# *Project 2*

## *Nikhil Guruji (Cohort 1)*

1)

It is observed that correlation of X and Y is approximately -0.7 which is as expected from the formula:

$$\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

Given that $\sigma_X = \sigma_Y = 1\ ;\ Cov(X,Y) = a = -0.7$

2)

Using Monte-Carlo simulation, the expected value is 1.438

3)

(a) The values are calculated in the R-code.

(b) The values of the last three integrals (for t=0.5, 3.2, 6.5) are almost equal to 1.

(c) When the antithetic variance reduction technique is used, it is observed that the expectation estimate has increased from 4.90 to 5.03 and the variance is almost halved from 0.0049 to 0.0024. Also, it is interestingly observed that all the three values for the second expectation are exactly equal (=1.0707) and the variance is as low as 0.0169

Note: The inbuilt R function var() uses the square of standard deviation. But I have used Variance = $\frac{\sigma^2}{n}$ so that the standard error is observed

(4)

(a) As calculated, the estimated call price from Monte Carlo simulation is 18.8226 and the variance is 1.111

(b) Using the Black Scholes formula:

$$c = S_0 N(d_1) - Xe^{-rt} N(d_2)$$

$$d_1 = \frac{log\left[\frac{S_0}{K}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

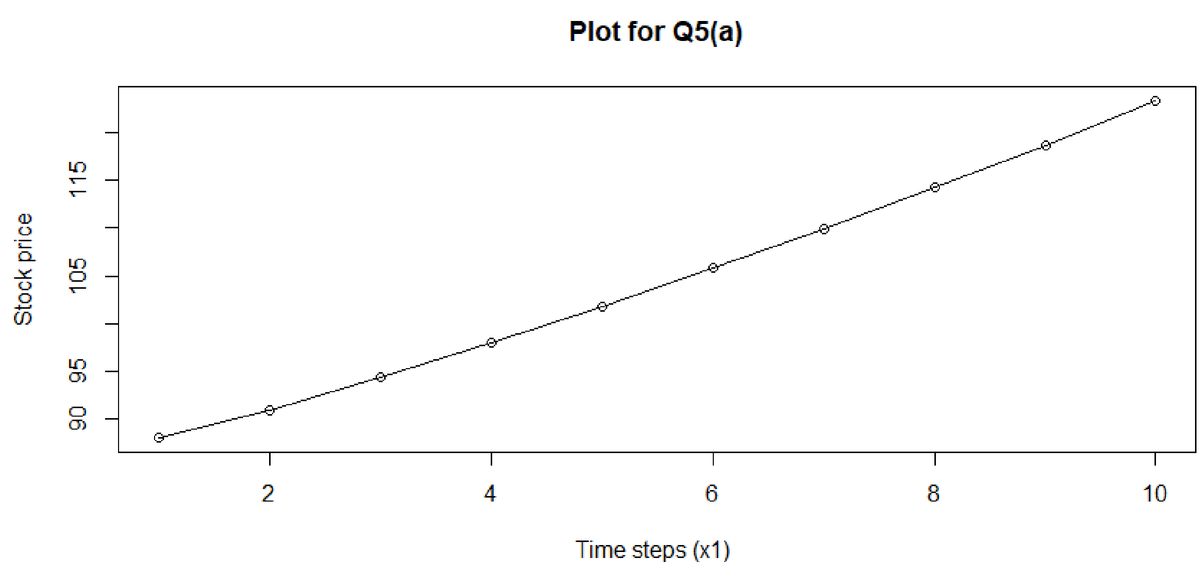
$$d_2 = d_1 - \sigma\sqrt{T}$$

The call option price is 18.2838.

On using the antithetic variance reduction technique, it was observed that the estimate (18.2804) was much closer to the actual value than the Monte Carlo estimate and the variance was also reduced to 0.051.

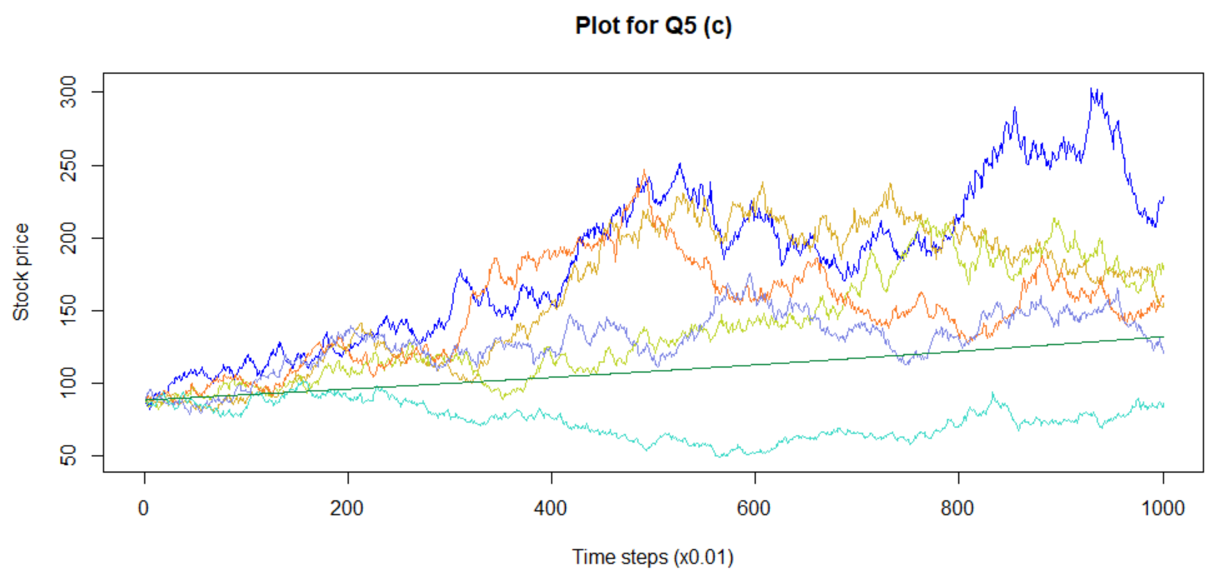
5)

(a) The following plot was observed:

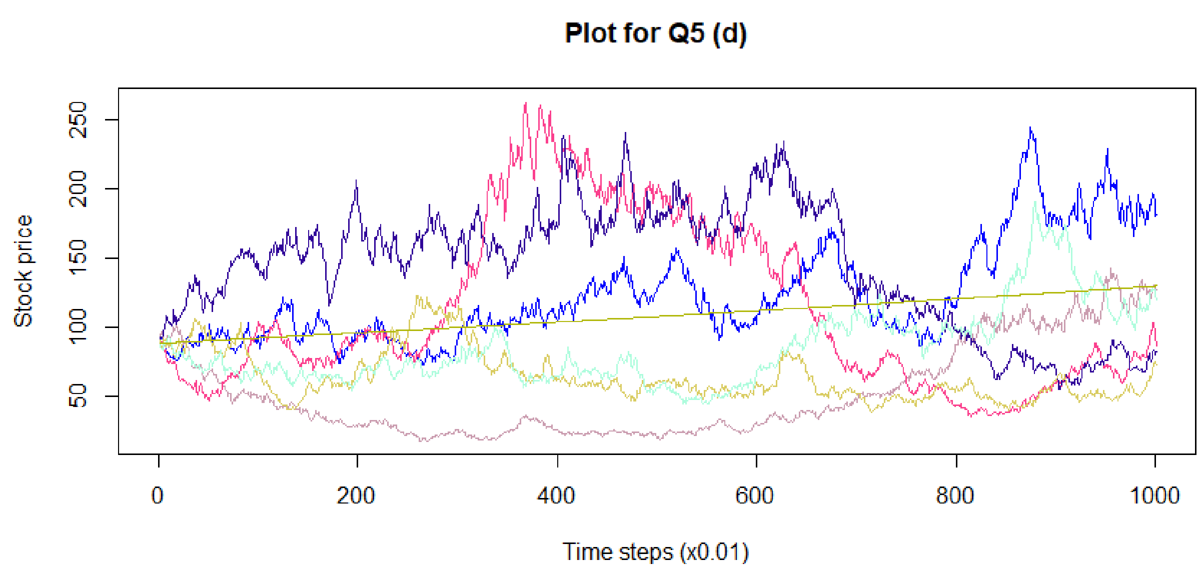



(b) The simulations were carried out in the R-code

(c) The following plot was observed (for $\sigma = 18\%$):




(d) The following plot will be observed if σ is changed to 35%:




It is observed that the stock paths are more volatile and more often below $S_0 = 88$ because the drift term $\left(r - \frac{\sigma^2}{2}\right)T$ reduces as σ increases.

Note: The straight line in the above two graphs is the expected value from part (a)

6)

(a) The trapezoidal method was used to calculate the integral numerically

(b) For Monte Carlo method of integration, the function $\sqrt{1 - x^2}$ was evaluated for 1000 uniform numbers between [0,1] and the mean was taken as the estimate for pi.

(c) In the variance reduction technique, importance sampling was used. The function t(x) was defined as given in the lecture notes.

The estimate of $\pi$ is calculated by: $4 \times \frac{1}{N} \sum_{i=1}^{N} \frac{g(U_i) f(U_i)}{t(U_i)}$

The variance from the formula from the lecture notes is 0.002 which is very small. This shows that the estimate is accurate.

For a total of 1000 simulations for each part, answer for part (a) was the closest to the actual value of π which is logical because it almost directly computes the value of the integral. The next closest was the answer for part (c).

The variance of answer for part (b) was found to be highest (0.7565)


References:

- Numerical Integration (Trapezoidal method)

  https://www.whitman.edu/mathematics/calculus_online/section08.06.html

- Importance Sampling in Monte Carlo Simulation of Rare transition events by Wei Cai:

  http://micro.stanford.edu/~caiwei/LLNL-CCMS-2005-Summer/Lecture_Notes/impsample_lecture_all.pdf