IDENTIFICATION OF INFLUENTIAL NODES IN SOCIAL NETWORKS

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Course Code: CSE3021 – Social and Information Networks

Slot: A2 + **TA2**

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1. Introduction: Social networks have an important feature of "influential" nodes which have great implications ranging from forecasting marketing, advertisement, recommendation, controlling the spread of information in a network, identifying vulnerabilities in the Internet back-bone or other physical networks, and measuring the impact of organizational financial performance within highly correlated financial networks. Degree centrality is a straightforward and efficient metric, however, it is less relevant since a node having a few high influential neighbours may have much higher influence than a node having a larger number of less influential neighbours. Although some well-known global metrics such as betweenness centrality and closeness centrality can give better results, due to the very high computational complexity, they are not easy to manage very large-scale online social webs. The local degree centrality which is a semi-local centrality measure between the low-relevant degree centrality and other time-consuming measures performs almost as good as closeness centrality measure.

Apart from these centrality measures, the algorithms such as K-shell decomposition and page-rank algorithms are used to identify the influential nodes. In Kshell algorithm nodes in a graph are computed by conducting the k-Shell decomposition. Viewed as nodes in a graph, the higher the k-shell level assigned, the closer the node is to the core of the graph. The assumption is that, if these nodes are users in a social network, the users in the higher k-shell levels are more influential in the network than users in lower k-shell levels. The k-shell decomposition algorithm groups all nodes in a network that have k (or less) connections or that are only connected to other nodes with k (or less) connections. Once a node has been identified, it is marked (and removed from the network for purposes of the algorithm) and the search continues until all nodes in shell k have been found. The process then moves to the next larger k-shell value (and continues until all nodes have been marked). In this basic algorithm, k-shell values are assigned in a linear fashion. That is, each k-shell value is equivalent to the analyzed connection count. The page rank algorithm is a link analysis algorithm which outputs a probability distribution. It can be applied to any kind of network. PageRank interprets a hyperlink from page x to page y as a vote, by page x, for page y. However, PageRank looks at more than the sheer number of votes; it also analyses the page that casts the vote. A hyperlink from a page to another page is an implicit conveyance of authority to the target page. Pages that point to page 'I' also have their own prestige scores. This algorithm is purely based on prestige scores of web pages.

The performance is evaluated by using the number of cascade iterations and number of infected nodes. The cascading process is applied to get the number of infected nodes. Simulations are done on various real networks to evaluate the performance of these algorithms efficiently.

2. Literature Review Summary Table

Authors and Year (Referen ce)	Title (Study)	Concept / Theoretical model/ Framework	Methodolog y used/ Implementa tion	Dataset details / Analysi s	Relevant Finding	Limitatio ns/ Future Research/ Gaps identified
A.Flexm an, A. Frieze and J. Vera in 2004	A geometrica 1 preferentia 1 attachment model of networks	The proportion of vertices of given degree follows an inverse power law	Probabilistic Method	A random graph with n vertices	Approach ing by random graphs also	Classical models of random graphs are not suitable for modelling these networks
Narayan am and Narahari in 2010	A Shapley value-based approach to discover influential nodes in social networks	Shapley value based approach	Greedy algorithm	Any social networ k	Discoveri ng influential nodes	Provides a solution to the cooperativ e game theory
D.Kemp e, J. Kleinber g and E. Tardosin 2003	Maximizin g the spread of influence through a social network	Framework based on submodular functions	Hill Climbing greedy algorithm	Choosi ng influent ial sets of individ ual as a proble m	Selecting most influential nodes	Implement ing in large collaborati on networks
S. Cheng, H. Shen, J. Huang, W.chen in 2014	Influence Maximizat ion via findingself consistent ranking	Two ways: Greedy approach, Heuristic approach	IM Ranking algorithm	Any social networ k	Finding a set of seed nodes maximizi ng influence	Finding influential individuals such as expert finding, online advertising
M.U. Ilyas, H. Radha in 2011	Identifying Influential Nodes in Online Social Networks Using	Principal component centrality(P CC)	Comparing PCC with eigen vector centrality (EVC)	Massiv e online social networ k	Identifyin g influential nodes in online social networks	To find the increase social hubs

T	Principal Componen t Centrality	Walaaa	Cont	Chani	T-1 1	D:0014 4-
J. Leskovec	Cost- effective	Works on submodular	Cost Effective	Choosi	Takes less time to	Difficult to overcome
, A.	outbreak	functions	Lazy	ng influent	achieve	hill-
Krause	detection	10110110115	Forward	ial sets	the	climbing
in 2007	in		algorithm	of	process	greedy
	networks		_	individ	_	approach
				ual as a		
				proble		
				m		
Chi K.	A network	Minima1	Finding	Stocks	Nodes are	To study
Tse, Jing	Perspectiv	Spanning	correlation	that		correlation
Liu and	e of the	Tree is used	between	were		s between
Francis	stock	for filtering	each pair of	traded		the closing
C.M.	market	networks		over		prices for
Lau in				two		all stocks
2010				periods		
				of time		

- **3. Objective of the project:** The main objective of this project is to identify the influential nodes in complex networks. Detection of influential nodes in networks is a key problem of prime importance for a plenty of important practical applications. Our intent is to determine the extent to which algorithms and relevant graph analysis techniques can be successfully leveraged across multiple domains to potentially limit the impact of an attack, quantify vulnerabilities, and potentially inform network design. In this paper, we measured influential node discovery techniques such as local degree centrality, K-shell decomposition and Page rank algorithm in the social circles using datasets from Facebook, google+ and twitter. We evaluated the performance of these techniques by using the spreading rate and total number of infected nodes.
- **4. Innovation component in the project:** Degree centrality is a straightforward and efficient metric, however, it is less relevant since a node having a few high influential neighbours may have much higher influence than a node having a larger number of less influential neighbours. Although some well-known global metrics such as betweenness centrality and closeness centrality can give better results, due to the very high computational complexity, they are not easy to manage very large-scale online social webs. In the project, we implemented a semi-locality measure, which is a trade-off between the low-relevant degree centrality measure and other time-consuming measures. This semi-locality measure (local degree centrality) includes the nearest and next nearest neighbours. For example, consider tow nodes 'A' and 'B'. Suppose node 'A' has small degree centrality but large local centrality, while node B has large degree centrality but small local centrality. Although node 'A' has less neighbours, these neighbours have

many connections with other nodes. So, if node 'A' is infected, it can affect more nodes through its neighbours. In contrast, if node 'B' connects many 1-degree nodes, the information cannot spread further. Thus, by considering the nearest and next nearest neighbours, we implemented a semi-centrality measure to identify the influential nodes. We compared the performance of this centrality measure with the K-shell and page rank algorithm by using number of cascade iterations and total number of infected nodes.

5. Work done and implementation

a. Methodology adapted:

Our methodology mainly consists of three phases:

Phase-1: Influential node detection

To determine the influential nodes, we executed the local degree centrality measure and algorithms such as K-shell decomposition and page-rank algorithm. We captured '10' most influential nodes as output in phase-1.

Phase-2: Cascading Execution

We leveraged the Independent Cascade (IC) algorithm to execute the cascade process across graphs with the '10' most influential nodes identified in Phase I as input and captured the total number of infected nodes, the number of cascade iterations, and the set of impacted nodes during each iteration.

Phase-3: Performance evaluation of these techniques

We evaluated the performance of these techniques by using total number of infected nodes and the number of cascade iterations obtained in phase-2. The more number of infected nodes represents the high performance of the technique.

We implemented the above mentioned algorithms in 'CPP' programming language and executed in Linux environment.

b. Dataset used:

- i) We used four datasets in this project. Three of them taken are taken from the Stanford's website and other one is randomly generated graph.
- ii) This project is based on the reference project present in Stanford website.
- iii) We implemented a semi-locality measure and algorithms like K-shell and Page rank algorithm which differs from the reference project.

c. Tools used:

i) SocNetV:

We selected this tool since it has many visualisation options and easy to get values for centrality measures. We used this tool to visualize the data sets. We obtained the influential nodes based on centrality measures present in the tool and compared with our 'semi-centrality measure'.

ii) Gephi:

We used this tool to visualize the social networks. We obtained the most influential nodes based on 'page-rank algorithm' present in the tool and verified with our implemented page-rank algorithm.

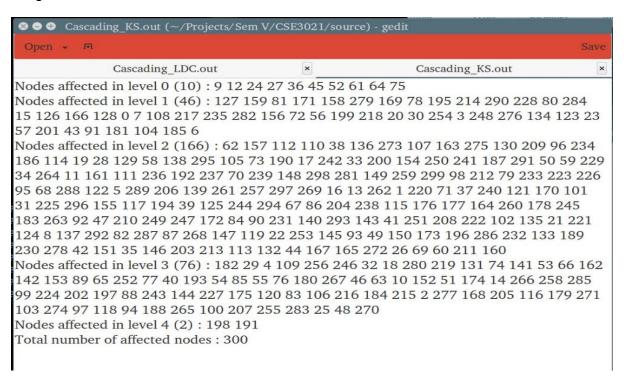
d. Screenshot and Demo:

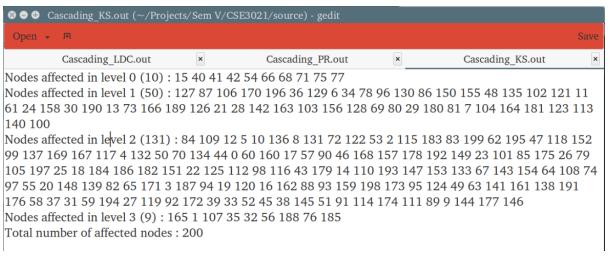
Code for K Shell Algorithm:

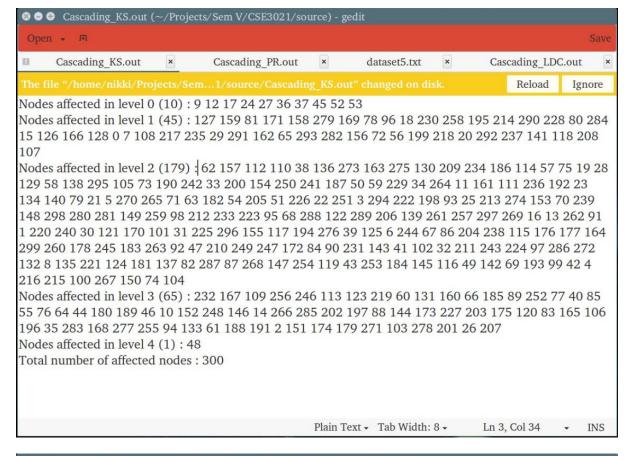
```
#include "bits/stdc++.h"
#include "socialnetwork.h"
using namespace std;
vector <int> seed points ks;
extern Graph Node_Properties[MAX_NODES];
int Adj_matrix[MAX_NODES][MAX_NODES];
vector <int> shell[MAX NODES];
vector <bool> incl(MAX NODES, false);
int ithshell(int x) {
  int i;
  for(i = 0; i < MAX NODES; ++i) {
    bool ok = false;
    for(int j = 0; j < MAX_NODES; ++j) {
      int cnt = 0;
      for(int k = 0; k < MAX_NODES; ++k) cnt += Adj_matrix[j][k];</pre>
      if(cnt <= x) {
        if(incl[j]) continue;
        incl[j] = true;
        shell[x].push_back(j);
        for(int k = 0; k < MAX_NODES; ++k) {
          if(Adj_matrix[j][k]) {
            ok = true;
            Adj_matrix[j][k] = Adj_matrix[k][j] = 0;
      }
    if(!ok) break;
void kshell_centrality() {
  for(int i = 0; i < MAX_NODES; ++i){
    for(int j = 0; j < Node_Properties[i].No_of_Neighbours(); ++j) {</pre>
     Adj_matrix[i][Node_Properties[i].Neighbour(j)] = 1;
      Adj_matrix[Node_Properties[i].Neighbour(j)][i] = 1;
```

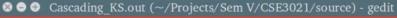
```
int last_shell = 0;
for(last_shell; last_shell < MAX_NODES; ++last_shell) {
   if(ithshell(last_shell) == -1) break;
}
last_shell--;
int k = TOP_K;
for(int i = last_shell; i >= 0 && k; --i) {
   for(int j : shell[i]) {
    if(k-- <= 0) break;
    seed_points_ks.push_back(j);
   }
}</pre>
```

Outputs for 4 Datasets:









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Nodes affected in level 0 (10): 9 12 17 24 27 36 37 45 52 53

Nodes affected in level 1 (45): 127 159 81 171 158 279 169 78 96 18 230 258 195 214 290 228 80 284 15 126 166 128 0 7 108 217 235 29 291 162 65 293 282 156 72 56 199 218 20 292 237 141 118 208 107

Nodes affected in level 2 (179): 62 157 112 110 38 136 273 163 275 130 209 234 186 114 57 75 19 28 129 58 138 295 105 73 190 242 33 200 154 250 241 187 50 59 229 34 264 11 161 111 236 192 23 134 140 79 21 5 270 265 71 63 182 54 205 51 226 22 251 3 294 222 198 93 25 213 274 153 70 239 148 298 280 281 149 259 98 212 233 223 95 68 288 122 289 206 139 261 257 297 269 16 13 262 91 1 220 240 30 121 170 101 31 225 296 155 117 194 276 39 125 6 244 67 86 204 238 115 176 177 164 299 260 178 245 183 263 92 47 210 249 247 172 84 90 231 143 41 102 32 211 243 224 97 286 272 132 8 135 221 124 181 137 82 287 87 268 147 254 119 43 253 184 145 116 49 142 69 193 99 42 4 216 215 100 267 150 74 104

Nodes affected in level 3 (65): 232 167 109 256 246 113 123 219 60 131 160 66 185 89 252 77 40 85 55 76 64 44 180 189 46 10 152 248 146 14 266 285 202 197 88 144 173 227 203 175 120 83 165 106 196 35 283 168 277 255 94 133 61 188 191 2 151 174 179 271 103 278 201 26 207

Nodes affected in level 4 (1): 48 Total number of affected nodes: 300

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Ln 3, Col 34

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Code for Cascading Algorithm:

```
void cascade ALG() {
 freopen("Cascading_ALG.out", "w", stdout);
 vector <int> affected[300];
 extern vector <int> seed_points_dc;
 vector <bool> visited(MAX NODES, false);
 affected[0] = seed points dc;
 for(int i : seed_points_dc) visited[i] = true;
 int lvl = 0;
 while(!affected[lvl].empty()) {
   for(int i : affected[lvl]) {
     for(int j = 0; j < Node_Properties[i].No_of_Neighbours(); ++j) {</pre>
        if(visited[Node_Properties[i].Neighbour(j)]) continue;
        visited[Node_Properties[i].Neighbour(j)] = true;
        affected[lvl + 1].push_back(Node_Properties[i].Neighbour(j));
     }
   lv1++;
  int sum = 0;
 for(int i = 0; i < lvl; ++i) {
   sum += affected[i].size();
   cout << "Nodes affected in level " << i << " (" << affected[i].size() << ") : ";</pre>
    for(int j : affected[i]) {
    cout << j << "
   cout << "\n";
 cout << "Total number of affected nodes : " << sum << "\n";</pre>
```

Code for Local degree centrality measure:

```
#include "bits/stdc++.h"
#include "socialnetwork.h"
using namespace std;
vector <int> seed_points_ldc;
extern Graph Node_Properties[MAX_NODES];
void local_degree_centrality() {
  vector <int> ldc[MAX_NODES];
  for(int i = 0; i < MAX_NODES; ++i) {
     vector <bool> vis(MAX_NODES, false);
     vector <int> tmp;
     for(int j = 0; j < Node_Properties[i].No_of_Neighbours(); ++j) {</pre>
       if(vis[Node_Properties[i].Neighbour(j)]) continue;
vis[Node_Properties[i].Neighbour(j)] = true;
ldc[i].push_back(Node_Properties[i].Neighbour(j));
       tmp.push_back(Node_Properties[i].Neighbour(j));
     for(int k : tmp) {
  for(int j = 0; j < Node_Properties[k].No_of_Neighbours(); ++j) {</pre>
         if(vis[Node_Properties[k].Neighbour(j)]) continue;
vis[Node_Properties[k].Neighbour(j)] = true;
          ldc[i].push back(Node Properties[k].Neighbour(j));
     Node_Properties[i].LDC_Value = ldc[i].size();
   vector <pair <double, int> > tmp;
  for(int i = 0; i < MAX_NODES; ++i) {
     tmp.push_back(make_pair(Node_Properties[i].LDC_Value, i));
  sort(tmp.rbegin(), tmp.rend());
for(int i = 0; i < TOP_K; ++i) {</pre>
     seed points ldc.push back(tmp[i].second);
}
```

Output for 4 Datasets:

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⊗ ● ⊕ Cascading_LDC.out (~/Projects/Sem V/CSE3021/source) - gedit

Save

Nodes affected in level 0 (10) : 284 228 57 269 290 5 129 236 200 297

Nodes affected in level 1 (111): 37 27 240 190 30 121 170 195 233 101 31 225 296 155 117 194 276 39 125 288 122 289 3 206 139 110 261 257 24 16 149 13 262 214 244 213 166 185 113 136 132 81 64 107 44 267 22 179 163 271 217 10 23 34 114 7 259 299 295 98 212 282 154 161 298 79 223 226 95 68 239 178 18 134 250 90 4 260 273 46 40 171 59 71 63 249 123 251 120 204 160 60 6 227 169 246 281 254 77 197 111 67 20 242 88 158 42 232 148 153 116

Nodes affected in level 2 (167): 29 291 162 65 293 15 126 128 159 108 41 150 140 147 49 266 235 61 173 196 21 174 55 231 252 72 85 141 62 56 237 118 70 199 33 253 263 103 176 51 287 73 2 135 105 186 91 192 238 157 181 14 193 243 69 131 133 268 278 183 151 28 137 109 83 87 66 94 172 294 142 124 82 32 205 210 86 216 119 234 84 230 202 285 112 211 92 224 280 127 264 275 221 80 8 152 220 241 189 115 229 50 168 104 54 0 89 274 182 258 177 130 75 38 283 219 138 209 9 96 45 19 201 43 74 53 256 208 187 184 143 164 100 36 52 11 58 203 144 76 78 47 247 222 218 277 106 1 286 245 102 17 270 265 35 146 279 255 25 198 180 97 26 12 99 167 292

Nodes affected in level 3 (12) : 272 165 248 48 215 145 93 156 188 175 191 207 Total number of affected nodes : 300

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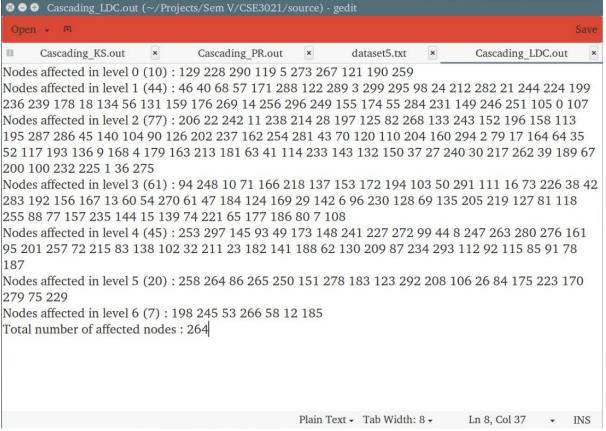
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Nodes affected in level 0 (10): 139 190 154 21 169 90 57 92 140 128

Nodes affected in level 1 (107): 12 94 105 11 31 109 64 124 2 46 16 163 104 80 166 44 95 98 49 29 25 82 54 183 63 141 193 161 198 79 102 199 186 43 135 177 173 32 100 194 133 138 119 167 184 18 66 182 159 152 67 172 60 148 85 117 179 10 36 110 170 97 111 59 99 84 26 146 112 58 125 176 150 33 123 61 126 113 27 115 144 13 108 132 81 53 195 191 151 107 50 9 147 142 149 22 137 77 189 168 121 71 129 88 6 39 78

Nodes affected in level 2 (83): 87 127 17 181 19 93 136 28 30 4 42 143 34 65 62 187 73 89 165 114 37 47 106 116 131 51 24 118 3 197 83 55 120 153 130 75 156 7 101 160 196 171 74 158 86 134 164 14 157 185 188 96 69 5 35 70 145 0 56 155 180 23 20 68 41 38 1 76 45 103 48 8 175 162 91 15 122 52 174 72 40 178 192

Total number of affected nodes: 200



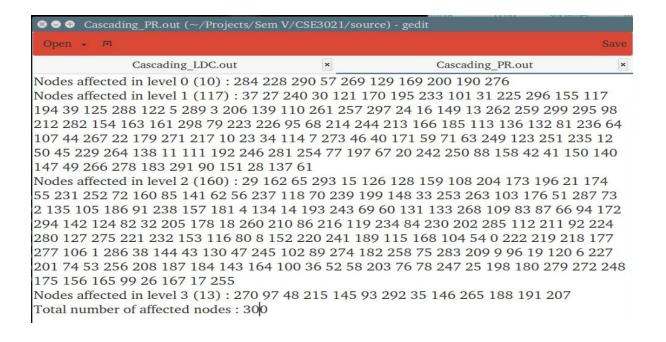


Code for Page Rank Algorithm:

```
#include "bits/stdc++.h"
#include "socialnetwork.h"
using namespace std;
vector (int) seed points pr;
extern Graph Node Properties [MAX_NODES];
double stochastic matrix[MAX_NODES][MAX_NODES];
const double damp factor = 0.85;
const double epsilon = 0.0005;
vector <double> mat_multiply(vector <double> x) {
  vector <double> ans(MAX_NODES, 0);
  for(int i = 0; i < MAX_NODES; ++i) {
    for(int j = 0; j < MAX_NODES; ++j) {</pre>
     ans[i] += stochastic_matrix[i][j] * x[j];
  return ans;
vector <double> power_iteration() {
  vector <double> prev(MAX_NODES, 1);
  vector <double> cm(MAX_NODES, 0.15);
  vector <double> curr(MAX_NODES), tmp;
  double nlp = 1, clp;
  while(1) {
    tmp = mat_multiply(prev);
    for(int i = 0; i < MAX NODES; ++i) {
    curr[i] = cm[i] + tmp[i];
    clp = -1.0;
    for(int i = 0; i < MAX NODES; ++i) {
    if(curr[i] > clp) clp = curr[i];
    for(int i = 0; i < MAX NODES; ++i) {
     curr[i] /= clp;
    if(abs(nlp - clp) < epsilon) break;
    prev = curr;
    nlp = clp;
```

```
if(abs(nlp - clp) < epsilon) break;</pre>
    prev = curr;
    nlp = clp;
  return curr;
void pagerank_centrality() {
  memset(stochastic_matrix, 0, sizeof(stochastic_matrix));
  for(int i = 0; i < MAX_NODES; ++i){</pre>
    for(int j = 0; j < Node_Properties[i].No_of_Neighbours(); ++j) {</pre>
      stochastic_matrix[i][Node_Properties[i].Neighbour(j)] = 1;
      stochastic matrix[Node Properties[i].Neighbour(j)][i] = 1;
  for(int i = 0; i < MAX_NODES; ++i) {</pre>
    for(int j = 0; j < MAX_NODES; ++j) {</pre>
      if(stochastic_matrix[i][j] > 0.5) stochastic_matrix[i][j] = 1.0 / Node_Properties[i].No_of_Neighbours();
  // Transpose
  for(int i = 0; i < MAX_NODES; ++i) {</pre>
    for(int j = i + 1; j < MAX_NODES; ++j) {
     swap(stochastic_matrix[i][j], stochastic_matrix[j][i]);
  // Multiply with damping factor
for(int i = 0; i < MAX_NODES; ++i) {</pre>
    for(int j = 0; j < MAX_NODES; ++j) {</pre>
     stochastic_matrix[i][j] *= damp_factor;
  vector <double> pagerank = power_iteration();
  priority_queue <pair<double, int> > pq;
  for(int i = 0; i < MAX NODES; ++i) {
    pq.push(make_pair(pagerank[i], i));
  int k = TOP K;
  while(k--) {
    seed_points_pr.push_back(pq.top().second);
    pq.pop();
```

Outputs for 4 Datasets:



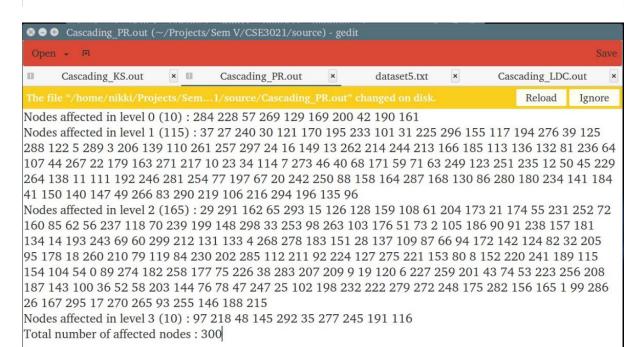


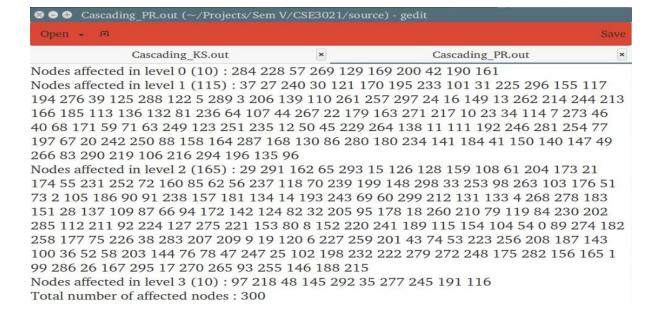
Nodes affected in level 0 (10): 92 190 140 139 21 169 154 90 128 142

Nodes affected in level 1 (107): 186 117 172 53 195 191 151 125 107 50 9 16 147 182 148 110 44 95 98 124 49 29 25 82 54 183 63 141 193 161 198 79 102 26 149 22 84 67 137 77 189 176 144 11 168 121 12 94 105 31 109 64 2 46 163 104 80 166 119 167 184 18 57 66 159 152 60 85 179 10 36 135 32 170 97 138 111 199 43 177 173 100 194 133 59 99 146 112 58 150 33 123 61 126 71 129 88 6 39 27 78 187 130 162 81 0 30

Nodes affected in level 2 (83): 165 13 62 96 86 196 160 69 158 136 113 91 143 37 87 131 108 72 35 73 157 197 55 5 52 181 20 24 118 3 83 23 7 155 145 132 175 19 89 171 74 134 115 114 47 28 164 14 51 185 188 70 116 153 93 56 180 34 156 68 41 120 127 17 1 38 65 4 174 42 76 106 75 101 8 45 15 103 48 122 40 178 192

Total number of affected nodes: 200





6. Results and discussion

Datasets	K Shell		Local Degree	Local Degree Centrality		Page Rank	
	Level 1	46	Level 1	111	Level 1	117	
Dataset 1	Level 2	166					
300 Nodes			Level 2	167	Level 2	160	
	Level 3	76					
			Level 3	12	Level 3	13	
	Level 4	2					
	Total Affected	290	Total Affected	290	Total Affected	290	
Dataset 2	Level 1	50	Level 1	107	Level 1	107	
200 Nodes	Level 2	131	Level 2	83	Level 2	8	
	Level 3	9	Level 3	83			
	Total	190	Total	190	Total	190	
	Affected		Affected		Affected		
Dataset 3	Level 1	45	Level 1	44	Level 1	115	
300 Nodes	Level 2	179	Level 2	77	Level 2	165	
	Level 3	65	Level 3	61	Level 3	10	
	Level 4	1	Level 4	45			
			Level 5	20			
			Level 6	7			
	Affected	290	Affected	254	Total Affected	290	
Dataset 4	Level 1	45	Level 1	111	Level 1	115	
300 nodes	Level 2	179	Level 2	167	Level 2	165	
	Level 3	65	Level 3	12	Level 3	10	
	Level 4	1					
	Affected	290	Affected	290	Affected	290	

First, we detected top 10 influential nodes using algorithms like page rank, local degree centrality and k shell. These nodes are considered to be present at level-0 and serves as input to cascading algorithm. Then, we applied independent cascading algorithm to find number of infected nodes and cascade iterations. In the above table, we compared those algorithms based on number of infected nodes and cascade iterations. We found that page rank algorithm performs better compared to other algorithms since the all the nodes are infected and number of levels obtained are less compared to other algorithms. Thus, we conclude that page rank algorithm suits better to detect influential nodes.

7. References

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