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Reconstruction and Repair Degree of Fractional Repetition Codes

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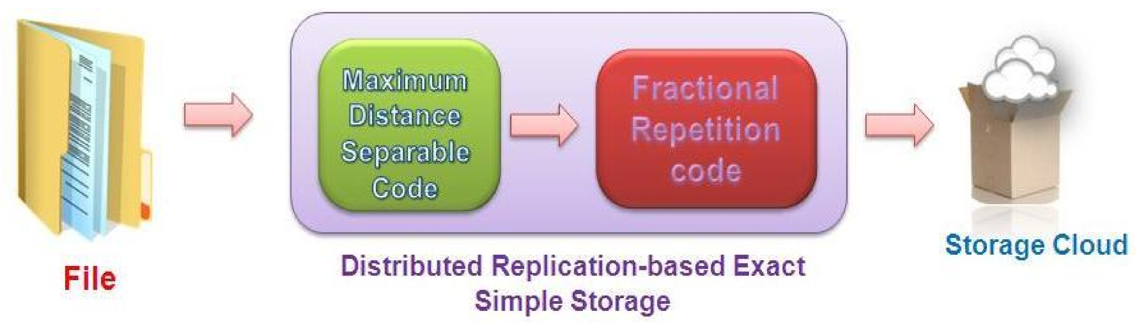
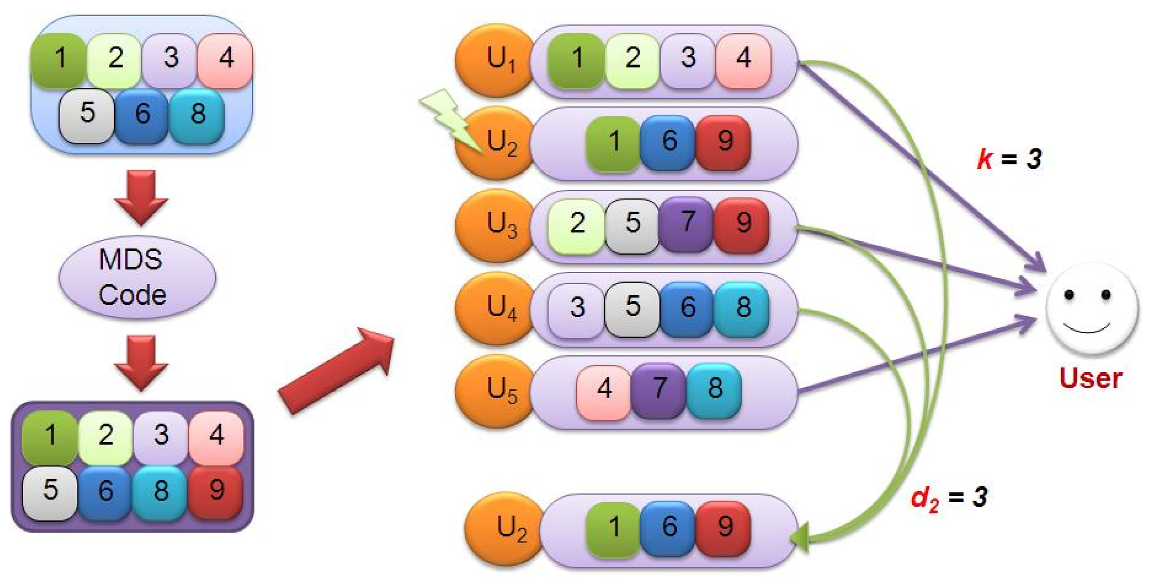
Abstract

Given a Fractional Repetition (FR) code, finding the reconstruction and repair degree in a Distributed Storage Systems (DSS), with parameters (n, k, d), is an important problem. In this work, we present algorithms for computing the reconstruction and repair degree of FR codes.



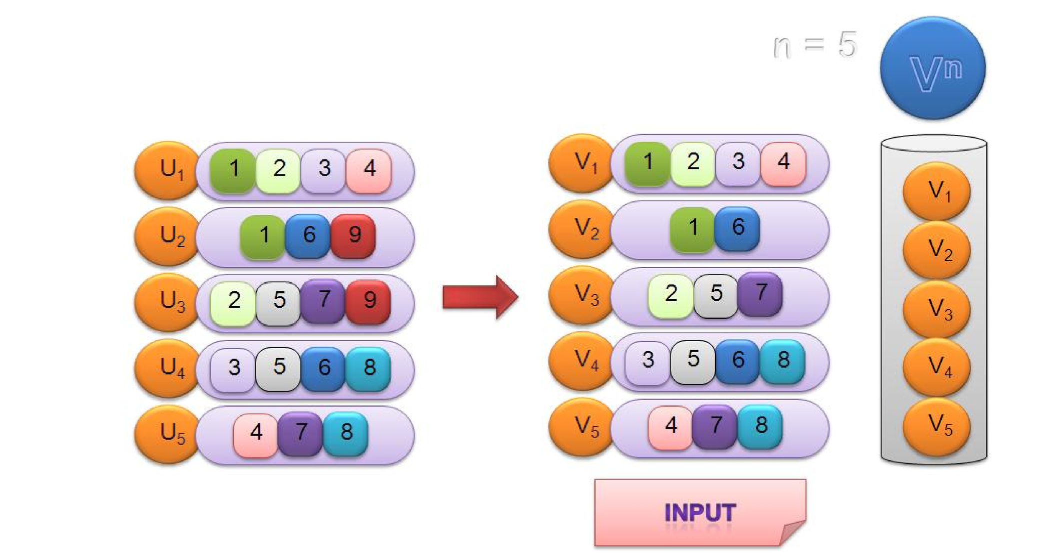


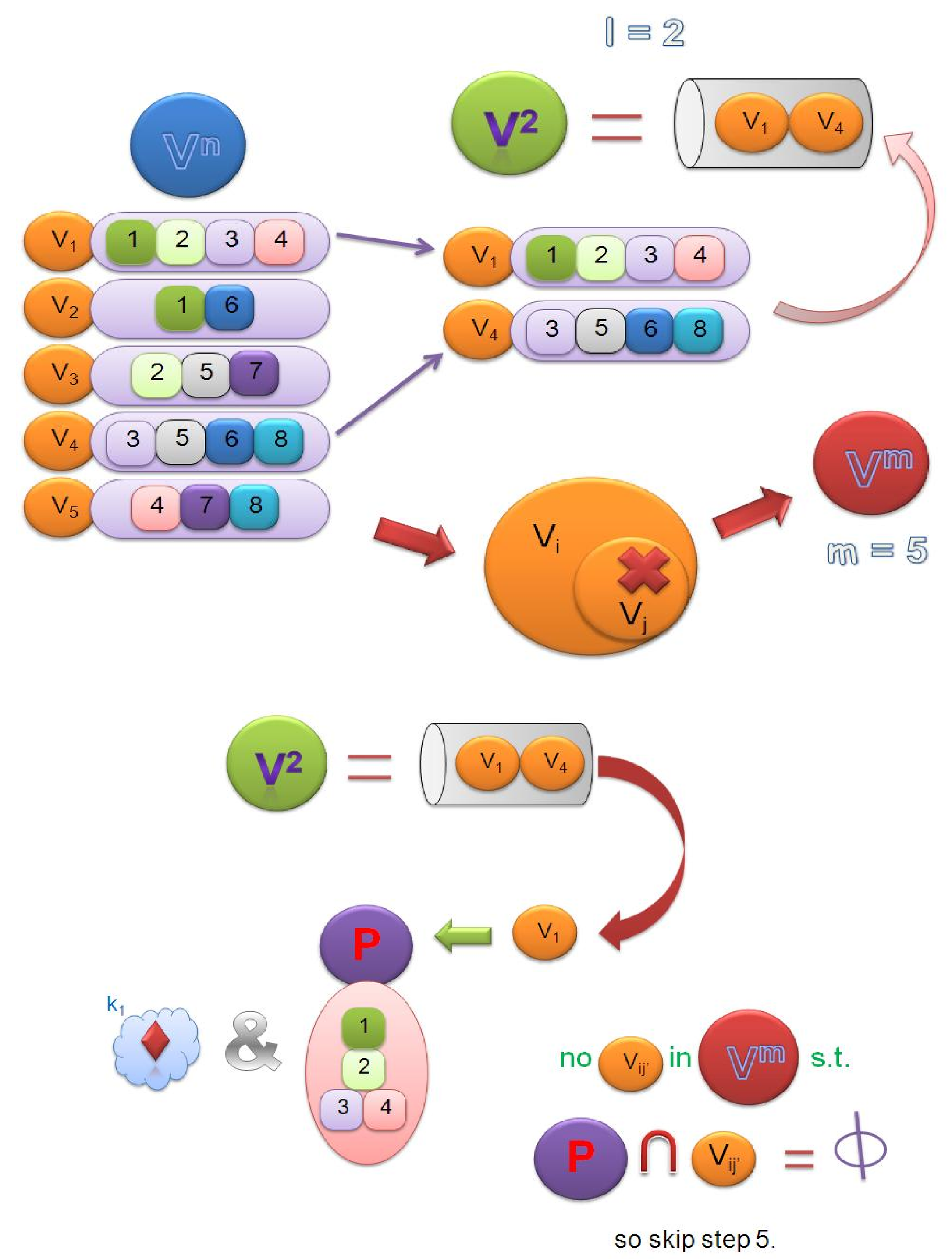
Algorithm for Reconstruction Degree $k^{\star}_{upp}$

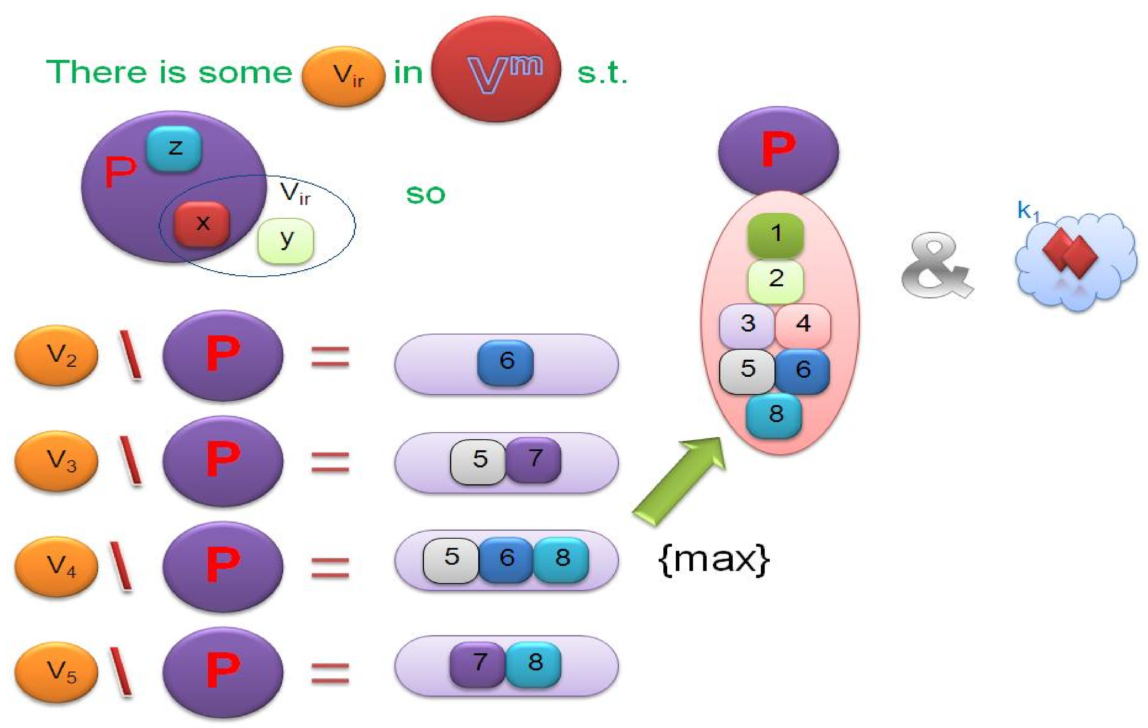
REQUIRE: Node packet distribution of FR code after removing the last packet θ from all n nodes of $V^n = \{V_1, V_2, ..., V_n\}$.
ENSURE: $k^{\star}_{upp}$ = Reconstruction degree

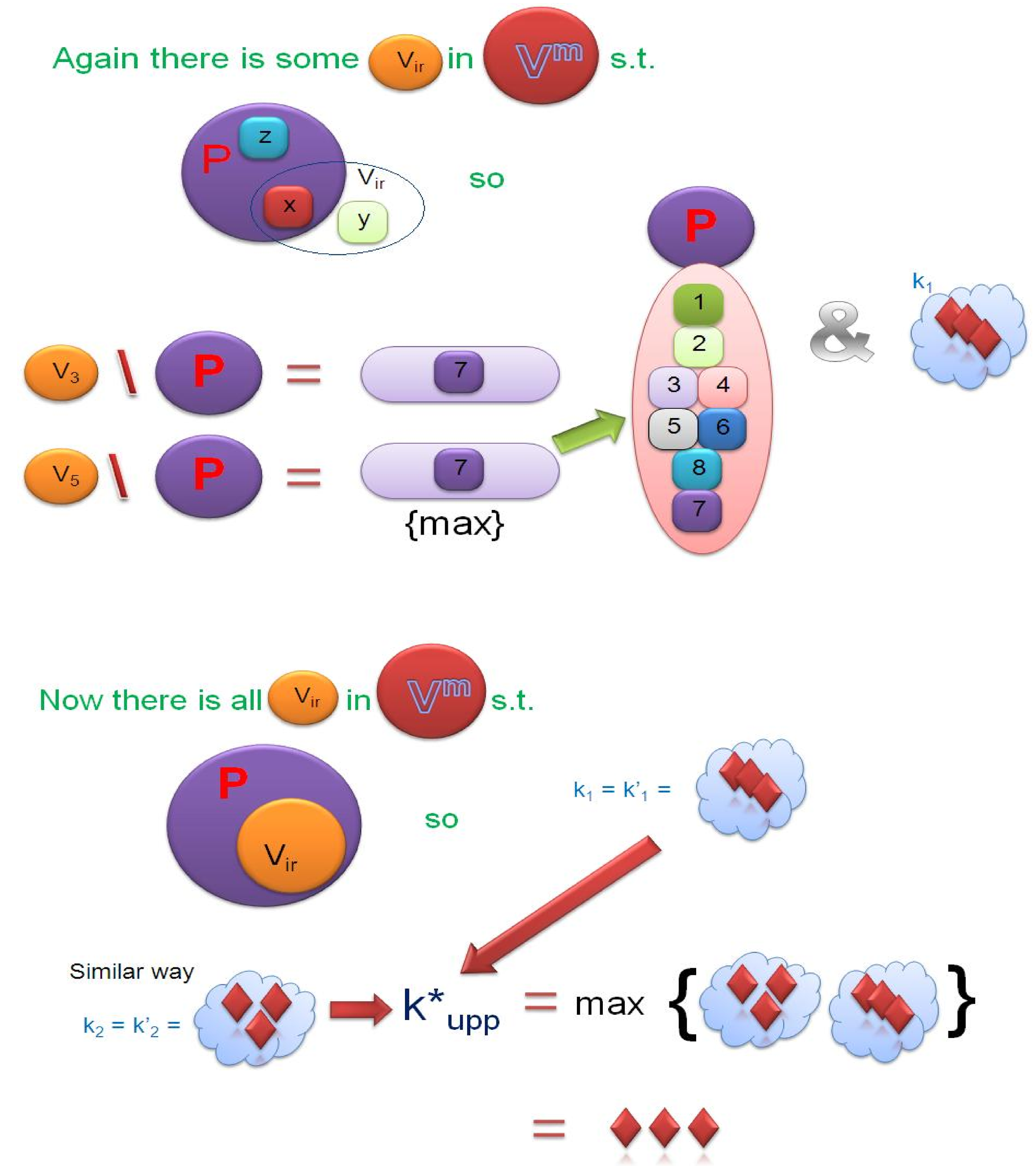
1. For $1 \leq i, j, m \leq n$, if $\exists\, V_i \,\&\, V_j \; s.t.\; V_j \subseteq V_i$ then delete all such V_j for all possible nodes V_i and list remaining collection of nodes as $V^m = \{V_{i_1}, V_{i_2}, ..., V_{i_m}\}, |V_{i_j}| = \alpha_{i_j} =$ number of packets in node V_{i_j}.
2. Let $V^l = \left\{V_{i_j} \in V^m | 1 \leq j \leq m \,\&\, |V_{i_j}| = \max\{\alpha_{i_j}\}\right\}$.
3. Pick an arbitary set $V_{i_j} \in V^l$, and call this set as P. Set the counter $k_\lambda = 1, 1 \leq k_\lambda \leq m$ and $1 \leq \lambda \leq |V^l| = l$.
4. If $\exists\, V_{i_{j'}} (1 \leq j' \leq m) \in V^m \; s.t.\; V_{i_{j'}} \bigcap P = \phi$ then go to step 5 otherwise jump to step 6.
5. Pick $V_{i_{j''}} (1 \leq j'' \leq m) \in V^m$ which has max cardinality among all $V_{i_{j''}}$ in V^m with $V_{i_{j''}} \bigcap P = \phi$. Update $P = P \bigcup V_{i_{j''}}$, update counter $k_\lambda = (k_\lambda + 1)$ and go to step 4.
6. If $\exists\, V_{i_r} (1 \leq r \leq m) \in V^m \; s.t.\; V_{i_r} \not\subset P$ then go to step 7 otherwise go to step 8.
7. Pick $V_{i_{r'}} (1 \leq r' \leq m) \in V^m$ which has maximum $\left|V_{i_{r'}} \backslash P\right|$ among all $V_{i_{r'}} \in V^m$ having the condition $V_{i_{r'}} \not\subset P$ then update $P = P \bigcup V_{i_{r'}}$, update counter $k_\lambda = (k_\lambda + 1)$ and go to step 6.
8. If $1 \leq \lambda < l$, then store k_λ in k'_λ and set $k_\lambda = k_{(\lambda+1)}$ and perform step 4 for $P = V_{i_{j'''}} (1 \leq j''' \leq m) \in V^l \; s.t. V_{i_{j'''}} \neq V_{i_j} \in V^l$, otherwise report $k^{\star}_{upp} = \min\left\{k'_\lambda\right\}_{\lambda=1}^{l}$.

Example







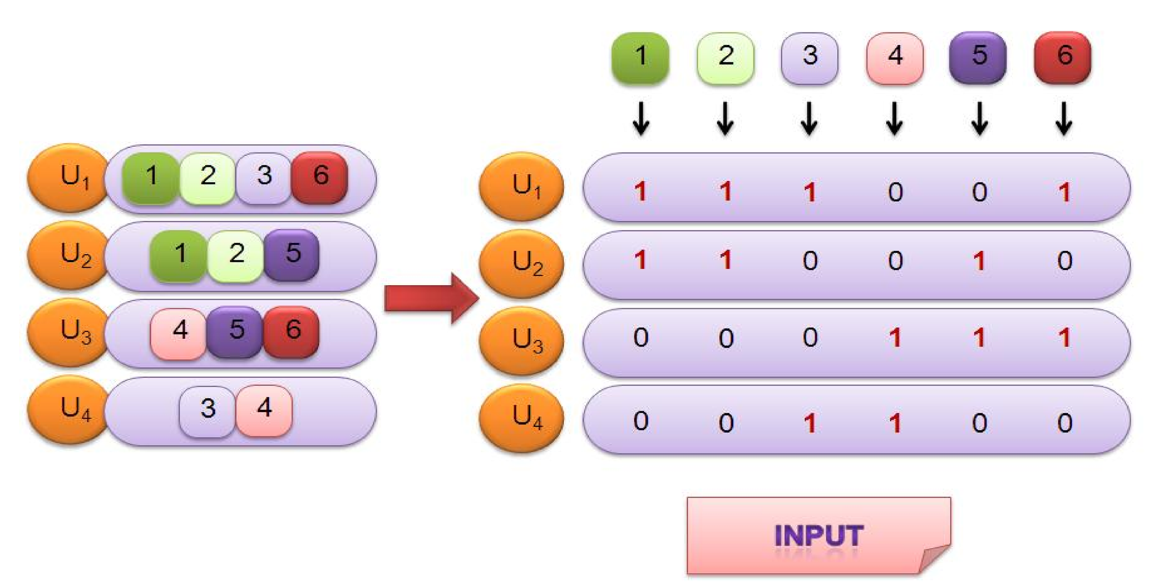


Algorithm to Compute Repair Degree d_i

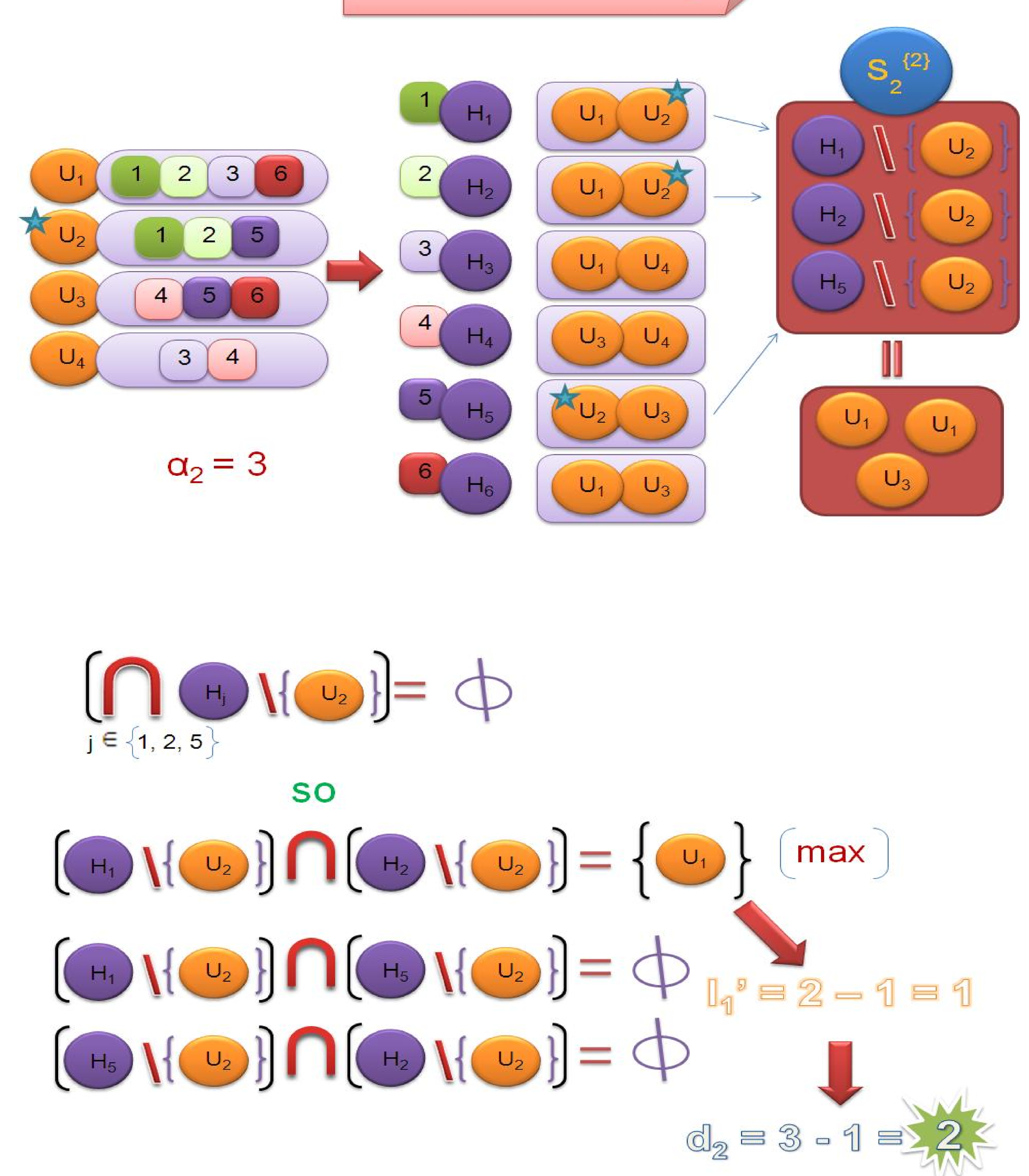
REQUIRE: Incidence matrix $M_{n \times \theta}$ of FR code and H_j.
ENSURE: Repair degree d_i for a node $U_i, 1 \leq i \leq n$.

1. For each node $i, 1 \leq i \leq n$ let $S_i^{\{i\}} = \{H_j \backslash \{i\} | i \in H_j, 1 \leq j \leq \theta\}$. Set $q = 1, 1 \leq q \leq n$.
2. Compute $T \subseteq \{1, 2, \ldots, \theta\} \; s.t.\; |T| > 1$ is maximum among all possible subsets and for $t \in T$, $H_t \backslash \{i\} \in S_i^{\{i\}}$, and $\bigcap H_t \backslash \{i\} \neq \phi$. Set counter $l_q (1 \leq q \leq n) = |T| - 1$. Store l_q in l'_q.
3. Update $S_i^{\{i\}} = S_i^{\{i\}} \backslash (H_t \backslash \{i\}), \forall t \in T$.
4. If $S_i^{\{i\}} = \phi$ or singleton set or $H_r \backslash \{i\} \cap H_s \backslash \{i\} \in S_i^{\{i\}} = \phi \; \forall 1 \leq r, s \leq n$ then $d_i = \alpha_i - \sum_{\lambda=1}^{q} l'_\lambda$, where $\alpha_i = |V_i|$, otherwise set $q = q + 1$ and go to step 2.

Example



Calculation for d_2



Remark

1. If $k^\star$ (spacific reconstruction degree of FR code) is the smallest set of nodes in a FR code, once contacted will reconstruct the whole data then

$$k^\star \leq k^\star_{upp} \text{ (Out put of algorithm 1).}$$

2. $k^\star \leq k_{FR}$, where k_{FR} (actual reconstruction degree of FR code) is the smallest set of **any** nodes, once contacted will reconstruct the entire data.

References

1. S. El Rouayheb and K. Ramchandran, *Fractional repetition codes for repair in distributed storage systems*, in Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on, Oct. 2010, pp. 1510 1517.
2. M. K. Gupta, A. Agrawal, and D. Yadav, *On weak dress codes for cloud storage*, CoRR, vol. abs/arXiv/1302.3681, 2013.
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