Empirical Methods in Finance
Homework 3

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Please use Matlab/R to solve these problems. You can just hand in one set of solutions that has all the names of the contributing students on it in each group.

[The quality of the write-up matters for your grade. Please imagine that you're writing a report for your boss at Goldman when drafting answers these questions. Try to be clear and precise.]

Principal Component Analysis

Download the 48 industry portfolio data (monthly) from Kenneth French's web site. Use the data from 1960 through 2015. Use the value-weighted returns. You may drop the industries that have missing values and are reported as -99.99. Also, download the 3 Fama-French factors from his web site. Use the monthly risk-free rate series provided by French in the same FF factor dataset to compute excess returns on these 48 portfolios.

1. Get the eigenvalues for the sample variance-covariance matrix of the excess returns to the 48 industries. Plot the fraction of variance explained by each eigenvalue in a bar plot.

2. Choose the 3 first (largest) principal components.

   (a) How much of the total variance do these 3 factors explain?

   (b) Give the mean sample return to these 3 factor portfolios, their standard deviation, and correlation.

(c) Consider the APT model using these three factors as pricing factors (with zero intercept (alpha) as the factors are traded). Plot the predicted return from this model for all the industries versus the realized average industry returns. Add a 45 degree line to this plot (as in the lecture note).

[Recall you get factor loadings (betas) from the eigenvectors. Or, if you like, you can run the time-series regression of each industry's return on the 3 factors. The result is the same.]

(d) Give the implied cross-sectional R^2 of the plot in c). That is, calculate:

$$R^2_{\text{cross-section}} = 1 - \frac{var\left(\bar{R}^{act} - \hat{R}^{pred}\right)}{Var\left(\bar{R}^{act}\right)}$$

where $\bar{R}^{act}$ is the $N \times 1$ vector of average industry excess returns. $\hat{R}^{pred}$ is the $N \times 1$ vector of predicted industry excess returns from the 3-factor model.

3. Now, download the 25 FF portfolios, same sample period.

(a) Get the eigenvalues for the sample variance-covariance matrix of the excess returns to these 25 F-F portfolios. Plot the fraction of variance explained by each eigenvalue in a bar plot.

(b) Given (a), how many factors does do you reckon you need to explain average returns to the 25 F-F portfolios?

Arbitrage Pricing in Factor Models

Assume there are two traded zero-investment factors with returns $R_{f1,t}$ and $R_{f2,t}$ and expected returns $E\left[R_{f1,t}\right] = \lambda_1 = 6\%$ and $E\left[R_{f2,t}\right] = \lambda_2 = -2\%$, respectively. Assume the APT with these two factors holds.

1. Assume a portfolio (A) has excess returns:

$$R^e_{A,t} = 1\% + 0.5 \times R_{f1,t} + 0.75 \times R_{f2,t}. \tag{1}$$

(a) There is a risk-free arbitrage opportunity in this economy. Give the trade you would implement to take advantage of this opportunity, as well as the profit you would realize.

(b) Write the excess return of portfolio A as

$$R^e_{A,t} = \delta + 1\% + 0.5 \times R_{f1,t} + 0.75 \times R_{f2,t}. \tag{2}$$

What must δ equal for there to be no arbitrage opportunities? Now, applying the no-arbitrage value of δ to the above equation, what is the expected return of $R^e_{A,t}$?