MATDIP401

Fourth Semester B.E. Degree Examination, June / July 2014 Advanced Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a Define direction cosine and direction ratio of a line. Hence show that 1² + m² + n² = 1.

 (06 Marks)
 - b. For any cube show that angle between any two diagonals is $\cos \frac{1}{3}$ (07 Marks)
 - c. Define plane. Derive equation of plane in general form. (07 Marks)
- 2 a. Find equation of plane passing through A(-1, 1, 1), B(1, -1, 1) and perpendicular to plane x + 2y + 2z 5 = 0 (06 Marks)
 - b. Show that the line $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{10}$ is parallel to plane 2x + 2y z = 6. Find distance between them.
 - c. Show that lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find point of intersection. (07 Marks)
- 3 a. Find sine and cosine of angle between the vectors 4i + 3j + k, 2i j + 2k. (06 Marks)
 - b. Show that points (4, 5, -1), (0, -1, -1), (3, 9, 4), (-4, 4, 4) are coplanar using vector method. (07 Marks)
 - c. Prove that $\begin{bmatrix} \vec{a} + \vec{b}, \ \vec{b} + \vec{c}, \ \vec{c} + \vec{a} \end{bmatrix} = 2 \begin{bmatrix} \vec{a}, \vec{b}, \vec{c} \end{bmatrix}$. (07 Marks)
- 4 a. A particle moves along the curve $x = t^3+1$, $y = t^2$, z = 2t + 5. Find components of its velocity and acceleration at t = 1 in the direction i + j + 3k (06 Marks)
 - b. Find directional derivative of $x^2 + y^2 + 4xyz$ at (1, -2, 2) in the direction 2i 2j + k.

 (07 Marks)
 - c. Show that $\operatorname{grad}\left(\frac{1}{r}\right) = -\frac{r}{r^2}$. (07 Marks)
- 5_{\triangle} a. For any scalar function ϕ show that $\operatorname{curl}(\operatorname{grad}\phi) = 0$. (06 Marks)
 - b. If $\vec{F} = \text{grad}\phi$, $\phi = x^2 + y^2 + z^2 + xyz$, find $\nabla \cdot (\vec{F})$ and $\nabla \times (\vec{F})$ at (1, 1, 1). (07 Marks)
 - c. Find a, b, c so that $\vec{F} = (x + y + az)i + (x + cy + 2z)j + (x + 2y z)k$ is irrotational. Find scalar function. (07 Marks)

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6 a. Find Laplace Transform if t^n and hence find $L\left(t^{\frac{1}{2}}\right)$.

(06 Marks)

b. Find $L[e^{2t}\cos 3t + e^{-t}\sin 2t + t\sin t]$.

(07 Marks)

c. Find $L\left[\frac{e^{t}(\cos 3t - \cos t)}{t}\right]$.

(07 Marks)

7 a. Find $L[\sin t \sin 2t \sin 3t]$.

(06 Marks)

b. Find L[f(t)] where $f(t) = \begin{cases} 1 & 0 < t \le 1 \\ t & 1 < t \le 2 \\ t^2 & t > 2 \end{cases}$

(07 Marks)

c. Find $L^{-1} \left\{ log \sqrt{\frac{s+a}{s-b}} \right\}$.

(07 Marks)

8 a. Find $L^{-1} \left\{ \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6} \right\}$.

- (10 Marks)
- b. Solve by Laplace transformation, $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 4e^{-3t}$, given y(0) = 0, y'(0) = -1.
 - (10 Marks)