

MATDIP401

## Fourth Semester B.E. Degree Examination, December 2010

## **Advanced Mathematics - II**

Time: 3 hrs.

Max. Marks 100

(07 Marks)

(06 Marks)

Note: Answer any FIVE full questions.

- 1 a. Find the ratio in which the line joining (2, 4, 16) and (3, 5, -4) is divided by the plane 2x 3y + z + 6 = 0.
  - b. Find the angle between the lines whose direction cosines are given by 3l + 3 + 5n = 0 and 6mn 2ln + 5lm = 0.
  - c. Derive the equation of the plane in the form  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ . (07 Marks)
- 2 a. Find the reflection of the point (1, 3, 4) in the plane 2x y + z + 3 = 0. (07 Marks)
  - b. Find the equation of the line through (1, 2, -1) and perpendicular to each of the lines  $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} \text{ and } \frac{x}{3} = \frac{y}{4} = \frac{z}{5}.$  (06 Marks)
  - c. Prove that the lines  $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{1+z}{7}$  and  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  intersect and find the coordinates of their point of intersection. (07 Marks)
- 3 a. If  $\vec{A} = \hat{i} + 2\hat{j} 3\hat{k}$  and  $\vec{B} = 3\hat{i} \hat{j} + 2\hat{k}$  then:
  - i) Show that A+B and A-B are orthogonal and
  - ii) Find the angle between  $2\vec{A} + \vec{B}$  and  $\vec{A} + 2\vec{B}$ .
  - b. Prove that  $[\vec{A} + \vec{B}, \vec{B} + \vec{C}, \vec{C} + \vec{A}] = 2[\vec{A} \vec{B} \vec{C}]$ .
  - c. If  $\vec{A} = 2\hat{i} \hat{j} + 3\hat{k}$ ,  $\vec{B} = -\hat{i} + 3\hat{j} + 3\hat{k}$  and  $\vec{C} = \hat{i} + \hat{j} 2\hat{k}$ , find the reciprocal triad  $(\vec{A}'\vec{B}'\vec{C}')$ . (07 Marks)
- 4 a. For the curve  $\vec{R} = a(\cos t \, \hat{i} + \sin t \, \hat{j} + t \tan \alpha \, \hat{k})$  where a and  $\alpha$  are constants, evaluate

$$\frac{d\vec{R}}{dt} \times \frac{d^2 \vec{R}}{dt^2}$$
 (06 Marks)

- b. The position vector of a moving particle at time t is  $\vec{R} = t^2 \hat{i} t^3 \hat{j} + t^4 \hat{k}$ ). Find the tangential and normal components of its acceleration at t = 1. (07 Marks)
- c. Find the directional derivative  $\phi = xyz$  along the direction of the normal to the surface  $x^2z + y^2x + z^2y = 3$  at the point (1, 1, 1). (07 Marks)

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be

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5 a. Show that 
$$\nabla^2(r^n) = n(n+1)r^{n-2}$$
 where  $r^2 = x^2 + y^2 + z^2$ .

(07 Marks)

b. If 
$$\vec{F} = e^{xyz}(\hat{i} + \hat{j} + \hat{k})$$
 find div  $\vec{F}$  and curl  $\vec{F}$ .

(06 Marks)

c. Prove that 
$$\nabla \times \nabla \times \vec{F} = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$$
.

(07 Marks)

6 a. Prove that 
$$L(\cos at) = \frac{s}{s^2 + a^2}$$
  $s > 0$ 

b. Find: i) 
$$L\{e^{-t} \sin^2 t\}$$
 ii)  $L\{te^{-t} \sin 3t\}$ 

iii) 
$$L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$$

(15 Marks)

7 a. IF 
$$f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t - 1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$$
, find L{f(t)}.

(07 Marks)

b. Find 
$$L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$$
.

(06 Marks)

c. Apply convolution theorem to evaluate 
$$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$$
.

(07 Marks)

8 a. If 
$$f'(t)$$
 is a continuous function and  $L\{f(t)\}=F(s)$  then prove that  $L\{f'(t)\}=sF(s)-f(0)$ .

(04 Marks)

b. Solve the following using Laplace transform:

$$y'' + 2y' - 3y = \sin t$$
, when  $y(0) = 0 = y'(0)$ .

(06 Marks)

c. Using Laplace transform method, solve the simultaneous equations:

$$\frac{dx}{dt} + 5x - 2y = t$$
;  $\frac{dy}{dt} + 2x + y = 0$ , given  $x = y = 0$ , when  $t = 0$ .

(10 Marks)