

### 13.3 Two-Period Panel Data Analysis

1. Consider the model for crime introduced earlier. Let us estimate the following functional form:

$$crm rte = \beta_0 + \beta_1 unem + u$$

Lets estimate an OLS for the model without any controls first:

```
> summary(lm(crmrte ~ unem, data = crime2, subset = (year == 87)))
```

Call:

```
lm(formula = crmrte ~ unem, data = crime2, subset = (year ==  
87))
```

Residuals:

Min	1Q	Median	3Q	Max
-57.55	-27.01	-10.56	18.01	79.75

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	128.378	20.757	6.185	1.8e-07 ***
unem	-4.161	3.416	-1.218	0.23

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 34.6 on 44 degrees of freedom

Multiple R-squared: 0.03262, Adjusted R-squared: 0.01063

F-statistic: 1.483 on 1 and 44 DF, p-value: 0.2297

We cannot infer the *unem* parameter because it is imprecisely estimated. But even if we did, it tells us that an increase in unemployment reduces crime – which is counter intuitive.

2. **Adding controls is one solution:** For instance, we could add *crmrte* from previous years as control with the argument that previous crime intensity could explain current crime intensity.

```
> mod1 = lm(log(crime2$crmrte) ~ + unem + log(crime2$lawexpc) , data = crime2, subset =
(year == 87))
> summary(mod1)
```

Call:

```
lm(formula = log(crime2$crmrte) ~ +unem + log(crime2$lawexpc),
    data = crime2, subset = (year == 87))
```

Residuals:

Min	1Q	Median	3Q	Max
-0.64786	-0.22955	-0.06368	0.22183	0.71164

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	3.34290	1.25053	2.673	0.0106 *
unem	-0.02900	0.03234	-0.897	0.3748
log(crime2\$lawexpc)	0.20337	0.17265	1.178	0.2453

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3231 on 43 degrees of freedom

Multiple R-squared: 0.05712, Adjusted R-squared: 0.01326

F-statistic: 1.302 on 2 and 43 DF, p-value: 0.2824

```
> nobs(mod1)
```

```
[1] 46
```

Adding past crime rate

```
> mod2 = lm(subset(log(crime2$crmrte),crime2$year==87) ~ subset(unem,crime2$year==87)
+ subset(log(crime2$lawexpc),crime2$year==87) + subset(log(crime2$crmrte), crime2$year==
82) , data = crime2)
> summary(mod2)
```

Call:

```
lm(formula = subset(log(crime2$crmrte), crime2$year == 87) ~
    subset(unem, crime2$year == 87) + subset(log(crime2$lawexpc),
        crime2$year == 87) + subset(log(crime2$crmrte), crime2$year ==
        82), data = crime2)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.48081	-0.12202	0.00659	0.14658	0.34428

Coefficients:

	Estimate	Std. Error	t value
(Intercept)	0.076450	0.821143	0.093
subset(unem, crime2\$year == 87)	0.008621	0.019517	0.442
subset(log(crime2\$lawexpc), crime2\$year == 87)	-0.139576	0.108641	-1.285
subset(log(crime2\$crmrte), crime2\$year == 82)	1.193923	0.132099	9.038

Pr(>|t|)

(Intercept)	0.926
subset(unem, crime2\$year == 87)	0.661
subset(log(crime2\$lawexpc), crime2\$year == 87)	0.206
subset(log(crime2\$crmrte), crime2\$year == 82)	2.1e-11 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1905 on 42 degrees of freedom

Multiple R-squared: 0.6798, Adjusted R-squared: 0.657

F-statistic: 29.73 on 3 and 42 DF, p-value: 1.799e-10

```
> nobs(mod2)
```

```
[1] 46
```

3. **Unobserved effects:** Alternatively, we could think about factors which affect the dependent variable to be categorized into two types: (a) a time varying factor  $d2_t$  which is a dummy variable that equals zero when  $t = 1$  and one when  $t = 2$  and (b)  $a_i$  which is a time-constant factor that affects  $y_{it}$ . The latter is called a **fixed effect or an unobserved effect**:

$$y_{it} = \beta_0 + \delta_0 d2_t + \beta_1 x_{it} + a_i + u_{it} \quad t = 1, 2$$

The  $u_{it}$  is called the **idiosyncratic error** or a time varying error because it represents unobserved factors that change over time and affect  $y_{it}$ .

4. **City fixed effects:** consider the model for crime which we used earlier, but now the unobserved effects version of the model:

$$crmrte_{it} = \beta_0 + \delta_0 d87_t + \beta_1 unem_{it} + a_i + u_{it}$$

Since the unit of analysis  $i$  is a city, the factors affecting city crime rates that do not change over time are captured in  $a_i$ . For instance, the area, location, weather, etc might be constant for a city in the dataset.

Are all factors constant in a city? How do we account for these factors?

5. **How do we estimate this model?** First we could pool these two years and run a pooled regression. But for pooled OLS to be consistent, we need to assume that  $a_i$  is uncorrelated with  $unem_{it}$  and  $d87_t$  when we run the following model:

$$crmrte_{it} = \beta_0 + \delta_0 d87_t + \beta_1 unem_{it} + v_{it}$$

where  $v_{it} = a_i + u_{it}$  is a form of composite error when we ignore the fixed effect. If  $a_i$  is correlated with the  $x_{it}$ , then the resulting bias is called the **heterogeneity bias**.



```
> library(wooldridge)
> summary(lm(crmrte ~ unem + d87, data = crime2))
```

Call:

```
lm(formula = crmrte ~ unem + d87, data = crime2)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-53.474	-21.794	-6.266	18.297	75.113

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	93.4202	12.7395	7.333	9.92e-11 ***
unem	0.4265	1.1883	0.359	0.720
d87	7.9404	7.9753	0.996	0.322

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.99 on 89 degrees of freedom

Multiple R-squared: 0.01221, Adjusted R-squared: -0.009986

F-statistic: 0.5501 on 2 and 89 DF, p-value: 0.5788

Pooling does not help us obtain trustworthy estimates

6. **First differencing methods:** consider the two time periods

$$crmrte_{i82} = \beta_0 + \beta_1 unem_{i82} + a_i + u_{i82}$$

$$crmrte_{i87} = \beta_0 + \delta_0 d87_t + \beta_1 unem_{i87} + a_i + u_{i87}$$

If we subtract first from second we get:

$$\begin{aligned}(crmrte_{i87} - crmrte_{i82}) &= \delta_0 + \beta_1(unem_{i87} - unem_{i82}) \\ &\quad + a_i + (u_{i87} - u_{i82})\end{aligned}$$

This is written as an estimable first difference equation:

$$\Delta crmrte_{it} = \delta_0 + \beta_1 \Delta unem_{it} + \Delta u_{it}$$

Thus this is a single equation where all variables are differenced over-time. Notice that the unobserved effect which was constant in both time periods gets canceled.

7. **Strict Exogeneity and First Difference:** Like previously, we need  $\Delta unem_{it}$  to be uncorrelated with  $\Delta u_{it}$ . This implies that unemployment in 82 does not affect  $\Delta u_{it}$  in 82 and 87.

When we observe the OLS estimate of the above equation, we call it the first differenced estimator.

**When would this fail?** (a) If law enforcement increases in cities where unemployment rate decreases. This could cause a negative correlation between  $\Delta u_{it}$  and  $\Delta unem_{it}$  and a biased OLS.

(b) Also,  $\Delta unem_{it}$  should have variation across  $i$ . If there is no variability, then the  $\beta$  would be under-estimated.

(c) We need to assume homoskedasticity. If it does not, we could also correct it for heteroskedasticity.

$$\widehat{\Delta crmrte} = 15.40 + 2.22 \Delta unem$$

$$(4.70) \quad (.88)$$

$$n = 46, R^2 = .127,$$

Consider the interpretation: Even if  $\Delta unem_{it} = 0$ , crime rate would increase by 15.40 per 1000 people every year. So there is a trend increase in time series.

8. **Limitations of First Differencing Methods:** (a) You might not have many data sets conducive to first differencing, especially a panel on Individual behavior might be really scarce. (b) First differencing could throw out  $a_i$  that is variation across individuals. We might not be left with a lot of variation across time to obtain small standard errors for our estimates. Even having a large sample might not provide a solution.

9. Explaining difference in wages across individuals using panel data: Consider the following model for wages:

$$\log(wage)_{it} = \beta_0 + \delta_0 d2_t + \beta_1 educ_{it} + a_i + u_{it}$$

Because, by definition, innate ability does not change over time, panel data methods seem ideally suited to estimate the return to education. This equation considers intercepts which change overtime represented by  $\delta_0$ .

To estimate it and remove  $a_i$  in the case when  $a_i$  is associated with  $educ_{it}$ , we could potentially difference the equation.

$$\Delta \log(wage)_{it} = \delta_0 + \beta_1 \Delta educ_{it} + \Delta u_{it}$$

But here, first differencing does not help as the variation in education in the sample is very less causing us to get imprecise estimates of  $\beta$ .

10. Adding several explanatory variables is straight forward. (Example 13.5: Biddle and Hamermesh, 1990 )

11. **Finite Distributed Lag Model:** Consider the model by Eddie (1994) to examine crime. His main idea is that the effect of convictions could take more than a year to impact the crime rates in the city. He calls the rate of conviction = “clear-up percentage”. He uses data for 1972 and 1978. He runs the following model:

$$\log(\text{crime})_{it} = \beta_0 + \delta_0 d78_t + \beta_1 \text{clrprc}_{i,t-1} + \beta_2 \text{clrprc}_{i,t-2} + a_t + u_{it}$$

where, he calculates the lag  $\text{clrprc}_{i,t-2}$  for the year 78 as  $= \text{clrprc}_{i,78} - \text{clrprc}_{i,76}$

### 13.3.1 Organizing Panel Data

Time.	
	$t_0$ $t_1$
$i_0$	Wide.
$i_1$	
$i_2$	
$i_3$	

$t$	
$i$	$t$
$i_0$	$t_0$
$i_0$	$t_1$
$i_0$	$t_2$
$i_0$	$\vdots$

Long form

Naturally, the organization of the data set depends on how you want to use it. For reading a panel data set, wide form is better. But for analysis, often long form is more conducive.