Homework - 9

1) i) $f(x,y) = x^2 + 2y^3$

$$\frac{df}{dx} = 2x \qquad \frac{df}{dy} = 6y$$

ii) $g(x,y) = 3xy - x^2y^2$

$$\frac{dg}{dx} = 3y - 2xy^2$$

$$\frac{dg}{dy} = 3x - 2x^2y$$

iii) $h(x,y,z) = x^3 + 2xyz - y^2z^3$

$$\frac{dh}{dx} = 3x^2 + 2yz^3$$

$$\frac{dh}{dy} = 2xz - 2yz^3$$

$$\frac{dh}{dz} = 2xy - 3y^2z^2$$

2)

x	0.5	2.3	2.9
y	1.4	1.9	3.2

initial slope - 1 # iterations - 3

initial intercept - 0 learning rate - 0.01

Cf - SSE $= \left(\Sigma (y\_pred)^2\right)$

$$f(x) = (y\_pred)^2 = (mx+b-y)^2$$

$$\frac{df}{dm} = 2(mx+b-y) \times x + 0 - 0$$

$$\frac{df}{dm} = 2(mx+b-y)x$$

$$\frac{df}{db} = 2(mx+b-y) \times 0+1-0$$

$$\frac{df}{db} = 2(mx+b-y)$$

iteration 1

$m = 1 \qquad b = 0$

X	y_pred = mx+b	Y
0.5	0.5	1.4
2.3	2.3	1.9
2.9	2.9	3.2

$$SSE_1 = \Sigma (y\_pred - y)^2$$
$$= (0.5 - 1.4)^2 + (2.3 - 1.9)^2 + (2.9 - 3.2)^2$$
$$= 1.06$$

$$\frac{df}{dm} = \Sigma 2 (y\_pred - y)\ x$$
$$= 2(0.5 - 1.4) \times 0.5 + 2(2.3 - 1.9) \times 2.3 + 2(2.9 - 3.2) \times 2.9$$
$$= -0.8$$

$$m_1 = m_0 - \alpha \times \frac{df}{dm}$$
$$= 1 - 0.01(-0.8)$$
$$= 1.008$$

$$\frac{df}{db} = \sum 2(y\_pred - y)$$

$$= 2(0.5 - 1.4) + 2(2.3 - 1.9) + 2(2.9 - 3.2)$$

$$= -1.6$$

$$b_1 = b_0 - \alpha \times \frac{df}{db}$$

$$= 0 - 0.01 \times (-1.6)$$

$$= 0.016$$

Iteration 2

$m = 1.008$ $b = 0.016$

x	y_pred	y
0.5	0.52	1.4
2.3	2.3364	1.9
2.9	2.9392	3.2

$$SSE_2 = \sum (y\_pred - y)^2$$

$$= (0.52 - 1.4)^2 + (2.33 - 1.9)^2 + (2.93 - 3.2)^2$$

$$= 1.03$$

$$\frac{df}{dm} = \sum 2(y\_pred - y)\, x$$

$$= 2(0.52 - 1.4)0.5 + 2(2.3344 - 1.9)2.3$$
$$+ 2(2.9392 - 3.2)2.9$$
$$= -0.3944$$

$$m_1 = m_0 - \alpha \times \frac{df}{dm}$$
$$= 1.008 - 0.01(-0.3944)$$
$$= 1.011944$$

$$\frac{df}{db} = \Sigma(Y\_pred - Y)$$
$$= 2(0.52 - 1.4) + 2(2.3344 - 1.9)$$
$$+ 2(2.932 - 3.2)$$
$$= -1.4128$$

$$b_1 = b_0 - \alpha \times \frac{df}{db}$$
$$= 0.016 - 0.01 \quad (-1.4128)$$
$$= 0.030128$$

Iteration 3

$m = 1.011964$ $\quad b = 0.030128$

x	Y_pred	y
0.5	0.5361	1.4
2.3	2.3575992	1.9
2.9	2.9647656	3.2

$$SSE_2 = \Sigma (Y\_pred - Y)^2$$
$$= (0.536 - 1.4)^2 + (2.358 - 1.9)^2 + (2.965 - 3.2)^2$$
$$= 1.011485$$

$SSE_1 = 1.06$

$SSE_2 = 1.03$

$SSE_3 = 1.011$

SSE is decreasing by every iteration