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SCHOOL of MANAGEMENT

IMPACT OF QUANTITATIVE TIGHTENING POLICY ON LEVEL, SLOPE AND CURVATURE OF THE YIELD CURVE: FINAL REPORT

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1 Executive Summary

In this project, we use the publication by Hamilton and Wu (2012) linking the quantitative easing (QE) with the level, slope, and curvature of the yield curve to study quantitative tightening (QT) in the same framework. The goal of the project is to study how different quantitative tightening policies might term structure factors, namely the level, slope and curvature.

Study the impact of quantitative tightening by the Federal Reserve on level, slope, and curvature of the yield curve.

In this project, we propose to use a model of risk-averse arbitrageurs to develop measures of how the maturity structure of debt held by the public might affect the pricing of level, slope, and curvature term structure risk. Hamilton and Wu find that these Treasury factors historically were quite helpful for predicting both yields and excess returns over 1990 to 2007. In the QE framework, the historical correlations are consistent with the claim that if in December 2006, the Fed were to have sold off all its Treasury holdings of less than 1-year maturity (about $400 billion) and use the proceeds to retire Treasury debt from the long end. Our project will not focus on the Zero Lower Bound environment since the Quantitative Tightening policies are implemented in a non-Zero Lower Bound environment.

Our goal in this project would be to study the effect of Quantitative Tightening using the benchmark of a maturity swap defined as selling long term (maturity more than 10 years) and buying short term (maturity less than 1 year). Moreover, we will also study the effect of Quantitative Tightening implemented in 2017 Fall as defined on the System Open Market Account [1] holdings by New York Fed.

[1] https://www.newyorkfed.org/markets/soma/sysopen_accholdings.html

2 Literature Review

Quantitative Easing

After the financial crisis in 2008, the Fed funds rate has been stuck at zero. As a result, it is difficult to further reduce the short-term interest rate using the traditional monetary policies. One alternative strategy is to influence the term structure of interest rates by changing the maturity composition of the public portfolio. Therefore, Hamilton and Wu implemented a maturity swap strategy that leads to the flattening of the yield curve which we will discuss further in Methodology section.

Quantitative Tightening

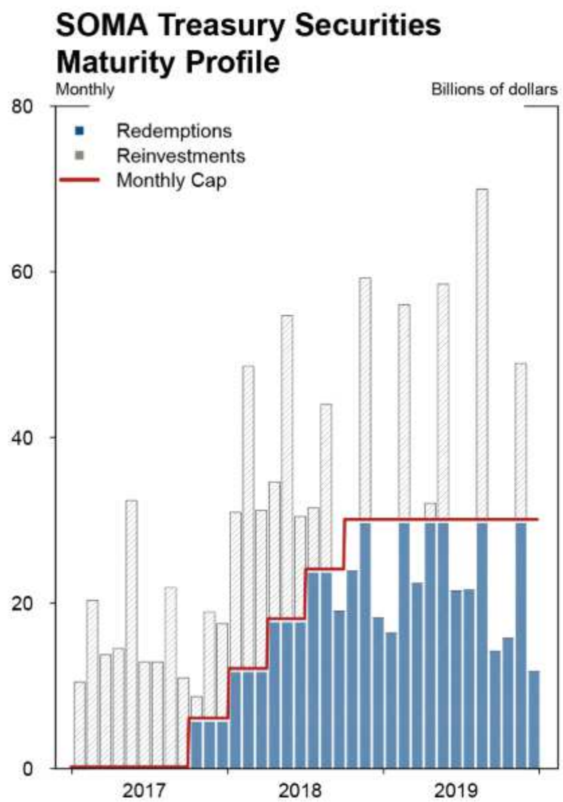
In June 2017, the Federal Reserve released a statement[2] on the Policy normalization and plans. The statement said:

The Committee intends to gradually reduce the Federal Reserve's securities holdings by decreasing its reinvestment of the principal payments it receives from securities held in the System Open Market Account. Specifically, such payments will be reinvested only to the extent that they exceed gradually rising caps.

For payments of principal that the Federal Reserve receives from maturing Treasury securities, the Committee anticipates that the cap will be $6 billion per month initially and will increase in steps of $6 billion at three-month intervals over 12 months until it reaches $30 billion per month.

The Committee also anticipates that the caps will remain in place once they reach their respective maximums so that the Federal Reserve's securities holdings will continue to decline in a gradual and predictable manner until the Committee judges that the Federal Reserve is holding no more securities than necessary to implement monetary policy efficiently and effectively.

[2] https://www.federalreserve.gov/newsevents/pressreleases/monetary20170614c.htm



Source: Federal Reserve Bank of New York and author calculations.

3 Data

From January 1990 to December 2009, we used the data used by Hamilton and Wu for the QE framework which is constructed as follows:

We use weekly observations y_{nt} as our baseline estimates , based on constant maturity Treasury yields as of Friday or the last business day of the week as reported in the FRED database of the Federal Reserve Bank of St. Louis[3]. In addition, we also include monthly analysis of security holding yields with nonstandard maturities by constructing constant maturity yields from the daily term structure of Gurkaynak, Sack, and Wright (2007) as of the last day of the month.[4] W.} We also constructed estimates of the face value of outright

[3]The 30-year yields are unavailable from February 19, 2002 to February 8, 2006. Over this interval, we alternatively used the 20-year rate minus 0.21, which is the amount by which the 20-year rate exceeded the 30-year rate before and after the gap.

[4]We calculate y_{nt} as $y_{nt} = \beta_{0t} + \beta_{1t}\tau_{1t}\frac{1-exp(-\frac{n}{\tau_1})}{n} + \beta_{2t}\tau_{1t}(1-(1+\frac{n}{\tau_{1t}})exp(-\frac{n}{\tau_{1t}})) + \beta_{3t}\tau_{2t}(1-(1+$

U.S. Treasury debt at each weekly maturity as of the end of each month from January 1990 to January 2011. For purposes of the study, we would like to take each semiannual coupon on a given bond as an individual zero-coupon bond (paying \$C at some time t + s) and compute the market value of the bond as the sum of the market value of each components. However, converting the face value into a market value by this device is missing our larger purpose of identifying exogenous sources of variation in the supply of outstanding debt at different maturities. The true market value of a given security would be highly endogenous with respect to changes in interest rates, but the face value is not.

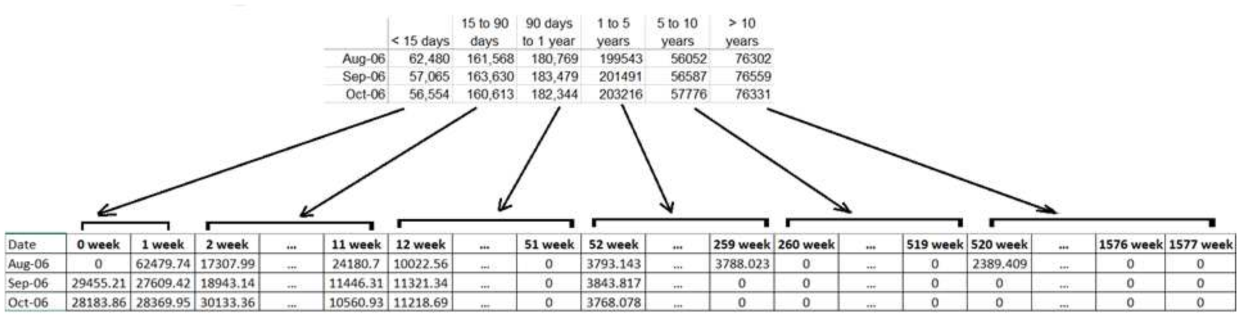
Following Hamilton and Wu's method, we downloaded Federal Reserve holding data from The Federal Reserve's weekly H41 reports that they release every Wednesday. The data is formed in six maturity breakdowns, which are less than 15 days, 16 to 90 days, 91 days to 1 year, 1 to 5 years, 5 to 10 years and over 10 years. And we take the last Wednesday value of the month as the monthly value in later calculation for public holdings. Another detail to mention is that, TIPS are removed during the estimation by assuming that Fed holdings of TIPS as a fraction of the Fed's holding of Notes and Bonds were the same across all maturity categories. For greater than 1 year maturities, we get an estimate of total TIPS holdings by multiplying this ratio for every maturity category greater than 1 year. We obtained TIPS holdings by Fed less than 1 year by multiplying this ratio with the ratio of the Fed's notes and bonds of maturity less than 1 year and the Fes's total Treasury securities less than 1 year. As a result, the nominal Fed holdings for each maturity category is estimated by removing the corresponding TIPS holdings within each maturity category. We then distributed this ratio proportionately across the total holdings of SOMA across every weekly maturity.

After December 2010, we obtain the data for Treasury holdings from the Center of Research in Security Prices (CRSP) and the data for Federal Reserve Holdings by maturity on the Federal Reserve Bank of New York website

$\frac{n}{\tau_{2t}})exp(-\frac{n}{\tau_{2t}}))$ with daily data for parameters $\{\beta_{0t}, \beta_{1t}, \beta_{2t}, \tau_{1t}, \tau_{2t}\}$ downloaded from om http://www.federalreservserve.gov/econresdata/ta/researchdata.htm.

under the System Open Market Account Holdings of Domestic Securities section with type as Quarterly Detail, Monthly Detail, and Weekly Aggregate Time Series [5].

One issues we ran into is that the Treasury outstandings is a $T \times 1577$ matrix whereas the Fed holdings is a $T \times 6$ matrix where T is the number of months from January 1990 to July 2018



	< 15 days	15 to 90 days	90 days to 1 year	1 to 5 years	5 to 10 years	> 10 years
Aug-06	62,480	161,568	180,769	199543	56052	76302
Sep-06	57,065	163,630	183,479	201491	56587	76559
Oct-06	56,554	160,613	182,344	203216	57776	76331

Date	0 week	1 week	2 week	...	11 week	12 week	...	51 week	52 week	...	259 week	260 week	...	519 week	520 week	...	1576 week	1577 week
Aug-06	0	62479.74	17307.99	...	24180.7	10022.56	...	0	3793.143	...	3788.023	0	...	0	2389.409	...	0	0
Sep-06	29455.21	27609.42	18943.14	...	11446.31	11321.34	...	0	3843.817	...	0	0	...	0	0	...	0	0
Oct-06	28183.86	28369.95	30133.36	...	10560.93	11218.69	...	0	3768.078	...	0	0	...	0	0	...	0	0

The SOMA matrix is expanded from $T \times 6$ to $T \times 1577$ using weights from the Treasury holdings (assumed to remain constant in the Fed holdings for the same month)

We then use the merged data from January 1990 to July 2018 to construct the fraction of public holdings (held by arbitrageurs) across each maturity by subtracting the Fed (or SOMA) holdings from the total treasury holdings and dividing each row by its sum to get a 342×1577 matrix. We call this matrix as z_{nt} as would be used later to calculate q_t.

[5] https://www.newyorkfed.org/markets/soma/sysopen_accholdings.html

4 Methodology

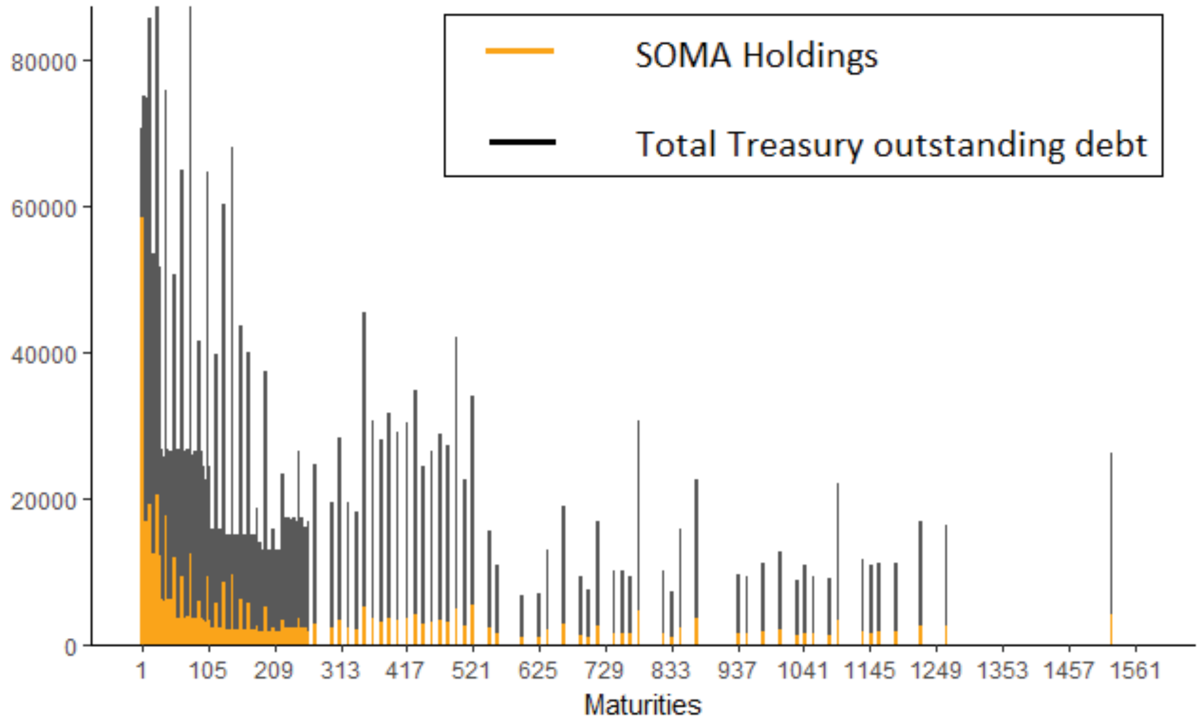
We use the parameters as estimated by Hamilton and Wu using the no arbitrage conditions as described in the Appendix.

The parameters that Hamilton and Wu obtain using the minimum-chi square technique are summarized in the table below:

Table 1: PARAMETER ESTIMATES FOR THE WEEKLY AFFINE TERM STRUCTURE MODEL

ρ ^ Q	0.9990	0.0094	-0.0140
	0.0027	0.9870	0.0330
	-0.0018	-0.0028	0.9867
$b_1 \times 5{,}200$	1.0345	-0.6830	0.6311
Σ	0.1094	0.0000	0.0000
	0.0360	0.1027	0.0000
	-0.0670	0.0025	0.0968

The Maturity structure of U.S. federal debt as of December 31, 2006 is plotted below. Horizontal axis: maturity in weeks. face value of marketable nominal Treasury securities of that maturity, in millions of dollars. Light bars: imputed holdings of the System Open Market Account of the U.S. Federal Reserve

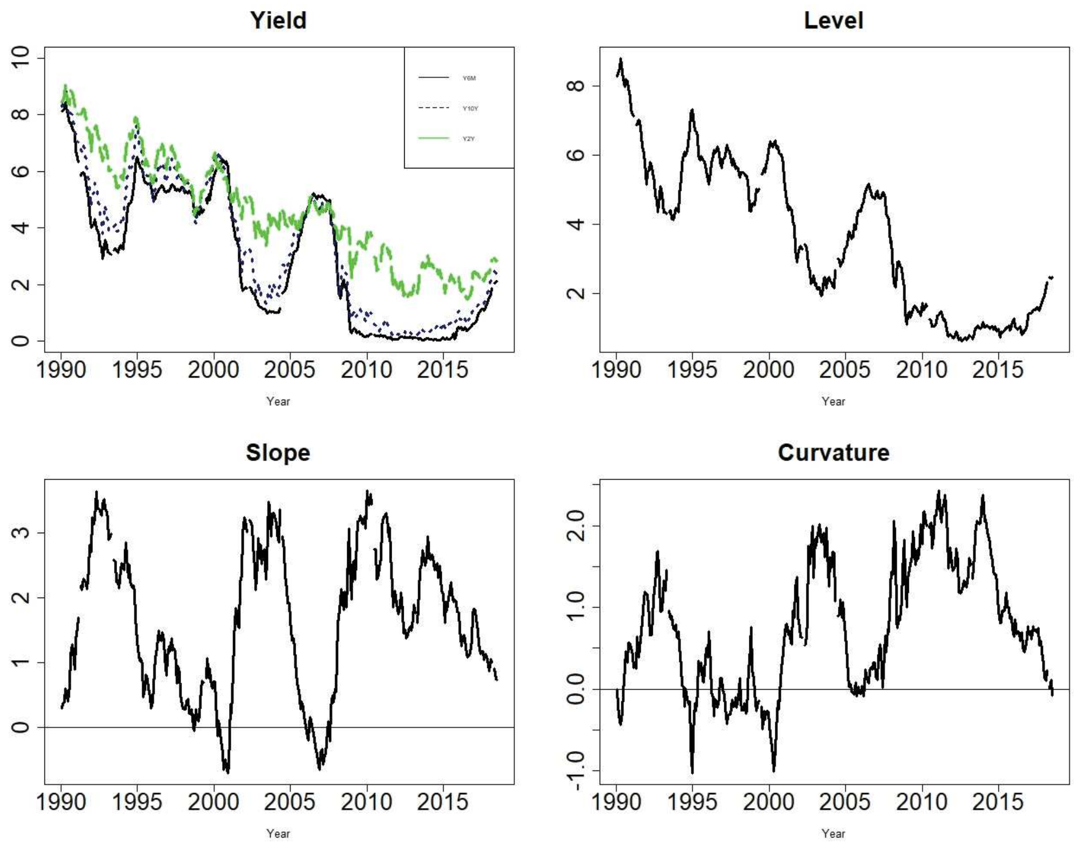


Factor Construction

The factors f_t are constructed using the data for 6-month, 2-year and 10-year Treasuries sampled weekly from January 1990 to the end of July 2007.

$$
\begin{aligned}
f_{1t} &= \frac{1}{3}(y_{26,t} + y_{104,t} + y_{520,t}) \\
f_{2t} &= y_{520,t} - y_{26,t} \\
f_{3t} &= y_{520,t} - 2y_{104,t} + y_{26,t}
\end{aligned}
$$

The following graphs plot the yields and the three factors:



The parameters $c^Q, \rho^Q, a_1, b_1, \Sigma_e$ are estimated from the equation (20)

So now, the matrix b_n in equation (19) becomes the following:

$$\begin{bmatrix} \hat{b}_{26} \\ \hat{b}_{104} \\ \hat{b}_{520} \end{bmatrix} = \begin{bmatrix} (1/3) & (1/3) & (1/3) \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -(1/2) & (1/6) \\ 1 & 0 & -(1/3) \\ 1 & (1/2) & (1/6) \end{bmatrix}$$

As a result of risk aversion of investors, there is a risk premium attached to the factors which is calculated as the difference between the expectations under the P and Q measure.

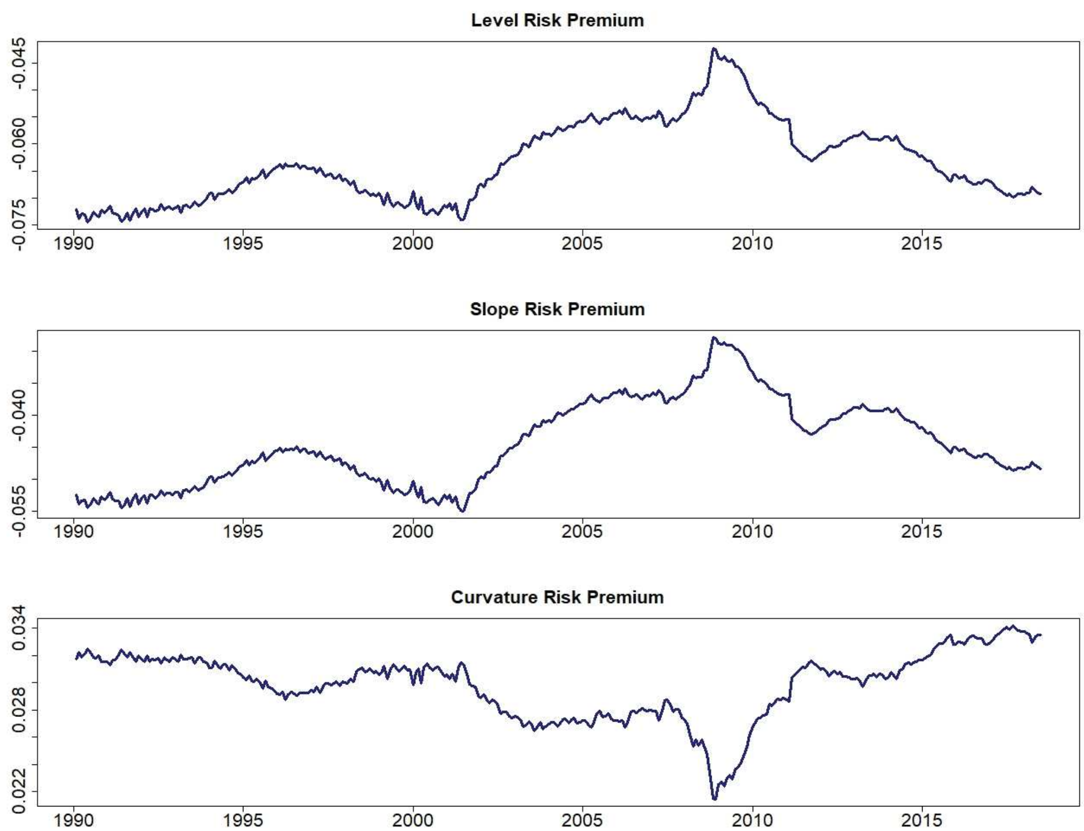
$$\mathbb{E}_t^{\mathbb{P}}(f_{t+1}) - \mathbb{E}_t^{\mathbb{Q}}(f_{t+1}) = \Sigma\lambda + \Sigma\Lambda f_t = \Sigma\lambda_t \tag{1}$$

Following the Vayanos-Vila framework, we started to consider how term structure risk factors would be priced under the following three important assumptions: (i), the preferred habitat sector is completely U.S. Treasury and Federal Reserve, (ii) the arbitrageurs comprise the entire private sectors and (iii) All U.S. Treasury debt is held by arbitrageurs. Extreme as it may sounds, these three assumptions do give a clear idea of how any variation in maturity structure of outstanding Treasury debt would affect the price of risk in one highly stylized case. With these assumptions, we will be able to measure the arbitrageurs' portfolio weights z_{nt} directly from ratio of debt held by the public of maturity n to the total public debt at the date.

Empirically, Hamilton and Wu found the term structure risk prices were given by

$$q_t = 100\Sigma\Sigma' \sum_{n=2}^{N} z_{nt}\bar{b}_{n-1} \tag{2}$$

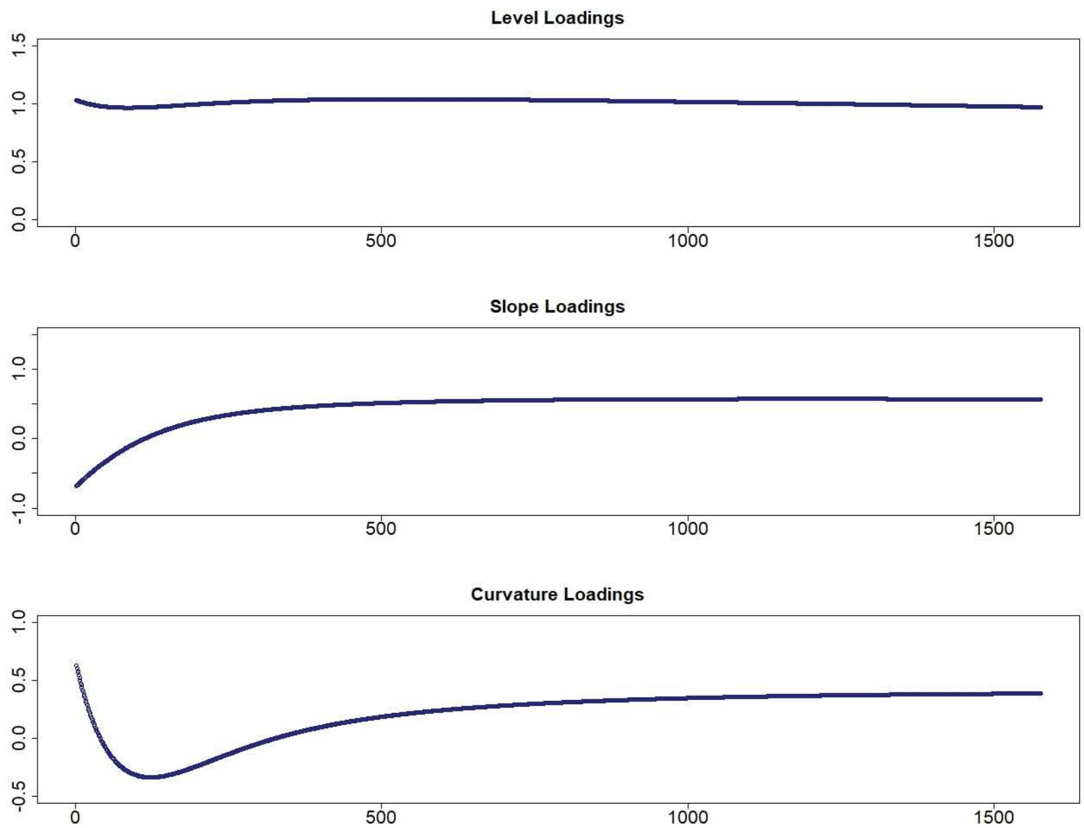
After estimation, the risk factors q_t are as plotted below:



These factors q_t are constructed by Hamilton and Wu in order to link the change in the yield curve factors to the maturity composition of the public debt. So, q_t summarizes the maturity structure by assigning loadings to each maturity and reflects the change in the maturity structure as one number.

It is observed that the three factors q_t are highly correlated whereas the loadings are not. It is difficult to draw any insight from the trends in these factors as they are merely a summary of the maturity structure.

The loadings ($\bar{b}_{nt}$) on the z_{nt} to construct q_t are plotted as follows:



In their paper, Hamilton and Wu study different factors like average maturity (z_t^A), principal components of the fraction of public holdings (z_t^{PC}), the Cohrance-Piazzesi factor (v_t) and concluded that the best R-squared obtained with minimum amount of variables is using the variables q_t.

Next, we would find the relation between these new factors q_t and the treasury factors f_t as given in the following equation:

$$f_{t+1} = c + \rho f_t + \phi q_t + \varepsilon \tag{3}$$

For Quantitative Easing, Hamilton and Wu consider the following exercise:

Suppose that at the end of month t, the Federal Reserve sells all its Treasury securities with maturity less than 1 year and use the proceeds to buy all of the outstanding nominal Treasury debt of maturity greater than n_{1t} , where n_{1t} would be determined by the size of the Federal Reserve's short-term holdings and outstanding long-term Treasuries at time t. For example, if implemented in December 2006, this would result in the Fed selling about $400 billion in short-term securities and buying about $400 billion in long-term securities, effectively retiring all the federal debt of 10-year and longer maturity. They then calculated what q_t^A would be under this counterfactual scenario, and calculated the average historical value of the difference between q_t^A and q_t which is represented by Δ

For Quantitative Tightening, we propose a similar approach as mentioned in Section 2 (Literature Review):

To implement the Quantitative tightening for three scenarios (QT ends in December 2019, QT ends in December 2020, QT ends in December 2021), we need to predict the Federal Reserve holdings after June 2018.

To predict Fed holdings for next month, the first step is to implement time decay of securities by removing first 4 columns of current holdings and shifting the other columns to the left (since holdings for first 4 columns will be retired after 1 month).

The second step is implementing reinvestments. The amount we reinvest will be total sum of retired securities minus the money cap for current month. And we will repurchase bonds with all maturities proportionally based on the proportions in the previous month. So, in the next month, the amount will be equal to the time decay of the securities from last month plus the re-investments done.

The next part is to predict the outstanding Treasury debt from June 2018 to the end of sample. We make an assumption that the Treasury debt for a given month is increased by 5% for the same month in the next year and the proportion across maturities remains constant for a given month every year.

Since not all maturities are available for repurchasing at the beginning of the month, the Federal reserve proposes to divide their repurchasing process into two parts: middle of the month and the end of the month. In our analysis, for simplicity, we assume all the proceeds are available at the beginning of the month and the re-investments are made immediately.

To get the Δ, we need a scenario without Quantitative Tightening.

To implement this scenario, we will need to undo the change in holdings from October 2017 to June 2018 as the Quantitative Tightening policy had already started in October 2017 and then forecast the holdings from July 2018 to the end of sample using the changed holdings for June 2018.

So, we add back the cap of \$ 6 billion proportionately to the federal reserve holdings from October 2017 to December 2017, add back \$ 12 billion and \$ 18 billion proportionately for federal reserve holdings from January 2018 to March 2018 and April 2018 to June 2018 respectively.

From June 2018 to the end of sample, we forecast the Federal Reserve Holdings in a similar fashion as mentioned before. But the major difference is that we don't include caps. We assume that all the proceeds obtained from short term debt retiring are fully re-invested proportionately across all maturities.

Next we get three sets of Δ for the three scenarios treating the period without QT as the benchmark. The results are summarized in the next section.

6 Results and Discussions

Table 2: RESULTS

	Factors	Ends Dec. 2019	Ends Dec. 2020	Ends Dec. 2021
Δ	Level	0.00144	0.00837	0.01646
	Slope	0.00112	0.00703	0.01378
	Curvature	-0.00071	-0.00316	-0.00625
$\phi_i{}'\Delta$	Level	0.01900	0.07000	0.13400
	Slope	-0.00200	-0.03500	-0.06700
	Curvature	-0.00800	-0.02400	-0.04500

As we predict, we observe a positive upward shift in the level of the yield curve. The level would shift by 1.88 bps, 6.99 bps, and 13.41 bps if the quantitative tightening goes on upto December 2019, December 2020, and December 2021 respectively.

The slope and curvature show a decrease which occurs because of a larger reduction in the dollar amount of the short term holdings compared to the long term holdings (since the Public holdings are more concentrated in the short term) through the procedure we follow.

In this strategy, our expectations are that the level would increase. As a result of reduction of re-investments by the Federal Reserve, the amount of treasury securities available to the arbitrageurs (public) increases. This increase in supply of securities creates a downward effect on the prices and hence an upward effect on the yield curve.

Moreover, since the short term rates are more sensitive to monetary policies and the long term rates are only affected by changes macro economic factors (which we don't consider in our analysis), the results are not a surprise.

7 References

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8 Appendix

No arbitrage parameters estimation

The investors are assumed to care only about the mean and variance of the return on their portfolios:

$$E_t(r_{t,t+1}) - (\gamma/2)Var_t(r_{t,t+1})y_{1t} = E_t(r_{i,t,t+1} - \gamma\theta_{it}) \tag{4}$$

such that θ_{it} is the derivative of total portfolio variance with respect to holdings of asset i

The yields and the factors are assumed to be affine functions of the factors f_t (the level, slope and the curvature)

$$p_{nt} = \bar{a_n} + \bar{b_n}' f_t \tag{5}$$

$$y_{1t} = a_1 + b_1' f_t \tag{6}$$

The one period holding period return associated with buying the n period bond at date t and selling the resulting (n-1) period bond at date t+1 given by

$$r_{n,t,t+1} = exp(\bar{a}_{n-1} + \bar{b}'_{n-1} f_{t+1} - \bar{a}_n - \bar{b}'_n f_t) - 1 \tag{7}$$

The factors are assumed to follow a VAR(1) process:

$$f_{t+1} = c + \rho f_t + \Sigma u_{t+1} \tag{8}$$

If z_{nt} is the fraction of the arbitrageurs (or the public investors) in the bond of maturity n, then the holding return from time t to t+1 is given by:

$r_{t,t+1} = \sum_{n=1}^{N} z_{nt} r_{n,t,t+1}$

Solving the arbitrageurs portfolio optimization problem (as done in the next section of the Appendix) results in the following implications for each maturity n:

$$\bar{a}_1 - \bar{b}_1' f_t = \bar{a}_{n-1} + \bar{b}_{n-1}'(c + \rho f_t) + \left(\frac{1}{2}\right) \bar{b}_{n-1}' \Sigma \Sigma' \bar{b}_{n-1} - \bar{a}_n - \bar{b}_n' f_t - \bar{b}_{n-1}' \Sigma \lambda_t \tag{9}$$

$$\lambda_t = \gamma \Sigma' d_t \tag{10}$$

$$d_t = \sum_{n-2}^{N} z_{nt} \bar{b}_{n-1} \tag{11}$$

The borrowing demand from the credit market participants as postulated by Vayanos and Vila (2009) are denoted as ξ_{nt} and is assumed to be a decreasing affine function relative to the wealth W_t as given:

$$\frac{\xi_{nt}}{W_t} = \zeta_{nt} - \alpha_n y_{nt} \tag{12}$$

Applying the no arbitrage condition such that the investment made by the Fed and the overall borrowing (demand) of the investors, we get

$$z_{nt} = \zeta_{nt} - \alpha_n y_{nt} \tag{13}$$

Assuming that ζ_{nt} is also an affine function of f_t, the equilibrium condition is:

$$\lambda_t = \lambda + \Lambda f_t \tag{14}$$

Next, we get the parameters under no arbitrage as follows:

$$\bar{b}'_n = \bar{b}'_{n-1}\rho^{\mathbb{Q}} - b'_1 \tag{15}$$

$$\rho^{\mathbb{Q}} = \rho - \Sigma\Lambda \tag{16}$$

$$\bar{a}_n = \bar{a}_{n-1} + \bar{b}'_{n-1}c^{\mathbb{Q}} + (\frac{1}{2})\bar{b}'_{n-1}\Sigma\Sigma'\bar{b}_{n-1} - a_1 \tag{17}$$

$$c^{\mathbb{Q}} = c - \Sigma\lambda \tag{18}$$

Estimation of the Affine Term Structure Models

Let y_{nt} denote the yield on an n-period pure discount bond

$$y_{nt} = a_n + b'_n f_t \tag{19}$$

After stacking this equation for M different observed yields, we add some measurement error and denote the regression as:

$$Y_{2t} = A + Bf_t + \Sigma_e u_t^e \tag{20}$$

The parameter estimates come from the minimum chi square estimation algorithm proposed by Hamilton and Wu(2010) that allows OLS to do the work of maximizing the joint likelihood function and uses the theoretical model to translate those OLS estimates back into the asset-pricing parameters of interest.

DETAILS OF THE ARBITRAGEURS' PORTFOLIO OPTIMIZATION PROBLEM

Let P_{nt} denote the price of a pure-discount n-period bond (with $P_{0t} = 1$), W_t the total wealth of the arbitrageurs, and z_{nt} the portion of their wealth allocated to each bond maturity. Then, the arbitrageurs' wealth evolves according to

$W_{t+1} = \sum_{n=1}^{N} z_{nt} \frac{P_{n-1,t+1}}{P_{nt}} W_t$

with associated rate of return

$r_{t,t+1} = \frac{W_{t+1} - W_t}{W_t} = \sum_{n=1}^{N} z_{nt} \left[\frac{P_{n-1,t+1}}{P_{nt}} - 1 \right]$

If the change in prices between t and t + 1 is small[6] the portfolio's mean return and variance can be approximated

$E_r r_{t,t+1} \approx -z_{1t}(\bar{a} + \bar{b}_1' f_t) + \sum_{n=2}^{N} z_{nt}[\bar{a}_{n-1} + \bar{b}_{n-1}'(c + \rho f_t) + (1/2)\bar{b}_{n-1}' \Sigma\Sigma' \bar{b}_{n-1} - \bar{a}_n - \bar{b}_n' f_t]$

$Var_t(r_{t,t+1}) \approx d_t' \Sigma\Sigma' d_t,$

where the $(J \times 1)$ vector d_t summarizes exposures to each of the J factor risks associated with holding the $(N \times 1)$ vector of bonds z_t . The arbitrageurs thus choose z_t so as to maximize (1) subject to the equations written above, and $\sum_{n=1}^{N} z_{nt} = 1$.

[6]Suppose that
$q_{n,t+1} \equiv \frac{(P_{n-1,t+1} - P_{nt})}{P_{nt}} = exp(\mu_n h + \sqrt{(h)} \varepsilon_{n,t+1}) - 1$
where $(??_{1,t+1}, ..., ??_{N,t+1})' \sim N(0, \Omega)$. Our approximation is derived from the limiting behavior as h becomes small, analogous to those obtained when considering a continuous-time representation of a discrete-time process. Thus, as in Merton(1969)
$E_t(\sum_{n=1}^{N} z_{nt} q_{n,t+1}) = \sum_{n=1}^{N} z_{nt}[\mu_n h + \Omega_{nn} h/2 + o(h)]$
$Var_t(\sum_{n=1}^{N} z_{nt} q_{n,t+1}) = z_t' \Omega z_t h + o(h)$
for Ω_{nn} the row n, column n element of Ω, and $z_t = (z_{1t}, ..., z_{Nt})'$. Equations (A1) and (A2) are obtained by setting $h = 1$ and $o(h) = 0$. Specifically,
$\frac{P_{n-1,t+1}}{P_{nt}} = exp(\bar{a}_{n-1} + \bar{b}_{n-1}' f_{t+1} - \bar{a}_n - \bar{b}_n' f)$
$\mu_n = \bar{a}_{n-1} + \bar{b}_{n-1}'(c + \rho f_t) - \bar{a}_n - \bar{b}_n' f_t$
$\Omega_{nn} = \bar{b}_{n-1}' \Sigma\Sigma' \bar{b}_{n-1}$