Project 6

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code quality, speed, and accuracy will determine the grades.

1. Consider a 12-month **Fixed Strike Lookback Call and Put** options, when the interest rate is 3% per annum, the current stock price is \$98 and the strike price is \$100. Use the MC simulation method to estimate the prices of Call and Put options for the following range of volatilities: from 12% to 48%, in increments of 4%.
The payoff of the Call option is $(S_{max} - X)^{+}$, where $S_{max} = \max\{S_t \colon t \in [0, T]\}$, and the payoff of the Put option is: $(X - S_{min})^{+}, where\ S_{min} = \min\ \{S_t : t \in [0, T]\}$.
Inputs: s*eed*
Outputs:

i. Graph: Call and put options prices as a function of the volatility. Put the Call graph in Proj6_1a.png and the Put graph in Proj6_1b.png.

2. Assume that the value of a collateral follows a jump-diffusion process:

$$\frac{dV_t}{V_t^-} = \mu dt + \sigma dW_t + \gamma dJ_t$$

where $\mu, \sigma,\ \gamma < 0$, and V_0 are given, J is a Poisson process, with intensity λ_1, independent of the Brownian Motion process W.
V_t^- is the value process before jump occurs at time t (if any).
Consider a collateralized loan, with a contract rate per period *r* and maturity *T* on the above-collateral, and assume the outstanding balance of that loan follows this process:

$$L_t = a - bc^{12t}$$

where $a > 0, b > 0\ , c > 1, and\ \ L_0$ are given. We have that $L_T = 0$.

Define the following stopping time:

$$Q = min\{t \geq 0\colon\ V_t \leq q_t L_t\}$$

This stopping time is the first time when the relative value of the collateral (with respect to the outstanding loan balance) crosses a threshold which will be viewed as the "optimal exercise boundary" of the option to default.

Define another stopping time, which is the first time an adverse event occurs:

$$S = min\{t \geq 0\colon N_t > 0\}$$

Assume that N_t is a Poisson process with intensity of λ_2.

Define $\tau = \min\{Q, S\}$.
We assume the embedded default option will be exercised at time τ, if and only if $\tau < T$.

If the option is exercised at time Q then the payoff to the borrower is $\left(L_Q - \epsilon V_Q\right)^{+}$.

If the option is exercised at time S then the payoff to the borrower is $abs(L_S - \epsilon V_S)$, where *abs(.)* is the absolute value function.

Notes:

1. If $\min\{Q, S\} > T$ then there is no default option exercise.

2. ϵ should be viewed as the recovery rate of the collateral, so $(1 - \epsilon)$ can be viewed as the legal and administrative expenses.

Assume J has intensity λ_1 and N has intensity λ_2. N is independent of J and W.

Assume the APR of the loan is $R = r_0 + \delta\lambda_2$ where r_0 is the "risk-free" rate, and δ is a positive parameter to measure the borrower's creditworthiness in determining the contract rate per period: *r*.
We have monthly compounding here, so $r = R/12$.

Assume that $q_t = \alpha + \beta t$, where $\beta > 0$, $\alpha < V_0/L_0$ and $\beta = \frac{\epsilon - \alpha}{T}$.

Use r_0 for discounting cash flows. Use the following base-case parameter values:

$V_0 = \$20{,}000, L_0 = \$22{,}000, \mu = -0.1,\ \sigma = 0.2,\ \gamma = -0.4,\ \lambda_1 = 0.2,\ T = 5$ years, $r_0 = 0.02, \delta = 0.25, \lambda_2 = 0.4,\ \alpha = 0.7,\ \epsilon = 0.95$. Notice that $PMT = \frac{L_0.r}{\left[1 - \frac{1}{(1+r)^n)}\right]}$, where $r = R/12$, $n = T * 12$, and $a = \frac{PMT}{r}, b = \frac{PMT}{r(1+r)^n}, c = (1+r)$. Notice that $q_T = \epsilon$.

Write the code as a function Proj6_2func.m that takes λ_1, λ_2 and T as parameters, setting defaults if these parameters are not supplied, and outputs the default option price, the default probability and the expected exercise time. Function specification:

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function [D, Prob, Et] = Proj6_2func(lambda1, lambda2, T)
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(a) Estimate the value of the default option for the following ranges of parameters:
λ_1 from 0.05 to 0.4 in increments of 0.05;
λ_2 from 0.0 to 0.8 in increments of 0.1;
T from 3 to 8 in increments of 1;

(b) Estimate the default probability for the following ranges of parameters:.

λ_1 from 0.05 to 0.4 in increments of 0.05;
λ_2 from 0.0 to 0.8 in increments of 0.1;
T from 3 to 8 in increments of 1;

(c) Find the Expected Exercise Time of the default option, conditional on $\tau < T$. That is, estimate $E(\tau \mid \tau < T)$ for the following ranges of parameters:.
λ_1 from 0.05 to 0.4 in increments of 0.05;
λ_2 from 0.0 to 0.8 in increments of 0.1;
T from 3 to 8 in increments of 1;

Inputs: s*eed*
Outputs:

i. Values: the default option D, the default probability Prob and the expected exercise time Et for parts (a), (b) and (c) with λ_1=.2, λ_2=0.4 and T=5.
ii. Graphs: For each of (a), (b) and (c) two graphs as a function of T, first with λ_1=0.2 and λ_2 from 0.0 to 0.8 in increments of 0.1, then with λ_2 = 0.4 and λ_1 from 0.05 to 0.4 in increments of 0.05. Put the two graphs in one .png file.

(d) ***[Optional – NOT for grading]*** Make additional assumptions as necessary to estimate the IRR of the investment.

Note: The drift of the V process should be a function of r_0, λ_1, σ under the risk-neutral measure, to be able to price the option, but not done so in this case.

3. ***[Optional – NOT for grading]***

Compute, via MC simulation, the prices of the following options using 50,000 simulations of paths of the underlying stock price process and dividing the time-interval into 50 equal parts:

(a) **Down-and-Out- Put**: $S(0) = 50,\ X = 50,\ r = 0.04,\ T =$ 2 months, $\sigma = 0.4$, and for the following range of S_b: from \$36 to \$46, in increments of \$1.
(b) **Down-and-In - Put**: $S(0) = 50,\ X = 50,\ r = 0.04,\ T =$ 2 months, $\sigma = 0.4$, and for the following range of S_b: from \$36 to \$46, in increments of \$1.

4. ***[Optional – NOT for grading]***

Compute the price of the **Asian Average Rate call** option by using:

(a) Standard MC method, where $S(0) = 50,\ X = 50,\ r = 0.04,\ T =$ 2 months, $\sigma = 0.4$.

(b) Halton's Low-discrepancy sequences to generate the paths of the stock price, where $S(0) = 50,\ X = 50,\ r = 0.04,\ T =$ 2 months, $\sigma = 0.4$.

5. ***[Optional – NOT for grading]***

Assume the stock price follows an Arithmetic Brownian Motion.

(a) Derive the formula for a price of a European call option, using all the other Black-Scholes assumptions.

(b) Use MC simulation techniques to compute the price and compare with the one implied by the formula derived.

(c) Now compare the prices of the options using this model with the Black-Scholes prices (using the same parameters). Vary your parameters over a wide range to compare the prices. Any observations?