Aim – Implement Union, Intersection, Complement and Difference operations on fuzzy sets

Theory –

Fuzzy set:

1. Fuzzy set is a set having degrees of membership between 1 and 0. Fuzzy sets are represented with tilde character (~). For example, Number of cars following traffic signals at a particular time out of all cars present will have membership value between [0,1].
2. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
3. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.

Mathematical logic:

A fuzzy set A~ in the universe of discourse, U, can be defined as a set of ordered pairs and it is given by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$

When the universe of discourse, U, is discrete and finite, fuzzy set A~ is given by where “n” is a finite value.

$$\tilde{A} = \sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(x_i)}{x_i} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \ldots + \frac{\mu_{\tilde{A}}(x_n)}{x_n}$$

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x}$$

Fuzzy sets also satisfy every property of classical sets.

Operations:

Common Operations on fuzzy sets: Given two Fuzzy sets A~ and B~

1. Union: Fuzzy set C~ is union of Fuzzy sets A~ and B~

$$\tilde{C} = \tilde{A} \cup \tilde{B}, \quad \mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

2. Intersection: Fuzzy set D~ is intersection of Fuzzy sets A~ and B~ **:**

$$\tilde{D} = \tilde{A} \cap \tilde{B} \quad \mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

3. Complement: Fuzzy set E~ is complement of Fuzzy set A~ :

$$\tilde{E} = \mathbb{C}_{\tilde{A}}X \qquad \mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x)$$

Code –

```
Honesty = {"Ayush": 0.2, "Sumit": 0.3, "Nikhil": 0.6, "Sarvar": 0.6, "Akash":
0.5}


Sincerity = {"Ayush": 0.9, "Sumit": 0.9, "Nikhil": 0.4, "Sarvar": 0.5,
"Akash": 0.4}


print('The Honesty of Students is represented as :', Honesty)
print('The Sincerity of Students is represented as :', Sincerity)


Union = dict()


for H, S in zip(Honesty, Sincerity):
    if Honesty[H] > Sincerity[S]:
        Union[H] = Honesty[H]
    else:
        Union[S] = Sincerity[S]

print('\nHonesty and Sincerity Union is :', Union)



Intersection = dict()
for H, S in zip(Honesty, Sincerity):
    if Honesty[H] < Sincerity[S]:
        Intersection[H] = Honesty[H]
    else:
        Intersection[S] = Sincerity[S]
print('\nHonesty and Sincerity Intersection is :', Intersection)



HonestyComplement = dict()
for H in Honesty:
    HonestyComplement[H] = 1-Honesty[H]
print('\nHonesty Complement is :', HonestyComplement)



SincerityComplement = dict()
for S in Sincerity:
    SincerityComplement[S] = 1 - Sincerity[S]
# print('\nSincerety Fuzzy Set Complement is :', SincerityComplement)



Difference = dict()
for H, SC in zip(Honesty, SincerityComplement):
    if Honesty[H] < SincerityComplement[S]:
        Difference[H] = Honesty[H]
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    else:
        Difference[SC] = SincerityComplement[SC]


print('\nHonesty and Sincerity Difference is :', Difference)
```

Output –

```
The Honesty of Students is represented as : {'Ayush': 0.2, 'Sumit': 0.3, 'Nikhil': 0.6, 'Sarvar': 0.6, 'Akash': 0.5}
The Sincerity of Students is represented as : {'Ayush': 0.9, 'Sumit': 0.9, 'Nikhil': 0.4, 'Sarvar': 0.5, 'Akash': 0.4}

Honesty and Sincerity Union is : {'Ayush': 0.9, 'Sumit': 0.9, 'Nikhil': 0.6, 'Sarvar': 0.6, 'Akash': 0.5}

Honesty and Sincerity Intersection is : {'Ayush': 0.2, 'Sumit': 0.3, 'Nikhil': 0.4, 'Sarvar': 0.5, 'Akash': 0.4}

Honesty Complement is : {'Ayush': 0.8, 'Sumit': 0.7, 'Nikhil': 0.4, 'Sarvar': 0.4, 'Akash': 0.5}

Honesty and Sincerity Difference is : {'Ayush': 0.2, 'Sumit': 0.3, 'Nikhil': 0.6, 'Sarvar': 0.5, 'Akash': 0.5}
```

Conclusion – In this experiment I learnt what are fuzzy sets, how it is used to, I used my own dataset for the experiment, I was able to perform various operations on fuzzy sets.