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MATDIP401

Fourth Semester B.E. Degree Examination, December 2011 Advanced Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions

- 1 a. If (ℓ, m, n) be the direction cosines of a line then prove that $\ell^2 + m^2 + n^2 = 1$. (06 Marks)
 - b. Find the angle between the two lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $2\ell + 2m nm = 0$. (07 Marks)
 - c. Show that the angle between any two diagonals of a cube is cos-1(1/3). (07 Marks)
- 2 a. Find the equation of the plane through the points (1, -2, 2), (-3, 1, -2) and perpendicular to the plane 2x y z + 6 = 0. (06 Marks)
 - b. Find the image of the point (1, 1, 2) in the plane 2x + y + z 3 = 0. (07 Marks)
 - c. Find the shortest distance and equation between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the x-axis.
- 3 a. Find the value of λ so that the vectors $\vec{a} = 2i 3j + k$, $\vec{b} = i + 2j 3k$ and $\vec{c} = j + \lambda k$ are coplanar. (06 Marks)
 - b. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$ and $\vec{b} \cdot (\vec{a} \times \vec{c})$, where $\vec{a} = i + j k$, $\vec{b} = 2i j + 2k$ and $\vec{c} = 3i j k$.
 - c. Show that the position vectors of the vertices of a triangle 2i j + k, i 3j 5k and
 - c. Show that the position vectors of the vertices of a triangle 2i j + k, i 3j 5k and 3i 4j 4k form a right angled triangle. (07 Marks)
- 4 a. Find the unit tangent vector to the space curve $x = \cos t^2$, $y = \sin t^2$ and z = 0. (06 Marks)
 - b. A particle moves along a curve with parametric equations $x = t \frac{t^3}{3}$, $y = t^2$ and $z = t + \frac{t^3}{3}$, where t is the time. Find the velocity and acceleration at any time t and also find their magnitudes at $t \neq 3$.
 - c. Find the angle between the surfaces $x^2yz + 3xz^2 = 5$ and $x^2yz^3 = 2$ at (1, -2, -1). (07 Marks)
- 5 a. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of i + j + 2k. (06 Marks)
 - b. Find the constants a, b, c such that the vector $\vec{F} = (\sin y + a z) i + (b x \cos y + z) j + (x + c y) k$ is irrotational. (07 Marks)
 - c. Prove that dir(curl A) = 0. (07 Marks)
- 6 a. Find the Laplace transform of tⁿ, where n is a +ve integer. (06 Marks)
 - b. Find L[t e^{-2t} cos 2t]. (07 Marks)
 - c. Find $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$. (07 Marks)

Important Note: I. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice

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7 Find the inverse Laplace transform for the following:

a.
$$\frac{s+2}{s^2+8s+25}$$

b.
$$\frac{2s-1}{s^2-5s+6}$$

c.
$$\frac{s}{\left(s^2+a^2\right)^2}$$
.

d.
$$\log \left(\frac{s+a}{s+b} \right)$$
.

(20 Marks)

8 a. Solve using Laplace transforms

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$$
, given that $y(0) = 0$ and $y'(0) = 0$.

(10 Marks)

b. Solve the simultaneous equations using Laplace transforms $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ subject to the conditions x(0) = 2 and y(0) = 0.