Reconstruction and Repair Degree of Fractional Repetition Codes

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***Abstract*—Given a Fractional Repetition (FR) code, finding the reconstruction and repair degree in a Distributed Storage Systems (DSS) is an important problem. In this work, we present algorithms for computing the reconstruction and repair degree of FR Codes.**

I. Introduction

We consider Distributed Storage Systems (DSSs) that use Distributed Replication-based Simple Storage (DRESS) codes consisting of an inner Fractional Repetition (FR) code and an outer Maximum Distance Separable (MDS) code to optimize various parameters of DSS [1], [2]. Constructing a FR code $\mathscr{C}(n,\theta,\alpha,\rho)$ for a given (n,k,d) DSS is an important problem addressed in [1]. On the other hand, given a FR code finding the reconstruction degree k (minimum number of nodes that needs to be contacted to reconstruct the entire data) and the repair degree d (number of nodes needs to be contacted in case of failure of a node) in such a (n,k,d) DSS has not been studied. Towards this end, for a given FR code, we define a reconstruction degree $k^\star$ as the smallest number of nodes one has to contact to recover the entire file. This gives us a lower bound on actual reconstruction degree k_{FR} defined as a degree such that user can get the entire data by contacting any (minimal) k_{FR} nodes. For weak dress codes [2], finding the repair degree is non-trivial problem. Algorithm 2 computes it.

II. algorithms

Let the column support (non-zero entries) of a column $M_j, 1 \leq j \leq \theta$ of incidence matrix M of a FR code $\mathscr{C}(n,\theta,\alpha,\rho)$ [3] is H_j and the repair degree of each node U_i is d_i.

Example 1. *Consider a FR Code $\mathscr{C}(5,9,4,2)$ (see [2]) with 9 packets and 5 nodes given as $U_1 = \{1,2,3,4\}, U_2 = \{1,6,9\}, U_3 = \{2,5,7,9\}, U_4 = \{3,5,6,8\}$ & $U_5 = \{4,7,8\}$. Algorithm 1 gives $k^\star_{upp} = 3$ and using algorithm 2, one finds the repair degree of node $U_2 = d_2 = 3$.*

Algorithm 1 Algorithm to compute reconstruction degree $k^\star$

Require: Node packet distribution of FR code after removing the last packet θ (as it can be recovered by parity) from all n nodes of $V^n = \{V_1, V_2, ..., V_n\}$.

Ensure: $k^\star_{upp}$ = Reconstruction degree

1 : For $1 \leq i,j,m \leq n$, if $\exists$ V_i & V_j $s.t.$ $V_j \subseteq V_i$ then delete all such V_j for all possible nodes V_i and list remaining collection of nodes as $V^m = \{V_{i_1}, V_{i_2}, ..., V_{i_m}\}, |V_{i_j}| = \alpha_{i_j} =$ number of packets in node V_{i_j}.

2 : Let $V^l = \left\{V_{i_j} \in V^m | 1 \leq j \leq m \text{ \& } |V_{i_j}| = \max\{\alpha_{i_j}\}\right\}$.

3 : Pick an arbitary set $V_{i_j} \in V^l$, and call this set as P. Set the counter $k_\lambda = 1, 1 \leq k_\lambda \leq m$ and $1 \leq \lambda \leq |V^l| = l$.

4 : If $\exists$ $V_{i_{j'}} (1 \leq j' \leq m) \in V^m$ $s.t.$ $V_{i_{j'}} \bigcap P = \phi$ then go to step 5 otherwise jump to step 6.

5 : Pick $V_{i_{j''}} (1 \leq j'' \leq m) \in V^m$ which has max cardinality among all $V_{i_{j''}}$ in V^m with $V_{i_{j''}} \bigcap P = \phi$. Update $P = P \bigcup V_{i_{j''}}$, update counter $k_\lambda = (k_\lambda + 1)$ and go to step 4.

6 : If $\exists$ $V_{i_r} (1 \leq r \leq m) \in V^m$ $s.t.$ $V_{i_r} \not\subset P$ then go to step 7 otherwise go to step 8.

7 : Pick $V_{i_{r'}} (1 \leq r' \leq m) \in V^m$ which has maximum $\left|V_{i_{r'}} \backslash P\right|$ among all $V_{i_{r'}} \in V^m$ having the condition $V_{i_{r'}} \not\subset P$ then update $P = P \bigcup V_{i_{r'}}$, update counter $k_\lambda = (k_\lambda + 1)$ and go to step 6.

8 : If $1 \leq \lambda < l$, then store k_λ in k'_λ and set $k_\lambda = k_{(\lambda+1)}$ and perform step 4 for $P = V_{i_{j'''}} (1 \leq j''' \leq m) \in V^l$ $s.t. V_{i_{j'''}} \neq V_{i_j} \in V^l$, otherwise report $k^\star_{upp} = \min\{k'_\lambda\}^l_{\lambda=1}$.

Algorithm 2 Algorithm to compute Repair Degree d_i

Require: Incidence matrix $M_{n\times\theta}$ of FR code and H_j.

Ensure: Repair degree d_i for a node $U_i, 1 \leq i \leq n$.

1 : For each node $i, 1 \leq i \leq n$ let $S_i^{\{i\}} = \{H_j \backslash \{i\} | i \in H_j, 1 \leq j \leq \theta\}$. Set $q = 1, 1 \leq q \leq n$.

2 : Compute $T \subseteq \{1, 2, \ldots, \theta\}$ $s.t.$ $|T| > 1$ is maximum among all possible subsets and for $t \in T$, $H_t \backslash \{i\} \in S_i^{\{i\}}$, and $\bigcap H_t \backslash \{i\} \neq \phi$. Set counter $l_q (1 \leq q \leq n) = |T| - 1$. Store l_q in l'_q.

3 : Update $S_i^{\{i\}} = S_i^{\{i\}} \backslash (H_t \backslash \{i\}), \forall t \in T$.

4 : If $S_i^{\{i\}} = \phi$ or singleton set or $H_r \backslash \{i\} \cap H_s \backslash \{i\} \in S_i^{\{i\}} = \phi$ $\forall 1 \leq r, s \leq n$ then $d_i = \alpha_i - \sum_{\lambda=1}^{q} l'_\lambda$, where $\alpha_i = |V_i|$, otherwise set $q = q + 1$ and go to step 2.

References

[1] S. El Rouayheb and K. Ramchandran, "Fractional repetition codes for repair in distributed storage systems," in *Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on*, Oct. 2010, pp. 1510 –1517.

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An extended abstract of the paper can be obtained from Arxiv or author's home page at http://www.guptalab.org