EC204	**Principles of Communication Systems**	**Even 2020**
Lab 2	**Signals and Systems**	

1. Continuous-time Signals : Sketch the following continuous-time signals in separate figures over the specified interval. For a complex valued signal, sketch magnitude and phase in the same figure separately.

   (a) For $t \in [-10\pi \quad 10\pi]$

   $$\begin{aligned} x(t) &= \frac{\sin(\pi t)}{\pi t} \quad \text{for} \quad t \neq 0 \\ &= 1 \qquad\quad \text{for} \quad t = 0 \end{aligned}$$

   (b) $x(t) = \prod(t)$ for $t \in [-2 \quad 2]$. The rect function is defined as $\prod(t) = 0$ for $|t| > 1/2$ and 1 for $|t| < 1/2$.

   (c) For 5 time periods

   $$x(t) = A_1 \cos(2\pi f_1 t + \theta_1) + A_1 \cos(2\pi f_2 t + \theta_1)$$

   Take $A_1 = 1$,$A_2 = 2$,$f_1 = 100$ Hz,$f_2 = 500$ Hz,$\theta_1 = \pi/6$ and $\theta_2 = \pi/4$.

   (d) $x(t) = u(t) - u(t-4)$ for $t \in [0 \quad 10]$ where $u(t)$ is the unit step signal.

   (e) $x(t) = (0.5)^t e^{j\pi t/2}$ for $t \in [-10 \quad 10]$

   You may use the following MATLAB functions figure, plot, subplot, sin, cos, find, abs, angle etc.

2. Discrete-time Signals and Systems

   (a) Generate a discrete-time signal $x[n] = \sin(2\pi n/50)$ for $0 \leq n < 100$. Sketch the sequence in figure 1.

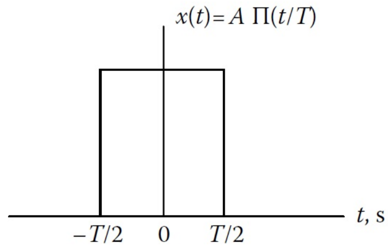

Figure 1: Figure for Problem 3a

(b) Generate a discrete-time signal $x[n] = n[u(n) - u(n-10)] - (20-n)[u(n-10) - u(n-20)]$. Sketch the sequence in figure 2.

(c) Write a function $y = \textbf{mysystem}(x)$ to implement the following discrete-time system

$$y(n) = x(n) - x(n-1)$$

Explain whether the system is linear, time-invariant, causal and stable.

(d) Find the output of the system by applying the signal generated in (a) to the system implemented in (c). Plot the input and output sequence in the same figure 3.

(e) Find the output of the system by applying the signal generated in (b) to the system implemented in (c). Plot the input and output sequence in the same figure 4.

(f) Compare the input and output signals in (d) and (e). What can you say about the functionality of the system implemented in (c) ?

You may use the following MATLAB functions figure,stem,subplot,sin etc.

3. Continuous-time Fourier transform (CTFT) : Consider a finite energy signal $x(t)$. The CTFT of the signal $x(t)$ is computed as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

(a) Find the analytical expression of the CTFT $X(f)$ of the continuous-time signal $x(t) = A\prod(t/T)$ as shown in Fig. 1. The rect function is defined as $\prod(t/T) = 0$ for $|t| > T/2$ and 1 for $|t| < T/2$. Take $A = 1, T = 4$.

(b) Sketch the magnitude $|X(f)|$ and phase spectrum $\angle X(f)$ of $x(t)$ in a single figure 1. Consider suitable range of frequencies (say -5 Hz to +5 Hz) on x-axis.

(c) Write a function $y = \mathbf{myinteg}(x, f, a, b)$ to numerically evaluate a definite integral using Trapezoidal Rule.

$$y = \int_a^b f(x)dx$$

where $f(x)$ is a complex valued function in general. Test your function for $f(x) = \sin^2(x)$ with $a = 0, b = \pi$.

(d) Compute the CTFT $X(f)$ of $x(t)$ by using the function implemented in (b). Sketch the magnitude $|X(f)|$ and phase spectrum $\angle X(f)$ of $x(t)$ in a single figure 2. Compare the result with that of (b).

(e) Now, consider the following infinite length signal

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

with $f_1 = 100$ Hz and $f_2 = 500$ Hz. Trunctae this signal for $0 \le t \le 0.05$ sec. Approximate the CTFT of $x(t)$ using the function implemented in (b). Sketch the magnitude $|X(f)|$ and phase spectrum $\angle X(f)$ of $x(t)$ in a single figure 3. Did you get the expected result ? If not then why ? Explain.

You may use the following MATLAB functions figure, plot, subplot, sin, cos, angle, abs, trapz etc.

4. Discrete-time Fourier transform (DTFT) : The discrete-time Fourier transform (DTFT) of a discrete-time signal $x[n]$ is defined as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

where $u[n]$ is the unit step sequence. In general $X(e^{j\omega})$ is a complex function of the real variable ω and can be written as $X(e^{j\omega}) = |X(e^{j\omega})|e^{\angle X(e^{j\omega})}$. The quantity $|X(e^{j\omega})|$ is called the magnitude spectrum and the $\angle X(e^{j\omega})$ is called the phase spectrum respectively. The DTFT $X(e^{j\omega})$ is a periodic and continuous function in ω with a period 2π. The inverse DTFT is given by

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

If $x(n)$ is of infinite duration, then MATLAB cannot be used directly to compute $X(e^{j\omega})$ from $x(n)$. However, we can use it to evaluate the expression $X(e^{j\omega})$ over $[0,\ \pi]$ frequencies and then plot its magnitude and phase. If $x(n)$ is of finite duration, then MATLAB can be used to compute $X(e^{j\omega})$ numerically at any frequency ω. The approach is to implement the analytical equation directly.

The DTFT is primarily useful for theoretical analysis but is not practical for numerical computation of signal spectrum of the discrete-time signal. This is because DTFT requires infinite sum computation and is a continuous function of discrete-time frequency ω. DFT is very often used as a practical approximation to the DTFT.

(a) Consider the discrete-time signal

$$x[n] = (0.5)^n u[n]$$

Does the DTFT exists for this sequence ? If yes, then find the analytical expression of the DTFT.

(b) Evaluate $X(e^{j\omega})$ in (a) at 501 equidistanced frequencies between $[0, \pi]$ and plot its magnitude and phase spectra.

(c) Consider a finite duration discrete-time signal $x[n] = \{1, 2, 3, 4, 5\}$. Numerically find the DTFT of the sequence at 501 equidistanced frequencies between $[0, \pi]$. You may use the following MATLAB functions figure,plot,subplot,angle,abs,sum etc.