Homework 3

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Principal Component Analysis

```
library(data.table)
library(zoo)
library(xts)
library(lubridate)
#Process Data
ffactor <- fread("https://raw.githubusercontent.com/ErinHu95/HW/master/F-F_Research_Data_Factors.CSV")
```

```
## Warning in fread("https://raw.githubusercontent.com/ErinHu95/HW/master/F-
## F_Research_Data_Factors.CSV"): Stopped reading at empty line 1103 but text
## exists afterwards (discarded): Annual Factors: January-December
```

```
indRtn <- fread("https://raw.githubusercontent.com/ErinHu95/HW/master/48_Industry_Portfolios.CSV")
```

```
## Warning in fread("https://raw.githubusercontent.com/ErinHu95/HW/master/
## 48_Industry_Portfolios.CSV"): Stopped reading at empty line 1111 but text
## exists afterwards (discarded): Average Equal Weighted Returns -- Monthly
```

```
ffactor$V1 <- paste(ffactor$V1,"01",sep = "")
indRtn$V1 <- paste(indRtn$V1,"01",sep = "")
ffactor$V1 <- ymd(ffactor$V1)
indRtn$V1 <- ymd(indRtn$V1)
ffactor <- xts(ffactor[,2:ncol(ffactor)],order.by = ffactor$V1)
indRtn <- xts(indRtn[,2:ncol(indRtn)],order.by = indRtn$V1)
ffactor <- ffactor["1960-01-01::2016-01-01"]
indRtn <- indRtn["1960-01-01::2016-01-01"]
indRtn[indRtn==-99.99] <- NA
na_flag <- apply(is.na(indRtn),2,sum)
indRtn <- indRtn[,which(na_flag == 0)]
ind_excess_rtn <- data.frame()
for(i in 1:ncol(indRtn)){
  ind_excess_rtn <- cbind(ind_excess_rtn,indRtn[,i]-ffactor[,"RF"])
}
```

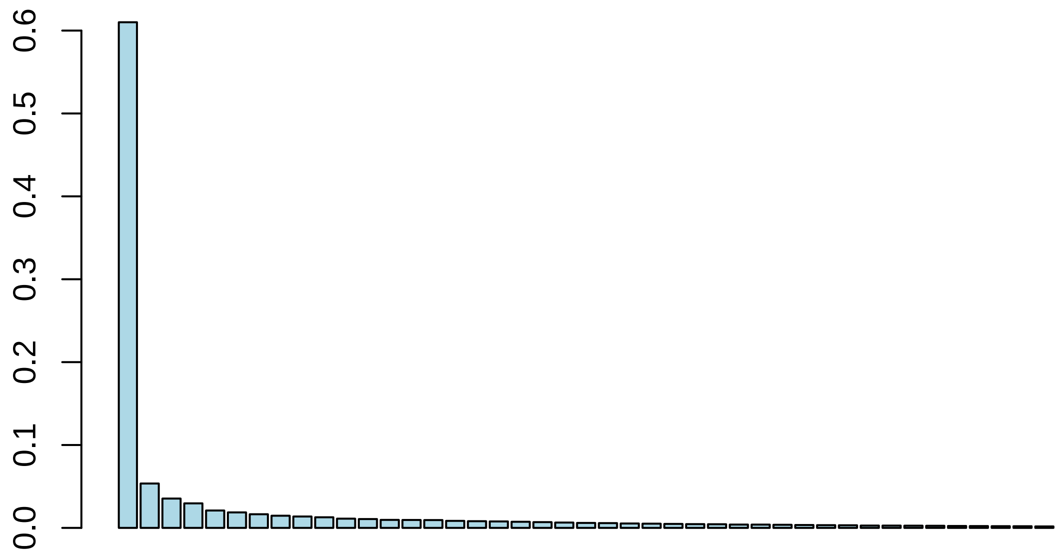
1.Get the eigenvalues for the sample variance-covariance matrix of the excess returns to the 48 industries. Plot the fraction of variance explained by each eigenvalue in a bar plot.

```
varMatrix <- cov(ind_excess_rtn)
ev <- eigen(varMatrix)
ev$vectors[,1]
```

```
##  [1] -0.13218934 -0.09600277 -0.10397116 -0.09439002 -0.17543607
##  [6] -0.19830249 -0.15424815 -0.11383266 -0.16354311 -0.12401897
## [11] -0.10483834 -0.14697589 -0.16034530 -0.17159369 -0.16840992
```

```
## [16] -0.18885078 -0.18048047 -0.17135353 -0.16740671 -0.16164015
## [21] -0.16480495 -0.16064922 -0.16257692 -0.17100871 -0.10231785
## [26] -0.06980616 -0.09634850 -0.16385114 -0.16702645 -0.15541600
## [31] -0.18483845 -0.18011721 -0.14024975 -0.13520850 -0.15264183
## [36] -0.15521607 -0.13622121 -0.15460720 -0.14394566 -0.13482909
## [41] -0.19040002 -0.16285429 -0.16881221
```

```r
evalue <- ev$values
frac <- evalue/sum(evalue)
barplot(frac,col = "lightblue")
```



2 Choose the 3 largest principal components.

a) How much of the total variance do these 3 factors explain?

```r
#2(a)
f3frac <- sum(frac[1:3])
```

b) Give the mean sample return to these 3 factor portfolios, their standard deviation, and correlation.

```r
#2(b)
pca <- prcomp(ind_excess_rtn,scale = T)
lds1 <- t(pca$rotation)
a <- t(ind_excess_rtn)
f3 <- lds1%*%a
f3mean <- apply(f3[1:3,],1,mean)
f3std <- apply(f3[1:3,],1,sd)
```

```r
f3cor <- cor(cbind(f3[1,],f3[2,],f3[3,]))
names(f3mean)=c("PC1","PC2","PC3")
names(f3std)=c("PC1","PC2","PC3")
rownames(f3cor)=c("PC1","PC2","PC3")
colnames(f3cor)=c("PC1","PC2","PC3")

f3mean
```

```
##        PC1        PC2        PC3
##  3.7785413  0.4216165 -0.2847471
```

```r
f3std
```

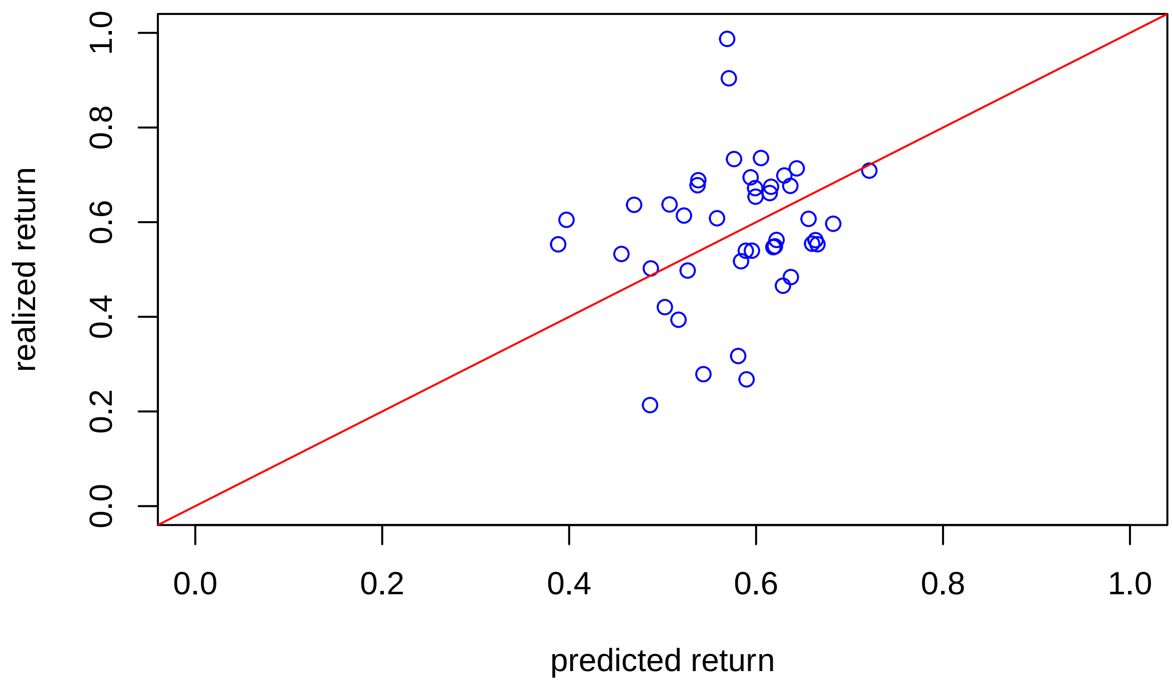
```
##       PC1       PC2       PC3
## 32.223893  8.944588  8.068516
```

```r
f3cor
```

```
##             PC1        PC2        PC3
## PC1  1.00000000 -0.3785867 0.06619521
## PC2 -0.37858674  1.0000000 0.13492464
## PC3  0.06619521  0.1349246 1.00000000
```

c) Consider the APT model using these three factors as pricing factors (with zero intercept (alpha) as the factors are traded). Plot the predicted return from this model for all the industries versus the realized average industry returns. Add a 45 degree line to this plot (as in the lecture note).

```r
#2(c)
pca3 <- prcomp(ind_excess_rtn,scale = T,rank. = 3)
lds <- pca3$rotation
pred_rtn <- (lds)%*%f3mean
rlz_rtn <- apply(ind_excess_rtn,2,mean)
plot(x = t(pred_rtn),y = rlz_rtn,xlim = c(0,1),ylim = c(0,1),xlab = "predicted return",ylab = "realized
abline(a = 0,b = 1,col = "red")
```



d) Give the implied cross-sectional R2 of the plot in c).

```
#2d

csRsqr <- 1-(var(rlz_rtn-pred_rtn)/var(rlz_rtn))
csRsqr
```

```
##             [,1]
## [1,] -0.04525831
```

3. Now, download the 25 FF portfolios, same sample period.

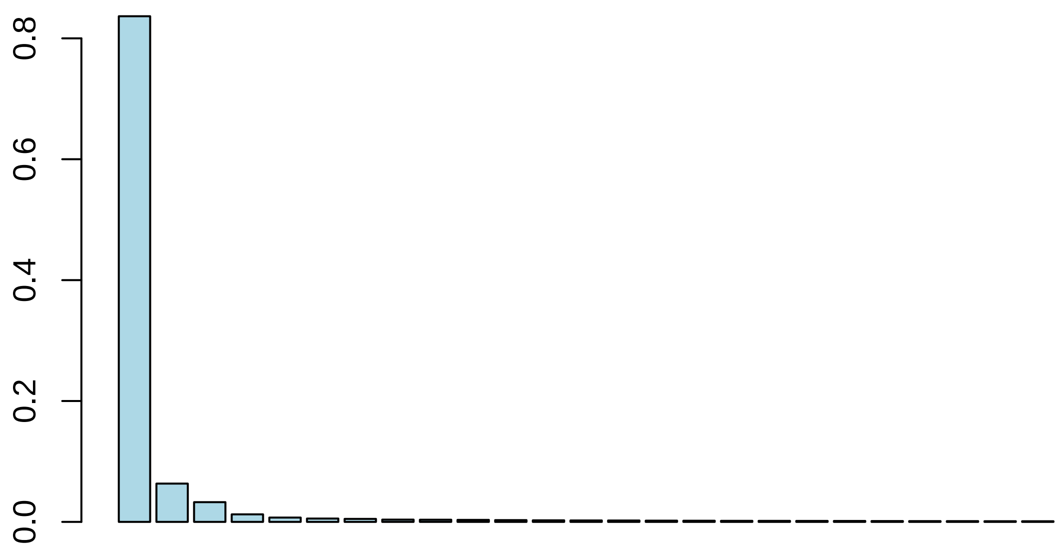
```
portRtn <- fread("https://raw.githubusercontent.com/ErinHu95/HW/master/25_Portfolios_5x5.CSV")
```

```
## Warning in fread("https://raw.githubusercontent.com/ErinHu95/HW/master/
## 25_Portfolios_5x5.CSV"): Stopped reading at empty line 1115 but text exists
## afterwards (discarded): Average Equal Weighted Returns -- Monthly
```

```
portRtn$V1 <- paste(portRtn$V1,"01",sep = "")
portRtn$V1 <- ymd(portRtn$V1)
portRtn <- xts(portRtn[,2:ncol(portRtn)],order.by = portRtn$V1)
portRtn <- portRtn["1960-01-01::2016-01-01"]
port_excess_rtn <- data.frame()
for(i in 1:ncol(portRtn)){
  port_excess_rtn <- cbind(port_excess_rtn,portRtn[,i]-ffactor[,"RF"])
}
```

a) Get the eigenvalues for the sample variance-covariance matrix of the excess re- turns to these 25 F-F portfolios. Plot the fraction of variance explained by each eigenvalue in a bar plot.

```
varMatrix3a <- cov(port_excess_rtn)
ev3a <- eigen(varMatrix3a)
evalue3a <- ev3a$values
frac3a <- evalue3a/sum(evalue3a)
barplot(frac3a,col = "lightblue")
```



b) Given (a), how many factors does do you reckon you need to explain average returns to the 25 F-F portfolios?

```
pct = sum(frac3a[1:5])
pct
```

```
## [1] 0.9522429
```

```
pct <- sum(frac3a[1:10])
pct
```

```
## [1] 0.9733475
```

```
pct = sum(frac3a[1:20])
pct
```

```
## [1] 0.993969
```

We need 5,10,20 factors to explain around 95%,97%, and 99% respectively, of the average returns of the 25 F-F portfolioss.

Arbitrage Pricing in Factor Models

a) There is a risk-free arbitrage opportunity in this economy. Give the trade you would implement to take advantage of this opportunity, as well as the profit you would realize.

Construct a portfolio B using the portfolios that 'mimick' the factors:
Invest 0.5 in factor 1 mimick portfolio
Invest 0.75 in factor 2 mimick portfolio
Invest 1-0.5-0.75 in risk-free rate
As a result, we copy a portfolio with the same factors as A. We could arbitrage by shorting B longing A to get risk-free alpha of 1%. The profit will be 1%.

b) Write the excess return of portfolio A as

$$R^e_{A,r} = \delta + 1\% + 0.5 \times R_{f1,t} + 0.75 \times R_{f2,t}$$

##What must δ equal for there to be no arbitrage opportunities? Now, applying the no-arbitrage value of δ to the above equation, what is the expected return of $R^e_{A,r}$?
δ should equal -1% due to the fact that the intercept term should be zero for no arbitrage. The expected return of $R^e_{A,r}$ is

```
exp_rtn <- 0.5*0.06+0.75*(-0.02)
exp_rtn
```

```
## [1] 0.015
```