Modeling Geomagnetic Disturbances Using Computer-Based Simulations

Nikhil Nayak

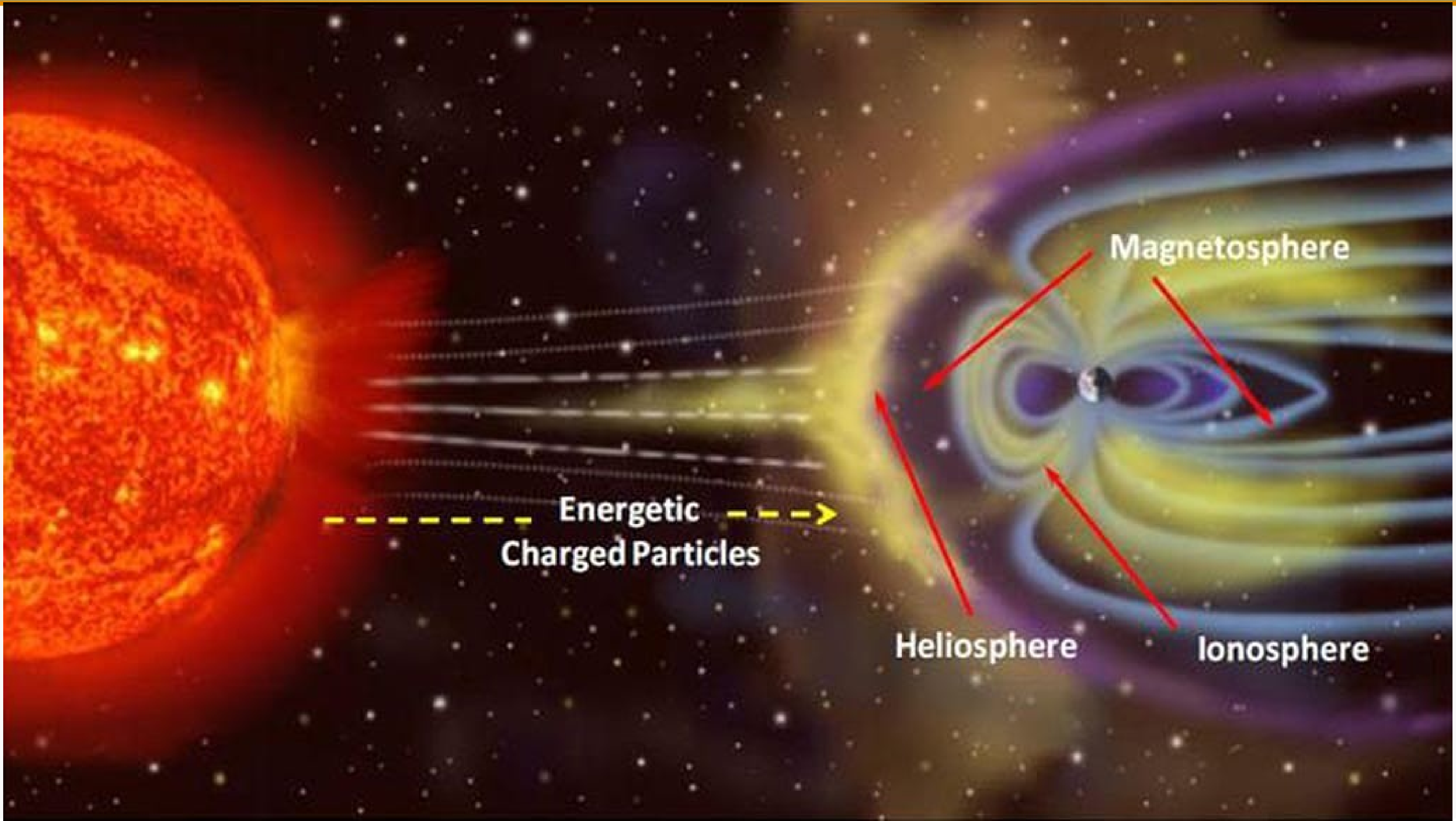
What is a Geomagnetic Disturbance (GMD)?

✨Space Weather✨

But what does that actually mean?

- A GMD is created from magnetic field emitted by the sun, causing Geomagnetic Induced Currents (GICs)
- When the GICs collides with the Earth’s magnetic field, it can cause disturbances in electrical systems in the affected areas
- These disturbances can include the fluctuation in the system’s voltage and current, which can overheat the system and cause power outages

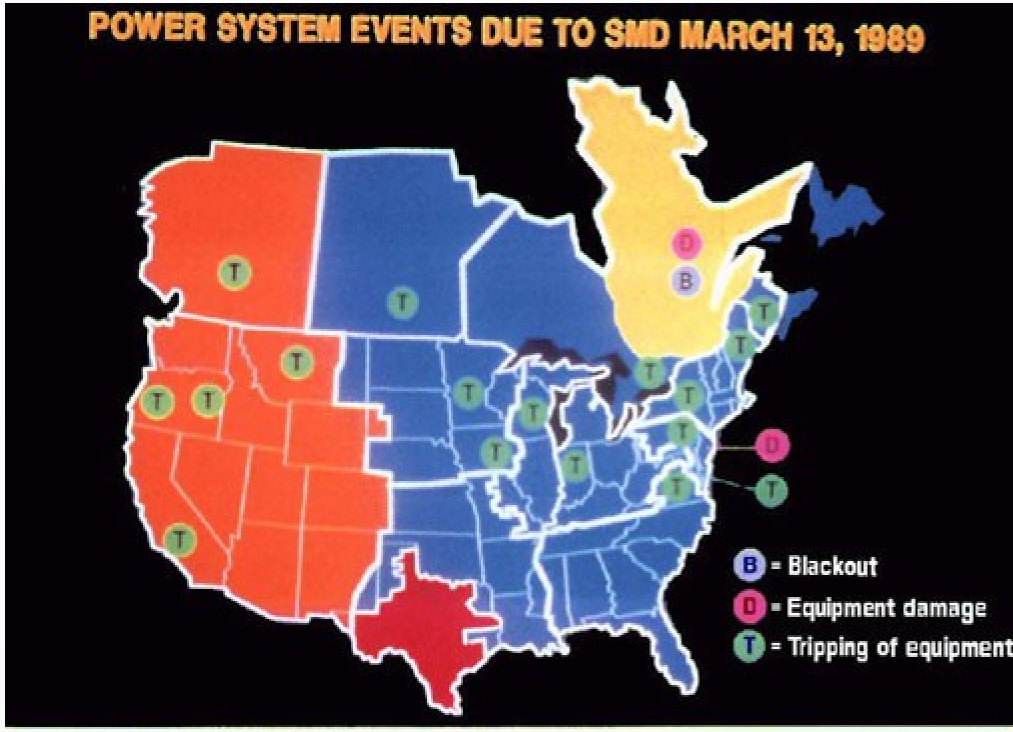
Diagram of GMDs



Why should you care about this?

This is a real issue, not just a theoretical problem. Take the Hydro-Quebec 1989 power outage, for example.

- On March 12, 1989, voltage fluctuations were seen on the Hydro-Quebec power grid
- One day later, the Earth's magnetic field started violently fluctuating, causing a province-wide blackout in less than one minute
- This was caused by a solar storm that astronomers witnessed on March 10, 1989, a storm that affected dozens of countries, from Canada to Europe



How can we solve this?

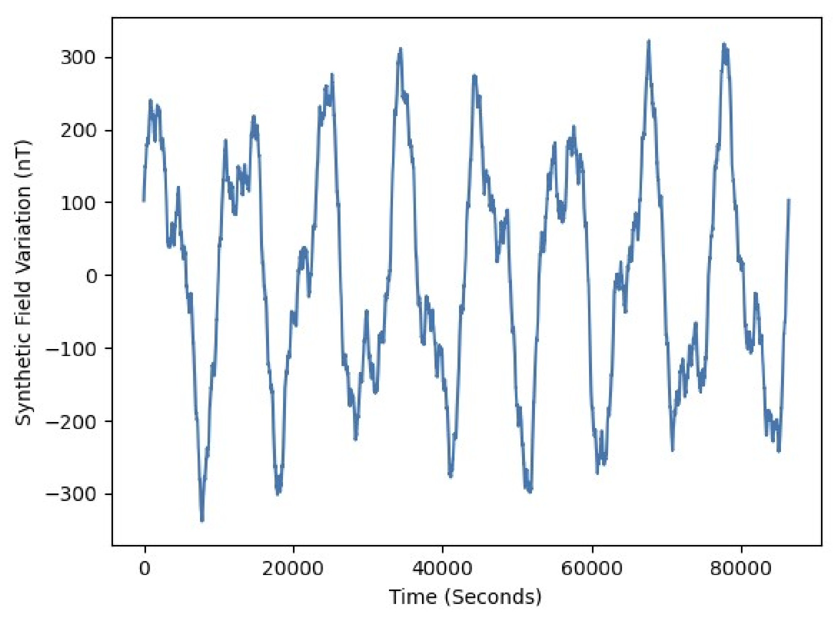
- We need to take our geomagnetic field recordings and use them to determine the geoelectric fields at the Earth’s surface
- To understand the system we’re looking at, we need to model it
- In this case, we can use a set of mathematical equations to model the GMD system

Simulating a GMD

- Before we try to model a real dataset, we can start with a simulated base case. Our dataset can be defined by *B(t):*

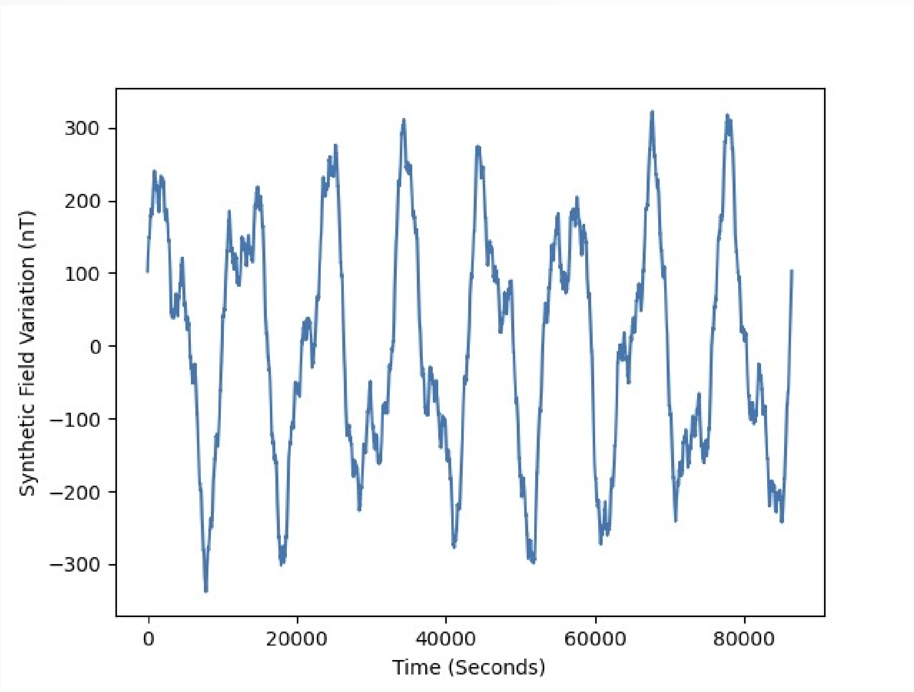
$$B(t) = \sum_{m=1}^{7} A_m \sin(2\pi f_m t + \Phi_m)$$

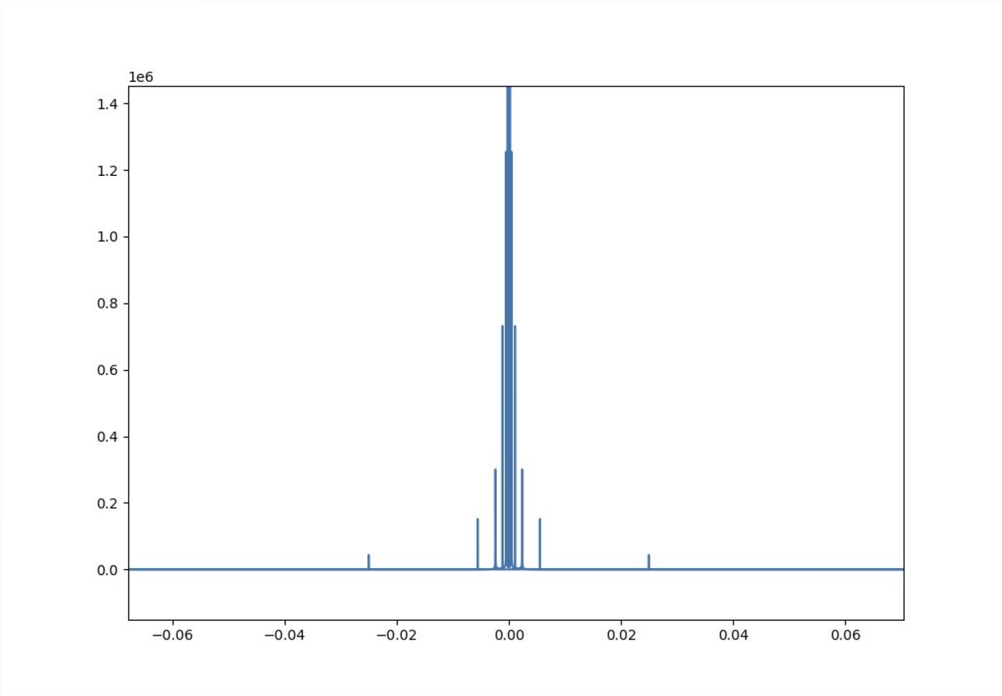
m	A_m (nT)	Φ_m (deg)	f_m (Hz)	$T_m = 1/f_m$ (min)
1	200	10	0.00009259	180
2	90	20	0.00020833	80
3	30	30	0.00047619	35
4	17	40	0.00111111	15
5	8	50	0.00238095	7
6	3.5	60	0.00555555	3
7	1	70	0.025	2/3



Discretization

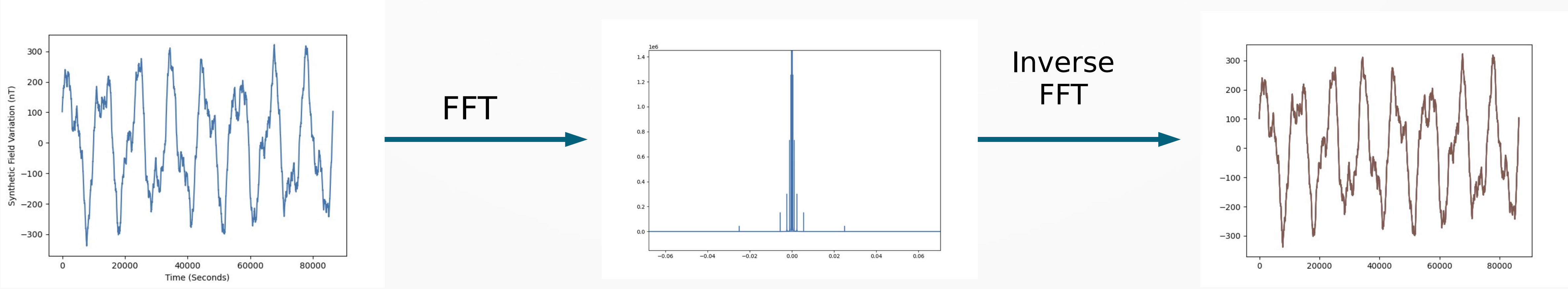
- Because we are implementing our model on a computer, we need to discretize the function into bins using a Fourier Transform





Discretization (Explained)

- FFTs are representations of a function using just *sin* and *cos*
- Each line of an FFT represents 1 function. When all the functions are added together, they resemble the original function
- This converts our data from the time domain to the frequency domain, making it easier to process the signals



Complex Numbers in FFTs

- To accurately represent a signal, you need to represent amplitude and phase
- Complex numbers are inherently good at representing waves because of Euler’s formula:

$$e^{ix} = \cos x + i \sin x$$

- Euler’s formula helps simplify the process of creating Fourier Transforms, making it possible to represent a wave using the sum of *cos* and *sin* functions
- Each complex number is just a real and imaginary component, or a *x* and *y*. x and y can make up a point on a plane, meaning the point can have a distance from origin (amplitude) and rotation from a horizontal line (phase)

Modeling the System

How are we going to model a GMD system?

- To model the GMD system, we are using a 5 layer model designed for the Hydro-Quebec incident. This model was created in the *Boteler 2019* research paper. Here's the model:

$$K_n = \eta_n \frac{K_{n+1}\left(1 + e^{-2k_n l_n}\right) + \eta_n\left(1 - e^{-2k_n l_n}\right)}{K_{n+1}\left(1 - e^{-2k_n l_n}\right) + \eta_n\left(1 + e^{-2k_n l_n}\right)}$$

- To understand what all those letters mean, we're going to need some more info

Modeling the System (cont.)

$$K_n = \eta_n \frac{K_{n+1}\left(1+e^{-2k_n l_n}\right) + \eta_n\left(1-e^{-2k_n l_n}\right)}{K_{n+1}\left(1-e^{-2k_n l_n}\right) + \eta_n\left(1+e^{-2k_n l_n}\right)}$$

$$K_N = \frac{i2\pi f}{k_N} = \sqrt{\frac{i2\pi f}{\mu_0 \sigma_N}}$$

- *K* is a recursive function, defined by the base case on the right
- The 5 layer model represents 5 layers of the Earth, from 15km to 200km
- When we use layer “1”, the recursive function calls each layer until it hits the base case, returning the modeled data.

Relevant Model Code (Julia)

```julia
Ls = [15, 10, 125, 200]*1e3
res = [20000, 200, 1000, 100]
Sigmas = [1/20000, 1/200, 1/1000, 1/100, 1/3]



N = 5 # number of layers in the model

mu = 4 * pi * 10^-7 # μ, defined below equation 2
i = 1im # √-1


function K_N(f) # top layer of the model, equation 16
    return sqrt((i * 2 * pi * f) / (mu * Sigmas[N]))
end


function k(n, f) # equation 4
    return sqrt(i * 2 * pi * f * mu * Sigmas[n])
end

function eta(n, f) # η, defined below equation 18
    return i * 2 * pi * f / k(n, f)
end
```

```julia
function eExpression(n, f) # e ^ (-2knln)
    return ℯ ^ (-2 * k(n, f) * Ls[n])
end

function K(n, f) # Equation 19
    #println("in layer ", n)
    if n == N
        return K_N(f)
    end

    K_NPrev = K(n+1, f)
    e = eExpression(n, f)

    num = K_NPrev * (1 + e) + eta(n, f) * (1 - e)
    den = K_NPrev * (1 - e) + eta(n, f) * (1 + e)

    return eta(n, f) * num / den
end
```

Benchmark

- To benchmark our computer implementation of the *Boteler 2019* model, we can run some test cases

TABLE 3. Transfer function K(f) for the frequencies in the synthetic test magnetic field variation for a multi-layer Earth (5-layer Québec model).

Boteler 2019 Results

m	f_m (Hz)	Amplitude, $\lvert K_m \rvert$ (mV/km/nT)	Phase, θ_m (deg)
1	0.000093	0.2188	77.15
2	0.000208	0.4480	73.76
3	0.000476	0.8681	67.17
4	0.001111	1.5392	62.08
5	0.002381	2.5935	60.58
6	0.005556	4.6625	54.97
7	0.025	9.6047	44.38

Computer Implemented Results

m	f_m	Amplitude	Phase
1	0.000093	0.219701	77.139393
2	0.000208	0.447422	73.771382
3	0.000476	0.867800	67.172638
4	0.001111	1.539096	62.080986
5	0.002381	2.593561	60.575656
6	0.005556	4.662704	54.969143
7	0.025000	9.604690	44.381495

TABLE 7. Parameters of electric field waveform for a multi-layer Earth (5-layer Québec model).

Boteler 2019 Results

m	f_m (Hz)	Amplitude, E_m (mV/km)	Phase, φ_m (deg)
1	0.000093	43.76735	87.15
2	0.000208	40.32326	93.76
3	0.000476	26.04161	97.17
4	0.001111	26.16634	102.08
5	0.002381	20.74819	110.58
6	0.005556	16.31864	114.97
7	0.025	9.60469	114.38

Computer Implemented Results

m	f_m	Amplitude	Phase
1	0.000093	43.940296	87.139393
2	0.000208	40.267957	93.771382
3	0.000476	26.033997	97.172638
4	0.001111	26.164632	102.080986
5	0.002381	20.748486	110.575656
6	0.005556	16.319466	114.969143
7	0.025000	9.604690	114.381495

TABLE 7. Parameters of electric field waveform for a multi-layer Earth (5-layer Québec model).

Boteler 2019 Results

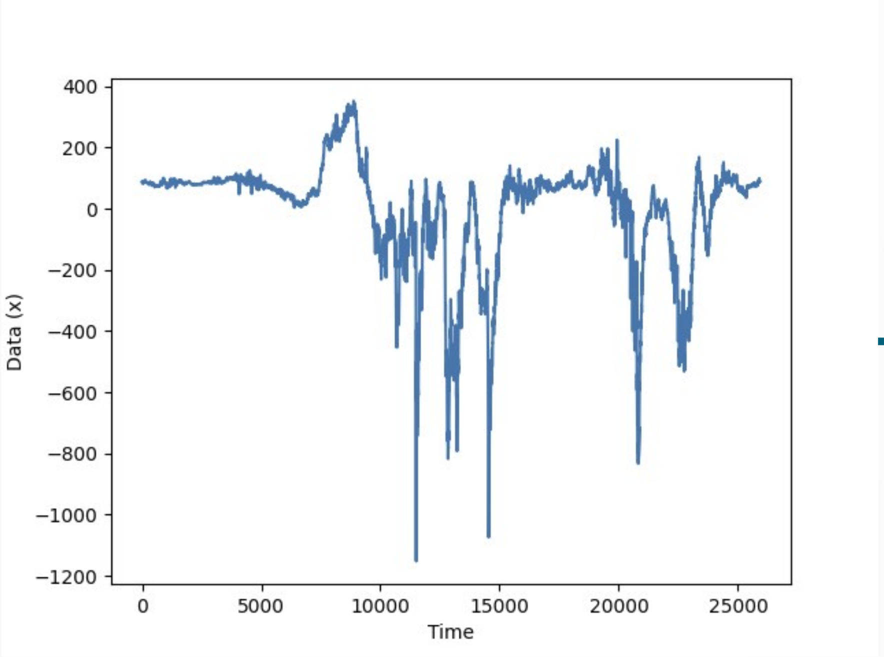
m	f_m (Hz)	Amplitude, E_m (mV/km)	Phase, φ_m (deg)
1	0.000093	43.76735	87.15
2	0.000208	40.32326	93.76
3	0.000476	26.04161	97.17
4	0.001111	26.16634	102.08
5	0.002381	20.74819	110.58
6	0.005556	16.31864	114.97
7	0.025	9.60469	114.38

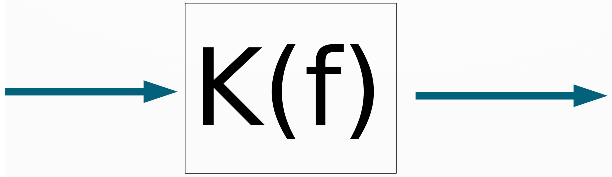
Computer Implemented Results

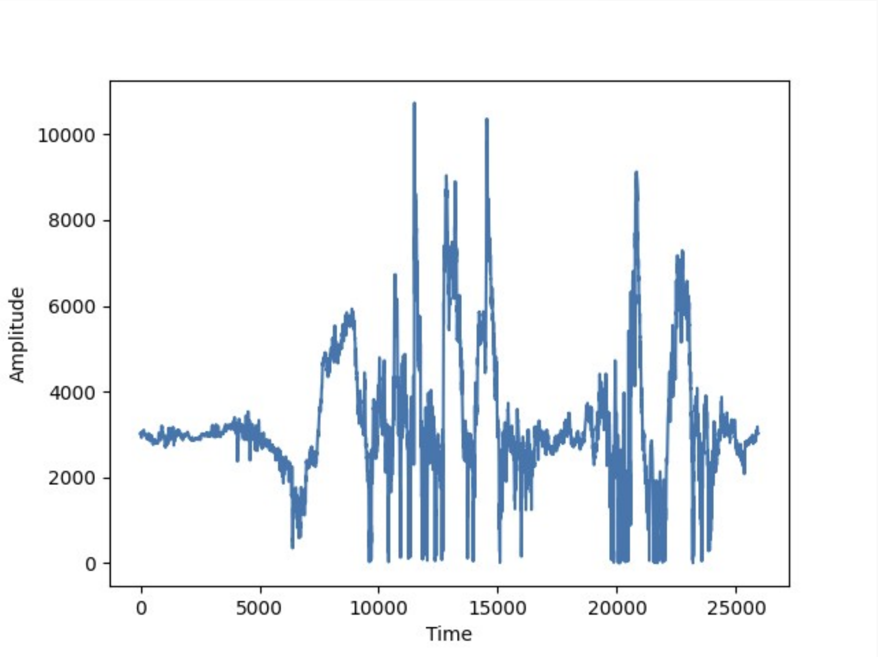
m	f_m	Amplitude	Phase
1	0.000093	43.940296	87.139393
2	0.000208	40.267957	93.771382
3	0.000476	26.033997	97.172638
4	0.001111	26.164632	102.080986
5	0.002381	20.748486	110.575656
6	0.005556	16.319466	114.969143
7	0.025000	9.604690	114.381495

Real World Application

- Data from Fort Simpson, Canada
- Calculated Geoelectric fields for Fort Simpson dataset, mV/km/nT







Conclusion

- In this project, a computer implementation of the Quebec 5-layer model was created
- Despite some challenges regarding creating a computer implementation of the simulation, I was able to create an accurate and efficient transfer function model
- In the future, I want to take the output of the model and calculate the actual currents and voltages in the system
- Additionally, I want to implement the 5 layer model in a performance-focused programming language; Julia had some issues with speed during development and testing due to its Just-In-Time compilation method

Bibliography

https://www.naes.com/news/what-is-a-geomagnetic-disturbance-and-how-can-it-affect-the-power-grid/

https://www.nasa.gov/mission_pages/sunearth/spaceweather/index.html

https://ieeexplore.ieee.org/abstract/document/8859181

https://towardsdatascience.com/fast-fourier-transform-937926e591cb#:~:text=As%20the%20name%20implies%2C%20the,)%20to%20O(NlogN)%20

http://www.librow.com/articles/article-10

https://wdc.bgs.ac.uk/catalog/format.html

Questions?