CS660 Summer 2023 CRN 40729
Homework 2
Due: June 19, 2023, 11:59 PM

Complete all **five** problems. Put your pages in order and scan your solutions and upload one PDF. I will not grade multiple files, jpegs, Mac Pages, or any other image files. Each problem is worth 4 points for a total of 20 points.

1. Consider $\mathbb{R}^3$ with the inner product

$$\langle \boldsymbol{x}, \boldsymbol{y} \rangle := \boldsymbol{x}^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \boldsymbol{y}$$

Furthermore, we define $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ as the standard/canonical basis in $\mathbb{R}^3$.

(a) Determine the orthogonal projection $\pi_U(\boldsymbol{e}_2)$ of $\boldsymbol{e}_2$ onto $U = span[\boldsymbol{e}_1, \boldsymbol{e}_3]$.

(b) Compute the distance $d(\boldsymbol{e}_2, U)$.

2. Let W be the subspace of $\mathbb{R}^3$ spanned by $\boldsymbol{u_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{u_2} = \begin{bmatrix} 7 \\ -2 \\ 11 \end{bmatrix}$.

$\left(\text{i. e.}, W = Span\left\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -11 \end{pmatrix}\right\}\right)$. Find the vector(s) that span(s) the orthogonal complement $W^\perp$ of W.

3. Let W be the subspace of $\mathbb{R}^5$ spanned by $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \\ 2 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 2 \\ 4 \\ 7 \\ 2 \\ -1 \end{bmatrix}$. Find a basis of the orthogonal complement $W^\perp$ of W.

4. Find the orthonormal basis for the subspace U of $\mathbb{R}^4$ spanned by the vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 2 \\ -4 \\ -3 \end{bmatrix}$$

5. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 4 \\ 7 & -5 & 2 \end{bmatrix}$$

(a) Are the rows of $\boldsymbol{A}$ orthogonal?
(b) Is $\boldsymbol{A}$ an orthogonal matrix?
(c) Are the columns of $\boldsymbol{A}$ orthogonal?

Provide support for your answers.