6.2 More on Functional Forms

6.2.1 Log Models

1. Interpreting log log models

$$\begin{aligned} log(price) =& \beta_0 + \beta_1 log(nox) + \beta_2 rooms + u \\ & 9.23(0.19) - 0.718(0.066) + 0.306(0.019) \end{aligned}$$

$100\hat{\beta}_2$: semi-elasticity of price with respect to rooms cause only y is in logs. However this is an approximation only for small changes in y i.e. $\Delta log(price)$. To track the exact partial effect in a semi-log model we use the following:

$$\%\Delta\hat{y} = 100 \cdot [exp(\hat{\beta}_2 \Delta\hat{x}_2) - 1]$$

With correction: a unit increase in number of rooms increases house price by 35.8%. (**check**!!) What is the coefficient when number of rooms reduce by one unit?

2. Prevalance of log models in empirical work

   (a) When $y > 0$, models using log(y) as the dependent variable often satisfy the CLM assumptions more closely than models using the level of y.

   (b) Taking the log of a variable often narrows its range hence be less sensitive to outliers.

3. Common rules:

    (a) A variable is a positive dollar amount (price/revenue) or large integer number (GDP/population), the log is often taken

    (b) Variables that are measured in years – such as education, experience, tenure, age, and so on – usually appear in their original form.

        Caution: Remember, if *unem* goes from 8 to 9, this is an increase of one percentage point, but a 12.5% increase from the initial unemployment level. Using the log means that we are looking at the percentage change in the unemployment rate: $log(9) - log(8) \approxeq 0.118$ or 11.8%, which is the logarithmic approximation to the actual 12.5% increase.

    (c) When variable contains few zeros, then use $log(1 + y)$ only when such instances are few (say 2% of total). For other cases use Tobit models

6.2.2 Model with Quadratics

1. **Quardratic transformations in linear equations**: are also used quite often in applied economics to capture decreasing or increasing marginal effects and to tease out a non-linear relation between y and the x.

$$wage = \beta_0 + \beta_1 exper + \beta_2 exper^2$$

What is the partial effect of $exper$?

$$\Delta\hat{y} \approxeq (\hat{\beta}_1 + 2\hat{\beta}_2 x)\Delta x$$

What does this imply? Level of experience matters in calculating the partial effects.

2. *Example*: The previous wage model is estimated to be:

$$\hat{wage} = 3.73 + 0.298exper - 0.0061exper^2$$

Interpretation: when $\beta_2 < 0$ when $\beta_1 > 0$ we know that experience has diminishing effect on wage. A second year of experience only gives an additional 28.6 cents in comparison with the first hour which gives 29.8 cents.

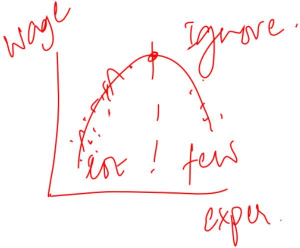
3. What if $\hat{\beta}_1 > 0$ while $\hat{\beta}_2 < 0$. The relation is parabolic and we are then bothered about the turning point (vertex) of the parabola.

   - Turning point is a value of x obtained from the regression exercise:

   $$x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$$

   - Turning point for experience comes out to be 24.4 years. Does this make sense? How do we interpret the economics underlying this estimate?

4. **First possibility**: If there are very few values on the RHS, then we can make a claim that the estimates are biased by outliers in data.



5. **Second possibility**: the estimated effect of *exper* on *wage* is biased because we have controlled for no other factors, or because the functional relationship between *wage* and *exper* in model equation is not entirely correct

6. What if the estimated β_1 and β_2 for a quardratic model are of same sign for the range of data?

7. Check out the example of house prices to understand application of quardratic effects.

8. Quardratic and log models can be combined to estimate models in which the elasticity is not constant at every point in data.
   Consider the house price data in which we take log-quardratic nitro-oxide as an explanatory variable.

$$\begin{aligned} log(price) &= \beta_0 + \beta_1 log(nox) + \beta_2 [log(nox)]^2 \\ &\quad + \beta_3 crime + \beta_4 rooms + \beta_5 rooms^2 + \beta_6 stratio + u \end{aligned}$$

6.2.3 Model with Interaction Terms

1. *Example*: Consider a model which tries to explain test scores in a final exam ($stndfnl$) in terms of percentage of classes attended, prior college grade point average, and ACT scores:

$$\begin{aligned} stndfnl = &\beta_0 + \beta_1 atndrte + \beta_2 priGPA + \beta_3 ACT + \beta_4 priGPA^2 \\ &+ \beta_5 ACT^2 + \beta_6 priGPA \cdot atndrte + u \end{aligned}$$

   We think that it is the combination of attendance as well as prior sincerity on students which determines their test scores.

2. Regression result:

Using the 680 observations in ATTEND, for students in a course on microeconomic principles, the estimated equation is

$$\begin{aligned}
\widehat{stndfnl} &= \underset{(1.36)}{2.05} - \underset{(.0102)}{.0067}\, atndrte - \underset{(.48)}{1.63}\, priGPA - \underset{(.098)}{.128}\, ACT \\
&+ \underset{(.101)}{.296}\, priGPA^2 + \underset{(.0022)}{.0045}\, ACT^2 + \underset{(.0043)}{.0056}\, priGPA \cdot atndrte \qquad [6.19] \\
&n = 680,\ R^2 = .229,\ \bar{R}^2 = .222.
\end{aligned}$$

3. **Caution while interpretation**:

- We cannot interpret β_1 in isolation. It does not make sense to say what would be the test score if a student attended the classes if his $priGPA = 0$.
- What are the β_1 and β_6 t-values? What does it tell me about β_1?
- What is the partial effect of atndrte on stndful?

- Partial effect of $atndrte$ on $stndful$ when $pri\bar{G}PA = 2.59$ is $-0.0067 + 0.0056(2.59) \approxeq 0.0078$: Because $atndrte$ is measured as a percentage, it means that a 10 percentage point increase in $atndrte$ increases $stndfnl$ by .078 standard deviations from the mean final exam score.

- Correct partial effect of $priGPA$

$$\frac{\partial\ stndfnl}{\partial\ priGPA} = -1.63 + 2(0.296)priGPA + 0.0056(atndrte)$$

- *Homework*: use the data in ATTEND to compute the partial effect of priGPA from the data.

6.2.4 Average Partial Effects

1. Model with interactions and quardratics have partial effects which have level variables such as the partial effect of x_2 in the following model:

$$y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2 + \beta_3 x_1 x_2$$

2. **Problem**: we do not have one number from regression estiamtes which give us the partial effect.

3. **Potential solution**: We compute partial effects across each unit of the sample and take its average – called the Average Partial Effect

$$\text{Partial Effect}: \frac{\partial y}{\partial x_2} = \beta_2 + \beta_3 x_1$$

$$\text{Average Partial Effect} := \beta_2 + \beta_3 \frac{1}{n} \sum_{i=0}^{n} x_{1i}$$