Project 7

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code quality, speed, and accuracy will determine the grades.

1. Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20$ per annum. Assume the prevailing risk-free rate is $r = 4$ per annum. Use the $X = ln(S)$ transformation of the Black-Scholes PDE, and $\Delta t = 0.002,$ with $\Delta X = \sigma\sqrt{\Delta t},$ *with* $\Delta X = \sigma\sqrt{3\Delta t}$ with $\Delta X = \sigma\sqrt{4\Delta t}$, and a *uniform* grid to price a European Put option with strike price of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$4 to \$16; using the specified methods below:

(a) Explicit Finite-Difference method,
(b) Implicit Finite-Difference method,
(c) Crank-Nicolson Finite-Difference method.

Inputs:

i. currPrice

Outputs:

ii. Values: Pa, Pb and Pc for the European Put option using each of the methods (a), (b) and (c), given *currPrice*.
iii. Writeup: compare the three methods from (a), (b) and (c) and comment. To compare, calculate the relative error with respect to the exact Black-Scholes-Merton formula value. Do this for current stock prices of \$4 to \$16 in \$1 increments and put in a table. Put the table and your comments in a .pdf file.

2. Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20$ per annum. Assume the prevailing risk-free rate is $r = 4$ per annum.
Use the Black-Scholes PDE (for S) to price American Call and American Put options with strike prices of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$4 to \$16; using the specified methods below:

(a) Explicit Finite-Difference method,
(b) Implicit Finite-Difference method,
(c) *Crank-Nicolson Finite-Difference method.*

Choose $\Delta t = 0.002$, with $\Delta S = 0.5$,or with$\Delta S = 1$,∨ $with\Delta S = 1.5$.

Inputs:

1. currPrice

Outputs:

i. Values: Ca, Cb, Cc, Pa, Pb and Pc and for the American call and put options using each of the methods (a), (b) and (c) given *currPrice*.
ii. Graphs: Plot the American Call option price as a function of the current stock price from \$4 to \$16 in \$1 increments for methods (a), (b) and (c) on the same graph.

Use a color legend or linestyles to differentiate the plots. Do the same for the American Put option in another graph. Place the two graphs in a .pdf file.

3. ***[Optional-NOT for grading]*** Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20$ per annum. Assume the prevailing risk-free rate is $r = 4$ per annum. Use the original Black-Scholes PDE

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P}{\partial S^2} + rS\frac{\partial P}{\partial S} - rP = 0$$

$\Delta t = 0.002$, $u = e^{\sigma\sqrt{0.25\Delta t}}$, $u = e^{\sigma\sqrt{\Delta t}}$, $u = e^{\sigma\sqrt{4\Delta t}}$,and a *binomial/trinomial* grid to price the specified options using the specified methods below:

Use the *Explicit Finite-difference method*, the *Implicit Finite-Difference Method*, and *Crank-Nicolson Finite-Difference Method* to price a American Put option with strike price of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$1 to \$20, in increments of about \$1.

4. ***[Optional-NOT for grading]*** Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20$ per annum. Assume the prevailing risk-free rate is $r = 4$ per annum. Use the transformation of the Black-Scholes PDE to the **Heat Equation**, $\Delta t = 0.002$, $\Delta X = \sqrt{4\Delta t}$ or $\Delta X = \sqrt{2\Delta t}$, $\Delta X = \sigma\sqrt{1.9\Delta t}$, and a *uniform* grid to price the following options using the specified methods below:

Use the *Explicit Finite-difference method*, the *Implicit Finite-Difference Method*, and *Crank-Nicolson Finite-Difference Method* to price an American Call and Put option with strike price of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$1 to \$20, in increments of about \$1.

5. ***[Optional-NOT for grading]*** Player A has \$1 and player B has \$2. In each play one player pays the other \$1. In each game, the chance that A wins is ¾. The players play until one is bankrupt. What is the probability of A winning?

6. ***[Optional-NOT for grading]*** Mr. A has \$50 and needs to double his money. He decides to invest in (European) roulette to double his money. He is considering two strategies:

   a. *Bet all \$50 on the "Red" at once;*

   b. *Bet \$1 at a time on the "Red" until he has won or lost \$50.*

   Which strategy would you suggest him to use and why?