X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} X_{jN}^{\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow \sigma}$ $+ \ \eta(\sigma) X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} + X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $+ X_{iR}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow d} X_{iN}^{d \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} - X_{iR}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow d} X_{iN}^{\sigma \leftarrow \sigma} X_{iN}^{d \leftarrow -\sigma}$ $-X_{iR}^{-\sigma \leftarrow -\sigma}X_{jR}^{\sigma \leftarrow d}X_{iN}^{-\sigma \leftarrow \sigma}X_{jN}^{d \leftarrow -\sigma} - \eta(\sigma)X_{iR}^{d \leftarrow d}X_{jR}^{0 \leftarrow -\sigma}X_{iN}^{d \leftarrow 0}X_{jN}^{0 \leftarrow \sigma}$ $+ \ \eta(\sigma) X_{iR}^{d \leftarrow d} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} + X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $-X_{iR}^{d\leftarrow d}X_{jR}^{\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}-X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow 0}$ $+\,X_{jR}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-\sigma}+\eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma}X_{iN}^{0\leftarrow0}$ $-\eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow-}$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,Ni}^{-}] = 0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{0\oplus}] = 0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{+\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{-\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,R}] = 0$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = & + \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow d}_{jN}X^{d\leftarrow-\sigma}_{iR} \\ & + X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow d}_{jN}X^{\sigma\leftarrow 0}_{iR} + \eta(\sigma)X^{d\leftarrow d}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} \\ & - X^{d\leftarrow d}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow 0}_{iR} + \eta(\sigma)X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow\sigma}_{iR} \\ & - X^{\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow 0}_{iR} + \eta(\sigma)X^{-\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR} \\ & + X^{-\sigma\leftarrow\sigma}_{jN}X^{0\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow 0}_{iR} \\ \end{split}$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{-\oplus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{-}] &= -X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow d} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow 0}X_{jN}^{0\leftarrow0} \\ &- X_{iN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow0} + X_{iN}^{\sigma\leftarrow 0}X_{iR}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jN}^{0\leftarrow0} - X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow \sigma}X_{iR}^{-\sigma\leftarrow0} - X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \\ &- \eta(\sigma)X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow0} + X_{iN}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow0} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow0} + X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0\oplus}_{t,cross}] = &-\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{0\leftarrow-\sigma}_{jN}\\ &-\eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{0\leftarrow\sigma}_{jN}\end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = &-X^{-\sigma\leftarrow0}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ &+X^{\sigma\leftarrow0}_{iN}X^{d\leftarrow d}_{iR}X^{0\leftarrow\sigma}_{jN} + X^{\sigma\leftarrow0}_{iN}X^{-\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ &-X^{\sigma\leftarrow d}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} + X^{-\sigma\leftarrow d}_{iN}X^{0\leftarrow0}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ &+X^{-\sigma\leftarrow d}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} + X^{-\sigma\leftarrow d}_{iN}X^{d\leftarrow-\sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{d\leftarrow d}_{iR} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{\sigma\leftarrow \sigma}_{iR} - \eta(\sigma)X^{d\leftarrow \sigma}_{iN}X^{0\leftarrow \sigma}_{jN}X^{\sigma\leftarrow -\sigma}_{iR} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iN}X^{d\leftarrow d}_{iR} - \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow -\sigma}_{iN}X^{-\sigma\leftarrow \sigma}_{iN} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iN}X^{-\sigma\leftarrow -\sigma}_{iR} + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{-\sigma\leftarrow -\sigma}_{iR}X^{0\leftarrow \sigma}_{jN} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{jN}X^{0\leftarrow \sigma}_{iR}X^{0\leftarrow \sigma}_{jN} + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{\sigma\leftarrow \sigma}_{iR}X^{0\leftarrow \sigma}_{jN} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{iR}X^{0\leftarrow \sigma}_{jN} + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{\sigma\leftarrow \sigma}_{iR}X^{0\leftarrow \sigma}_{jN} \\ & + \eta(\sigma)X^{d\leftarrow -\sigma}_{iN}X^{0\leftarrow \sigma}_{iR}X^{0\leftarrow \sigma}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{-\oplus}_{t,cross}] = & -X^{\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{iR}X^{0\leftarrow \sigma}_{jN}\\ & -X^{\sigma\leftarrow d}_{iN}X^{d\leftarrow 0}_{iR}X^{0\leftarrow \sigma}_{jN} \end{split}$$

$$\begin{split} |[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,cross}^{+}], \hat{H}_{t,I}] &= + X_{t,N}^{d-1} X_{t,N}^{0} - X_{t,R}^{0} - X_{t,R}^{0}$$

$$[[\hat{H}_{t,Ni}^+,\hat{H}_{t,R}],\hat{H}_{t,Ni}^0]=0$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,R}], \hat{H}_{t,Ni}^+] = 0$$

$$[[\hat{H}_{t,Ni}^+,\hat{H}_{t,R}],\hat{H}_{t,Ni}^-]=0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,R}], \hat{H}_{t,cross}^{+\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,R}], \hat{H}_{t,cross}^{-\oplus}] = 0$$

$$[[\hat{H}_{t,Ni}^{+}, \hat{H}_{t,R}], \hat{H}_{t,R}] = 0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,Ni}^{0}] = & + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0} \\ & + X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow0} \\ & - X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow\sigma} + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{0\leftarrow0}X_{iR}^{-\sigma\leftarrow0} \\ & - X_{iN}^{\sigma\leftarrow d}X_{jN}^{0\leftarrow0}X_{iR}^{d\leftarrow\sigma} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow0} \\ & + X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \\ & + X_{iN}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,Ni}^{+}] &= -X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow\sigma} \\ &+ X_{iN}^{0\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow\sigma} - \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow\sigma} - X_{iN}^{0\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow\sigma} \\ &+ \chi_{iN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow\sigma}X_{jN}^{\sigma\leftarrow\sigma} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jN}^{\sigma\leftarrow-\sigma} - X_{jN}^{d\leftarrow}X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jN}^{\sigma\leftarrow-\sigma} - X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} - X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} + X_{iN}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow0} \\ &+ \chi_{iN}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow0} \end{split}$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,Ni}^{-}]=0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,cross}^{0\oplus}] = & -X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow0}X_{jN}^{-\sigma\leftarrow d} \\ & -X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow0}X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\oplus}],\hat{H}_{t,cross}^{0\ominus}] &= + \, \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} \\ &\quad + \, \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow d} + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{-\sigma\leftarrow-\sigma} \\ &\quad + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow d} + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{o\leftarrow0} X_{iR}^{-\sigma\leftarrow d} \\ &\quad + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow d} + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow d} \\ &\quad + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow0} X_{jN}^{-\sigma\leftarrow0} - \eta(\sigma) X_{iN}^{-\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} \\ &\quad + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ &\quad + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{-}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = & -X^{0\leftarrow-\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{\sigma\leftarrow0}_{jN} \\ & +X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{\sigma\leftarrow0}_{jN} + X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow d}_{iR}X^{\sigma\leftarrow0}_{jN} \\ & -X^{d\leftarrow\sigma}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{-\sigma\leftarrow d}_{jN} + X^{d\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{-\sigma\leftarrow d}_{jN} \\ & +X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow0}_{iR}X^{-\sigma\leftarrow d}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\oplus}], \hat{H}_{t,cross}^{-\oplus}] = & - \eta(\sigma) X_{iN}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} \\ & - \eta(\sigma) X_{iN}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} \end{split}$$

$$\begin{split} [[\tilde{H}_{t,Nl},\tilde{H}_{t,cross}],\tilde{H}_{t,Rl}] &= + \eta(\sigma) X_{1N}^{s,r,r} X_{jR}^{s,r-d} X_{lR}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{1N}^{s,r,r} X_{jN}^{s,r-d} X_{lR}^{s,r-d} X_{jR}^{s,r-d} \\ &+ X_{il}^{s,r,r} X_{jN}^{s,r-d} X_{lR}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{il}^{s,r,r} X_{jN}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} \\ &+ X_{il}^{s,r,r} X_{jN}^{s,r-d} X_{iR}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{il}^{s,r,r} X_{jN}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} \\ &+ X_{il}^{s,r,r} X_{j}^{s,r-d} X_{iR}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{il}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} \\ &+ X_{il}^{s,r,r-d} X_{jR}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{il}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d} \\ &+ X_{il}^{s,r,r-d} X_{jR}^{s,r-d} X_{il}^{s,r-d} X_{jR}^{s,r-d} + \eta(\sigma) X_{il}^{s,r-d} X_{jR}^{s,r-d} X_{iR}^{s,r-d} X_{jR}^{s,r-d} X_{jR}^{s,r-d}$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,Ni}^{+}] = 0$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,Ni}^{-}] = 0$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,cross}^{0\ominus}]=0$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,cross}^{+\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,cross}^{-\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,R}] = 0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{0}] = & + \eta(\sigma)X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow0} \\ & + X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma} + \eta(\sigma)X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-d} \\ & - X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{d\leftarrow-d}X_{iR}^{\sigma\leftarrow-d} \\ & - X_{iN}^{-\sigma\leftarrow0}X_{jN}^{d\leftarrow}X_{iR}^{0\leftarrow-\sigma} + \eta(\sigma)X_{iN}^{\sigma\leftarrow-d}X_{jN}^{d\leftarrow0}X_{iR}^{-\sigma\leftarrow-d} \\ & + X_{iN}^{\sigma\leftarrow-d}X_{jN}^{d\leftarrow0}X_{iR}^{0\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{+}] &= -X_{iN}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow \sigma}\\ &+ X_{iN}^{d\leftarrow \sigma}X_{iR}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow -\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma}X_{jN}^{\sigma\leftarrow \sigma}\\ &+ \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow \sigma}X_{jN}^{\sigma\leftarrow -\sigma} - X_{iN}^{d\leftarrow \sigma}X_{iR}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma}\\ &+ \chi_{iN}^{d\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow -\sigma} + \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma}\\ &+ X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma} - X_{iN}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow -\sigma}\\ &- \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow -\sigma}X_{jN}^{0\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow -\sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{o\leftarrow d}\\ &- \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{0\leftarrow -\sigma} + X_{jN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{o\leftarrow d}\\ &+ \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{jN}^{0\leftarrow 0}X_{iR}^{0\leftarrow -\sigma} + X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}\\ &+ X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow \sigma}X_{iR}^{o\leftarrow d} \end{split}$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{-}] = 0$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{0\oplus}] &= -X_{iN}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &+ X_{iN}^{d\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d} + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow d} \\ &- X_{iN}^{0\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} + X_{iN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow d}X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{0\ominus}] = & -\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d} \\ & -\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{+\ominus}] = & -X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{\sigma\leftarrow0} \\ & -X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{-\ominus}] &= + \, \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow d} X_{iR}^{-\sigma\leftarrow-\sigma} \\ &\quad + \, \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow d} X_{iR}^{0\leftarrow0} + \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} \\ &\quad + \eta(\sigma) X_{jN}^{\sigma\leftarrow0} X_{iN}^{-\sigma\leftarrow d} X_{iR}^{0\leftarrow0} + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow d} X_{jN}^{\sigma\leftarrow0} \\ &\quad + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} - \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{-\sigma\leftarrow d} \\ &\quad + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{d\leftarrow d} X_{jN}^{-\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{\sigma\leftarrow d} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ &\quad + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{d\leftarrow d} X_{jN}^{-\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} \\ &\quad + \eta(\sigma) X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow-\sigma} X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{I,N}^{-},\hat{H}_{t,cross}^{+}],\hat{H}_{i,R}] &= + \chi_{j}^{\alpha-0} - \chi_{j}^{\alpha-0} - \chi_{i}^{-\alpha-0} - \chi_{j}^{\alpha-0} - \chi_{j}^{-\alpha-1} - \chi_{j}^{-\alpha-1$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{+}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{-}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{0\ominus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{-\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,R}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,R}], \hat{H}_{t,Ni}^{+}] = 0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{-}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}]=0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}] = 0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,cross}^{-\oplus}] = 0$$

$$[[\hat{H}_{t,Ni}^{-},\hat{H}_{t,R}],\hat{H}_{t,R}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{0}] &= +X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} + X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow}X_{jN}^{\sigma\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma} \\ &- X_{iN}^{\sigma\leftarrow}X_{iR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow\sigma} \\ &- X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} + X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &- X_{iN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma} + X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma} - X_{jN}^{-\sigma\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &- X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} - X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &- X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} + X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &- X_{jN}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} + X_{jN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{o\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} + X_{jN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma} - \sigma \end{split}$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{+}_{t,Ni}] = & + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & - \eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{\sigma\leftarrow\sigma}_{iR}X^{d\leftarrow\sigma}_{jN} + \eta(\sigma)X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow-d}_{iR}X^{d\leftarrow-\sigma}_{jN} \\ & - \eta(\sigma)X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow-\sigma}_{iR}X^{d\leftarrow\sigma}_{jN} - \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow\sigma}_{jN}X^{\sigma\leftarrow-\sigma}_{iR} \\ & + \eta(\sigma)X^{d\leftarrow-\sigma}_{iN}X^{0\leftarrow\sigma}_{iN}X^{0\leftarrow\sigma}_{iR} + \eta(\sigma)X^{d\leftarrow-\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{0\leftarrow\sigma}_{iR} \\ & - \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{jN}X^{0\leftarrow-\sigma}_{iR} - \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{jN}X^{-\sigma\leftarrow-\sigma}_{iR} \\ & + \eta(\sigma)X^{d\leftarrow\sigma}_{jN}X^{0\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{iR} - \eta(\sigma)X^{d\leftarrow\sigma}_{iN}X^{0\leftarrow-\sigma}_{jN}X^{-\sigma\leftarrow-\sigma}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{-}] &= -\eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{jN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\oplus}_{t,cross}] = &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & -X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma} \\ & -\eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow d} \\ & -X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma} - \eta(\sigma)X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow d} \\ & +\eta(\sigma)X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,cross}^{+\ominus}] = & - \eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} \\ & - \eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus}, \hat{H}_{t,cross}^{0\ominus}], \hat{H}_{t,cross}^{-\oplus}] = & + \eta(\sigma) X_{iN}^{\sigma \leftarrow d} X_{iR}^{-\sigma \leftarrow 0} \\ & - \eta(\sigma) X_{iN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow 0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,R}] &= +X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{o\leftarrow0}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma} + \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{o\leftarrow\sigma} \\ &+ X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow0} \\ &- X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &+ X_{iR}^{d\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &+ X_{iR}^{d\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - \eta(\sigma)X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma} - \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{+}]=0$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{-}]=0$$

$$[[\hat{H}^{0\oplus}_{t,cross},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{0\oplus}_{t,cross}]=0$$

$$[[\hat{H}_{t,cross}^{0\oplus}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$[[\hat{H}_{t.cross}^{0\oplus},\hat{H}_{t.cross}^{+\ominus}],\hat{H}_{t.cross}^{-\oplus}]=0$$

$$[[\hat{H}_{t.cross}^{0\oplus},\hat{H}_{t.cross}^{+\ominus}],\hat{H}_{t.R}]=0$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = &-X^{0\leftarrow-\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{-\sigma\leftarrow d}_{jN} \\ &-X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow0}_{iR}X^{\sigma\leftarrow d}_{jN} + X^{\sigma\leftarrow d}_{iN}X^{0\leftarrow\sigma}_{jN}X^{d\leftarrow0}_{iR} \\ &+X^{-\sigma\leftarrow d}_{iN}X^{0\leftarrow-\sigma}_{jN}X^{d\leftarrow0}_{iR} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{+}] = &-\eta(\sigma)X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow0}X_{jN}^{0\leftarrow\sigma} \\ &+\eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jN}^{0\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{-}] = & + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow 0} \\ & - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{jN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow 0} \end{split}$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{0\ominus}] = & + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow 0} \\ & - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow 0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{+\ominus}] = & -\eta(\sigma)X_{iN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} \\ & +\eta(\sigma)X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} \end{split}$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{-\oplus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,R}] &= -X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0} \\ &- \eta(\sigma)X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma} - X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{d\leftarrow\sigma} + \eta(\sigma)X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow d} \\ &+ X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow d} - \eta(\sigma)X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d} \\ &+ X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d} - \eta(\sigma)X_{jR}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}^{\text{Pi}}_{t,cross},\hat{H}_{t,R}], \hat{H}^{0}_{t,N}] &= X_{1N}^{\text{Pi}} - X_{N}^{\text{pi}} - X_{N}^{\text{pi}} - X_{N}^{\text{de}} -$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{+}]=0$$

 $[[\hat{H}_{t.cross}^{0\oplus},\hat{H}_{t.R}],\hat{H}_{t.Ni}^{-}] = + \, \eta(\sigma) X_{iN}^{0\leftarrow 0} X_{iR}^{\sigma\leftarrow\sigma} X_{jR}^{\sigma\leftarrow 0} X_{iN}^{-\sigma\leftarrow d}$ $- \ \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow 0} X_{jN}^{-\sigma \leftarrow 0} + X_{iN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d}$ $-~X_{iN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow0}+\eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow0}X_{jN}^{-\sigma\leftarrow0}$ $- \ \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0} X_{jN}^{-\sigma \leftarrow 0} - X_{iN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d}$ $+ \ X_{iN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow 0} + \eta(\sigma) X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{jN}^{-\sigma \leftarrow d} X_{jN}$ $- \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{jN}^{-\sigma \leftarrow 0} + X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow d}$ $-~X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow d}X_{jR}^{d\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow 0}+X_{iN}^{0\leftarrow 0}X_{iR}^{d\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow d}$ $- \ X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{-\sigma \leftarrow 0} - \eta(\sigma) X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow d}$ $+ \ \eta(\sigma) X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} - \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $+ \ \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} - \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $+ \ \eta(\sigma) X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} - X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{jN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $+ \ X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} - X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $+ \ X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} + X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $-~X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma}X_{iN}^{0\leftarrow d}X_{jN}^{-\sigma\leftarrow0}-\eta(\sigma)X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow d}X_{iN}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow d}$ $+ \ \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} + \eta(\sigma) X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $- \eta(\sigma) X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow d} X_{jN}^{-\sigma \leftarrow 0} - X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow 0} X_{jN}^{-\sigma \leftarrow d}$ $+~X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{iN}^{0\leftarrow d}X_{jN}^{-\sigma\leftarrow0}-\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}$ $- \ X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}$ $+ \ X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $- \ X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $+~\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d}+\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow 0}X_{iR}^{0\leftarrow 0}$ $+~\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}+X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}$ $+ \ X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $+~\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d}-\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow -\sigma}$ $+ X_{iN}^{\sigma \leftarrow \sigma} X_{iN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iN}^{\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow 0}$ $+ \ X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0}$ $- \ X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $+ \ X_{iN}^{-\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow d} X_{iR}^{d\leftarrow d} X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{-\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow d} X_{iR}^{d\leftarrow 0} X_{jR}^{0\leftarrow -\sigma}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0}$ $-\eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow-\sigma}-X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow 0}$ $-~X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow-\sigma}+X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow d}X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma}$ $- X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow}$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\oplus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}] &= + \eta(\sigma) X_{iN}^{0\leftarrow0} X_{iR}^{o\leftarrowd} X_{jR}^{o\leftarrow0} \\ &- X_{iN}^{0\leftarrow0} X_{iR}^{o\leftarrowd} X_{jR}^{d\leftarrow\sigma} + \eta(\sigma) X_{iN}^{0\leftarrow0} X_{iR}^{d\leftarrow\sigma} X_{jR}^{0\leftarrow-\sigma} \\ &- X_{iN}^{0\leftarrow0} X_{iR}^{o\leftarrowd} X_{jR}^{o\leftarrowd} - X_{jR}^{o\leftarrow0} - \chi_{iN}^{o\leftarrow0} X_{iN}^{0\leftarrow0} X_{iN}^{o\leftarrow0} X_{iN}^{o\leftarrow0} \\ &- \chi_{iR}^{0\leftarrow0} X_{iR}^{o\leftarrow0} X_{iR}^{o\leftarrow0} - \chi_{iN}^{o\leftarrow0} - \chi_{iN}^{o\leftarrow0} X_{iN}^{o\leftarrow0} \\ &- \eta(\sigma) X_{jR}^{o\leftarrow0} X_{iR}^{o\leftarrow0} X_{iN}^{o\leftarrow0} - \eta(\sigma) X_{jR}^{d\leftarrow\sigma} X_{iR}^{0\leftarrow\sigma} X_{iN}^{o\leftarrow0} \\ &- X_{jR}^{d\leftarrow\sigma} X_{iR}^{o\leftarrowd} X_{iN}^{0\leftarrow0} + \eta(\sigma) X_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow0} \\ &- X_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrowd} X_{iN}^{o\leftarrow0} + \eta(\sigma) X_{jR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrowd} X_{iN}^{o\leftarrow0} \\ &- X_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrowd} X_{iN}^{o\leftarrow0} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &- \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow0} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow0} \\ &- X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &- \chi_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} \\ &+ \chi_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - \chi_{iR}^{o\leftarrow\sigma} - \chi_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} \\ &- \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} \\ &- \eta(\sigma) X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} - X_{iN}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o\leftarrow\sigma} X_{iR}^{o$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}_{t,R}],\hat{H}^{-\oplus}_{t,cross}] = &-\eta(\sigma)X_{iN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0} \\ &+X_{iN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow 0}X_{jR}^{d\leftarrow \sigma}+X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow 0} \\ &+\eta(\sigma)X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma}+\eta(\sigma)X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow 0}X_{iN}^{0\leftarrow d} \\ &+X_{jR}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{iN}^{0\leftarrow d}-\eta(\sigma)X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow 0}X_{iN}^{0\leftarrow d} \\ &+X_{jR}^{d\leftarrow \sigma}X_{iR}^{\sigma\leftarrow 0}X_{iN}^{0\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}^{0\oplus}_{t,cross},\hat{H}_{t,R}],\hat{H}_{t,R}] &= + X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{jR}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{jR}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} - X_{jR}^{d\leftarrow} \\ &+ X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{\sigma\leftarrow} - X_{jR}^{0\leftarrow} - X_{iN}^{o\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{d\leftarrow} \\ &+ X_{iN}^{0\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{\sigma\leftarrow} - X_{jR}^{o\leftarrow} - X_{iR}^{o\leftarrow} X_{iR}^{\sigma\leftarrow} X_{jR}^{\sigma\leftarrow} \\ &- \eta(\sigma) X_{iN}^{0\leftarrow} X_{iR}^{d\leftarrow} X_{jR}^{\sigma\leftarrow} - X_{jR}^{o\leftarrow} - \eta(\sigma) X_{iN}^{0\leftarrow} X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - \eta(\sigma) X_{iN}^{0\leftarrow} X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - \eta(\sigma) X_{iN}^{0\leftarrow} - X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - \eta(\sigma) X_{iN}^{0\leftarrow} - X_{iR}^{o\leftarrow} - X_{jR}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - X_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - X_{iN}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - X_{iN}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{jR}^{o\leftarrow} - X_{iN}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{iN}^{o\leftarrow} - X_{iN}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \eta(\sigma) X_{iR}^{d\leftarrow} - X_{iN}^{o\leftarrow} - \chi_{iN}^{o\leftarrow} - \chi_{$$

$$\begin{split} [[\hat{H}^{0\ominus}_{t,cross},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{0}_{t,Ni}] = & + X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{\sigma\leftarrow0} \\ & + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{-\sigma\leftarrow0} - X_{iN}^{-\sigma\leftarrow0}X_{jN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow d} \\ & - X_{iN}^{\sigma\leftarrow0}X_{jN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{+}] = & + \eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma} \\ & - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow d}X_{jN}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{-}] = &-\eta(\sigma)X_{jN}^{\sigma\leftarrow0}X_{iN}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow0}\\ &+\eta(\sigma)X_{jN}^{-\sigma\leftarrow0}X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{0\ominus}] = & + \eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow-\sigma} \\ & - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow\sigma} \end{split}$$

$$[[\hat{H}_{t,cross}^{0\ominus}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{0\ominus}] = 0$$

$$[[\hat{H}^{0\ominus}_{t,cross}, \hat{H}^{+\ominus}_{t,cross}], \hat{H}^{+\ominus}_{t,cross}] = 0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus}, \hat{H}_{t,cross}^{+\ominus}], \hat{H}_{t,cross}^{-\ominus}] = & - \eta(\sigma) X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} \\ & + \eta(\sigma) X_{iN}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,R}] = &+ \eta(\sigma)X_{iN}^{d\leftarrow 0}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma}\\ &+ X_{iN}^{d\leftarrow 0}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d} - \eta(\sigma)X_{iN}^{d\leftarrow 0}X_{iR}^{0\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma}\\ &+ X_{iN}^{d\leftarrow 0}X_{iR}^{0\leftarrow \sigma}X_{jR}^{\sigma\leftarrow d} - X_{iR}^{\sigma\leftarrow d}X_{jR}^{0\leftarrow \sigma}X_{iN}^{d\leftarrow 0}\\ &- \eta(\sigma)X_{iR}^{\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow 0} - X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow -\sigma}X_{iN}^{d\leftarrow 0}\\ &+ \eta(\sigma)X_{iR}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow d}X_{iN}^{d\leftarrow 0} \end{split}$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{0}]=0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,Ni}^{+}]=0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{-}]=0$$

$$[[\hat{H}^{0\ominus}_{t,cross},\hat{H}^{-\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = 0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{0\ominus}]=0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{-\oplus}]=0$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,R}]=0$$

$$\begin{split} [[\hat{H}^{b}_{t,cross},\hat{H}_{t,R}], \hat{H}^{0}_{t,N}] &= +X^{ar-a}_{t,N}X^{0}_{t,R} - X^{0}_{t,R} -$$

$$\begin{split} [[\tilde{H}^{b}]_{coross}^{b,s}, \tilde{H}_{t,N}] &= + \eta(\sigma) X_{N}^{a,\sigma} X_{R}^{b,m} X_{R}^{b,m} X_{N}^{b,m} X_{N}^{d,m} - \sigma \\ &- \eta(\sigma) X_{N}^{c,s} - \sigma X_{R}^{b,c} X_{R}^{b,c} X_{N}^{b,c} - X_{N}^{c,s} - X_{R}^{b,c} X_{R}^{b,c} X_{N}^{c,m} - X_{R}^{b,c} - X_{N}^{c,c} X_{R}^{b,c} X_{N}^{c,c} - X_{R}^{c,c} X_{N}^{b,c} - X_{N}^{c,c} X_{R}^{b,c} X_{N}^{c,c} - X_{N}^{c,c} X_{R}^{b,c} X_{N}^{c,c} - X_{N}^{c,c} X_{R}^{b,c} X_{N}^{c,c} - X_{N}^{c,c} X_{R}^{c,c} X_{N}^{c,c} X_{N}^{c,c}$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{-}]=0$$

$$\begin{split} [[\hat{H}^{0\ominus}_{t,cross},\hat{H}_{t,R}],\hat{H}^{0\ominus}_{t,cross}] &= -X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0} \\ &+ X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{-\sigma\leftarrow0}X_{jR}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{-\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma} \\ &+ \chi_{iR}^{d\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma} \\ &+ \eta(\sigma)X_{iR}^{-\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}] = &+ X_{iN}^{d\leftarrow 0} X_{iR}^{\sigma\leftarrow d} X_{jR}^{0\leftarrow \sigma} \\ &+ \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{\sigma\leftarrow d} X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow \sigma} \\ &+ X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{0\leftarrow -\sigma} \\ &+ X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{0\leftarrow -\sigma} \\ &+ X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{\sigma\leftarrow d} - \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{0\leftarrow -\sigma} \end{split}$$

$$[[\hat{H}_{t.cross}^{0\ominus}, \hat{H}_{t.R}], \hat{H}_{t.cross}^{-\oplus}] = 0$$

$$\begin{split} [[\hat{H}^{0\ominus}_{t,cross},\hat{H}_{t,R}],\hat{H}_{t,R}] &= + X_{iN}^{\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{0\leftarrow \sigma} X_{jR}^{-\sigma} - X_{iN}^{\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma} - X_{iN}^{-\sigma} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma \leftarrow \sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma \leftarrow \sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma \leftarrow \sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma \leftarrow \sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{-\sigma \leftarrow \sigma} + X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{0\leftarrow \sigma} X_{jR}^{0\leftarrow \sigma} - X_{jR}^{0\leftarrow \sigma} - X_{iR}^{0\leftarrow \sigma} X_{iR}^{0\leftarrow \sigma} - X_{iR}^{0\leftarrow \sigma} X_{iR$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,Ni}^{0}] &= + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jN}^{-\sigma\leftarrow d} \\ &+ X_{iN}^{d\leftarrow \sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jN}^{\sigma\leftarrow d} + X_{iN}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow 0} X_{jN}^{-\sigma\leftarrow d} \\ &+ X_{iN}^{d\leftarrow \sigma} X_{iR}^{0\leftarrow 0} X_{jN}^{\sigma\leftarrow d} - X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jN}^{d\leftarrow -\sigma} \\ &+ X_{iN}^{d\leftarrow \sigma} X_{iR}^{0\leftarrow 0} X_{jN}^{\sigma\leftarrow d} - X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jN}^{d\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow 0} X_{iR}^{d\leftarrow d} X_{jN}^{0\leftarrow -\sigma} - X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow -\sigma} X_{jN}^{d\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow 0} X_{iR}^{\sigma\leftarrow \sigma} X_{jN}^{0\leftarrow -\sigma} + X_{iN}^{\sigma\leftarrow d} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jN}^{d\leftarrow -\sigma} \\ &+ X_{iN}^{\sigma\leftarrow 0} X_{iR}^{-\sigma\leftarrow \sigma} X_{jN}^{0\leftarrow -\sigma} - X_{iN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow d} X_{jN}^{d\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} X_{iN}^{d\leftarrow d} + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &+ X_{jN}^{-\sigma\leftarrow d} X_{jN}^{d\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} - X_{iN}^{\sigma\leftarrow d} X_{jN}^{d\leftarrow -\sigma} X_{iR}^{o\leftarrow 0} \\ &+ X_{jN}^{-\sigma\leftarrow 0} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} - X_{iN}^{\sigma\leftarrow d} X_{jN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &+ X_{jN}^{-\sigma\leftarrow 0} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{d\leftarrow d} + X_{jN}^{-\sigma\leftarrow 0} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} X_{iN}^{o\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{jN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{jN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{\sigma\leftarrow d} X_{iN}^{\sigma\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow -\sigma} X_{iR}^$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{+}] &= + \, \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jN}^{0\leftarrow \sigma} \\ &- \eta(\sigma) X_{iN}^{d\leftarrow \sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jN}^{0\leftarrow -\sigma} + \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow 0} X_{jN}^{0\leftarrow \sigma} \\ &- \eta(\sigma) X_{iN}^{d\leftarrow \sigma} X_{iR}^{0\leftarrow 0} X_{jN}^{0\leftarrow -\sigma} + \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{jN}^{0\leftarrow \sigma} X_{iR}^{d\leftarrow d} \\ &+ \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{jN}^{0\leftarrow \sigma} X_{iR}^{\sigma\leftarrow \sigma} - \eta(\sigma) X_{jN}^{d\leftarrow -\sigma} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow \sigma} \\ &+ \eta(\sigma) X_{iN}^{d\leftarrow -\sigma} X_{jN}^{0\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow \sigma} - \eta(\sigma) X_{jN}^{d\leftarrow \sigma} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{d\leftarrow d} \\ &- \eta(\sigma) X_{jN}^{d\leftarrow \sigma} X_{iN}^{0\leftarrow -\sigma} X_{iR}^{\sigma\leftarrow \sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,Ni}^{-}] &= -\eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow d}X_{jN}^{\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{d\leftarrow d}X_{jN}^{\sigma\leftarrow d} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow d} - \eta(\sigma)X_{jN}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow 0} \\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{\sigma\leftarrow d} - \eta(\sigma)X_{jN}^{\sigma\leftarrow 0}X_{iN}^{-\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow \sigma} \\ &- \eta(\sigma)X_{jN}^{\sigma\leftarrow 0}X_{iN}^{-\sigma\leftarrow d}X_{iR}^{0\leftarrow 0} + \eta(\sigma)X_{jN}^{-\sigma\leftarrow 0}X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow \sigma} \\ &+ \eta(\sigma)X_{jN}^{-\sigma\leftarrow 0}X_{iN}^{\sigma\leftarrow d}X_{iR}^{0\leftarrow 0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus}, \hat{H}_{t,cross}^{-\ominus}], \hat{H}_{t,cross}^{0\ominus}] = & - \eta(\sigma) X_{iN}^{0 \leftarrow -\sigma} X_{iR}^{d \leftarrow \sigma} \\ & - \eta(\sigma) X_{iN}^{0 \leftarrow -\sigma} X_{iR}^{d \leftarrow \sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus}, \hat{H}_{t,cross}^{-\oplus}], \hat{H}_{t,cross}^{0\ominus}] = & + \eta(\sigma) X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} \\ & - \eta(\sigma) X_{iN}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,cross}^{+\ominus}] = & + X_{iN}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d} \\ & + \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma} - \eta(\sigma)X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow -\sigma} \\ & + X_{iN}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma} \\ & + \eta(\sigma)X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,cross}^{-\oplus}] &= -X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow0} + \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow0} \\ &- X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,R}] &= + \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow 0} \\ &+ X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow \sigma}X_{jR}^{\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d} - X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{o\leftarrow 0}X_{jR}^{0\leftarrow \sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d} + X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{0\leftarrow \sigma} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d} - \eta(\sigma)X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0} \\ &+ X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0} - \eta(\sigma)X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0} \\ &+ X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0}X_{iN}^{-\sigma\leftarrow 0} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow -\sigma} - \eta(\sigma)X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0}X_{iN}^{d\leftarrow d} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow -\sigma} - X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d} \\ &- X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{\sigma\leftarrow -\sigma} + X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow \sigma}X_{iR}^{-\sigma\leftarrow -\sigma} \\ &+ \eta(\sigma)X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow -\sigma} + \eta(\sigma)X_{jR}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma}X_{iN}^{-\sigma\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow -\sigma} - X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow \sigma}X_{iN}^{-\sigma\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{iN}^{-\sigma\leftarrow -\sigma} - X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow \sigma}X_{iN}^{-\sigma\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow \sigma}X_{iN}^{o\leftarrow -\sigma} - X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{o\leftarrow -\sigma}X_{iN}^{-\sigma\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow \sigma}X_{iN}^{o\leftarrow -\sigma} - X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow \sigma}X_{iN}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} - X_{jR}^{o\leftarrow 0}X_{iR}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow \sigma}X_{iN}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} - X_{jR}^{o\leftarrow 0}X_{iR}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} \\ &+ X_{jR}^{d\leftarrow -\sigma}X_{iR}^{o\leftarrow \sigma}X_{iN}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma} - X_{iR}^{o\leftarrow 0}X_{iR}^{o\leftarrow -\sigma}X_{iN}^{o\leftarrow -\sigma}X$$

 $[[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{0}] = + \, \eta(\sigma) X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma}$ $+ \ \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} + X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} X_{jN}^{d \leftarrow -\sigma}$ $+ \ X_{iN}^{d \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma}$ $- \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} + X_{iN}^{d \leftarrow d} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} X_{jN}^{d \leftarrow d} X_{jN}^{-\sigma \leftarrow \sigma} X_{jN}^{\sigma \leftarrow d} X_{jN}^{d \leftarrow d} X_{jN}^{\sigma \leftarrow d} X_{jN$ $+ \ X_{iN}^{d \leftarrow 0} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{d \leftarrow d} X_{iR}^{0 \leftarrow 0} X_{jR}^{0 \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma}$ $+~\eta(\sigma)X_{iN}^{d\leftarrow0}X_{iR}^{0\leftarrow0}X_{jR}^{0\leftarrow\sigma}X_{jN}^{0\leftarrow-\sigma}+X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow0}X_{jR}^{-\sigma\leftarrow d}X_{jN}^{d\leftarrow-\sigma}$ $+~X_{iN}^{d\leftarrow0}X_{iR}^{0\leftarrow0}X_{jR}^{-\sigma\leftarrow d}X_{jN}^{0\leftarrow-\sigma}-X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow d}X_{jR}^{-\sigma\leftarrow0}X_{jN}^{d\leftarrow-\sigma}$ $-\,X_{iN}^{d \leftarrow 0}X_{iR}^{0 \leftarrow d}X_{jR}^{-\sigma \leftarrow 0}X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma)X_{iN}^{d \leftarrow d}X_{iR}^{0 \leftarrow d}X_{jR}^{d \leftarrow \sigma}X_{jN}^{d \leftarrow -\sigma}$ $+~\eta(\sigma)X_{iN}^{d\leftarrow0}X_{iR}^{0\leftarrow d}X_{jR}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow-\sigma}-\eta(\sigma)X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{d\leftarrow d}X_{jN}^{d\leftarrow-\sigma}$ $-\eta(\sigma)X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow-\sigma}-X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d}X_{iN}^{d\leftarrow-\sigma}$ $- \ X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iR}^{d \leftarrow d} X_{jR}^{0 \leftarrow \sigma} X_{iN}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma}$ $-~\eta(\sigma)X_{iR}^{d\leftarrow d}X_{jR}^{0\leftarrow\sigma}X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow-\sigma}-X_{iR}^{d\leftarrow d}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d}X_{jN}^{d\leftarrow-\sigma}$ $- \ X_{iR}^{d \leftarrow d} X_{jR}^{-\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma}$ $-~\eta(\sigma)X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow-\sigma}X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow-\sigma}+X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow d}X_{iN}^{d\leftarrow d}X_{jN}^{d\leftarrow-\sigma}$ $+ X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow -\sigma} - X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{0 \leftarrow d} X_{iN}^{d \leftarrow d} X_{iN}^{d \leftarrow -\sigma}$ $-\,X_{jR}^{-\sigma\leftarrow0}X_{iR}^{0\leftarrow d}X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow-\sigma}+\eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{iN}^{d\leftarrow d}X_{jN}^{d\leftarrow-\sigma}$ $+ \ \eta(\sigma) X_{jR}^{d\leftarrow\sigma} X_{iR}^{0\leftarrow d} X_{iN}^{d\leftarrow 0} X_{jN}^{0\leftarrow -\sigma} - \eta(\sigma) X_{iN}^{-\sigma\leftarrow -\sigma} X_{jN}^{d\leftarrow -\sigma} X_{iR}^{-\sigma\leftarrow -\sigma} X_{jR}^{0\leftarrow \sigma}$ $- \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma} X_{jR}^{-\sigma$ $- \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}$ $- \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{jR}^{-\sigma \leftarrow d} + X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow d} X_{jR}^{-\sigma \leftarrow 0}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow d} X_{jR}^{d \leftarrow \sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow \sigma}$ $+ \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jR}^{0 \leftarrow \sigma}$ $+ \ X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}$ $-~X_{iN}^{-\sigma \leftarrow -\sigma}X_{jN}^{d \leftarrow -\sigma}X_{iR}^{-\sigma \leftarrow \sigma}X_{jR}^{\sigma \leftarrow d} + X_{iN}^{-\sigma \leftarrow -\sigma}X_{jN}^{d \leftarrow -\sigma}X_{jR}^{-\sigma \leftarrow 0}X_{iR}^{0 \leftarrow d}$ $- \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow d} - \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}$ $- \ X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}$ $- \ X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}$ $-X_{iN}^{\sigma\leftarrow-\sigma}X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow0}X_{iR}^{-\sigma\leftarrow d}+X_{iN}^{\sigma\leftarrow-\sigma}X_{iN}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow0}$ $- \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow d} X_{jR}^{d \leftarrow \sigma} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow \sigma}$ $+ X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{0 \leftarrow \sigma}$ $+ \ X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}$ $- \ X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} + X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{iR}^{-\sigma \leftarrow 0} X_{iR}^{0 \leftarrow d}$ $- \, \eta(\sigma) X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow \sigma} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow d}$

$$[[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{+}]=0$$

$$\begin{split} |[\hat{H}_{1,cross}^{A}, \hat{H}_{l,R}], \hat{H}_{l,N}^{-}]| &= + X_{lN}^{Ad-} X_{lR}^{ac} - S_{lR}^{ac} - S_{lR}^{ac}$$

$$[[\hat{H}_{t.cross}^{+\ominus}, \hat{H}_{t.R}], \hat{H}_{t.cross}^{0\oplus}] = 0$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}] = &+ X_{iN}^{d\leftarrow 0} X_{iR}^{-\sigma\leftarrow d} X_{jR}^{0\leftarrow -\sigma} \\ &- \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{-\sigma\leftarrow d} X_{jR}^{\sigma\leftarrow d} + \eta(\sigma) X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{0\leftarrow \sigma} \\ &+ X_{iN}^{d\leftarrow 0} X_{iR}^{0\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow d} + X_{iR}^{\sigma\leftarrow d} X_{jR}^{0\leftarrow \sigma} X_{iN}^{d\leftarrow 0} \\ &+ \eta(\sigma) X_{iR}^{\sigma\leftarrow d} X_{jR}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow 0} + X_{iR}^{-\sigma\leftarrow d} X_{jR}^{0\leftarrow -\sigma} X_{iN}^{d\leftarrow 0} \\ &- \eta(\sigma) X_{iR}^{-\sigma\leftarrow d} X_{jR}^{\sigma\leftarrow d} X_{iN}^{d\leftarrow 0} \end{split}$$

$$[[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{-\ominus}] &= -X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{0\leftarrow -\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{\sigma\leftarrow d} - X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{d\leftarrow \sigma} - X_{iR}^{d\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{d\leftarrow \sigma} - X_{iR}^{d\leftarrow 0}X_{jR}^{0\leftarrow \sigma}X_{iN}^{d\leftarrow d} \\ &- \eta(\sigma)X_{iR}^{i\leftarrow 0}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d} - \eta(\sigma)X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma}X_{iN}^{d\leftarrow d} \\ &- X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d}X_{iN}^{d\leftarrow d} - X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}X_{iN}^{d\leftarrow d} \\ &+ \eta(\sigma)X_{iR}^{d\leftarrow \sigma}X_{jR}^{o\leftarrow d}X_{iN}^{d\leftarrow d} - X_{iR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow -\sigma}X_{iR}^{d\leftarrow d} \\ &+ \eta(\sigma)X_{jR}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow -\sigma}X_{iN}^{d\leftarrow d} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow d}X_{jR}^{-\sigma\leftarrow 0} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{-\sigma\leftarrow +\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{d\leftarrow \sigma} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow -\sigma} + \chi_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow -\sigma} + \chi_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow -\sigma}$$

$$\begin{split} [[\hat{H}^{+\ominus}_{t,cross},\hat{H}_{t,R}],\hat{H}_{t,R}] &= +X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow0} \\ &- X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow d} + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow-\sigma} \\ &- X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow d} + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow0} \\ &- X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow d} + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow0} \\ &+ X_{iN}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{o\leftarrow \sigma} + \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{0\leftarrow-\sigma}X_{jR}^{0\leftarrow0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{0\leftarrow-\sigma}X_{jR}^{-\sigma\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{o\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{0\leftarrow-\sigma}X_{jR}^{0\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow-\sigma}X_{jR}^{o\leftarrow d} - X_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow \sigma}X_{iN}^{0\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma} - X_{iR}^{-\sigma\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iR}^{d\rightarrow\sigma}X_{jR}^{0}X_{iN}^{d\leftarrow-\sigma} - X_{iR}^{-\sigma\leftarrow d}X_{jR}^{o\leftarrow\sigma}X_{iN}^{d\leftarrow-\sigma} \\ &+ X_{iR}^{-\sigma\leftarrow d}X_{jR}^{0\leftarrow d}X_{iN}^{0\leftarrow-\sigma}X_{iN}^{d\leftarrow-\sigma} - X_{iR}^{-\sigma\leftarrow d}X_{iN}^{o\leftarrow-\sigma}X_{iN}^{o\leftarrow-\sigma}X_{iN}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{jR}^{d\leftarrow d}X_{iR}^{0\leftarrow\sigma}X_{iN}^{d\leftarrow-\sigma} - X_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{jR}^{d\leftarrow d}X_{iR}^{0\leftarrow\sigma}X_{iN}^{d\leftarrow-\sigma} - \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iN}^{d\leftarrow-\sigma} \\ &- \chi_{iR}^{o\leftarrow d}X_{jR}^{-\sigma}X_{iN}^{0\leftarrow-\sigma}X_{iN}^{o\leftarrow-\sigma} - \eta(\sigma)X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow-\sigma} \\ &- \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow-\sigma} + \eta(\sigma)X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{jR}^{o\leftarrow d}X_{iR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow\sigma}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{jR}^{o\leftarrow d}X_{iR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &+ \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \chi_{iR}^{o\leftarrow d}X_{jR}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iR}^{o\leftarrow d}X_{iN}^{o\leftarrow d}X_{iN}^{o\leftarrow-\sigma} + \eta(\sigma)X_{jR}^{o\leftarrow d}X_{iR}^{o\leftarrow d}X_{iN}^{o$$

$$\begin{split} ||\hat{H}_{t,cross}^{\perp,\oplus,cross},\hat{H}_{t,R}|, \hat{H}_{0,N}^{\perp}|| &= + \eta(\sigma) X_{iN}^{\perp,-\sigma-e-} X_{iR}^{\perp,d-d} X_{jR}^{\sigma-d} X_{jN}^{\sigma-d} + X_{iR}^{-\sigma-e-} X_{iR}^{\perp,d-d} X_{jR}^{\perp,e-d} X_{jN}^{-d-d} + X_{iR}^{-\sigma-e-} X_{iR}^{\perp,d-d} X_{jR}^{\perp,e-d} X_{jN}^{-d-d} + X_{iN}^{-\sigma-e-} X_{iR}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jN}^{\perp,e-d} - X_{jN}^{-\sigma-d} + X_{iN}^{-\sigma-e-} X_{iR}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jN}^{\perp,e-d} - X_{jN}^{-\sigma-d} + X_{iN}^{-\sigma-e-} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} - X_{jN}^{-\sigma-d} X_{iN}^{\perp,e-d} X_{jN}^{\perp,e-d} - \eta(\sigma) X_{iN}^{-\sigma-e-} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{iN}^{-\sigma-e-d} X_{iN}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{iN}^{-\sigma-e-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \chi_{iN}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jN}^{\perp,e-d} + \chi_{iN}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jR}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{iN}^{-\sigma-e-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{jN}^{-\sigma-e-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{jR}^{-\sigma-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{jR}^{-\sigma-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-d} + \eta(\sigma) X_{jR}^{\perp,e-d} X_{iR}^{\perp,e-d} X_{jN}^{\perp,e-d} X_{jN}^{\perp,e-$$

 $[[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{+}] = + X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $-~X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow d}X_{jR}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow-\sigma}+\eta(\sigma)X_{iN}^{-\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow d}X_{jR}^{d\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma}$ $-~\eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{0\leftarrow-\sigma}X_{jN}^{0\leftarrow-\sigma}-X_{iN}^{-\sigma\leftarrow-\sigma}X_{iR}^{d\leftarrow0}X_{jR}^{\sigma\leftarrow d}X_{jN}^{0\leftarrow\sigma}$ $+ \ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{d\leftarrow0} X_{jR}^{\sigma\leftarrow d} X_{jN}^{0\leftarrow-\sigma} + X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{\sigma\leftarrow\sigma} X_{jR}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow0} X_{jN$ $- \ X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow 0} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $- \ \eta(\sigma) X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} X_{jN}^{0 \leftarrow -\sigma} + X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0} X_{jN}^{0 \leftarrow \sigma}$ $- \ X_{iN}^{-\sigma \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} X_{jN}^{0 \leftarrow \sigma}$ $+~\eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{d\leftarrow\sigma}X_{jN}^{0\leftarrow-\sigma}-X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{-\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}$ $+\,X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{iN}^{-\sigma\leftarrow\sigma}X_{jN}^{0\leftarrow-\sigma}-X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{iN}^{-\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma}$ $+ \ X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $+ \ \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $+ \ \eta(\sigma) X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} + \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $- \eta(\sigma) X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $+ \ X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} + X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma}$ $- \left. X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow -\sigma} X_{iN}^{0 \leftarrow \sigma} \right.$ $+ \ \eta(\sigma) X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} X_{iN}^{-\sigma \leftarrow \sigma} X_{jN}^{0 \leftarrow -\sigma} - X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $- \, \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} - X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{\sigma \leftarrow \sigma} X_{jR}^{\sigma \leftarrow 0}$ $-~\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow \sigma}X_{iR}^{\sigma\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma}-X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow \sigma}X_{iR}^{\sigma\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0}$ $+~\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jR}^{d\leftarrow\sigma}+X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow\sigma}X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}$ $+ \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} + \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow 0}$ $+ \ \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} - \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $+ \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{iR}^{d \leftarrow 0} X_{jR}^{\sigma \leftarrow d} - X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow}$ $+ \ \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow -\sigma} + X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{\sigma \leftarrow 0}$ $+ \ \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow d} X_{jR}^{d \leftarrow -\sigma} + \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $-X_{jN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d}+X_{jN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow 0}$ $+ \ \eta(\sigma) X_{jN}^{d\leftarrow\sigma} X_{iN}^{0\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jR}^{d\leftarrow-\sigma} + X_{jN}^{d\leftarrow\sigma} X_{iN}^{0\leftarrow d} X_{iR}^{\sigma\leftarrow-\sigma} X_{jR}^{-\sigma\leftarrow0}$ $- \, \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} - X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow 0}$ $- \ X_{jN}^{d\leftarrow\sigma} X_{iN}^{0\leftarrow d} X_{jR}^{\sigma\leftarrow 0} X_{iR}^{-\sigma\leftarrow -\sigma} - \eta(\sigma) X_{jN}^{d\leftarrow\sigma} X_{iN}^{0\leftarrow d} X_{jR}^{d\leftarrow -\sigma} X_{iR}^{0\leftarrow 0}$ $- \ \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow -\sigma} + \eta(\sigma) X_{jN}^{d \leftarrow \sigma} X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}$ $-\,X_{jN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow 0}X_{jR}^{\sigma\leftarrow d}+X_{jN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow}$ $-\eta(\sigma)X_{iN}^{d\leftarrow\sigma}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}$

$$[[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,Ni}^{-}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\oplus}] &= + \, \eta(\sigma) X_{iN}^{0\leftarrow d} X_{iR}^{-\sigma\leftarrow 0} X_{jR}^{\sigma\leftarrow 0} \\ &\quad + X_{iN}^{0\leftarrow d} X_{iR}^{-\sigma\leftarrow 0} X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow \sigma} X_{jR}^{\sigma\leftarrow 0} \\ &\quad + \eta(\sigma) X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow \sigma} X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow 0} \\ &\quad - \eta(\sigma) X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow 0} \\ &\quad - \eta(\sigma) X_{iN}^{0\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{d\leftarrow \sigma} \end{split}$$

$$[[\hat{H}_{t.cross}^{-\oplus},\hat{H}_{t.R}],\hat{H}_{t.cross}^{0\ominus}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}] &= + \eta(\sigma)X_{iN}^{-\sigma} \overset{\leftarrow}{\sim} X_{iR}^{d-\sigma} X_{jR}^{0\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{d\leftarrow-\sigma} X_{jR}^{0\leftarrow-\sigma} - X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{d\leftarrow\sigma} X_{jR}^{\sigma\leftarrow} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{a\leftarrow} X_{jR}^{\sigma\leftarrow} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{a\leftarrow} X_{jR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{a\leftarrow} X_{jR}^{\sigma\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{o\leftarrow-d} X_{jR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-d} X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{o\leftarrow-d} X_{jR}^{-\sigma\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{o\leftarrow-d} X_{jR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &- X_{iN}^{-\sigma\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} - \eta(\sigma)X_{jR}^{o\leftarrow-\sigma} X_{iR}^{-\sigma\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &+ X_{jR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} + \eta(\sigma)X_{jR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &- \chi_{jR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} - \chi_{iR}^{d\leftarrow\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow-\sigma} \\ &+ X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-d} X_{iN}^{o\leftarrow-\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} \\ &- X_{jR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} \\ &- X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} \\ &- X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} - \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow-d} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} - X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} - X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} - X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} - X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma$$

$$[[\hat{H}_{t.cross}^{-\oplus}, \hat{H}_{t.R}], \hat{H}_{t.cross}^{-\oplus}] = 0$$

$$\begin{split} [[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,R}] &= + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{\sigma\leftarrow \sigma} + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{d\leftarrow d} \\ &- X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow -\sigma} + X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{d\leftarrow d} \\ &+ X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow -\sigma} - X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow \sigma} X_{jR}^{o\leftarrow -\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow -\sigma} + \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow d} X_{jR}^{o\leftarrow -\sigma} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow 0} X_{jR}^{d\leftarrow d} - \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow d} X_{jR}^{o\leftarrow 0} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} - \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow d} X_{jR}^{o\leftarrow 0} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} - \eta(\sigma) X_{iN}^{\sigma\leftarrow -\sigma} X_{iR}^{o\leftarrow d} X_{jR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} - \eta(\sigma) X_{iN}^{\sigma\leftarrow -\sigma} X_{iR}^{o\leftarrow d} X_{iN}^{o\leftarrow d} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow 0} X_{iN}^{o\leftarrow d} + X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow d} \\ &- X_{iR}^{d\leftarrow \sigma} X_{jR}^{o\leftarrow 0} X_{iN}^{-\sigma\leftarrow d} + X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow d} \\ &- X_{iR}^{d\leftarrow \sigma} X_{jR}^{o\leftarrow -\sigma} X_{iN}^{-\sigma\leftarrow d} + X_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow d} \\ &+ \eta(\sigma) X_{jR}^{o\leftarrow -\sigma} X_{iN}^{o\leftarrow 0} - \chi_{iN}^{o\leftarrow d} + \eta(\sigma) X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow d} \\ &+ \eta(\sigma) X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \eta(\sigma) X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \eta(\sigma) X_{jR}^{o\leftarrow 0} X_{iR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \eta(\sigma) X_{iR}^{o\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \chi_{iR}^{d\leftarrow -\sigma} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} - \eta(\sigma) X_{jR}^{o\leftarrow 0} X_{iR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &- \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &- \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} + \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}^{o\leftarrow 0} \\ &+ \chi_{iR}^{d\leftarrow 0} X_{jR}^{o\leftarrow 0} X_{iN}$$