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Mid Term Examination
Third Semester (B.Tech), September - 2016
Applied Mathematics III (ETMA-201)

Time: 1.5 hours

Max. Marks: 30

Note: Attempt three questions in all. Question 1 is compulsory. All questions carry equal marks.

1. Attempt any four parts. Each part carries 2.5 marks.

- Derive Euler's formulae for expansion of a function into Fourier series in the interval $(C, C + 2\pi)$
- State giving reasons whether $\operatorname{cosec} x$ can be expanded into Fourier series in the interval $(-\pi, \pi)$.
- Find the Fourier transform of the function $f(x) = e^{-a|x|}$, $-\infty < x < \infty$
- Solve the difference equation $y_{x+3} - 3y_{x+1} + 2y_x = 0$
- Find the Z transform of $u(n) = n + 2 \sin n\theta + 3$

2. a) Find the Fourier series expansion of the following periodic function $f(x) = |x|$, $-\pi < x < \pi$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (5)

b) Following table gives the variation of a periodic current over a period T (5)

t (sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A (amp)	1.98	1.30	1.05	1.3	-0.88	-0.25	1.98

Express A in Fourier series upto first harmonic.

3. a) Find the Fourier transform of the function $f(x) = e^{-\frac{x^2}{2}}$ (4)

b) The temperature distribution $u(x, t)$ in a semi-infinite rod is determined by the P.D.E.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x < \infty, \text{ subject to conditions}$$

i. $u(x, 0) = 0, \quad x \geq 0$ (6)

ii. $\frac{\partial u}{\partial x} = -\mu$ (a constant), when $x = 0, \quad t > 0$

Determine the temperature formula.

4. a) Form a difference equation by eliminating arbitrary constants a and b ,

$$y_x = a \cos x\theta + b \sin x\theta \quad (4)$$

b) Solve the difference equation $y_{n+1} - 2y_n = n^2 3^n$ (6)

or

4. a) State and Prove Convolution theorem for Z transforms (6)

b) Find the inverse Z transform of $\frac{z}{6z^2 - 5z + 1}$ (4)