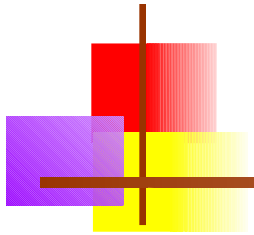
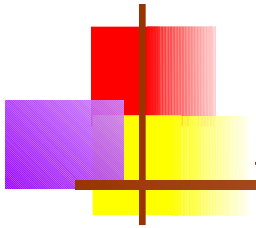
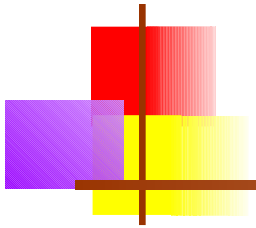


Backtracking



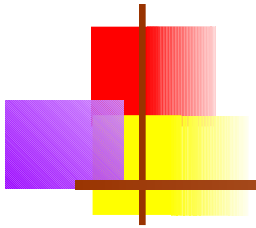
A short list of categories

- Algorithm types we will consider include:
  - Simple recursive algorithms
  - Backtracking algorithms
  - Divide and conquer algorithms
  - Dynamic programming algorithms
  - Greedy algorithms
  - Branch and bound algorithms
  - Brute force algorithms
  - Randomized algorithms



Backtracking

- Suppose you have to make a series of *decisions*, among various *choices*, where
  - You don't have enough information to know what to choose
  - Each decision leads to a new set of choices
  - Some sequence of choices (possibly more than one) may be a solution to your problem
- Backtracking is a methodical way of trying out various sequences of decisions, until you find one that "works"

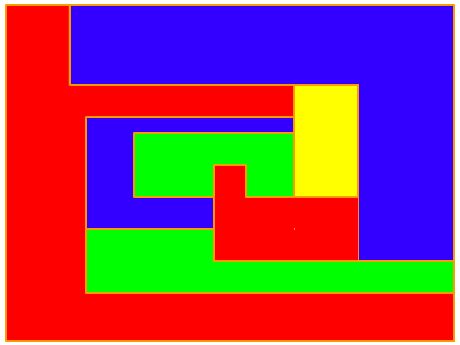


Solving a maze

- Given a maze, find a path from start to finish
- At each intersection, you have to decide between three or fewer choices:
  - Go straight
  - Go left
  - Go right
- You don’t have enough information to choose correctly
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many types of maze problem can be solved with backtracking

Coloring a map

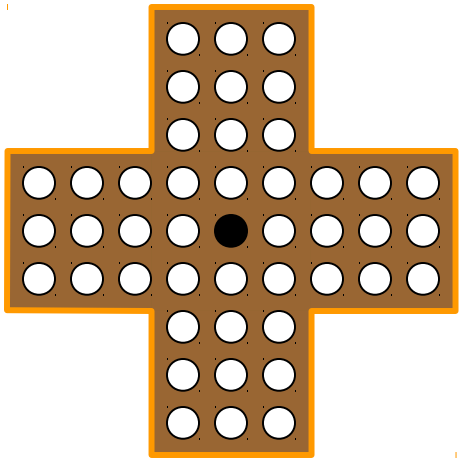
- You wish to color a map with not more than four colors
  - red, yellow, green, blue
- Adjacent countries must be in different colors



- You don’t have enough information to choose colors
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many coloring problems can be solved with backtracking

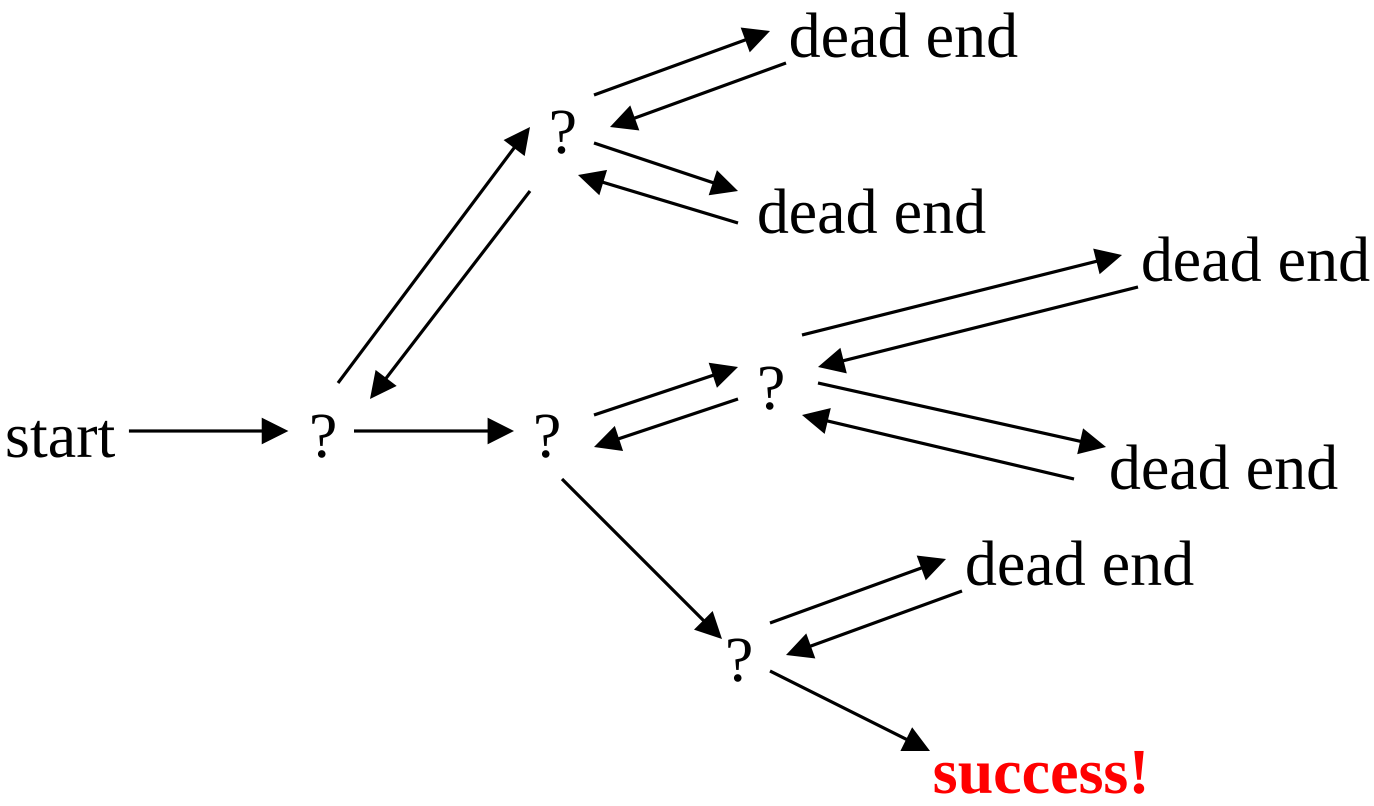
Solving a puzzle

- In this puzzle, all holes but one are filled with white pegs
- You can jump over one peg with another
- Jumped pegs are removed
- The object is to remove all but the last peg



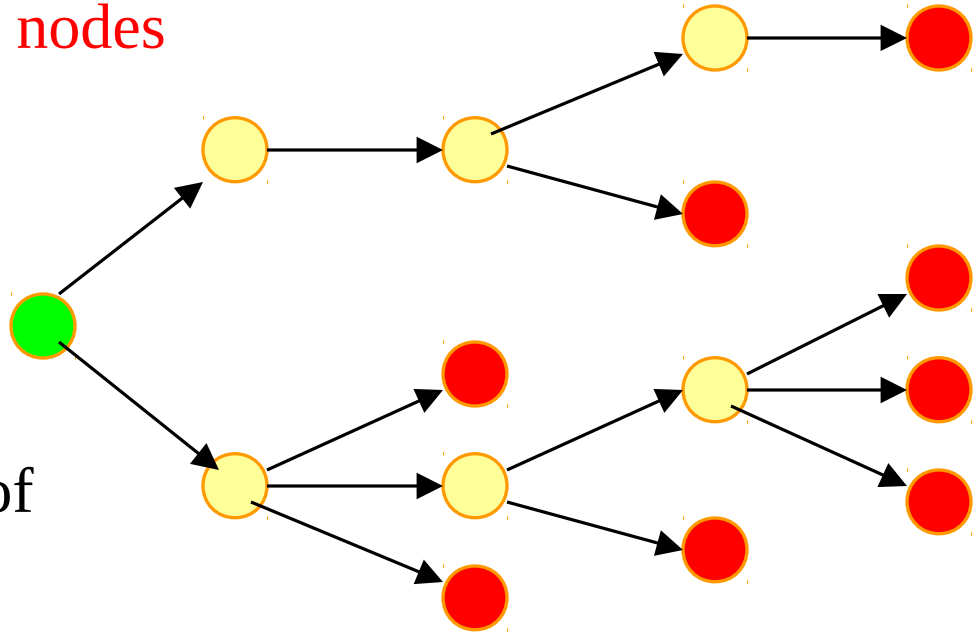
- You don’t have enough information to jump correctly
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many kinds of puzzle can be solved with backtracking

Backtracking (animation)



Terminology I

A tree is composed of nodes



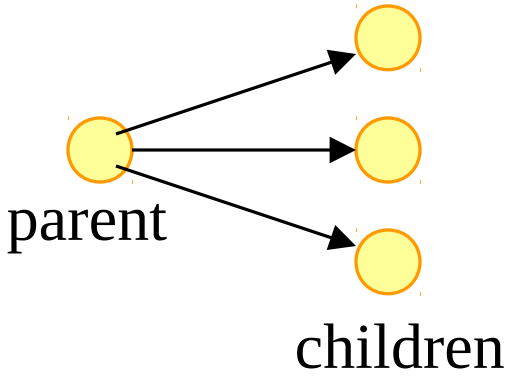
There are three kinds of nodes:

- The (one) root node
- Internal nodes
- Leaf nodes

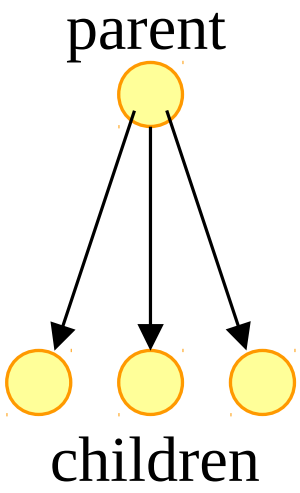
Backtracking can be thought of as searching a tree for a particular "goal" leaf node

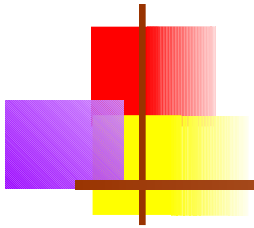
Terminology II

- Each non-leaf node in a tree is a parent of one or more other nodes (its children)
- Each node in the tree, other than the root, has exactly one parent



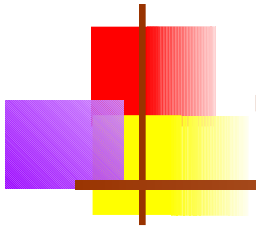
Usually, however, we draw our trees *downward*, with the root at the top





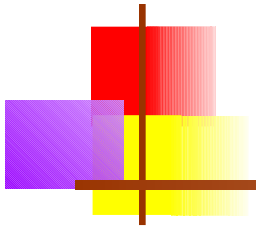
Real and virtual trees

- There is a type of data structure called a tree
  - But we are **not** using it here
- If we diagram the sequence of choices we make, the diagram looks like a tree
  - In fact, we did just this a couple of slides ago
  - Our backtracking algorithm “sweeps out a tree” in “problem space”



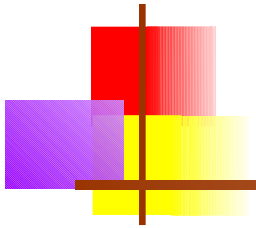
The backtracking algorithm

- Backtracking is really quite simple--we “explore” each node, as follows:
- To “explore” node N:
    1. If N is a goal node, return “success”
    2. If N is a leaf node, return “failure”
    3. For each child C of N,
        3.1. Explore C
            3.1.1. If C was successful, return “success”
    4. Return “failure”



Full example: Map coloring

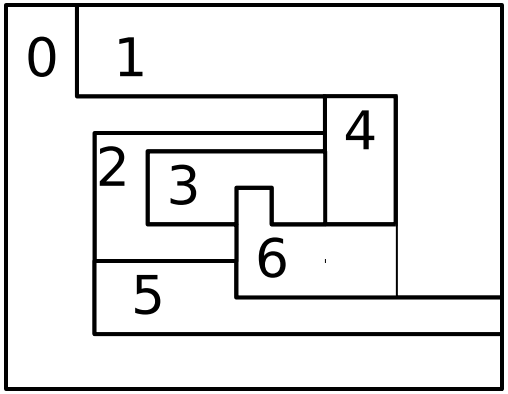
- The Four Color Theorem states that any map on a plane can be colored with no more than four colors, so that no two countries with a common border are the same color
- For most maps, finding a legal coloring is easy
- For some maps, it can be fairly difficult to find a legal coloring
- We will develop a complete Java program to solve this problem



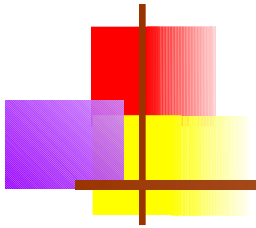
Data structures

- We need a data structure that is easy to work with, and supports:
  - Setting a color for each country
  - For each country, finding all adjacent countries
- We can do this with two arrays
  - An array of "colors", where `countryColor[i]` is the color of the i^{th} country
  - A ragged array of adjacent countries, where `map[i][j]` is the j^{th} country adjacent to country i
    - Example: `map[5][3]==8` means the 3^{th} country adjacent to country 5 is country 8

Creating the map

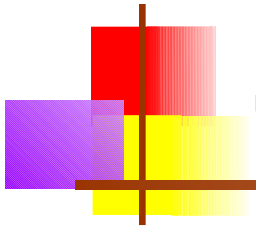



```
int map[][];

void createMap() {
    map = new int[7][];
    map[0] = new int[] { 1, 4, 2, 5 };
    map[1] = new int[] { 0, 4, 6, 5 };
    map[2] = new int[] { 0, 4, 3, 6, 5 };
    map[3] = new int[] { 2, 4, 6 };
    map[4] = new int[] { 0, 1, 6, 3, 2 };
    map[5] = new int[] { 2, 6, 1, 0 };
    map[6] = new int[] { 2, 3, 4, 1, 5 };
}
```



Setting the initial colors

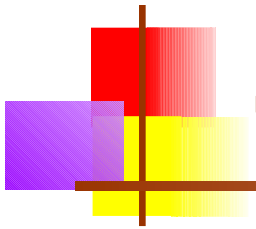
```
static final int NONE = 0;
static final int RED = 1;
static final int YELLOW = 2;
static final int GREEN = 3;
static final int BLUE = 4;

int mapColors[] = { NONE, NONE, NONE, NONE,
                    NONE, NONE, NONE };
```



The main program

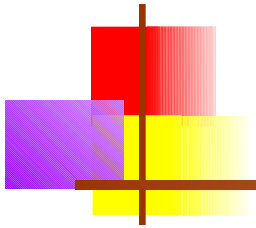
(The name of the enclosing class is ColoredMap)

```
public static void main(String args[]) {
    ColoredMap m = new ColoredMap();
    m.createMap();
    boolean result = m.explore(0, RED);
    System.out.println(result);
    m.printMap();
}
```



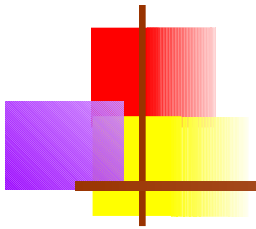
The backtracking method

```
boolean explore(int country, int color) {
    if (country >= map.length) return true;
    if (okToColor(country, color)) {
        mapColors[country] = color;
        for (int i = RED; i <= BLUE; i++) {
            if (explore(country + 1, i)) return true;
        }
    }
    return false;
}
```



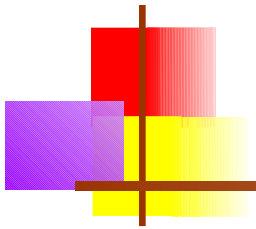
Checking if a color can be used

```
boolean okToColor(int country, int color) {
    for (int i = 0; i < map[country].length; i++) {
        int ithAdjCountry = map[country][i];
        if (mapColors[ithAdjCountry] == color) {
            return false;
        }
    }
    return true;
}
```



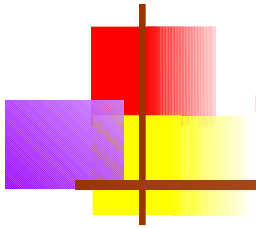
Printing the results

```
void printMap() {
    for (int i = 0; i < mapColors.length; i++) {
        System.out.print("map[" + i + "] is ");
        switch (mapColors[i]) {
            case NONE:   System.out.println("none");   break;
            case RED:    System.out.println("red");    break;
            case YELLOW: System.out.println("yellow"); break;
            case GREEN:  System.out.println("green");  break;
            case BLUE:   System.out.println("blue");   break;
        }
    }
}
```



Recap

- We went through all the countries recursively, starting with country zero
- At each country we had to decide a color
  - It had to be different from all adjacent countries
  - If we could not find a legal color, we reported failure
  - If we could find a color, we used it and recurred with the next country
  - If we ran out of countries (colored them all), we reported success
- When we returned from the topmost call, we were done



The End