term-3-0E

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{+}], \hat{H}_{t,Ni}^{-}] = 0$$

$$[[\hat{H}_{t,Ni}^{0}, \hat{H}_{t,Ni}^{-}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = &+ X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} \\ &+ X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} + X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow-\sigma} \\ &+ X_{iN}^{\sigma\leftarrow0} X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} + X_{iN}^{\sigma\leftarrow0} X_{iR}^{0\leftarrow\sigma} X_{jN}^{0\leftarrow\sigma} \\ &+ X_{iN}^{\sigma\leftarrow0} X_{jN}^{-\sigma\leftarrow\sigma} X_{iR}^{0\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow0} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{d\leftarrow-\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow d} X_{iR}^{0\leftarrow0} X_{jN}^{d\leftarrow-\sigma} - X_{iN}^{-\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow-\sigma} X_{jN}^{0\leftarrow\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow d} X_{iR}^{o\leftarrow\sigma} X_{jN}^{d\leftarrow-\sigma} - X_{iN}^{-\sigma\leftarrow0} X_{iR}^{\sigma\leftarrow-\sigma} X_{jN}^{0\leftarrow\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow d} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{d\leftarrow-\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{0\ominus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = & + X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iR}^{0\leftarrow0} \\ & + X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow\sigma} - X_{jN}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow\sigma} \\ & + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow\sigma} X_{iR}^{0\leftarrow0} - X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} \\ & + X_{jN}^{-\sigma\leftarrow d} X_{iN}^{d\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} - X_{jN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ & + X_{jN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} X_{jN}^{\sigma\leftarrow\sigma} + X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ & + X_{iN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} X_{jN}^{\sigma\leftarrow0} + X_{iN}^{0\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow\sigma} X_{jN}^{\sigma\leftarrow0} \\ & + X_{iN}^{0\leftarrow\sigma} X_{iR}^{d\leftarrow d} X_{jN}^{\sigma\leftarrow0} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{+\ominus}_{t,cross}],\hat{H}^{-\ominus}_{t,cross}] = &+ X_{iN}^{-\sigma \leftarrow d} X_{iR}^{d \leftarrow d} X_{jN}^{d \leftarrow -\sigma} \\ &+ X_{iN}^{-\sigma \leftarrow d} X_{iR}^{\sigma \leftarrow \sigma} X_{jN}^{d \leftarrow -\sigma} - X_{iN}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} \\ &+ X_{iN}^{\sigma \leftarrow 0} X_{iR}^{d \leftarrow d} X_{jN}^{0 \leftarrow \sigma} - X_{iN}^{\sigma \leftarrow d} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} \\ &+ X_{iN}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow -\sigma} X_{jN}^{0 \leftarrow \sigma} - X_{iN}^{\sigma \leftarrow -\sigma} X_{jN}^{d \leftarrow -\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{0}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = &+ X^{\sigma\leftarrow0}_{jN} X^{0\leftarrow\sigma}_{iN} X^{d\leftarrow d}_{iR} \\ &- X^{\sigma\leftarrow0}_{jN} X^{0\leftarrow-\sigma}_{iN} X^{-\sigma\leftarrow\sigma}_{iR} + X^{\sigma\leftarrow0}_{jN} X^{0\leftarrow\sigma}_{iN} X^{-\sigma\leftarrow\sigma}_{iR} \\ &+ X^{-\sigma\leftarrow d}_{jN} X^{d\leftarrow-\sigma}_{iN} X^{d\leftarrow d}_{iR} + X^{-\sigma\leftarrow d}_{jN} X^{d\leftarrow-\sigma}_{iN} X^{\sigma\leftarrow\sigma}_{iR} \\ &- X^{-\sigma\leftarrow d}_{jN} X^{d\leftarrow\sigma}_{iN} X^{\sigma\leftarrow-\sigma}_{iR} \end{split}$$

$$[[\hat{H}^0_{t,Ni},\hat{H}_{t,R}],\hat{H}^0_{t,Ni}]=0$$

$$[[\hat{H}^0_{t,Ni},\hat{H}_{t,R}],\hat{H}_{t,R}] = 0$$

$$[[\hat{H}_{t,Ni}^+,\hat{H}_{t,Ni}^-],\hat{H}_{t,Ni}^0]=0$$

```
[[\hat{H}_{t,Ni}^{+},\hat{H}_{t,Ni}^{-}],\hat{H}_{t,R}] = + X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{\sigma \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}
                                                                                                        +~\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{iR}^{\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d}+\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma}
                                                                                                        + \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        +~\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{jR}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow d}+\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{jR}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow \sigma}
                                                                                                        + \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow d} + X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{0 \leftarrow 0}
                                                                                                        - \ \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}
                                                                                                        + \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} + X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                        -~\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow d}-\eta(\sigma)X_{iN}^{d\leftarrow d}X_{jN}^{0\leftarrow 0}X_{jR}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow -\sigma}
                                                                                                        + \ X_{iN}^{d \leftarrow d} X_{jN}^{0 \leftarrow 0} X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow d} - X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}
                                                                                                        -~\eta(\sigma)X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow0}X_{jR}^{-\sigma\leftarrow d}-\eta(\sigma)X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{0\leftarrow\sigma}
                                                                                                        - \ X_{iN}^{d\leftarrow 0} X_{jN}^{0\leftarrow d} X_{iR}^{d\leftarrow -\sigma} X_{jR}^{-\sigma\leftarrow d} - X_{iN}^{d\leftarrow 0} X_{jN}^{0\leftarrow d} X_{jR}^{\sigma\leftarrow 0} X_{iR}^{0\leftarrow d} X_{iR}^{\sigma\leftarrow 0} X_{iR}^{0\leftarrow d} X_{iR}^{\sigma\leftarrow 0} X_{iR}^{0\leftarrow 0} X_{iR}
                                                                                                        - \ \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        - \ X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow d} - X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}
                                                                                                        + \ \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{iR}^{d \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}
                                                                                                         -X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow d}-X_{iN}^{d\leftarrow 0}X_{jN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow -\sigma}
                                                                                                        + \ \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{d \leftarrow 0} X_{jN}^{0 \leftarrow d} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                        -~X_{iN}^{d\leftarrow0}X_{jN}^{0\leftarrow d}X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}+X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}
                                                                                                        +~\eta(\sigma)X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow 0}X_{iR}^{\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d}+\eta(\sigma)X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow 0}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow \sigma}
                                                                                                        + \ X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} + X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        + \ \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} + \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        +\,X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow 0}X_{jR}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}+X_{jN}^{d\leftarrow d}X_{iN}^{0\leftarrow 0}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{0\leftarrow -\sigma}
                                                                                                        - \ \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{\sigma \leftarrow d} - \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{iR}^{d \leftarrow \sigma} X_{jR}^{0 \leftarrow -\sigma}
                                                                                                        + \ X_{jN}^{d\leftarrow d} X_{iN}^{0\leftarrow 0} X_{iR}^{d\leftarrow \sigma} X_{jR}^{\sigma\leftarrow d} + X_{jN}^{d\leftarrow d} X_{iN}^{0\leftarrow 0} X_{jR}^{-\sigma\leftarrow 0} X_{iR}^{0\leftarrow -\sigma}
                                                                                                        - \ \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} - \eta(\sigma) X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                        + \ X_{jN}^{d \leftarrow d} X_{iN}^{0 \leftarrow 0} X_{jR}^{d \leftarrow \sigma} X_{iR}^{\sigma \leftarrow d} - X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{iR}^{\sigma \leftarrow 0} X_{jR}^{0 \leftarrow \sigma}
                                                                                                        -~\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{\sigma\leftarrow0}X_{jR}^{-\sigma\leftarrow d}-\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow-\sigma}X_{jR}^{0\leftarrow\sigma}
                                                                                                        - \ X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{iR}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} - X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        -~\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow d}-\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{jR}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow\sigma}
                                                                                                        - \ X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{-\sigma \leftarrow d} - X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{iR}^{-\sigma \leftarrow 0} X_{jR}^{0 \leftarrow -\sigma}
                                                                                                        +~\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{\sigma\leftarrow d}+\eta(\sigma)X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow-\sigma}
                                                                                                        -~X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jR}^{\sigma\leftarrow d}-X_{jN}^{d\leftarrow 0}X_{iN}^{0\leftarrow d}X_{jR}^{-\sigma\leftarrow 0}X_{iR}^{0\leftarrow}
                                                                                                        + \ \eta(\sigma) X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} + \eta(\sigma) X_{jN}^{d \leftarrow 0} X_{iN}^{0 \leftarrow d} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                        -~X_{jN}^{d\leftarrow0}X_{iN}^{0\leftarrow d}X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}-X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}
                                                                                                        - \ \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{iR}^{\sigma \leftarrow 0} X_{jR}^{-\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow -\sigma} X_{jR}^{0 \leftarrow \sigma}
                                                                                                        - \ X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow d} - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{0 \leftarrow \sigma}
                                                                                                        - \ \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{\sigma \leftarrow 0} X_{iR}^{-\sigma \leftarrow d} - \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow -\sigma} X_{iR}^{0 \leftarrow \sigma}
                                                                                                         -X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow-\sigma}X_{iR}^{-\sigma\leftarrow d}-X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{0\leftarrow-\sigma}
                                                                                                        +~\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow0}X_{jR}^{\sigma\leftarrow0}+\eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow-\sigma}
                                                                                                        - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{iR}^{d \leftarrow \sigma} X_{jR}^{\sigma \leftarrow d} - X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{-\sigma \leftarrow 0} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                        + \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{3 - \sigma \leftarrow 0} X_{iR}^{\sigma \leftarrow d} + \eta(\sigma) X_{iN}^{\sigma \leftarrow \sigma} X_{jN}^{-\sigma \leftarrow -\sigma} X_{jR}^{d \leftarrow \sigma} X_{iR}^{0 \leftarrow -\sigma}
                                                                                                         -X_{iN}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}-X_{jN}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow0}X_{jR}^{0\leftarrow\sigma}
```

$$\begin{split} [[\hat{H}_{t,Ni}^{+},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,cross}^{-\oplus}] = & -X_{iN}^{-\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma} \\ & +X_{iN}^{\sigma\leftarrow0}X_{iR}^{d\leftarrow d}X_{jN}^{0\leftarrow\sigma} + X_{iN}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma} \end{split}$$

$$\begin{split} [[\hat{H}^{+}_{t,Ni},\hat{H}^{-\oplus}_{t,cross}],\hat{H}^{0\ominus}_{t,cross}] = &-X^{-\sigma\leftarrow0}_{iN}X^{\sigma\leftarrow-\sigma}_{iR}X^{0\leftarrow\sigma}_{jN} \\ &+X^{\sigma\leftarrow0}_{iN}X^{d\leftarrow d}_{iR}X^{0\leftarrow\sigma}_{jN} + X^{\sigma\leftarrow0}_{iN}X^{-\sigma\leftarrow-\sigma}_{iN}X^{0\leftarrow\sigma}_{jN} \end{split}$$

$$[[\hat{H}_{t,Ni}^+, \hat{H}_{t,R}^-], \hat{H}_{t,Ni}^-] = 0$$

$$\begin{split} [[\hat{H}^{-}_{t,Ni},\hat{H}^{0\oplus}_{t,cross}],\hat{H}^{+\ominus}_{t,cross}] = &-X^{0\leftarrow -\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{\sigma\leftarrow0}_{jN} \\ &+X^{0\leftarrow\sigma}_{iN}X^{-\sigma\leftarrow\sigma}_{iR}X^{\sigma\leftarrow0}_{jN} + X^{0\leftarrow\sigma}_{iN}X^{d\leftarrow d}_{iR}X^{\sigma\leftarrow0}_{jN} \end{split}$$

$$\begin{split} [[\hat{H}_{t,Ni}^{-},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,cross}^{0\oplus}] = & -X_{iN}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{-\sigma\leftarrow d} \\ & +X_{iN}^{d\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow d} + X_{iN}^{d\leftarrow-\sigma}X_{iR}^{0\leftarrow0}X_{jN}^{-\sigma\leftarrow d} \end{split}$$

$$[[\hat{H}_{t,Ni}^{-}, \hat{H}_{t,R}], \hat{H}_{t,Ni}^{+}] = 0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,Ni}^{0}] &= + X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{\sigma\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{jN}^{-\sigma\leftarrow0} + X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow d}X_{jN}^{\sigma\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow d}X_{jN}^{-\sigma\leftarrow0} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow0} \\ &+ X_{iN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow d}X_{jN}^{-\sigma\leftarrow0} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma} \\ &- X_{iN}^{\sigma\leftarrow d}X_{iR}^{0\leftarrow\sigma}X_{jN}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow0}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{0\leftarrow\sigma} \\ &- X_{iN}^{\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow-\sigma}X_{jN}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma} \\ &+ X_{iN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma} + X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow0} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow-\sigma} - X_{jN}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{o\leftarrow-\sigma} \\ &- X_{iN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow-\sigma} - X_{jN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &- X_{iN}^{-\sigma\leftarrow0}X_{jN}^{0\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow-\sigma} + X_{jN}^{\sigma\leftarrow0}X_{jN}^{0\leftarrow-\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &- X_{jN}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} + X_{jN}^{\sigma\leftarrow0}X_{iN}^{d\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &- X_{jN}^{-\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} + X_{jN}^{\sigma\leftarrow0}X_{iN}^{d\leftarrow\sigma}X_{iR}^{d\leftarrow-\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow0}X_{iN}^{d\leftarrow\sigma}X_{iR}^{o\leftarrow-\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{d\leftarrow-\sigma}X_{iR}^{o\leftarrow-\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{o\leftarrow-\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X_{iR}^{\sigma\leftarrow-\sigma} \\ &+ X_{jN}^{\sigma\leftarrow0}X_{iN}^{0\leftarrow\sigma}X_{iR}^{\sigma\leftarrow-\sigma} - X_{jN}^{\sigma\leftarrow-\sigma}X_{iN}^{\sigma\leftarrow-\sigma}X$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{0\ominus}],\hat{H}_{t,R}] &= + X_{iN}^{0\leftarrow0} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} X_{jR}^{0\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iN}^{0\leftarrow0} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} + \eta(\sigma) X_{iN}^{0\leftarrow0} X_{iR}^{d\leftarrow-\sigma} X_{jR}^{0\leftarrow\sigma} \\ &+ X_{iN}^{0\leftarrow0} X_{iR}^{d\leftarrow-\sigma} X_{jR}^{-\sigma\leftarrowd} - X_{iN}^{\sigma\leftarrow\sigma} X_{iR}^{0\leftarrow\sigma} X_{jR}^{\sigma\leftarrow0} \\ &- \eta(\sigma) X_{iN}^{\sigma\leftarrow\sigma} X_{iR}^{0\leftarrow\sigma} X_{jR}^{0\leftarrow-\sigma} - \eta(\sigma) X_{iN}^{\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} \\ &- X_{iN}^{\sigma\leftarrow\sigma} X_{iR}^{-\sigma\leftarrow} X_{jR}^{0\leftarrow\sigma} + \eta(\sigma) X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} X_{jR}^{o\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{d\leftarrow-\sigma} + X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{0\leftarrow-\sigma} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} + X_{iN}^{\sigma\leftarrow\sigma} X_{jR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow-\sigma} \\ &+ \eta(\sigma) X_{iN}^{-\sigma\leftarrow\sigma} X_{iR}^{\sigma\leftarrow0} X_{jR}^{\sigma\leftarrow0} X_{iN}^{\sigma\leftarrow\sigma} + \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{jR}^{0\leftarrow\sigma} X_{iN}^{o\leftarrow\sigma} \\ &+ \eta(\sigma) X_{iR}^{\sigma\leftarrow0} X_{jR}^{-\sigma\leftarrow0} X_{iN}^{\sigma\leftarrow\sigma} - X_{jR}^{\sigma\leftarrow0} X_{iN}^{0\leftarrow\sigma} X_{iN}^{o\leftarrow\sigma} \\ &+ X_{iR}^{d\leftarrow-\sigma} X_{jR}^{-\sigma\leftarrow0} X_{iN}^{\sigma\leftarrow\sigma} - \chi_{jR}^{\sigma\leftarrow0} X_{iR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow0} \\ &- X_{jR}^{\sigma\leftarrow0} X_{iR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow\sigma} - \eta(\sigma) X_{jR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- \eta(\sigma) X_{jR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow\sigma} X_{iN}^{o\leftarrow0} - \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{iR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow-\sigma} X_{iR}^{\sigma\leftarrow0} X_{iN}^{o\leftarrow0} + \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow-\sigma} X_{jR}^{\sigma\leftarrow0} X_{iN}^{o\leftarrow0} + \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow-\sigma} X_{jR}^{\sigma\leftarrow0} X_{iN}^{o\leftarrow0} + \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow-\sigma} X_{jR}^{\sigma\leftarrow0} X_{iN}^{o\leftarrow0} + \eta(\sigma) X_{iR}^{d\leftarrow-\sigma} X_{jR}^{o\leftarrow-\sigma} X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow-\sigma} X_{jR}^{\sigma\leftarrow0} X_{iN}^{o\leftarrow-\sigma} X_{iN}^{$$

$$[[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,cross}^{+\ominus}],\hat{H}_{t,Ni}^{-}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}] &= + \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow}dX_{jR}^{-\sigma\leftarrow0}\\ &- X_{iN}^{0\leftarrow0}X_{iR}^{\sigma\leftarrow}dX_{jR}^{d\leftarrow\sigma} + \eta(\sigma)X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}\\ &- X_{iN}^{0\leftarrow0}X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow}d - X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}\\ &- X_{iN}^{0\leftarrow0}X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow}d - X_{jR}^{o\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}\\ &- \eta(\sigma)X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow}dX_{iN}^{0\leftarrow0} - \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}\\ &- X_{jR}^{d\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow}dX_{iN}^{0\leftarrow0} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}\\ &- X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow}dX_{iN}^{0\leftarrow0} + \eta(\sigma)X_{jR}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}\\ &- X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow}dX_{iN}^{0\leftarrow0} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &- X_{iR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}dX_{iN}^{0\leftarrow0} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow0}\\ &- X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{d\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma} - X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &- \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow$$

$$[[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,cross}^{-\oplus}],\hat{H}_{t,Ni}^{+}]=0$$

$$\begin{split} [[\hat{H}_{t,cross}^{0\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{0\ominus}] &= -X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma} + \eta(\sigma)X_{iN}^{\sigma\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,Ni}^{0}] = &+ X_{iN}^{d\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{jN}^{-\sigma\leftarrow d} \\ &+ X_{iN}^{d\leftarrow \sigma}X_{iR}^{-\sigma\leftarrow -\sigma}X_{jN}^{\sigma\leftarrow d} + X_{iN}^{d\leftarrow -\sigma}X_{iR}^{0\leftarrow 0}X_{jN}^{-\sigma\leftarrow d} \\ &+ X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow 0}X_{jN}^{\sigma\leftarrow d} - X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow d}X_{jN}^{d\leftarrow -\sigma} \\ &+ X_{iN}^{d\leftarrow \sigma}X_{iR}^{0\leftarrow 0}X_{jN}^{\sigma\leftarrow d} - X_{iN}^{-\sigma\leftarrow d}X_{iR}^{d\leftarrow \sigma}X_{jN}^{d\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{d\leftarrow d}X_{jN}^{0\leftarrow -\sigma} - X_{iN}^{-\sigma\leftarrow d}X_{iR}^{\sigma\leftarrow \sigma}X_{jN}^{d\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow 0}X_{iR}^{\sigma\leftarrow \sigma}X_{jN}^{0\leftarrow -\sigma} + X_{iN}^{\sigma\leftarrow d}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{d\leftarrow -\sigma} \\ &+ X_{iN}^{\sigma\leftarrow 0}X_{iR}^{-\sigma\leftarrow \sigma}X_{jN}^{0\leftarrow -\sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,cross}^{-\ominus}],\hat{H}_{t,R}] &= + \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{\sigma\leftarrow 0} \\ &+ X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{0\leftarrow -\sigma} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{\sigma\leftarrow d} + X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow \sigma}X_{jR}^{\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow \sigma}X_{jR}^{d\leftarrow -\sigma} + X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow 0} \\ &- \eta(\sigma)X_{iN}^{d\leftarrow d}X_{iR}^{0\leftarrow -\sigma}X_{jR}^{d\leftarrow -\sigma} - \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow -\sigma}X_{jR}^{0\leftarrow -\sigma} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{d\leftarrow 0}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow 0} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow 0} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow \sigma} + X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow 0} \\ &- X_{iN}^{-\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow 0}X_{jR}^{-\sigma\leftarrow d} - \eta(\sigma)X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow 0} \\ &+ \eta(\sigma)X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow \sigma} \\ &+ X_{iN}^{\sigma\leftarrow -\sigma}X_{iR}^{-\sigma\leftarrow d}X_{jR}^{d\leftarrow \sigma} \end{split}$$

$$\begin{split} [[\hat{H}_{t,cross}^{+\ominus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{-\ominus}] &= -\eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}\\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{-\sigma\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{0\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}\\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow}A + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}\\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow}A + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}\\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{o\leftarrow}A - X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}\\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}X_{jR}^{o\leftarrow\sigma}A + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}AX_{jR}^{d\leftarrow\sigma}A + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}AX_{jR}^{d\leftarrow\sigma}A + \eta(\sigma)X_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &+ X_{iN}^{o\leftarrow\sigma}X_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A + X_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &+ \chi_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A + \chi_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &+ \chi_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A + \eta(\sigma)X_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &+ \chi_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}A + \eta(\sigma)X_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{jR}^{o\leftarrow\sigma}A \\ &+ \chi_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}A + \eta(\sigma)AX_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}A + \eta(\sigma)AX_{iN}^{o\leftarrow\sigma}AX_{iR}^{o\leftarrow\sigma}A + \eta(\sigma)AX_{iN}^{o\leftarrow\sigma}A + \eta(\sigma)AX_{iN}^{o\leftarrow\sigma}A + \eta($$

$$\begin{split} [[\hat{H}_{t,cross}^{-\oplus},\hat{H}_{t,R}],\hat{H}_{t,cross}^{+\ominus}] &= + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma} \\ &- \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma} - X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrowd} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrowd} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrowd}X_{jR}^{\sigma\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow\sigma} + \eta(\sigma)X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{-\sigma\leftarrow0} \\ &+ X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow\sigma} - X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{iN}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow d}X_{jR}^{d\leftarrow\sigma} - \chi_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &+ X_{jR}^{\sigma\leftarrow0}X_{iR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} - \eta(\sigma)X_{jR}^{\sigma\leftarrow0}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- \eta(\sigma)X_{jR}^{d\leftarrow\sigma}X_{iR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} + \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{iR}^{-\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} - X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- \eta(\sigma)X_{iR}^{d\leftarrow\sigma}X_{jR}^{0\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} - X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &+ X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} + \eta(\sigma)X_{jR}^{-\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &+ X_{iR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} + \eta(\sigma)X_{jR}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} + \eta(\sigma)X_{jR}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{-\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{jR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} + \eta(\sigma)X_{jR}^{-\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{jR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} \\ &- X_{iR}^{d\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma} - X_{iR}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{iN}^{\sigma\leftarrow\sigma}X_{$$