

Homework 3 Part-6
CS 6347: Statistical Methods in AI / ML

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3. Run the two programs on the six PGMs given on the class web page. Try the following values for $N = \{100, 1000, 10000, 20000\}$ and $w = \{1, 2, 3, 4, 5\}$ and run each algorithm at least 10 times using different random seeds. For each run, compute the log-relative error between the exact partition function and the approximate one computed by your algorithm. Namely, compute

$$\frac{\log(Z) - \log(\hat{Z})}{\log(Z)}$$

where Z is the exact answer and \hat{Z} is the approximate answer.

For each PGM, report your results in a table such as the one given below and describe your findings in a few sentences (e.g., which method is better and why?; how N and w impact the accuracy, time-complexity and variance; etc.).

Grids 14[illegible]

Grids 15

Uniform Proposal									
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	12.98456	0.81170	14.22946	0.77736	51.79010	1.10969	69.00254	2.84359
	Error	0.21050	0.00377	0.20960	0.00210	0.20994	0.00108	0.20936	0.00078
2	Time	13.59513	0.41008	15.13700	0.35572	55.58910	1.39042	64.82802	2.50103
	Error	0.15303	0.00217	0.15272	0.00161	0.15261	0.00078	0.15252	0.00046
3	Time	13.39354	0.53801	15.68721	0.48922	57.75350	0.62797	63.56381	2.15619
	Error	0.12909	0.00128	0.12882	0.00169	0.12872	0.00106	0.12851	0.00073
4	Time	14.19907	0.51813	16.37880	0.33048	61.76112	2.62992	68.92888	3.07526
	Error	0.09282	0.00105	0.09250	0.00146	0.09236	0.00058	0.09221	0.00053
5	Time	14.03924	0.44085	16.48849	0.47860	60.48807	0.65914	65.28044	2.23847
	Error	0.08851	0.00150	0.08849	0.00087	0.08871	0.00122	0.08841	0.00062
Adaptive Proposal									
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	13.13566	0.62197	15.68355	0.70445	76.647489	1.367121	87.38087	4.26002
	Error	0.21050	0.00377	0.20856	0.00193	0.209201	0.0019174	0.20894	0.00112
2	Time	13.57791	0.50901	16.94662	0.50335	80.491335	2.1118595	83.73162	1.56300
	Error	0.15303	0.00217	0.15239	0.00222	0.152337	0.0012168	0.15252	0.00061
3	Time	13.64972	0.52143	17.40589	0.48360	83.4180357	1.6194427	84.82784	2.95905
	Error	0.12909	0.00128	0.12855	0.00216	0.128105	0.0012869	0.12832	0.00070
4	Time	14.12202	0.62845	18.14237	0.30070	86.72922	2.7926123	91.99552	3.23653
	Error	0.09282	0.00105	0.09220	0.00196	0.09242609	0.00102567	0.09209	0.00082
5	Time	14.06389	0.67700	8.19688	0.39947	86.1091635	0.874323	85.83850	2.98927
	Error	0.08851	0.00150	0.08850	0.00082	0.0885693	0.0017827	0.08841	0.00074

Grids 16

		Uniform Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	12.70084	0.40533	14.17422	0.26268	31.79894	1.83697	58.25438	0.34247
	Error	0.31748	0.00342	0.32100	0.00326	0.31961	0.00097	0.31956	0.00053
2	Time	12.89572	0.40164	15.18509	0.53336	35.92221	1.53650	62.15402	0.70716
	Error	0.23934	0.00236	0.23942	0.00129	0.23847	0.00099	0.23918	0.00082
3	Time	14.13040	1.15405	15.64980	0.38137	37.01291	1.25019	63.67980	0.23878
	Error	0.20216	0.00203	0.20162	0.00159	0.20110	0.00054	0.20090	0.00077
4	Time	13.90162	0.86591	16.25230	0.41016	30.40179	1.43200	67.15102	0.63989
	Error	0.14488	0.00158	0.14524	0.00115	0.14434	0.00102	0.14430	0.00061
5	Time	8.45996	0.19320	16.24533	0.64757	30.44755	1.12156	68.00786	0.57872
	Error	0.13887	0.00189	0.13792	0.00177	0.14434	0.00102	0.13887	0.00105
		Adaptive Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	13.08727	0.63412	16.05559	0.39165	36.08068	0.46785	76.64235	0.53045
	Error	0.31748	0.00342	0.32061	0.00343	0.31923	0.00116	0.31956	0.00055
2	Time	13.28263	0.39002	16.90528	0.43903	40.74443	1.73558	80.28029	0.70036
	Error	0.23934	0.00236	0.23887	0.00120	0.23830	0.00100	0.23901	0.00056
3	Time	14.08321	1.06004	17.39697	0.47631	42.59476	1.82960	82.31472	0.47984
	Error	0.20216	0.00203	0.20130	0.00157	0.20082	0.00062	0.20080	0.00095
4	Time	14.22736	1.19051	18.06893	0.37182	46.34456	2.18024	86.01524	0.92262
	Error	0.14488	0.00158	0.14508	0.00130	0.13871	0.00079	0.14421	0.00078
5	Time	8.53728	0.20398	18.35216	0.99505	45.98814	1.93373	88.83060	1.52459
	Error	0.13887	0.00189	0.13777	0.00194	0.138567	0.0008167	0.13877	0.00080

Grids 17

		Uniform Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	7.80487	0.35670	14.40713	0.59843	31.49817	1.61044	58.33358	0.42207
	Error	0.36328	0.00230	0.36332	0.00213	0.362936	0.00087	0.36246	0.00055
2	Time	8.30943	0.23854	14.49025	0.28153	34.24105	0.33110	62.07245	0.30365
	Error	0.27336	0.00203	0.27192	0.00108	0.27300	0.00113	0.27313	0.00057
3	Time	8.18793	0.42552	14.69727	0.21496	35.96965	0.45128	67.55829	3.47254
	Error	0.22997	0.00241	0.23050	0.00181	0.23017	0.00137	0.23035	0.00049
4	Time	8.65441	0.14386	16.36419	1.06627	38.51256	0.20357	75.61580	2.95295
	Error	0.16551	0.00166	0.16473	0.00143	0.16545	0.00079	0.16553	0.00081
5	Time	8.71387	0.14830	15.59889	0.25797	39.29034	0.48344	75.80373	2.94582
	Error	0.15903	0.00187	0.15913	0.00109	0.15862	0.00068	0.15849	0.00025
		Adaptive Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	7.95292	0.48725	15.89828	0.60996	35.870589	0.820793	79.34540	1.03125
	Error	0.36328	0.00230	0.36279	0.00223	0.362861	0.000863	0.36225	0.00058
2	Time	8.25069	0.21671	16.29362	0.22828	39.075134	0.340692	83.29002	0.82864
	Error	0.27336	0.00203	0.27175	0.00102	0.272909	0.0011101	0.27302	0.00053
3	Time	8.12772	0.26367	16.53824	0.25091	41.1990585	0.44908199	93.13686	5.50595
	Error	0.22997	0.00241	0.23050	0.00197	0.2301109	0.0013059	0.23027	0.00050
4	Time	8.77779	0.00166	18.14608	1.13872	54.001236	0.4455143	102.33740	7.22988
	Error	0.16551	0.00166	0.16460	0.00150	0.1653303	0.0008609	0.16546	0.00080
5	Time	8.83040	0.14119	17.36095	0.31801	44.44011	0.5528137	99.00629	3.69922
	Error	0.15903	0.00187	0.15898	0.00101	0.158416	0.00076828	0.15832	0.00024

Grids 18

		Uniform Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	7.75453	0.20151	13.67330	0.13451	30.78043	0.48896	59.69402	0.91661
	Error	0.37867	0.00202	0.37835	0.00246	0.37803	0.00105	0.37797	0.00071
2	Time	8.24715	0.39151	14.74582	0.54727	34.47783	0.45294	67.91980	2.25421
	Error	0.28510	0.00264	0.28497	0.00178	0.28494	0.00070	0.28420	0.00067
3	Time	8.56193	0.14645	15.03276	0.39265	35.74513	0.38764	70.71509	11.12475
	Error	0.24224	0.00189	0.24048	0.00147	0.24053	0.00113	0.24049	0.00063
4	Time	9.15021	0.37830	15.40818	0.13821	38.22187	0.40342	68.33980	2.95773
	Error	0.17346	0.00234	0.17333	0.00104	0.17303	0.00081	0.17257	0.00062
5	Time	8.79262	0.51749	15.86281	0.30934	39.18690	1.25385	73.20098	2.93993
	Error	0.16616	0.00221	0.16609	0.00185	0.16596	0.00074	0.16580	0.00061
		Adaptive Proposal							
w		100		200		500		1000	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Time	7.77546	0.06727	15.44677	0.27734	35.80433	0.84681	85.02631	1.95773
	Error	0.37867	0.00202	0.37817	0.00255	0.37785	0.00098	0.37782	0.00073
2	Time	8.37887	0.25429	16.37211	0.52676	49.34107	0.50563	96.49801	3.35566
	Error	0.28510	0.00264	0.28498	0.00168	0.28468	0.00077	0.28357	0.00068
3	Time	8.53447	0.12278	16.70971	0.53472	50.79606	0.39644	133.85116	3.40635
	Error	0.24224	0.00189	0.24034	0.00153	0.24046	0.00113	0.24045	0.00072
4	Time	9.12969	0.45923	17.21367	0.33170	53.25125	0.56880	129.57928	2.41593
	Error	0.17346	0.00234	0.17329	0.00102	0.17299	0.00080	0.17103	0.00063
5	Time	8.71393	0.51321	17.66062	0.32931	54.49364	1.55968	132.77801	2.52593
	Error	0.16616	0.00221	0.16602	0.00186	0.16596	0.00073	0.16573	0.00068

Report

Due, to the high processing time by the algorithm on higher sample sizes, I restricted the N values to 1000. The code ran for around 20 hours altogether.

As you can see above, higher w values gave better results as the error, reduced. This is because the number of variables used for sampling is reduced. At the same time, time taken to run the program increased.

With higher N values, the error rate reduced very slightly for most cases but sometimes it even increased. However, the standard deviation kept getting smaller and smaller as N increases, i.e. variance reduces with higher values of N .

The time taken by the script also increases with higher values of N .

When comparing adaptive proposal and uniform proposal distributions we can see that adaptive proposal does better than uniform as the value of N increases but takes longer as it is necessary to calculate the new proposal distribution after every 100 steps. Hence it takes time even after parallelizing.

The best parameters for reducing error and time would be to use the adaptive proposal distribution model with $N=100$ and $w=5$ for all PGMs.