CS660 Summer 2023 CRN 40729
Homework 1
Due: June 12, 2023, 11:59 PM

Complete all five problems. Put your pages in order and scan your solutions and upload one PDF. I will not grade multiple files, jpegs, Mac Pages, or any other image files. Each problem is worth 4 points for a total of 20 points.

1. Compute the following matrix products, if possible. If the product is not possible, state why.

(a)

$$\begin{bmatrix} 1 & 5 \\ 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 4 & 6 \\ 7 & 2 & 5 \\ 9 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ 7 & 2 & 5 \\ 9 & 8 & 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 4 & 1 & -4 & -1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 2 & 2 \\ 4 & 2 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 4 & 1 & -4 & -1 \end{bmatrix}$$

2. Write $\boldsymbol{y} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$ as a linear combination of $\boldsymbol{x_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{x_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{x_3} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

3. Consider two subspaces $U_1$and U_2 where U_1 is the solution space of the homogeneous equations system $A_1\boldsymbol{x} = 0$ and U_1 is the solution space of the homogeneous equations system $A_2\boldsymbol{x} = 0$.

$$A_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 3 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad and\ A_2 = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 2 \\ 6 & -4 & 2 \\ 2 & -1 & 3 \end{bmatrix}$$

(a) Determine the dimension of U_1, U_2.
(b) Determine bases of U_1 and U_2.
(c) Determine a basis for $U_1 \cap U_2$.

4. Suppose $S = \{v_1, v_2, \ldots, v_m)$ spans a vector space V. Prove:

(a) If $w \in V, then\ \{w, v_1, v_2, \ldots, v_m\}$ is linearly dependent and spans V.
(b) If $\boldsymbol{v_i}$ is a linear combination of $\{v_1, v_2, v_{i-1}\}$, then S without v_i spans V.

5. Consider the basis

$$\boldsymbol{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$. Find the change of bases matrix $\boldsymbol{P}$ from the standard basis $\{\boldsymbol{e_1}, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ to the basis **B.**