

MATDIP401

## Fourth Semester B.E. Degree Examination, June 2012 Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

## Note: Answer any FIVE full questions.

- 1 a. Find the angles between any two diagonals of a cube. (06 Marks)
  - b. Find the equations of two planes, which bisect the angles between the planes 3x 4y + 5z = 3, 5x + 3y 4z = 9.
  - c. Find the image of the point (1, 2, 3) in the line  $\frac{x+1}{2} = \frac{y-3}{3} = 2$  (07 Marks)
- 2 a. Find the equation of the plane through the point (1, -1, 0) and perpendicular to the line 2x + 3y + 5z 1 = 0 = 3x + y z + 2. (06 Marks)
  - b. Find the value of k such that the line  $\frac{x}{k} = \frac{y-2}{2} = \frac{z+3}{3}$  and  $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$  are coplanar. For this k find their point of intersection. (07 Marks)
  - c. Find the distance of the point (1, -2, 3) from the plane x y + z = 5 measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ . (07 Marks)
- 3 a. Show that the position vectors of the vertices of a triangle  $\vec{a} = 3(\sqrt{3}\hat{i} \hat{j})$ ,  $\vec{b} = 6\hat{j}$ ,  $\vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$  form an isosceles triangle. (06 Marks)
  - b. Find the unit normal to both the vectors  $4\hat{\mathbf{i}} \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$  and  $-2\hat{\mathbf{i}} + \hat{\mathbf{j}} 2\hat{\mathbf{k}}$ . Find also the sine of the angle between them.
  - c. Prove that the position vectors of the points A, B, C and D represented by the vectors  $-\hat{j} \hat{k}$ ,  $4\hat{i} + 5\hat{j} + \hat{k}$ ,  $3\hat{i} + 9\hat{j} + 4\hat{k}$  and  $-4\hat{i} + 4\hat{j} + 4\hat{k}$ , respectively are coplanar. (07 Marks)
- a. Find the value of λ so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, λ, 1) may lie on one plane.
  - b. If  $\vec{a}, \vec{b}, \vec{c}$  are the position vectors of points A, B, C, prove that  $(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$  is a vector perpendicular to the plane of triangle ABC. (07 Marks)
  - c. Find a set of vectors reciprocal to the set  $2\hat{i} + 3\hat{j} \hat{k}$ ,  $\hat{i} \hat{j} 2\hat{k}$ ,  $\hat{i} + 2\hat{j} + 2\hat{k}$ . (07 Marks)
- 5 a. Find the maximum directional derivative of  $log(x^2 + y^2 + z^2)$  at (1, 1, 1). (06 Marks)
  - b. Find the unit normal vector to the curve  $\vec{r} = 4 \sin t \hat{i} + 4 \cos t \hat{j} + 3t \hat{k}$ . (07 Marks)
  - c. Show that  $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$  is both solenoidal and irrotational. (07 Marks)
- a. Find the Laplace transforms of  $\sin^2 3t$  and  $\sqrt{t}$ . (06 Marks)
  - b. Find L[f(t)], given that  $f(t) = \begin{cases} t-1 & 0 < t < 2 \\ 3-t & t > 2 \end{cases}$  (07 Marks)

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c. Find the Laplace transform of e<sup>2t</sup> cos t + t e<sup>-t</sup> sin 2 t.

(07 Marks

7 a. Find the Laplace transform of  $\int \cos 2(t-u)\cos 3u du$ .

(06 Marks)

- b. Find the inverse Laplace transform of
  - i)  $\frac{s+1}{s^2-s+1}$
- ii)  $\frac{1}{s(s^2+a^2)}$ .

- (14 Marks)
- 8 a. Find the inverse Laplace transform by using convolution theorem of  $\frac{1}{(s^2 + a^2)^2}$ . (10 Marks)
  - b. By applying Laplace transform, solve the differential equation  $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 5e^{2t}$ . Subject to the conditions y(0) = 2, y'(0) = 1.