Project 7

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Q1-

(i) For initial stock price = $10 and other given parameters:

	Explicit Finite Difference Method	Implicit Finite Difference Method	Crank Nicolson Method
multiplier=1	0.4641108	0.4645212	0.4648002
multiplier=3	0.4647152	0.4641563	0.4644360
multiplier=4	0.4690281	0.4684822	0.4687553

(ii)

The errors or deviation from Black Scholes values (Estimated Values – Black Scholes Values) are summarized as follows:

For $\Delta X=\sigma\sqrt{\Delta t}$,

	Explicit Method	Implicit Method	Crank Nicolson Method
4	3.823070e-02	3.400707e-02	3.400295e-02
5	2.833845e-02	2.695718e-02	2.695224e-02
6	3.556168e-02	3.231219e-02	3.230217e-02
7	3.869763e-02	3.769573e-02	3.761812e-02

	Explicit Method	Implicit Method	Crank Nicolson Method
8	6.917359e-02	6.817478e-02	6.800426e-02
9	2.928325e-02	2.705521e-02	2.707976e-02
10	-5.837630e-04	-1.733651e-04	1.057070e-04
11	2.849895e-04	-1.562213e-04	1.151767e-06
12	-8.125694e-05	7.267623e-05	4.182273e-05
13	-4.002024e-05	1.188376e-04	3.707275e-05
14	-7.683900e-05	7.659731e-05	2.236712e-05
15	-5.175463e-05	3.224649e-05	8.693956e-06
16	-6.975808e-06	1.094352e-05	2.959839e-06

For $\Delta X=\sigma\sqrt{3\Delta t}$,

	Explicit Method	Implicit Method	Crank Nicolson Method
4	4.630368e-02	4.630673e-02	4.630522e-02
5	4.799678e-02	4.799945e-02	4.799813e-02

	Explicit Method	Implicit Method	Crank Nicolson Method
6	6.973083e-02	6.973651e-02	6.973361e-02
7	1.103881e-01	1.104897e-01	1.104390e-01
8	6.776456e-02	6.810065e-02	6.793287e-02
9	5.942486e-02	5.942805e-02	5.942613e-02
10	2.068983e-05	-5.382242e-04	-2.585537e-04
11	-9.181814e-05	-4.065652e-04	-2.494244e-04
12	2.668567e-05	8.859961e-05	5.742729e-05
13	-5.082038e-05	1.129493e-04	3.125184e-05
14	7.873586e-06	1.162051e-04	6.223119e-05
15	-1.315811e-06	4.573511e-05	2.224417e-05
16	5.333398e-07	1.654347e-05	8.502223e-06

For $\Delta X=\sigma\sqrt{4\Delta t}$,

	Explicit Method	Implicit Method	Crank Nicolson Method
4	7.232112e-02	7.231559e-02	7.231838e-02
5	5.101740e-02	5.101820e-02	5.101781e-02
6	6.117049e-02	6.117787e-02	6.117412e-02
7	1.046190e-01	1.047222e-01	1.046707e-01
8	1.358921e-01	1.361944e-01	1.360436e-01
9	7.488428e-02	7.490904e-02	7.489635e-02
10	4.333562e-03	3.787622e-03	4.060782e-03
11	1.315507e-04	-1.827712e-04	-2.584288e-05
12	-1.245453e-05	4.988326e-05	1.849975e-05
13	7.312980e-05	2.360753e-04	1.547887e-04
14	1.868029e-05	1.267442e-04	7.290303e-05
15	1.190934e-05	5.900805e-05	3.549301e-05

	Explicit Method	Implicit Method	Crank Nicolson Method
16	2.940782e-06	1.899031e-05	1.092964e-05

Errors are usually largest for Explicit Finite difference method (especially for the case where $\Delta X = \sigma\sqrt{\Delta t}$). This indicates that the explicit method does not converge optimally for $\Delta X < \sigma\sqrt{3\Delta t}$.

However, the errors are quite low for both the Implicit Finite difference and the Crank – Nicolson methods, which converge no matter what the value of ΔX is.

Thus, the LAX Equivalence Theorem is successfully observed.

Q2- (i) For $\Delta S = 0.5$,

	Call (Explicit Method)	Call (Implicit Method)	Call (Crank Nicolson Method)	Put (Explicit Method)	Put (Implicit Method)	Put (Crank Nicolson Method)
4	5.667881e-08	7.820757e-08	6.687000e-08	6.0000000000	6.0000000000	5.9920861365
5	7.231184e-06	8.684637e-06	7.940903e-06	5.0000000000	5.0000000000	4.9920940105
6	3.305650e-04	3.609128e-04	3.456601e-04	4.0000000000	4.0000000000	3.9924317297
7	6.162843e-03	6.362796e-03	6.263032e-03	3.0000000000	3.0000000000	2.9983491021
8	5.310236e-02	5.345394e-02	5.327830e-02	2.0000000000	2.0000000000	1.9453643699
9	2.429922e-01	2.428488e-01	2.429201e-01	1.0913221936	1.0906986195	1.0750061929
10	6.876371e-01	6.870062e-01	6.873219e-01	0.4979138708	0.4969976546	0.4794079981
11	1.392309e+00	1.391946e+00	1.392127e+00	0.1795101941	0.1790459561	0.1742133352
12	2.268203e+00	2.268241e+00	2.268222e+00	0.0615770165	0.0615991524	0.0603075849
13	3.225214e+00	3.225369e+00	3.225292e+00	0.0175941509	0.0177594702	0.0173780176
14	4.212402e+00	4.212508e+00	4.212455e+00	0.0045453211	0.0046677938	0.0045410597
15	5.208995e+00	5.209038e+00	5.209017e+00	0.0010867505	0.0011463458	0.0011028036
16	6.208164e+00	6.208171e+00	6.208168e+00	0.0002449879	0.0002681958	0.0002537585

For $\Delta S = 1$,

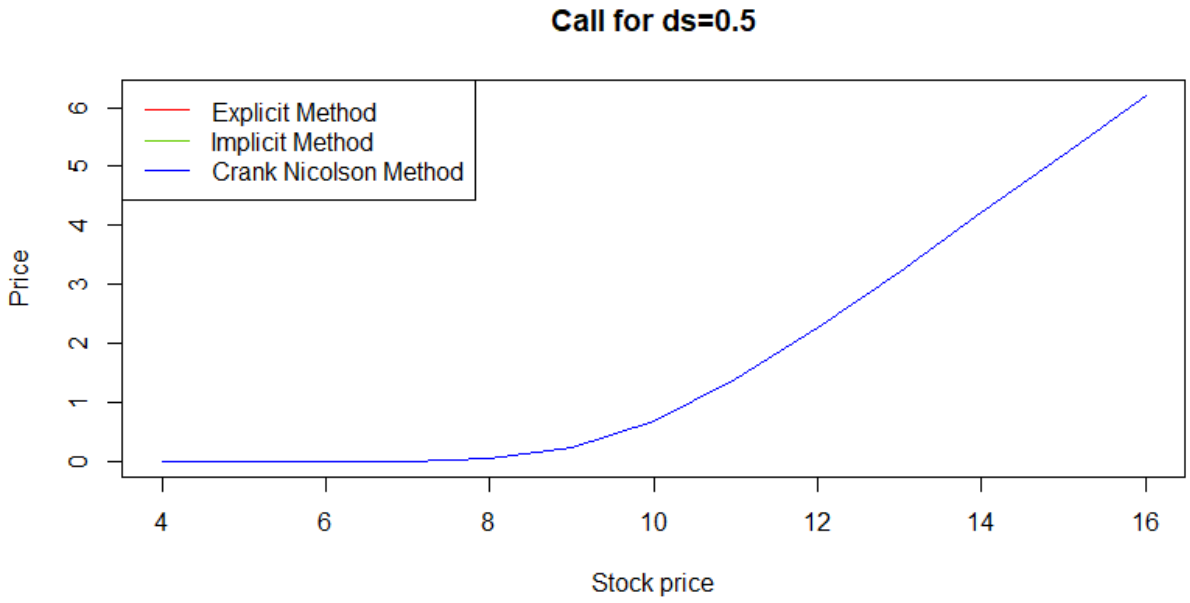
	Call (Explicit Method)	Call (Implicit Method)	Call (Crank Nicolson Method)	Put (Explicit Method)	Put (Implicit Method)	Put (Crank Nicolson Method)
4	4.171085e-06	4.684283e-06	4.423657e-06	6.0000000000	6.0000000000	5.9821898299
5	9.235844e-05	9.945828e-05	9.588245e-05	5.0000000000	5.0000000000	4.9822812887
6	1.264977e-03	1.318960e-03	1.291928e-03	4.0000000000	4.0000000000	3.9834773342
7	1.112847e-02	1.134334e-02	1.123607e-02	3.0000000000	3.0000000000	2.9934214758
8	6.458407e-02	6.494150e-02	6.476304e-02	2.0000000000	2.0000000000	1.9469484467
9	2.533123e-01	2.532145e-01	2.532629e-01	1.0874514657	1.0868849704	1.0754483013
10	6.939852e-01	6.931868e-01	6.935865e-01	0.4930028053	0.4920271095	0.4757718986
11	1.404674e+00	1.404322e+00	1.404497e+00	0.1820974810	0.1816983559	0.1766825228
12	2.283255e+00	2.283315e+00	2.283285e+00	0.0670004173	0.0670616459	0.0564699599
13	3.238978e+00	3.239133e+00	3.239056e+00	0.0216068392	0.0217768666	0.0182412905
14	4.224293e+00	4.224401e+00	4.224347e+00	0.0065953428	0.0067223094	0.0065325228
15	5.219726e+00	5.219777e+00	5.219752e+00	0.0019375897	0.0020060998	0.0019371545
16	6.218368e+00	6.218382e+00	6.218375e+00	0.0005544521	0.0005857475	0.0005606772

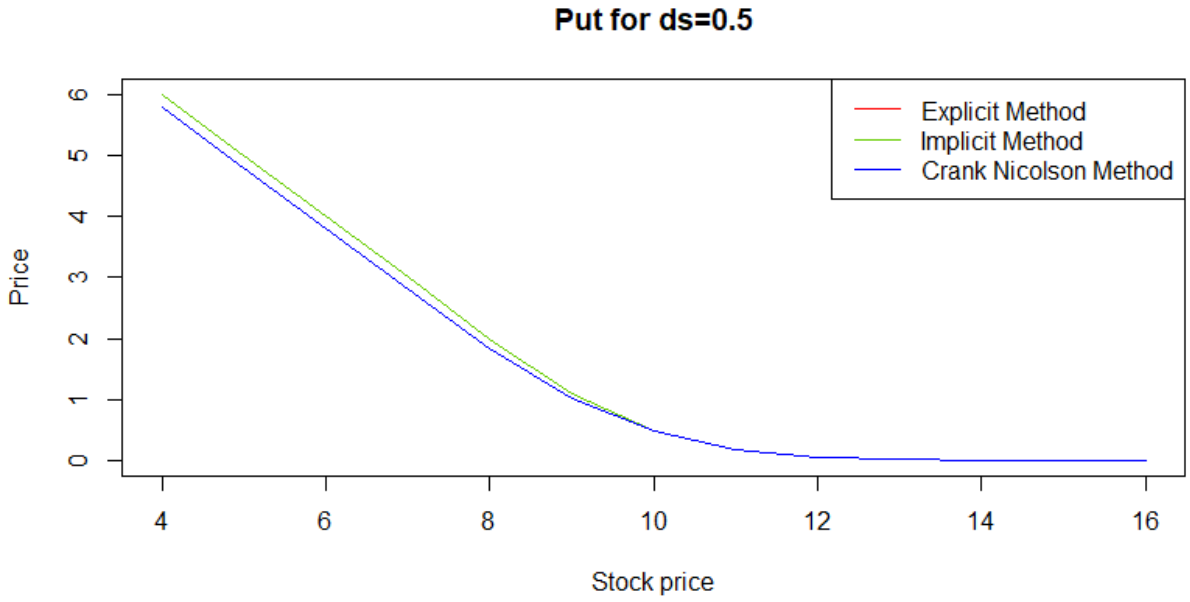
For $\Delta S = 1.5$,

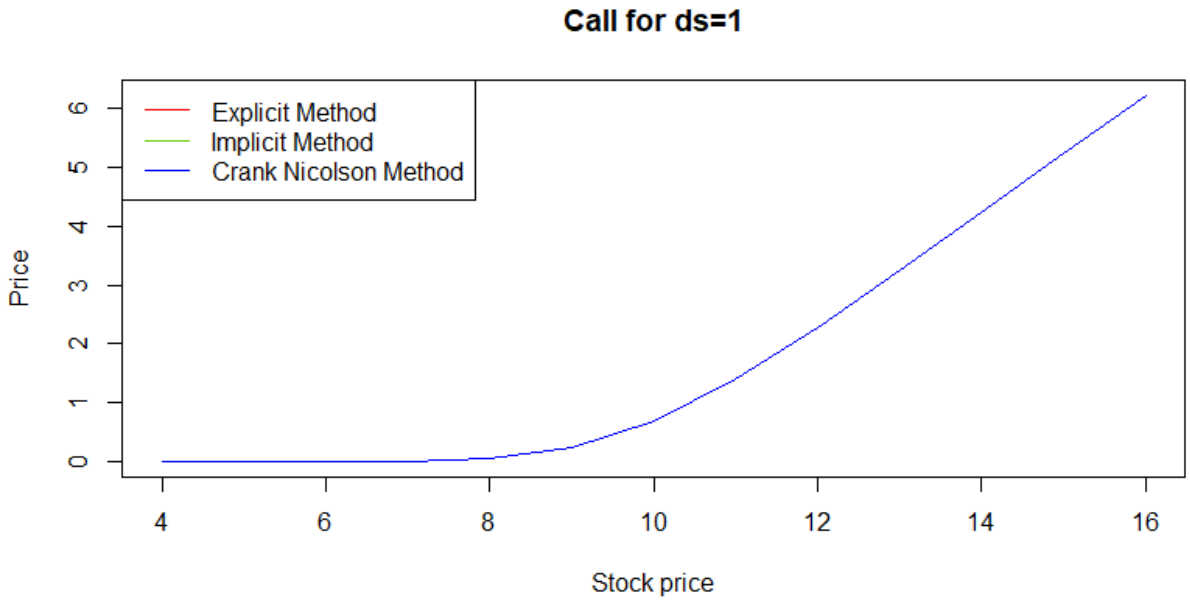
	Call (Explicit Method)	Call (Implicit Method)	Call (Crank Nicolson Method)	Put (Explicit Method)	Put (Implicit Method)	Put (Crank Nicolson Method)
4	1.342982e-05	1.432714e-05	1.387509e-05	6.0000000000	6.0000000000	5.9920999447
5	9.408997e-05	9.791948e-05	9.599795e-05	5.0000000000	5.0000000000	4.9119833943
6	4.686909e-03	4.773080e-03	4.729997e-03	4.0000000000	4.0000000000	3.9770147403
7	1.073385e-02	1.088776e-02	1.081087e-02	3.0000000000	3.0000000000	2.8028969387
8	4.805405e-02	4.826467e-02	4.815947e-02	2.0000000000	2.0000000000	1.9600468634
9	3.107023e-01	3.106061e-01	3.106541e-01	1.1029173384	1.1026728437	1.0829388505
10	6.326768e-01	6.319044e-01	6.322904e-01	0.4739576326	0.4730661586	0.4543764451
11	1.355715e+00	1.355498e+00	1.355606e+00	0.1728483497	0.1726498285	0.1704937183
12	2.318169e+00	2.318152e+00	2.318160e+00	0.0611462777	0.0611556386	0.0504449987
13	3.224922e+00	3.225041e+00	3.224982e+00	0.0175110793	0.0176504003	0.0170676499
14	4.194653e+00	4.194705e+00	4.194679e+00	0.0065664824	0.0066348957	0.0065661406
15	5.232276e+00	5.232342e+00	5.232309e+00	0.0045679193	0.0046522355	0.0045937436
16	6.208490e+00	6.208496e+00	6.208493e+00	0.0005800288	0.0006036543	0.0005791807

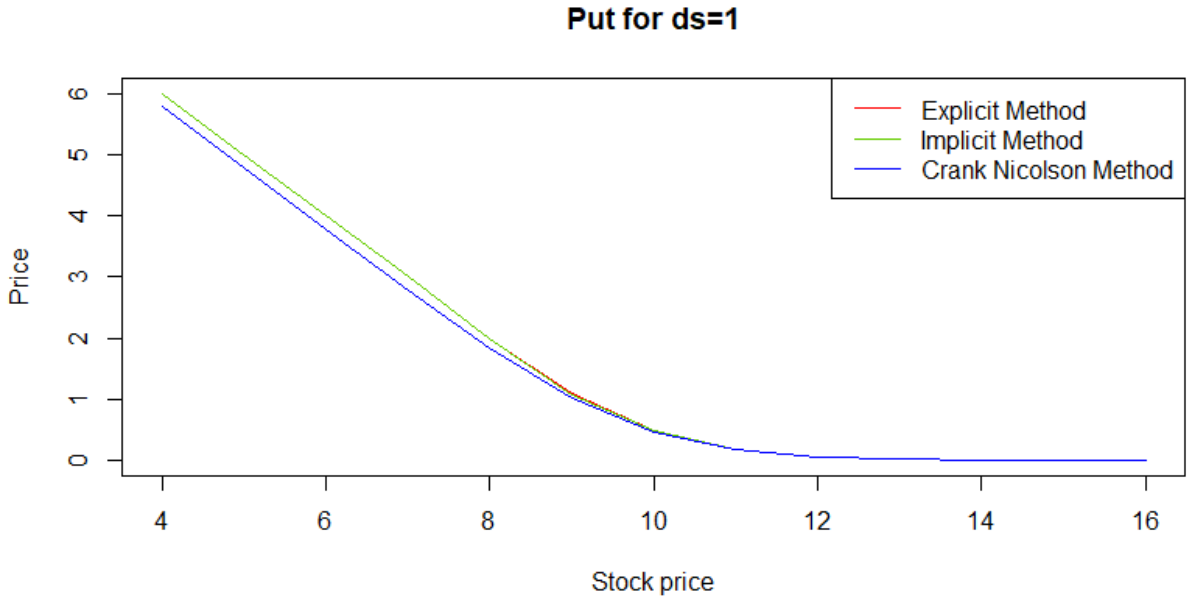
It is observed from the above tables that the accuracy decreases as we increase the ΔS. The Crank – Nicolson method is still the closest and the most efficient out of all the methods.

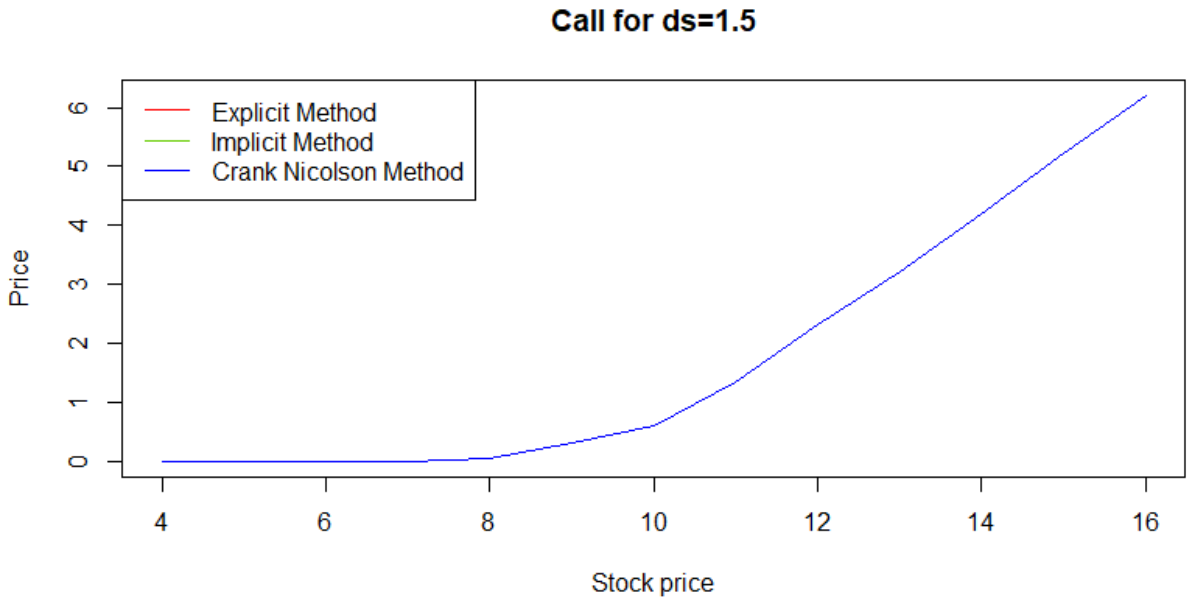
(ii)

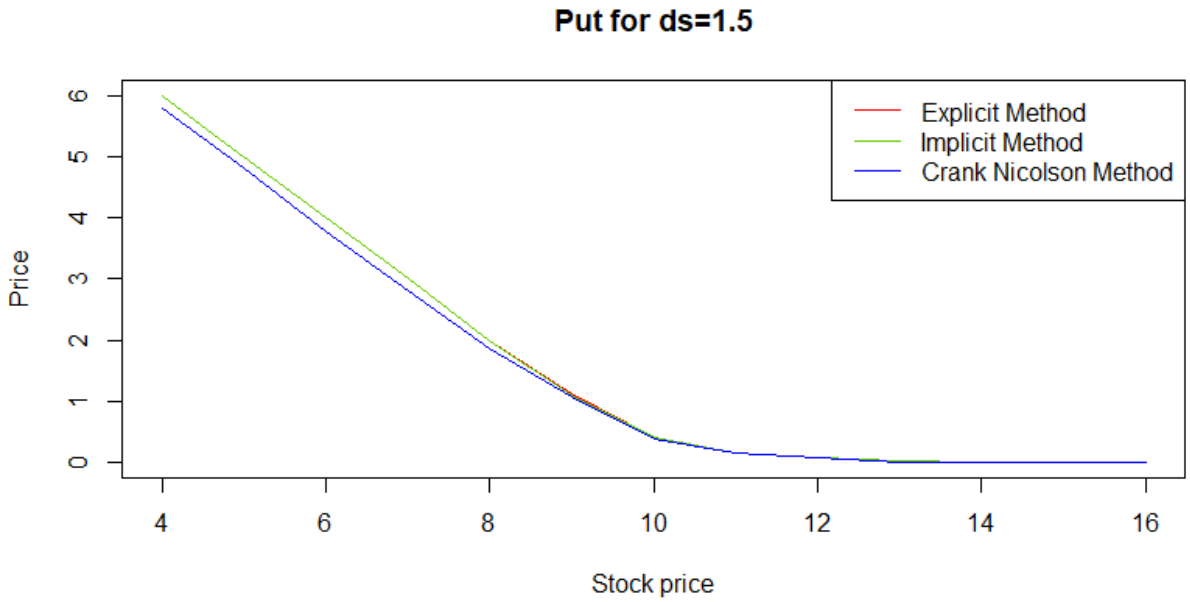





Call for ds=1
Explicit Method
Implicit Method
Crank Nicolson Method
Price
0
1
2
3
4
5
6
4
6
8
10
12
14
16
Stock price


Put for ds=1
Explicit Method
Implicit Method
Crank Nicolson Method
Price
0
1
2
3
4
5
6
4
6
8
10
12
14
16
Stock price





Note: The graphs of call for all the three methods have approximately the same value up to 4 decimal places. Which is why they cannot be distinguished on the graph.