USN 1CR11MEC4 10

MATDIP401

Fourth Semester B.E. Degree Examination, June 2012
Advanced Mathematics - II

Time: 3 hrs.

Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the angles between any two diagonals of a cube. **(06 Marks)**

b. Find the equations of two planes, which bisect the angles between the planes $3x - 4y + 5z = 3$, $5x + 3y - 4z = 9$. **(07 Marks)**

c. Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$ **(07 Marks)**

2 a. Find the equation of the plane through the point (1, -1, 0) and perpendicular to the line $2x + 3y + 5z - 1 = 0 = 3x + y - z + 2$. **(06 Marks)**

b. Find the value of k such that the line $\frac{x}{k} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. For this k find their point of intersection. **(07 Marks)**

c. Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\hat{i} - \hat{j})$, $\vec{b} = 6\hat{j}$, $\vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$ form an isosceles triangle. **(06 Marks)**

b. Find the unit normal to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find also the sine of the angle between them. **(07 Marks)**

c. Prove that the position vectors of the points A, B, C and D represented by the vectors $-\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$, respectively are coplanar. **(07 Marks)**

4 a. Find the value of λ so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, λ, 1) may lie on one plane. **(06 Marks)**

b. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C, prove that $(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})$ is a vector perpendicular to the plane of triangle ABC. **(07 Marks)**

c. Find a set of vectors reciprocal to the set $2\hat{i} + 3\hat{j} - \hat{k}$, $\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$. **(07 Marks)**

5 a. Find the maximum directional derivative of $\log(x^2 + y^2 + z^2)$ at (1, 1, 1). **(06 Marks)**

b. Find the unit normal vector to the curve $\vec{r} = 4\sin t\hat{i} + 4\cos t\hat{j} + 3t\hat{k}$. **(07 Marks)**

c. Show that $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

6 a. Find the Laplace transforms of $\sin^2 3t$ and $\sqrt{t}$. **(06 Marks)**

b. Find L[f(t)], given that $f(t) = \begin{cases} t-1 & 0 < t < 2 \\ 3-t & t > 2 \end{cases}$. **(07 Marks)**

c. Find the Laplace transform of $e^{2t}\cos t + t\,e^{-t}\sin 2t$. **(07 Marks)**

7 a. Find the Laplace transform of $\int_0^t \cos 2(t-u)\cos 3u\,du$. **(06 Marks)**

b. Find the inverse Laplace transform of

i) $\dfrac{s+1}{s^2-s+1}$ ii) $\dfrac{1}{s(s^2+a^2)}$. **(14 Marks)**

8 a. Find the inverse Laplace transform by using convolution theorem of $\dfrac{1}{(s^2+a^2)^2}$. **(10 Marks)**

b. By applying Laplace transform, solve the differential equation $\dfrac{d^2y}{dt^2}+5\dfrac{dy}{dt}+6y=5e^{2t}$.

Subject to the conditions $y(0)=2$, $y'(0)=1$. **(10 Marks)**

D.D = $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\therefore |\nabla\phi| = \sqrt{(\nabla\phi)}$

*****

4

a) Find λ so that $A(-1,4,3)$ $B(3,2,-5)$ $C(-3,8,-5)$ $D(-3,\lambda,1)$ may lie on one plane.

$\overrightarrow{OA} = -i + 4j - 3k$

$\overrightarrow{OB} = 3i + 2j - 5k$

$\overrightarrow{OC} = -3i + 8j - 5k$

$\overrightarrow{OD} = -3i + \lambda j + k$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4i - 2j - 2k$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2i + 4j - 2k$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -2i + (\lambda - 4)j + 4k$

Since $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar, $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$

$$\begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & \lambda-4 & 4 \end{vmatrix} = 0$$

$\therefore 12\lambda - 24 = 0 \quad \therefore \boxed{\lambda = 2}$

b) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C prove that $(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$ is perpendicular to ~~each one of vectors~~ plane of triange ABC.

To show $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$ is perpendicular to each one of the vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$

$\overrightarrow{AB} = \bar{b} - \bar{a}$, $\overrightarrow{BC} = \bar{c} - \bar{b}$ $\overrightarrow{CA} = \bar{a} - \bar{c}$

$$(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})\cdot(\bar{b}-\bar{a})$$

$$=(\bar{a}\times\bar{b})\cdot(\bar{b}-\bar{a})+(\bar{b}\times\bar{c})\cdot(\bar{b}-\bar{a})+(\bar{c}\times\bar{a})\cdot(\bar{b}-\bar{a})$$

$$=(\bar{a}\times\bar{b})\cdot\bar{b}-(\bar{a}\times\bar{b})\cdot\bar{a}+(\bar{b}\times\bar{c})\cdot\bar{b}-(\bar{b}\times\bar{c})\cdot\bar{a}$$
$$+(\bar{c}\times\bar{a})\cdot\bar{b}-(\bar{c}\times\bar{a})\cdot\bar{a}$$

$$=[\bar{a}\bar{b}\bar{b}]-[\bar{a}\bar{b}\bar{a}]+[\bar{b}\bar{c}\bar{b}]-[\bar{b}\bar{c}\bar{a}]+[\bar{c}\bar{a}\bar{b}]-[\bar{c}\bar{a}\bar{a}]$$

$$=0-0+0-[\bar{b}\bar{c}\bar{a}]+[\bar{b}\bar{c}\bar{a}]+0$$ (a scalar triple product of 2 equal vectors is zero)

$$=0$$

Similarly $(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})\cdot(\bar{c}-\bar{b})=0$

and $(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})\cdot(\bar{a}-\bar{c})=0$

3) Find a set of vectors reciprocal to the set $2i+3j-k,\ i-j-2k,\ -i+2j+2k$

Let $\bar{a}=2i+3j-k$

$\bar{b}=i-j-2k$

$\bar{c}=-i+2j+2k$

$$[\bar{a}\bar{b}\bar{c}]=\begin{vmatrix}2&3&-1\\1&-1&-2\\-1&2&2\end{vmatrix}=3$$

$$\bar{a}\times\bar{b}=\begin{vmatrix}i&j&k\\2&3&-1\\1&-1&-2\end{vmatrix}=-7i+3j-5k,\quad \bar{b}\times\bar{c}=2i-k,\quad \bar{c}\times\bar{a}=-8i+3j-7k$$

$$\therefore \bar{a'}=\frac{\bar{b}\times\bar{c}}{[\bar{a}\bar{b}\bar{c}]}=\frac{2i-k}{3}=\frac{2}{3}\hat{i}-\frac{1}{3}k,\quad \bar{b'}=\frac{-8}{3}i+j-\frac{7}{3}k$$

$$\bar{c'}=\frac{-7}{3}i+j-\frac{5}{2}k$$

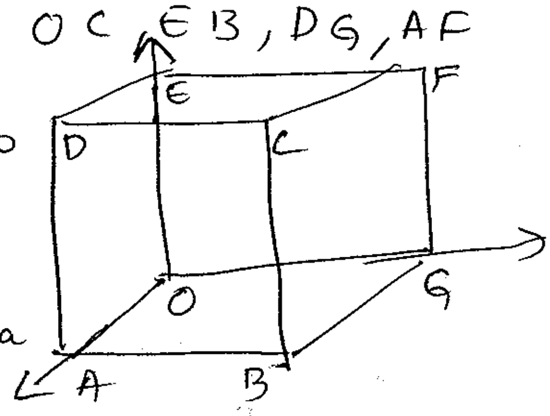
(1)

(a) Find angles between any two diagonals of a cube.

Let 'a' be the side of a cube.
Let us consider the coordinate system such that O is origin, x axis is along OA, y axis is along OG, and z axis is along OE

$O(0,0,0)$ $A(a,0,0)$ $B(a,a,0)$ $C(a,a,a)$
$D(a,0,a)$ $E(0,0,a)$ $F=(0,a,a)$ $G=(0,a,0)$

Four diagonals are OC, EB, DG, AF



D.R's of OC are $a-0, a-0, a-0$
$= a, a, a$

D.R's of E.B are $a-0, a-0, 0-a$
$= a, a, -a$

$\therefore$ Angle between OC and EB is

$$\cos\theta = \frac{a(a)+a(a)+a(-a)}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+(-a)^2}} = \frac{1}{3}$$

$$\theta = \cos^{-1}(1/3)$$

(b) Find equation of two planes which bisect the angles between the planes $3x-4y+5z=3$, $5x+3y-4z=9$

Eq of planes is given by

$$\frac{3x-4y+5z-3}{\sqrt{3^2+(-4)^2+5^2}} = \pm \frac{5x+3y-4z-9}{\sqrt{5^2+3^2+(-4)^2}}$$

$$\frac{3x-4y+5z-3}{\sqrt{50}} = \pm \frac{5x+3y-4z-9}{\sqrt{50}}$$

$3x-4y+5z-3 = 5x+3y-4z-9$ and $3x-4y+5z-3 = -(5x+3y-4z-9)$

$-2x-7y+9z+6=0$ and $8x-y+z-12=0$

(c) Find image of $(1,2,3)$ in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$

Let $Q(\alpha,\beta,\gamma)$ be image of $P(1,2,3)$ in line AB and R midpoint of PQ

$R = \left(\frac{\alpha+1}{2}, \frac{\beta+2}{2}, \frac{\gamma+3}{2}\right)$

Eq of AB is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z}{-1}$, D.R's of AB are $2,3,-1$

D.R's of PQ are $\alpha-1, \beta-2, \gamma-3$.

Since AB is perpendicular to PQ, $2(\alpha-1)+3(\beta-2)-1(\gamma-3)=0$

$2\alpha+3\beta-\gamma-5=0 \rightarrow (1)$

Any point on AB is $(2\gamma-1, 3\gamma+3, -\gamma)$

$\frac{\alpha+1}{2} = 2\gamma-1$, $\quad \frac{\beta+2}{2} = 3\gamma+3$, $\quad \frac{\gamma+3}{2} = -\gamma$

$\alpha = 4\gamma-3$ $\quad \beta = 6\gamma+4$ $\quad \gamma = -2\gamma-3$

$\therefore (1)$ becoms $2(4\gamma-3)+3(6\gamma+4)-(-2\gamma-3)-5=0$

$\boxed{\gamma = -1/7}$

$(\alpha,\beta,\gamma) = \left(-25/7, 22/7, -19/7\right)$

2

a Find Equation of plane through the point $(1,-1,0)$ and perpendicular to the line $2x+3y+5z-1=0=3x+y-z+2$

Equation of plane through $(1,-1,0)$

is $a(x-1)+b(y+1)+c(z-0)=0 \rightarrow (1)$

This is perpendicular to the line

$2x+3y+5z-1=0=3x+y-z+2$

$\therefore$ It is perpendicular to the planes

$2x+3y+5z-1=0$ and $3x+y-z+2=0$

$2a+3b+5c=0 \rightarrow (2)$ $\quad 3a+b-c=0 \rightarrow (3)$

Eliminating a, b, c from (1), (2), (3)

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 3 & 5 \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$(x-1)(-3-5)-(y+1)(-2-15)+z(2-9)=0$

$-8x+17y-7z+25=0$ is reqd plane

(b) Find k such that lines $\frac{x}{k}=\frac{y-2}{2}=\frac{z+3}{3}$

and $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ are coplanar. Find point of intersection.

From condition of coplanarity

$$\begin{vmatrix} 2-0 & 6-2 & 3+3 \\ k & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\therefore 2k - 2 = 0$

$\boxed{k = 1}$

$\therefore \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = t$

$\therefore x = t, \; y = 2t + 2, \; z = 3t - 3 \rightarrow (1)$

Also $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} \longrightarrow (2)$

Sub (1) in (2)

$$\frac{t-2}{2} = \frac{2t+2-6}{3} = \frac{3t-3-3}{4}$$

$\therefore \frac{t-2}{2} = \frac{2t-4}{3} \quad \therefore \boxed{t = 2} \quad \therefore (x, y, z) = (2, 6, 3)$

(c) Find distance of $(1,-2,3)$ from plane $x-y+z=5$ measured parallel to line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

Equation of line thru $A(1,-2,3)$ and parallel to $\frac{x}{2} = \frac{y}{3}, \frac{z}{-6}$ is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$

Any point on this line is $(2r+1, 3r-2, -6r+3)$

If this lies on plane $x - y + z = 5$ then

$2r + 1 - (3r-2) + (-6r+3) = 5 \quad \therefore \boxed{r = 1/7}$

P is $\left(9/7, -11/7, 15/7\right)$ A is $(1,-2,3)$

$$AP = \sqrt{\left(1-9/7\right)^2 + \left(-2+\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2} = 1 \text{ unit}$$

3\.a) S.T the position vectors of the vectors of a $\triangle$ $\vec{a} = 3(\sqrt{3}\hat{i} - \hat{j})$ $\vec{b} = 6\hat{j}$ $\vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$ form an isosceles $\triangle$

Let A, B, C be the vertices of $\triangle$

$\vec{OA} = 3(\sqrt{3}\hat{i} - \hat{j})$ $\quad \vec{OB} = 6\hat{j}$ $\quad \vec{OC} = 3(\sqrt{3}\hat{i} + \hat{j})$

$\vec{AB} = \vec{OB} - \vec{OA} = -3\sqrt{3}\hat{i} + 9\hat{j}$

$\vec{BC} = \vec{OC} - \vec{OB} = 3\sqrt{3}\hat{i} \cancel{+} 3\hat{j}$

$\vec{CA} = \vec{OA} - \vec{OC} = -6\hat{j}$

$|\vec{AB}| = \sqrt{(3\sqrt{3})^2 + 9^2} = \sqrt{108}$

$|\vec{BC}| = \sqrt{(3\sqrt{3})^2 + (-3)^2} = 6$

$|\vec{CA}| = \sqrt{(-6)^2} = 6$

$\triangle ABC$ is an isosceles $\triangle$

b) Find the ~~a~~ unit normal to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find also the sine of the angle bet them.

Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ $\quad \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = \hat{i}(2-3) - \hat{j}(-8+6) + \hat{k}(4-2)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k}$$

$|\vec{a} \times \vec{b}| = \sqrt{1+4+4} = 3$

$\therefore$ Unit normal vector to both the vectors is

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3}$$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|} = \frac{3}{\sqrt{16+1+9}\,\sqrt{4+1+4}} = \frac{1}{\sqrt{26}}$$

$$\sin\theta = \frac{1}{\sqrt{26}}$$

c) P.T the position vectors of the pts A, B, C and D are represented by the vectors $-\hat{j}-\hat{k}$ $4\hat{i}+5\hat{j}+\hat{k}$ $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ resp are coplanar

Let $\vec{OA} = -\hat{j}-\hat{k}$ $\quad \vec{OB} = 4\hat{i}+5\hat{j}+\hat{k}$

$\vec{OC} = 3\hat{i}+9\hat{j}+4\hat{k}$ $\quad \vec{OD} = -4\hat{i}+4\hat{j}+4\hat{k}$

$\vec{AB} = \vec{OB}-\vec{OA} = 4\hat{i}+6\hat{j}+2\hat{k}$

$\vec{AC} = \vec{OC}-\vec{OA} = 3\hat{i}+10\hat{j}+5\hat{k}$

$\vec{AD} = \vec{OD}-\vec{OA} = -4\hat{i}+5\hat{j}+5\hat{k}$

$$\vec{AB}\cdot(\vec{AC}\times\vec{AD}) = \begin{vmatrix} 4 & 6 & 2 \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix}$$

$$4(50-25)-6(15+20)+2(15+40) = 0$$

$\therefore$ $\vec{AB}$ $\vec{AC}$ $\vec{AD}$ are coplanar

$\vec{OA}$ $\vec{OB}$ $\vec{OC}$. $\vec{OD}$ are coplanar

5 ⓐ Find the maximum directional derivative of $\log(x^2+y^2+z^2)$ at $(1,1,1)$.

<u>Solution</u>: WKT The max directional derivative of ϕ is $|\nabla\phi|$

Let $\phi = \log(x^2+y^2+z^2)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} \Rightarrow$$

$$= \frac{1}{x^2+y^2+z^2} 2x i + \frac{1}{x^2+y^2+z^2} 2y j + \frac{1}{(x^2+y^2+z^2)} 2z k$$

$$[\nabla\phi]_{(1,1,1)} = \frac{2i}{3} + \frac{2}{3}j + \frac{2}{3}k \Rightarrow \frac{2}{3}(i+j+k)$$

$$\therefore |\nabla\phi| = \sqrt{\frac{4}{9}(1+1+1)} = \sqrt{\frac{4}{9}(3)} = \sqrt{\frac{12}{9}} = \sqrt{\frac{4}{3}}$$

5 ⓑ Find the unit normal vector to the curve $\vec{r} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\,\hat{k}$.

<u>solution</u>: $\vec{T} = \frac{d\vec{r}}{dt} = 4\cos t\, i - 4\sin t\, j + 3k$

$$|\vec{T}| = \sqrt{(4\cos t)^2 + (-4\sin t)^2 + 3^2}$$

$$= \sqrt{16(\cos^2 t + \sin^2 t) + 9}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

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EVEN SEMESTER 2011
TIME TABLE
DEPARTMENT OF MCA

FACULTY NAME: Ms. Pillai Anitha

Day \ Time	8:00 To 8:50	8:50 To 9:40	9:40 To 10:30	10:30 To 10:40	10:40 To 11:30	11:30 To 12:20	12:20 To 01:10	01:10 To 02:00	02:00 To 02:50	2:50 To 3:40
Monday				COFFEE BREAK			LUNCH BREAK			
Tuesday								WEB LAB(B2)		
Wednesday								WEB LAB(B1)		
Thursday										
Friday										
Saturday										

Work Load (in units)

Web Programming Laboratory	3 * 2	06
Project Work	3	03
Total Units		09

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$$\therefore \quad \hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{4\cos t\, i - 4\sin t\, j + 3k}{5}$$

$$\frac{d\hat{T}}{dt} = \frac{1}{5}\left(-4\sin t\, i - 4\cos t\, j\right)$$

$$\therefore \quad \frac{d\hat{T}}{ds} = \frac{d\hat{T}/dt}{ds/dt} = \frac{d\hat{T}/dt}{|d\vec{r}/dt|}$$

$$= \frac{\frac{1}{5}\left(-4\sin t\, i - 4\cos t\, j\right)}{5}$$

$$\frac{d\hat{T}}{ds} = \frac{-4}{25}\sin t\, i - \frac{4}{25}\cos t\, j$$

$$|d\hat{T}/ds| = \sqrt{\frac{16}{625}\sin^2 t + \frac{16}{625}\cos^2 t} = \frac{4}{25}$$

$$\therefore \quad \hat{n} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|} = \frac{-(4/25)\sin t\, i - (4/25)\cos t\, j}{4/25}$$

$$\hat{n} = -\sin t\, i - \cos t\, j$$

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EVEN SEMESTER 2011

TIME TABLE

DEPARTMENT OF MCA

FACULTY NAME: Mrs. B. Nithya Ramesh

Day \ Time	8:00 To 8:50	8:50 To 9:40	9:40 To 10:30	10:30 To 10:40	10:40 To 11:30	11:30 To 12:20	12:20 To 01:10	01:10 To 02:00	02:00 To 02:50	2:50 To 3:40
Monday				COFFEE BREAK			LUNCH BREAK	ADA LAB(B1)		
Tuesday	ADA									
Wednesday			ADA					ADA LAB(B2)		
Thursday	ADA									
Friday		ADA								
Saturday										

Work Load (in units)

Design and Analysis of Algorithms	4 * 2	08
Algorithms Laboratory	3 * 2	06
Project Work	3	03
Total Units		17

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5 (b) Evaluate $\int_0^\infty x^2 e^{-4x}\, dx$ by using gamma function

<u>Solution:</u> $I = \int_0^\infty e^{-4x} x^2\, dx$

Put $4x = t \Rightarrow x = \frac{t}{4} \Rightarrow dx = \frac{dt}{4}$

If $x = 0 \Rightarrow t = 0$; if $x = \infty \Rightarrow t = \infty$

$\therefore I = \int_0^\infty \left(\frac{t}{4}\right)^2 e^{-t} \frac{dt}{4}$

$= \frac{1}{(4)^3} \int_0^\infty t^2 e^{-t}\, dt$

$= \frac{1}{(4)^3} \Gamma(2+1) \Rightarrow \frac{\Gamma(3)}{64} \Rightarrow \frac{2!}{64}$

$\therefore I = \frac{2!}{64}$

5 (c) Solve $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$ interms of gamma function.

<u>Solution:</u> WKT $\beta(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$

Then $m = \frac{5}{2}$ & $n = \frac{3}{2}$

$\therefore \beta\left(\frac{5}{2}, \frac{3}{2}\right) = \frac{\Gamma\left(\frac{5}{2}\right) \cdot \Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{5}{2} + \frac{3}{2}\right)}$

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EVEN SEMESTER 2011

TIME TABLE

DEPARTMENT OF MCA

FACULTY NAME: Prof. RACHNA SHARMA

Time / Day	8:00 To 8:50	8:50 To 9:40	9:40 To 10:30	10:30 To 10:40	10:40 To 11:30	11:30 To 12:20	12:20 To 01:10	01:10 To 02:00	02:00 To 02:50	2:50 To 3:40
Monday				COFFEE BREAK	JAVA		LUNCH BREAK			
Tuesday					JAVA					
Wednesday					JAVA					
Thursday					JAVA					
Friday										
Saturday										

Work Load (in units)

Topics in Enterprise Architectures-I	4 * 2	08
Project Work	3	03
Total Units		11

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DIRECTOR

$$\beta\left(\frac{5}{2}, \frac{3}{2}\right) = \frac{\Gamma\left(\frac{5}{2}\right) \cdot \Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{8}{2}\right)} = \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(4)}$$

$$= \frac{\left\{\frac{3}{2} \cdot \frac{1}{2} \cdot \Gamma(1/2)\right\} \cdot \left\{\frac{1}{2}\, \Gamma(1/2)\right\}}{3 \cdot 2 \cdot 1.}$$

$$= \frac{\left(\frac{3}{4}\sqrt{\pi}\right)\left(\frac{1}{2}\sqrt{\pi}\right)}{3 \cdot 2 \cdot 1} = \frac{\frac{3}{8}(\pi)}{3 \cdot 2 \cdot 1}$$

$$\therefore \beta\left(\frac{5}{2}, \frac{3}{2}\right) = \frac{\frac{3}{8}(\pi)}{3 \cdot 2 \cdot 1}$$

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CMR INSTITUTE OF TECHNOLOGY

EVEN SEMESTER 2011

TIME TABLE

DEPARTMENT OF MCA

FACULTY NAME: Ms. Pratima V. Patil

Day \ Time	8:00 To 8:50	8:50 To 9:40	9:40 To 10:30	10:30 To 10:40	10:40 To 11:30	11:30 To 12:20	12:20 To 01:10	01:10 To 02:00	02:00 To 02:50	2:50 To 3:40
Monday				COFFEE BREAK		DS	LUNCH BREAK			
Tuesday	DS	DS		COFFEE BREAK			LUNCH BREAK			
Wednesday				COFFEE BREAK	DS		LUNCH BREAK			
Thursday				COFFEE BREAK			LUNCH BREAK	DS LAB(B3)	DS LAB(B3)	DS LAB(B3)
Friday		DS		COFFEE BREAK	DS LAB(B2)	DS LAB(B2)	LUNCH BREAK	DS LAB(B1)	DS LAB(B1)	DS LAB(B1)
Saturday				COFFEE BREAK			LUNCH BREAK			

Work Load (in units)

Data Structure Using C	4 * 2	08
Data Structure Using C Laboratory	3 * 3	09
Total Units		**17**

TIME TABLE COORDINATOR | HOD | DIRECTOR

5 (c) Show That $\vec{F} = \dfrac{xi+yj}{x^2+y^2}$ is both solenoidal & irrotational

<u>Solution:</u> $\vec{F} = \dfrac{x}{x^2+y^2} i + \dfrac{y}{x^2+y^2} k$

$$\nabla . F = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)$$

$$= \frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2+x^2-y^2}{(x^2+y^2)^2} = 0$$

$\therefore$ $\vec{F}$ is solenoidal.

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix}$$

$$= i\left(0 - \frac{\partial}{\partial z}\left(\frac{y}{x^2+y^2}\right)\right) - j\left(0 - \frac{\partial}{\partial z}\left(\frac{x}{x^2+y^2}\right)\right) +$$
$$k\left(\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)\right)$$

$$= i(0) + j(0) + k\left[\frac{-2xy}{(x^2+y^2)} + \frac{2xy}{(x^2+y^2)^2}\right]$$

$\nabla \times \vec{F} = 0$

Hence $\vec{F}$ is irrotational.

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CMR INSTITUTE OF TECHNOLOGY

EVEN SEMESTER 2011

TIME TABLE

DEPARTMENT OF MCA

FACULTY NAME: Mrs. Neha Agrawal

Day \ Time	8:00 To 8:50	8:50 To 9:40	9:40 To 10:30	10:30 To 10:40	10:40 To 11:30	11:30 To 12:20	12:20 To 01:10	01:10 To 02:00	02:00 To 02:50	2:50 To 3:40
Monday				COFFEE BREAK	C++		LUNCH BREAK			
Tuesday						C++				
Wednesday	C++									
Thursday								C++ LAB(B2)		
Friday			C++		C++ LAB(B1)			C++ LAB(B3)		
Saturday										

Work Load (in units)

Object Oriented Programming with C++	4 * 2	08
Object Oriented Programming with C++ Laboratory	3 * 3	09
Total Units		17

TIME TABLE COORDINATOR

HOD

DIRECTOR

6 (a) Find the L.T of $\sin^2 3t$ and $\sqrt{t}$.

Solution: (i) Let $f(t) = \sin^2 3t$

WKT $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\therefore f(t) = \frac{1}{2}(1 - \cos 6t)$

Therefore, $L[f(t)] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+36}\right]$

(ii) Let $f(t) = \sqrt{t} = t^{1/2}$

$$L[f(t)] = \frac{\Gamma\left(\frac{1}{2}+1\right)}{s^{(1/2+1)}}$$

$\Gamma(n+1) = n\Gamma(n)$

$\therefore \Gamma(1/2) = \sqrt{\pi}$

$$= \frac{\frac{1}{2}\Gamma(1/2)}{s^{3/2}} = \frac{\sqrt{\pi}}{2s^{3/2}}$$

6 (b) Find $L[f(t)]$, given that $f(t) = \begin{cases} t-1 & 0<t<2 \\ 3-t & t>2 \end{cases}$

Solution: $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$$= \int_0^2 e^{-st}(t-1)\,dt + \int_2^\infty e^{-st}(3-t)\,dt$$

$$= \left.(t-1)\frac{e^{-st}}{-s}\right]_0^2 - \int_0^2 1 \cdot \frac{e^{-st}}{-s}\,dt + \left.(3-t)\frac{e^{-st}}{-s}\right]_2^\infty -$$

$$\int_2^\infty (-1)\frac{e^{-st}}{-s}\,dt$$

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CMR INSTITUTE
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ICP SCHEDULE

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

ICP Schedule for academic year 2011-2012(EVEN Semester)

IVA

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	27/02/2012	SS	3.00-4.00pm	Ms. Rashmi K.V
2	06/03/2012	MC	2:10-3:00pm	Mrs. S.Sujatha
3	09/03/2012	HDL	1:20-2:10pm	Mrs. Shobha
4	20/03/2012	LIC	2:10-3:00pm	Mrs. Chitra L
5	21/03/2012	CS	1:20-2:10pm	Mrs. Richa Tengashe
6	02/04/2012	MC	2:10-3:00pm	Mrs. S.Sujatha
7	12/04/2012	HDL	1:20-2:10pm	Mrs. Shobha
8	17/04/2012	SS	3.00-4.00pm	Ms. Rashmi K.V
9	25/04/2012	CS	1:20-2:10pm	Mrs. Richa Tengashe

IV B

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	07/03/2012	MC	1:20-2:10pm	Mr. Naveen Kumar C J
2	15/03/2012	HDL	1:20-2:10pm	Mr. Umesh G.B
3	20/03/2012	SS	2:10-3:00pm	Mrs. Richa Tengashe
4	20/03/2012	CS	3.00-4.00pm	Mr. Kishore D.V
5	27/03/2012	LIC	2:10-3:00pm	Mrs. Archana A Nawandhar
6	03/04/2012	HDL	1.20-2.10pm	Mr. Umesh G.B
7	12/04/2012	MC	2:10-3:00pm	Mr. Naveen Kumar C J
8	16/04/2012	SS	2:10-3.00pm	Mrs. Richa Tengashe
9	20/04/2012	LIC	2.10-3.00pm	Mrs. Archana A Nawandhar
10	23/04/2012	CS	2.10-3.00pm	Mr. Kishore D.V

$f(t) = e^{-t}\, t \sin 2t$

$L[f(t)] = L[e^{-t}\, t \sin 2t]$

$L[t \sin 2t] = (-1) \frac{d}{dt}\left[\frac{2}{s^2+4} \right]$

$$= \frac{(-1)(0) + (-1)[2 \cdot 2s]}{s^2+4}$$

$$= \frac{-4s}{s^2+4}$$

$L[e^{-t}\, t \sin 2t] = e^{-t}\left[\frac{-4s}{s^2+4} \right]_{s \to s+1}$

$$= \frac{-4(s+1)}{(s+1)^2+4} \rightarrow (2)$$

Adding (1) & (2) $L[f(t)] =$

$$= \frac{s-2}{(s-2)^2+1^2} + \frac{(-4)(s+1)}{(s+1)^2+4}$$

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ICP SCHEDULE

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

ICP Schedule for academic year 2011-2012(ODD Semester)

III A

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	19/08/11	NA	2:10-3:00pm	Mr. Kishore D.V
2	22/08/11	AEC	1:20-2:10pm	Mrs. Anitha.P
3	23/08/11	FT	2:10-3:00pm	Mr. Ravisha
4	30/08/11	EI	1:20-2:10pm	Mrs. S.Sujatha
5	02/09/11	LD	2:10-3:00pm	Mrs. Chitra L
6	19/09/11	NA	2:10-3:00pm	Mr. Kishore D.V
7	30/09/11	FT	2:10-3:00pm	Mr. Ravisha
8	21/10/11	AEC	1:20-2:10pm	Mrs. Anitha.P

III B

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	24/08/11	AEC	2:10-3:00pm	Mrs. Archana A Nawandhar
2	29/08/11	LD	1:20-2:10pm	Mr. Kishore D.V
3	13/09/11	NA	2:10-3:00pm	Mrs. Chitra L
4	16/09/11	FT	1:20-2:10pm	Mrs. Remya
5	17/10/11	AEC	2:10-3:00pm	Mrs. Archana A Nawandhar
6	18/10/11	FT	1.20-2.10pm	Mrs. Remya
7	20/10/2011	NA	2:10-3:00pm	Mrs. Chitra L

V A

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	24/08/11	CMOS	1:20-2:10pm	Mrs. Sophiya Susan S
2	29/08/11	DSP	2:10-3:00pm	Ms. Rashmi K.V
3	30/08/11	AC	2:10-3:00pm	Mrs. Richa Tengashe
4	12/09/11	MWR	2:10-3:00pm	Mrs. Meenu Verma
5	13/09/11	DSP	2:10-3:00pm	Mrs. Rashmi K.V.
6	15/09/11	CMOS	1.20-2.10pm	Mrs. Sophiya Susan S
7	24/10/2011	CMOS	1.20-2.10pm	Mrs. Sophiya Susan S
8	02/11/2011	AC	2:10-3:00pm	Mrs. Richa Tengashe

$$= \frac{-1}{s}\left(1\cdot e^{-2s} - (-1)e^{-0}\right) + \frac{1}{s}\left.\frac{e^{-st}}{-s}\right]_0^2 - \frac{1}{s}\left(0 - 1\cdot e^{-2s}\right) - \frac{1}{s}\left.\frac{e^{-st}}{(-s)}\right]_2^\infty$$

$$= \frac{-1}{s}\left(e^{-2s}+1\right) + \frac{1}{s}e^{-2s} - \frac{1}{s^2}\left(e^{-2s}-1\right) - \frac{1}{s^2}e^{-2s}$$

$$= \frac{-e^{-2s} - 1 + e^{-2s}}{s} + \frac{(-e^{-2s}) + 1 - e^{-2s}}{s^2}$$

$$L[f(t)] = \frac{-1}{s} + \frac{1 - 2e^{-2s}}{s^2}.$$

we have $f(0) = 0 - 1 = -1$

6(c) Find L.T of $e^{2t}\cos t + te^{-t}\sin 2t$

<u>Solution</u> $f(t) = e^{2t}\cos t$

$$L[f(t)] = e^{2t} L[\cos t]$$

$$= e^{2t}\left[\frac{s}{s^2+1^2}\right]_{s \to s-2}$$

$$L[f(t)] = \frac{s-2}{(s-2)^2+1^2} \longrightarrow (1)$$

$f(t) = te^{-t}\sin 2t$

IV C

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	28/02/2012	SS	1:20-2:10pm	Ms. Rashmi K.V
2	07/03/2012	MC	1:20-2:10pm	Mr. Naveen Kumar C J
3	20/03/2012	LIC	2:10-3:00pm	Mrs. Chitra L
4	15/03/2012	HDL	1.20-2.10pm	Mr. Umesh G.B
5	20/03/2012	CS	3.00-4.00pm	Mr. Kishore D.V
6	03/04/2012	HDL	1.20-2.10pm	Mr. Umesh G.B
7	12/04/2012	SS	1.20-2.10pm	Ms. Rashmi K.V
8	12/04/2012	MC	2.10-3.00pm	Mr. Naveen Kumar C J
9	16/04/2012	LIC	2.10-3.00pm	Mrs. Chitra L
10	23/04/2012	CS	2.10-3.00pm	Mr. Kishore D.V

VI A

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	12/03/2012	ITC	2.10-3.00pm	Mrs. Raja Thejashwini
2	14/03/2012	TLA	2.10-3.00pm	Mr. Abhishek Javali
3	23/03/2012	DC	1.20-2.10pm	Mr. Rakesh
4	26/03/2012	MP	2:10-3:00pm	Mrs. Archana A Nawandhar
5	05/04/2012	C++	2.10-3.00pm	Mrs. Pooja Mohnani
6	17/04/2012	TLA	2:10-3:00pm	Mr. Abhishek Javali
7	19/04/2012	MP	2:10-3:00pm	Mrs. Archana A Nawandhar
8	20/04/2012	LP	2:10-3:00pm	Mrs. Shobha
9	23/04/2012	DC	1.20-2.10pm	Mr. Rakesh
10	23/04/2012	SC	2.10-3.00pm	Dr. Fathima Jabeen

VI B

Sl.No	Dates	Subjects	Time	Name of the Faculty
1	12/03/2012	MP	2.10-3.00pm	Mrs. Sophiya Susan S
2	14/03/2012	TLA	2.10-3.00pm	Mrs. Raja Thejashwini
3	20/03/2012	DC	1.20-2.10pm	Mrs. Chitra L
4	03/04/2012	TLA	2.10-3.00pm	Mrs. Raja Thejashwini
5	05/04/2012	C++	2.10-3.00pm	Mrs. Gayathri A.P
6	13/04/2012	MP	2.10-3.00pm	Mrs. Sophiya Susan S
7	23/04/2012	SC	1.20-2.10pm	Prof. V. N Dabade
8	23/04/2012	DC	2.10-3.00pm	Mrs. Chitra L

7 (a) Find the L.T of $\int_0^t \cos 2(t-u) \cdot \cos 3u \cdot du$

if $L^{-1}[\bar{f}(s)] = f(t)$ $L^{-1}[\bar{g}(s)] = g(t)$ then

$$L^{-1}[\bar{f}(s) \cdot \bar{g}(s)] = \int_0^t f(u) \cdot g(t-u) \cdot du$$

$$\Rightarrow L\left[\int_0^t f(u) \cdot g(t-u)\, du\right] = \bar{f}(s) \cdot \bar{g}(s).$$

(or) $$L\left[\int_0^t f(t-u) \cdot g(u) \cdot du\right] = \bar{f}(s) \cdot \bar{g}(s)$$

Here $f(t-u) = \cos 2(t-u) \Rightarrow f(t) = \cos 2t$

$g(u) = \cos 3u \Rightarrow g(t) = \cos 3t.$

Now $L[f(t)] = \bar{f}(s)$

$$\Rightarrow L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[g(t)] = \bar{g}(s)$$

$$\Rightarrow L[\cos 3t] = \frac{s}{s^2+9}$$

$$\therefore L\left[\int_0^t \cos 2(t-u) \cdot \cos 3u\, du\right] = \bar{f}(s) \cdot \bar{g}(s)$$

$$= \frac{s}{s^2+4} \cdot \frac{s}{s^2+9}$$

$$= \frac{s^2}{(s^2+4)(s^2+9)}$$

7 b

i) we need to find $L^{-1}\left[\frac{s+1}{s^2-s+1}\right]$

now $F(s) = \frac{s+1}{s^2-s+1} = \frac{s+1}{s^2-2s\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$

$$= \frac{s+1}{\left(s-\frac{1}{2}\right)^2-\frac{1}{4}+1} = \frac{s+1}{\left(s-\frac{1}{2}\right)^2+3/4}$$

$\therefore \quad F(s) = \frac{s+1}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}}$

$$L^{-1}\{F(s)\} = L^{-1}\left[\frac{s+1}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}}\right] = L^{-1}\left[\frac{\left(s-\frac{1}{2}\right)+\frac{3}{2}}{\left(s-\frac{1}{2}\right)^2+3/4}\right]$$

$$= e^{1/2 t}\cdot L^{-1}\left[\frac{s+\frac{3}{2}}{s^2+\frac{3}{4}}\right]$$

$$= e^{t/2} L^{-1}\left[\frac{s}{s^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{3}{2}\cdot\frac{1}{s^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

$$= e^{t/2}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{3}{2}\frac{1}{\sqrt{3}/2}\sin\frac{\sqrt{3}}{2}t\right]$$

$$= e^{t/2}\cdot\left[\cos\frac{\sqrt{3}}{2}t + \sqrt{3}\sin\frac{\sqrt{3}}{2}t\right],$$

7(b) ii) we need to find $L^{-1}\left[\frac{1}{s(s^2+a^2)}\right]$

we know that $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$

Also if $L^{-1}[\bar{f}(s)] = f(t)$ then

$$L^{-1}\left[\frac{\bar{f}(s)}{s}\right] = \int_0^t f(t)\cdot dt$$

$\therefore$ Here $\bar{f}(s) = \frac{1}{s^2+a^2}$

$$\therefore L^{-1}\left[\frac{1}{s(s^2+a^2)}\right] = L^{-1}\left[\frac{\bar{f}(s)}{s}\right] = \int_0^t \frac{1}{a}\sin at\, dt$$

$$= \frac{1}{a}\left[\frac{-\cos at}{a}\right]_0^t$$

$$= \frac{1}{a^2}\left[1-\cos at\right]$$

$$= \frac{1-\cos at}{a^2}$$

8 a

Given $\frac{1}{(s^2+a^2)^2} = \frac{1}{(s^2+a^2)} \cdot \frac{1}{(s^2+a^2)}$

$= \bar{f}(s) \cdot \bar{g}(s)$.

$L^{-1}[\bar{f}(s)] = f(t) \Rightarrow L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a} \cdot \sin at$

$L^{-1}[\bar{g}(s)] = g(t) \Rightarrow L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a} \sin at$.

By using convolution theorem we get

$L^{-1}[\bar{f}(s) \cdot \bar{g}(s)] = \int_0^t f(u) \cdot g(t-u) \cdot du$

$\therefore L^{-1}\left[\frac{1}{s^2+a^2} \cdot \frac{1}{s^2+a^2}\right] = \int_0^t \frac{1}{a} \cdot \sin au \cdot \frac{1}{a} \sin a(t-u)\, du$

$= \frac{1}{a^2}\int_0^t \sin au \cdot \sin a(t-u)\, du$

$= \frac{1}{2a^2} \int_0^t 2 \sin au \cdot \sin a(t-u) \cdot du$

$\boxed{\begin{aligned} &\cos(A-B) - \cos(A+B) \\ &= 2 \sin A \sin B \end{aligned}}$

$= \frac{1}{2a^2} \int_0^t [\cos(au - at + au) - \cos(au + at - au)]\, du$

$= \frac{1}{2a^2} \cdot \int_0^t [\cos(2au - at) - \cos at]\, du$

$$= \frac{1}{2a^2}\left\{ \int_0^t \cos(2au - at)\, du - \int_0^t \cos at \, du \right\}$$

$$= \frac{1}{2a^2}\left\{ \left[\frac{\sin(2au - at)}{2a}\right]_0^t - \cos at \cdot [u]_0^t \right\}$$

$$= \frac{1}{2a^2}\left\{ \frac{[\sin(2at - at) - \sin(0 - at)]}{2a} - \cos at\,[t - 0] \right\}$$

$$= \frac{1}{2a^2}\left\{ \frac{\sin at + \sin at}{2a} - t\cos at \right\}$$

$$= \frac{1}{2a^2} \cdot \frac{\cancel{2}\sin at}{\cancel{2}a} - \frac{1}{2a^2} \cdot t\cos at$$

$$= \frac{1}{2a^3} \cdot \sin at - \frac{1}{2a^2} \cdot t\cos at$$

8(B) Given $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 5e^{2t}$

$y(0) = 2 ; y'(0) = 1$

$\therefore y''(t) + 5y'(t) + 6y(t) = 5e^{2t}$.

Taking Laplace transforms on both sides.

$L[y''(t)] + 5L[y'(t)] + 6 \cdot L[y(t)] = 5L[e^{2t}]$

$\Rightarrow s^2 L[y] - s\,y(0) - y'(0) + 5\{s L(y) - y(0)\} + 6L[y] = 5 \cdot \frac{1}{s-2}$

$\Rightarrow s^2 L[y] - s(2) - 1 + 5\{s L(y) - 2\} + 6L(y) = \frac{5}{s-2}$

$\Rightarrow (s^2 + 5s + 6) L(y) - 2s - 1 - 10 = \frac{5}{s-2}$

$(s^2 + 5s + 6) L(y) = \frac{5}{s-2} + 2s + 11$

$L(y) = \frac{2s^2 + 7s - 17}{(s-2)(s+2)(s+3)}$

$L(y) = \frac{2s^2 + 7s - 17}{(s+3)(s-2)(s+2)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s-2}$

$$2s^2+7s-17 = A(s+3)(s-2)+B(s+2)(s-2)+c(s+2)(s+3)$$

$\therefore$ put $s=2 \Rightarrow c=\frac{1}{4}$

$s=-2 \Rightarrow A=\frac{23}{4}$

$s=3 \Rightarrow B=-4$

$$\therefore \quad L[y] = \frac{2s^2+7s-17}{(s+2)(s+3)(s-2)} = \frac{\frac{23}{4}}{s+2} - \frac{4}{s+3} + \frac{\frac{1}{4}}{s-2}$$

$$\therefore y = L^{-1}\left[\frac{23}{4}\cdot\frac{1}{s+2} - 4\cdot\frac{1}{s+3} + \frac{1}{4}\cdot\frac{1}{s-2}\right]$$

$$= \frac{23}{4}\cdot e^{-2t} - 4e^{-3t} + \frac{1}{4}\cdot e^{2t}$$