STAT 149 Generalized Linear Models: Homework 1

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Question 1

(a) Number of app downloads out of 100 iPhone users: $Y \sim Bin(100, p)$ with p unknown

(b) Babies born on a single day in Boston: $Y \sim Po(\mu)$ with μ unknown

(c) Initial weight of male enrolling in diet:

(d) Initial weight of person enrolling in diet:

(e) Person liked/disliked star wars movie: $Y \sim Bern(p)$ with p unknown

(f) Coin flips with 1,2 as outcome set: alternative distribution

(g) Num hairs in a random Big Mac: $Y \sim N(\mu, \sigma^2)$ with μ, σ^2 unknown

(h) At least 1 hair in a random Big Mac: $Y \sim Bern(p)$ with p unknown

(i) Actual amount of Pepsi in a can: $Y \sim N(\mu, \sigma^2)$ with μ, σ^2 unknown

(j) Num times Trump says 'MAGA' in first three months: $Y \sim Po(\mu)$ with μ unknown

(k) Salary of quant on wall street: $Y \sim N(\mu, \sigma^2)$ with μ, σ^2 unknown but μ likely to be very large!

(l) Num hospital workers out of 20 who have had the flu: alternative (cannot assume trial of each person is independent as flu can be spread by contact with other people)

Question 2

(a) Log likelihood:

$$l(\mu) = -2\mu + (y_1 + y_2)log(\mu) - log(y_1! + y_2!)$$

First derivative:

$$l'(\mu) = -2 + (y_1 + y_2)\frac{1}{\mu}$$

Second derivative:

$$l''(\mu) = -(y_1 + y_2)\frac{1}{\mu^2}$$

(b) Maximum Likelihood estimate:

$$\mu = \frac{y_1 + y_2}{2}$$

$$l''(\frac{y_1 + y_2}{2}) = -\frac{4}{y_1 + y_2}$$

As the Poission distribution is positive valued $y_1, y_2 > 0$, so $l''(\mu) < 0$

(c) Newton Raphson Update formula:

$$\mu^{(i+1)} = \mu^{(i)} - \frac{l'(\mu)}{l''(\mu)}$$

$$\mu^{(i+1)} = \mu^{(i)}) - \frac{-2 + (y_1 + y_2)\frac{1}{\mu^{(i)}}}{-(y_1 + y_2)\frac{1}{(\mu^{(i)})^2}}$$

(d) Fisher Scoring Update formula:

$$\mu^{(i+1)} = \mu^{(i)} - \frac{l'(\mu)}{E[l''(\mu)]}$$

$$\mu^{(i+1)} = \mu^{(i)} - \frac{-2 + (y_1 + y_2)\frac{1}{\mu^{(i)}}}{\frac{-2}{\mu^{(i)}}}$$

(e) MLE Estimate using $y_1 = 3$ and $y_2 = 5$:

$$\mu = \frac{8}{2} = 4$$

Newton Raphson converges after 7 iterations with $\mu^{(0)} = 2.0$.
Fisher scoring converges in one iteration.

Question 3

Table 1: Newton Raphson Convergence

iteration	μ	l'	l''
1	2		
2	3	2	-2
3	3.75	0.6666666667	-0.8888888889
4	3.984375	0.1333333333	-0.5688888889
5	3.999938965	0.007843137255	-0.503929258
6	3.999999999	0.00003051804379	-0.5000152591
7	4	0.0000000004656612873	-0.5000000002
8	4	0	-0.5
9	4	0	-0.5
10	4	0	-0.5

Table 2: Fisher Scoring Convergence

iteration	μ	l'	l''
1	2		
2	4	2	-1