Homework 3

1)

$x = 7 \qquad p = 0.8 \qquad q = 0.2 \qquad n = 10$

$$B(x; n, p) = \binom{n}{x} p^x \cdot q^{(n-x)}$$

$$= \binom{10}{7} \cdot (0.8)^7 \cdot (0.2)^{(10-7)}$$

$$= {}^{10}C_7 \cdot (0.8)^7 \cdot (0.2)^3$$

$$= \frac{10!}{7! \cdot (10-7)!} \cdot (0.8)^7 \cdot (0.2)^3$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} \cdot (0.8)^7 \cdot (0.2)^3$$

$$= 120 \cdot 0.2907 \cdot 0.008 = \boxed{0.2013}$$

2) $n = 5 \qquad x = 3 \qquad p = 0.75$

$q = 0.25$

$$B(x; n, p) = \binom{n}{x} p^x q^{(n-x)} = \binom{n}{x} p^x (1-p)^{(n-x)}$$

$$= {}_5C_3 \times (0.75)^3 \times (0.25)^2$$

$$= \frac{5!}{3!(5-3)!} \times (0.75)^3 \times (0.25)^2$$

$$= \frac{5!}{3! \times 2!} \times (0.75)^3 \times (0.25)^2$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1} \times (0.75)^3 \times (0.25)^2$$

$$= 10 \times 0.4218 \times 0.0625$$

$= 0.2636$

3) $x = 50$ $\quad \lambda = \dfrac{\text{total no. of decays}}{\text{total no. of days}}$

$$P(x;\lambda) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

$$\frac{1000000 - 977287}{365} = \frac{22713}{365}$$

$$= 62.2273 \Rightarrow \lambda = 62.2273$$

$$P(50; 62.2273) = \frac{62.2273^{50} \times e^{-62.2273}}{50!}$$

$$= \frac{5.000\,896687 \times 10^{89} \times 9.64195603 \times 10^{-28}}{50!}$$

$$= \frac{4.72144282 \times 10^{62}}{50!} = \boxed{0.0155}$$

4) $\mu = 400$ days probability > 500 days

$$\lambda = \frac{1}{\mu} = \frac{1}{400} = 0.0025$$

$x = 500$

$$P(x > 500) = 1 - P(500)$$

$$= 1 - e^{-0.0025x}$$

$$= 1 - e^{-0.0025 \times 500}$$

$$= 1 - e^{-1.25}$$

$$= 0.7134$$

$$1 - 0.7134 = \boxed{0.286}$$