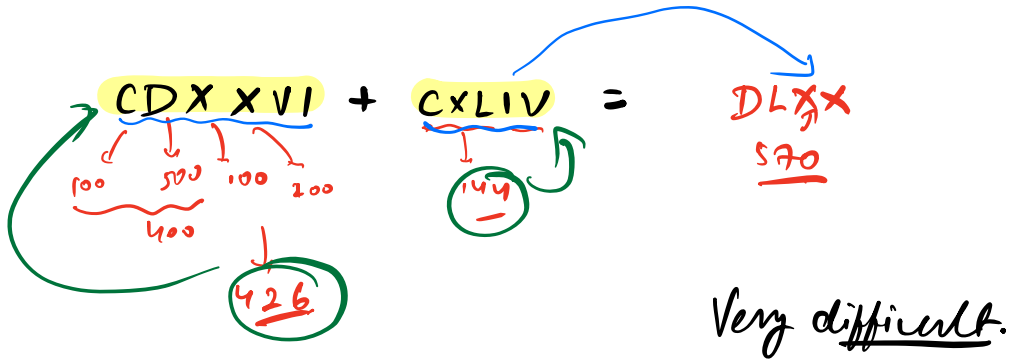
Bit Manipulation - 1

Jun 6, 2022

AGENDA

- What are number systems?
- Base of a number system
- Binary representation
  - Properties of a binary no.
  - Addition of binary numbers
- Bitwise operators
- An interesting problem :)

Q. CDX XVI + CXLIV = DLXX



Very difficult.

* Number systems are important

$$\begin{array}{r} {}^{1} \\ 426 \\ +\ 144 \\ \hline 570 \end{array}$$

(*) Expand 1397.

$1397 = 1000 + 300 + 90 + 7$

$1397 = 1 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 7 \times 10^0$

Expanding ang.

397

1397.

3197 =

Place value / Face value.

1 3 9 7 5

Face value → 9

Place value → 3000

Face value ? 9

Place value ? 900

$1397 = 1 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 7 \times 10^0$

1, 2, 3, 4, 5, ... 9, 0

$\alpha\beta\gamma\delta$

× 10 round no. / For easy calculation:

(ABCDE.... Z)

A - 1
B - 2
⋮
Z - 26
AA - 27

$(BCZ)_{26} = B \times 26^2 + C \times 26^1 + Z \times 26^0$

Base of a no. system = No. of symbols that I have in that no. system

1 - 1
2 - 2
3 - 3
4 - 4
5 - 5
6 → (11) → $1 \times 5^1 + 1 \times 5^0 = 6$

$(1307)_{25} = 1 \times 25^3 + 3 \times 25^2 + 0 \times 25^1 + 7 \times 25^0$

$(1307)_{25} = (17507)_{10}$

Octal no. system

↘ 8 symbols.

0, 1, 2, 3, 4, 5, 6, 7

$(78)_8$ = X

* Convert a no. from any no. system to decimal no. system.

$(abcde)_K = e \cdot K^0 + d \cdot K^1 + c \cdot K^2 + b \cdot K^3 + a \cdot K^4$

* $(02101)_3 = 0 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$

$= 54 + 9 + 0 + 1$

$= 64$

* $(131)_8 = 1 \times 8^2 + 3 \times 8^1 + 1 \times 8^0$

$= 64 + 24 + 1$

$= 89$

Computers.

Base 0 ? — No symbols!

Base 1 ?

0 ← 1	1 * ← 1
00 ← 2	11 ← 2
000 ← 3	111 ← 3
0000 ← 4	1111 ← 4
00000 ← 5	11111 ← 5
000000 ← 6	

× 1 million ← 000000 . . . .

Huge storage

Base 2 ?

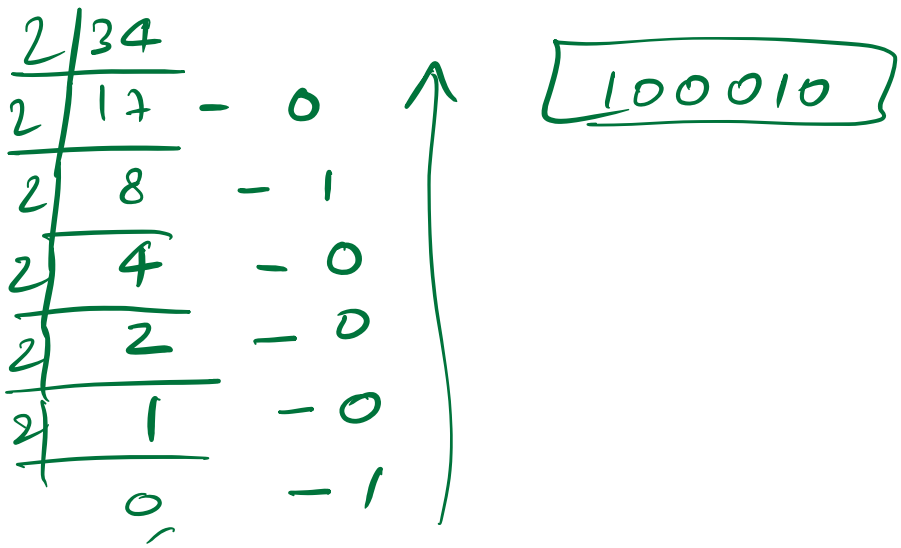
High	Low
~5 V	(~2.5 V)
On	Off
1	0
True	False

→ Base-2 no. system.

$(1011)_2 = 1\times2^3 + 1\times2^1 + 1\times2^0$

$= 8 + 2 + 1$

$= (11)_{10}$



$(1037)_{10} = (\ldots fedcba)_2$

5 4 3 2 1 0

a, b, c, d... are either 1 or 0.

$\Rightarrow 1037 = a \times 2^0 + b \times 2^1 + c \times 2^2 + d \times 2^3 + e \times 2^4 + \ldots$

$a \times 2^0$: either 1 or 0

$b \times 2^1 + c \times 2^2 + d \times 2^3 + e \times 2^4 + \ldots$: Divisible by 2

* Divide both sides by 2,

$$\frac{1037}{2} =$$

$1037 \% 2 = a \times 2^0$

$= a$

* $a = 1037 \% 2$

$1037 = a \times 2^0 + b \times 2^1 + c \times 2^2 + d \times 2^3 + \cdots$

a (0 or 1) $/2$

$\frac{1037}{2} = \frac{0 \text{ or } 1}{2} + b \times 2^0 + c \times 2^1 + d \times 2^2 + \cdots$

Quotient

$\frac{0 \text{ or } 1}{2} = 0$

$\frac{1037}{2}\ \% = b \times 2^0 + c \times 2^1 + d \times 2^2 + \cdots$

$b = \frac{1037}{2}\ \% 2$

$1037\ \%2$ ① $a = 1$

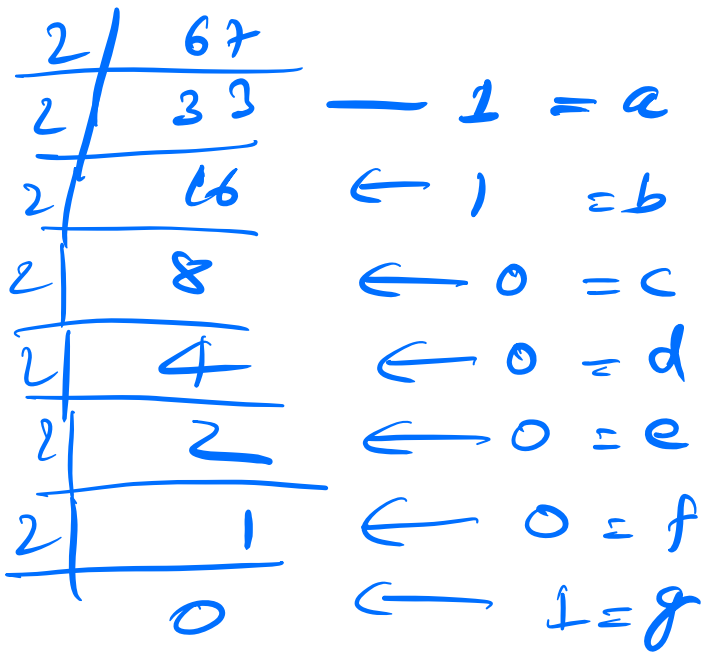
$\frac{1037}{2} = 518$

$518\ \%2$ $b = 0$

$\frac{518}{2} = 259$

$c = 1$

⋮



gfedcba

(1000011)

* Given a decimal no., return an array containing the binary representation.

```
// n
vector<int> vec(0);
while(n > 0){

    r = n % 2          front
    vec.push_back(r);
    n = n/2
} // Reverse the vector.
return vec.
```

* Decimal ⟵ Base 10 (Humans)
* Binary ⟵ Base 2 Computers!

1 0 0 1 1 0 0 0 1 0 0 1

* Hexadecimal ⟸ Base 16
* Octal ⟵ Base 8

How many no. using 4 bits ? = 16

0000 ⟵ 0
0001 ⟵ 1
0002 ⟵ 2
⋮
1111 ⟵ 15

10 ⟵ A
11 ⟵ B
12 ⟵ C
13 ⟵ D
14 ⟵ E
15 ⟵ F

5 15 14

5-15-14
5-1-5-1-4
5-1-5-14

4×6

#FF0083 ← color

R G B

R → 8 bit
G ← 8 bit
B ← 8 bit

00101100 00100000 00110011

#FFA01D

00000011110000

Numbers using 3 bits → 8

13

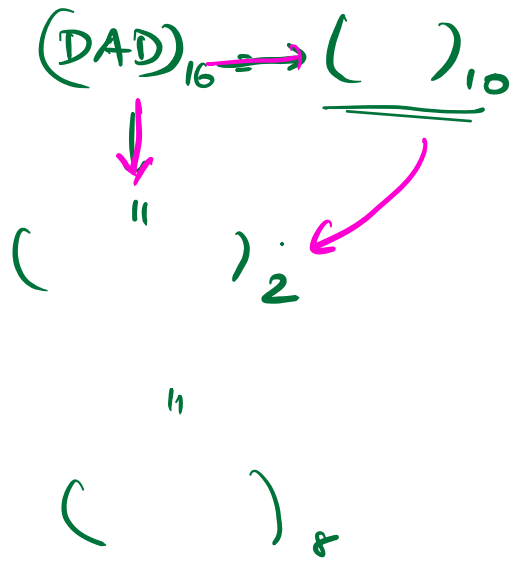
3 → 8
4 ← 16
5 ← 32

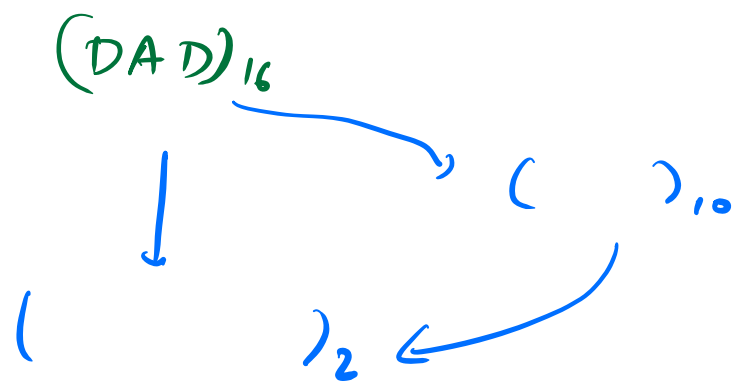
Break till 10:13 pm

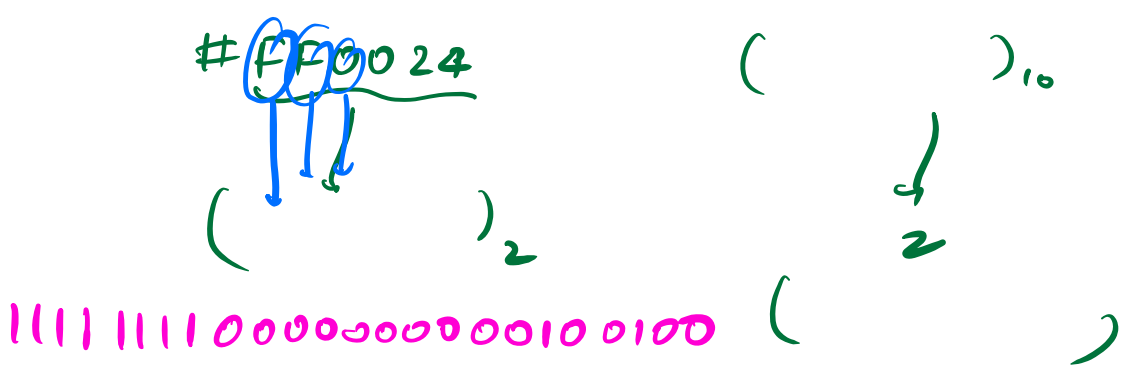
→ $(1010001101)_2 = (5065)_8$

5 0 6 5

0 → 0
1 → 1
2 → 2
3 ← 3
4 → 4
5 → 5
6 → 6
7 → 7
8 → 8
9 → 9
10 ← A
11 ← B
12 ← C
13 ← D
14 ← E
15 ← F







$(734)_8$

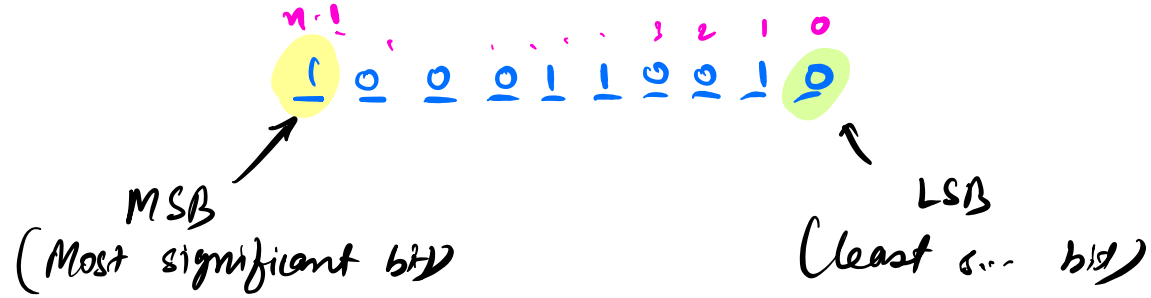
$= (111011100)_2$

$(DAD)_{16}$

$(110110101101)_2$

A - 10
11
12
D - 13

Properties of a binary no.



* In a n-digit binary no.?

Place value of MSB = 2^{n-1}

5023

5000 (5×10^3)

* In a N-digit binary no. whose MSB is 1 rest are 0?

what is the decimal value?

1000000....0

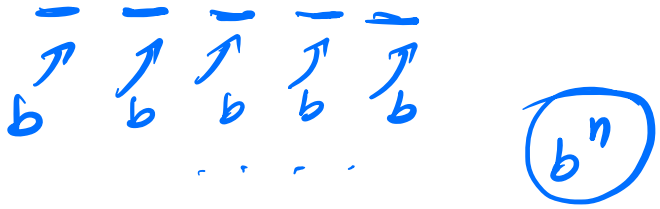
$= 2^{n-1}$

* You have a N digit no. in base 'b'. How many such nos. are possible?

5 digit no. in base 10?

→ 10^5

0 - 99999



32 bit nos. in base 2 ← 2^{32} such nos.

$0 \Rightarrow^{to} 2^{32}-1$

char
int
short
long

→ $0 \to 2^8-1$ (int)

$0 \to 2^{32}-1$ → int → 32 bits

$0 \to 2^{64}-1$ → long ← 64 bits

Addition

(1) 1
  1 3 7
+ 2 5 6
---
  3 9 3

1
1 3 7
2 5 6
---
3 9 3

7+6 = 13
13 % 10 = 3
13 / 10 = 1

Base 5:

  1 1
  1 3 4
(+) 1 2 3
---
  3 1 2

5+3+1 = 9

4+3 = 7
7 % 5 = 2
7 / 5 = 1

1+3+2 = 6
6 % 5 = 1
6 / 5 = 1

  1 1 1
  1 0 1 0 0 1 1 1 1
(+) 0 1 0 0 1 0 1 1 0
---
1 1 1 1 0 0 1 0 1

0+0 = 0
0+1 = 1
1+0 = 1
1+1 = 1 0
↑ ↑
carry forward

2 % 2 = 0
2 / 2 = 1

3 % 2 = 1
3 / 2 = 1

Quiz.

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 10011 \\ (+)\ 01001 \\ \hline 11100 \end{array}$$

Multiplication

(Ternary) → Base 3.

$$\begin{array}{r} 21121 \\ \times\ \ 2012 \\ \hline 12001 2 \\ 21121\,X \\ 0000\,XX \\ 12001\,22XXX \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 21121 \\ \hline 120012 \end{array}$$

$5/3 = 2$

$5\%3 = 1$

(*) $53 = 5\times10^1 + 3\times10^0$

530

$5\times10^2 + 3\times10^1 + 0\times10^0$

$53\,0$

Given, $(34)_{11} = (37)_{10}$

$(340)_{11} = 37 \times 11 = 407$

52
520 ×10

$34 \rightarrow 3\times11^1 + 4\times11^0$

$340 \rightarrow 3\times11^2 + 4\times11^1$

$101 =$

1010 ✓ Multiplied by 2

52
520 ← Multiplied by 10

$(101)_2$

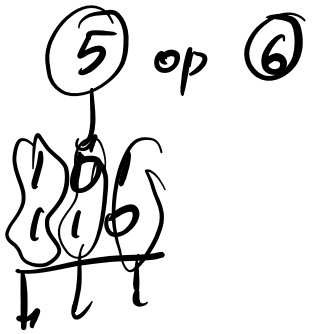
$(1010)_2$ ← Multiplied by 2.

Operators.

Decimal no. system → + , - , * , / , %

In binary, there are some more operators.

Bitwise operators → Acts on bit by bit.



Unary → ++, --

Binary. → 2 operands.
+ , - , * % , /

Bitwise operators.

① NOT (!, ~)

↳ flips the bit.

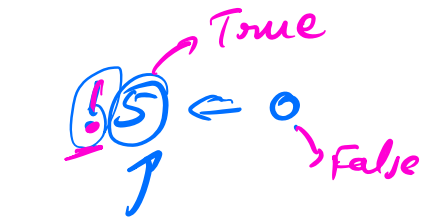
~0 = 1
~1 = 0

Unary

~5 = 2

~101

010



~5 = 2

↑
Bitwise

identity of an operator.

a + ? = a → 0

a * ? = a → 1

NOT has no identity.
identity is not defined.
Because it is unary.

OR

Either A or B.

x	y	x | y
0	0	0
0	1	1
1	0	1
1	1	1

a = 13 → 1101
b = 10 → 1010

1111

a | b = 15

Identity ?

a | ? = a

X a | 1 = a ← Is it always true ?

X

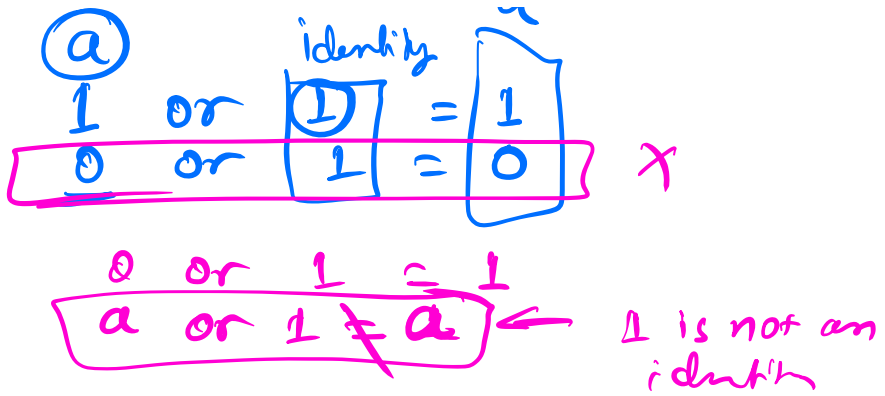
1 is not an identity.

0 | 1 = 1

Identity is 0.

a + ? = a

5 + 0 = 5
6 + 0 = 6
8 + 0 = 8



AND.

Both a and b.

x	y	x & y
0	0	0
0	1	0
1	0	0
1	1	1

a = 13
b = 10

```
 1101
 1010
------
 1000
```

a & b = 8

Identity of AND ?

a & ? = a

1 is an identity.

0 & 1 = 0
1 & 1 = 1

- **XOR**

↳ Exclusive or.
A or B but not both

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

a = 13
b = 10

a ^ b = 7.

```
  1101
^ 1010
------
  0111
```

Identity of XOR

$$a \wedge ? = a$$

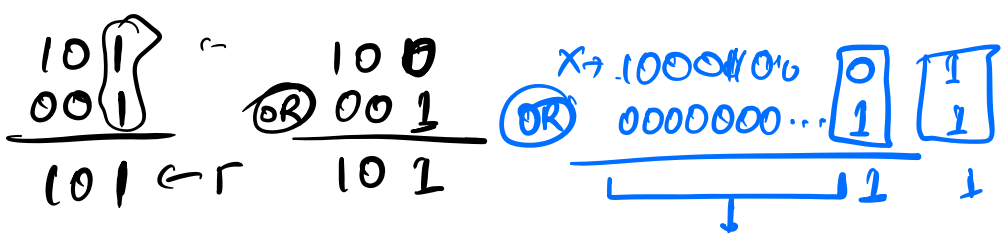
Identity of ^ is 0.

0 ^ 0 = 0
1 ^ 0 = 1

$$x \wedge x = 0$$

* XOR cancels itself out.

op →	OR	AND	XOR
0 op X	X	0	X
00001 op X	X → odd X+1 → even	1 → odd 0 → even	X+1 → even X−1 → odd
...111111... op X	...111111...	X	~X
X op X	X	X	0



{ If last bit is 1 → no. is odd,
" is 0 → no. is even.

a & 1010011100
00000000...1
00000000 1/or 0

1/0

(XOR)

1010 00110 . . .   0   1
0000000000   1   1

1010 00110   1   0

x+1 | x-1

1010100011 1
(OR)
1111111111111 1

1111 1111111111

X 10101000100001
111111111111111111
x 10101

x & 1 = x

0 ^ 1 = 1
1 ^ 1 = 0

→ if (x % 2 == 0) ← even
→ if (x & 1 == 1) ← odd

Q. You are given N numbers, where every no. appears twice, and one no. appears once. Find that no.

Doubt session

Programming languages have truthy and falsy values.

{ 1 → Truth
  0 → false

>1 → Truthy ∈ most languages.

NULL / None ∈ falsy ∈ most languages =

Check with SS.

$(DAD)_{16} = 13 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$

$= 3501$ ∈ Check.

1101 1010 1101

D A D

6655

0, 1, 2

$(34)_{11} = (37)_{10}$

$(52)_{10}$

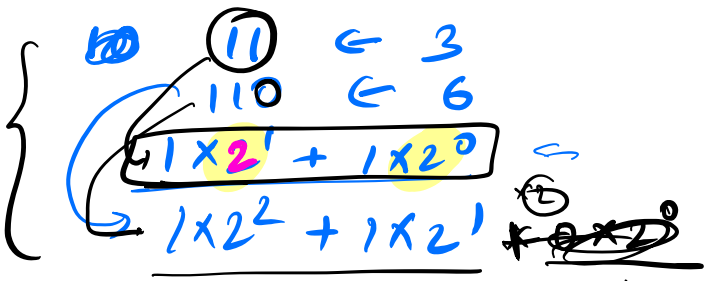
$(520)_{10}$

Multiplied by 10 (Base)

$(34)_{11} = (37)_{10}$

$(340)_{11} = 37 \times 11$

$= 407$



$4_{11} = (37)_{10}$

$3 \times 11^1 + 4 \times 11^0 = 37$

$(340)_{11}$

$3 \times 11^2 + 4 \times 11^1 = 3 \times 11$

1
01
01
01
11

(3)

3 % 2 = 1
3 / 2 = 1

1+1 = 10

+1

10
1
11

5
8
9
22

5+8 = 13
9
22

(22)

22 % 10 = 2
22 / 10 = 2