USN

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines (3, -1, 1) an (-3, 2, 4). **(06 Marks)**

b. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a line, then prove the following:

i) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

ii) $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1$ **(07 Marks)**

c. Find the projection of the line AB on the line CD where A = (1, 2, 3), B = (1, 1, 1), C = (0, 0, 1), D = (2, 3, 0). **(07 Marks)**

2 a. Find the equation of the plane through (1, -2, 2), (-3, 1, -2) and perpendicular to the plane $2x-y-z+6=0$. **(06 Marks)**

b. Find the image of the point (1, -2, 3) in the plane $2x+y-z=5$. **(07 Marks)**

c. Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. **(07 Marks)**

3 a. Find the constant 'a' so that the vectors $2i-j+k$, $i+2j-3k$ and $3i+aj+5k$ are coplanar. **(06 Marks)**

b. Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right]$. **(07 Marks)**

c. Find the unit normal vector to both the vectors $4i-j+3k$ and $-2i+j-2k$. Find also the sine of the angle between them. **(07 Marks)**

4 a. A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+5$ where t is the time. Find the components of its velocity and acceleration at time t = 1 in the direction of $2i+3j+6k$. **(06 Marks)**

b. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at the point (2, -1, 2). **(07 Marks)**

c. Find the directional derivative of $\phi=xy^2+yz^3$ at the point (1, -2, -1) in the direction of the normal to the surface $x\log z-y^2=-4$ at (-1, 2, 1). **(07 Marks)**

5 a. Prove that $\operatorname{div}(\operatorname{curl}\vec{A})=0$. **(06 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$. **(07 Marks)**

c. Show that the vector $\vec{F}=(3x^2-2yz)i+(3y^2-2zx)j+(3z^2-2xy)k$ is irrotational and find ϕ such that $\vec{F}=\operatorname{grad}\phi$. **(07 Marks)**

6 a. Find: $L\{\cos t\ \cos 2t\ \cos 3t\}$. (06 Marks)

b. Find: i) $L\{e^{-t}\cos^2 t\}$, ii) $L\{te^{-t}\sin 3t\}$. (07 Marks)

c. Find: $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$. (07 Marks)

7 a. Find: $L^{-1}\left\{\dfrac{4s+5}{(s-1)^2(s+2)}\right\}$. (06 Marks)

b. Find: i) $L^{-1}\left\{\dfrac{s+2}{s^2-4s+13}\right\}$, ii) $L^{-1}\left\{\log\left(\dfrac{s+a}{s+b}\right)\right\}$ (07 Marks)

c. Find: $L^{-1}\left\{\dfrac{1}{s^2(s+1)}\right\}$. (07 Marks)

8 a. Using Laplace transforms, solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$. (10 Marks)

b. Using Laplace transformation method solve the differential equation $y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$. (10 Marks)