Elliptic flow analysis with non-hydro mode in viscous hydrodynamics

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- Non-abelian gauge theory of strong interactions.

QCD lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \, i \, \gamma^{\mu} \left(\partial_{\mu} \delta_{ij} - ig \, (T_a)_{ij} \, \mathcal{A}^a_{\mu} \right) \psi_j - m \, \bar{\psi}_i \psi_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \tag{1}$$

where,

γ – Dirac matrices
λ_a – Gell-Mann matrices($a = 1 \ldots 8$)
$G^a_{\mu\nu}$ – gauge invariant gluon field strength tensor
$\mathcal{A}^a_{\mu}$ – 8 Gluon fields
ψ_i – Quark field

- Color charge is conserved. This local symmetry is governed by SU(3) guage group.
- Strong interaction does not discriminate between flavors – approximate flavor symmetry – also follows SU(3) group.
- Quarks are never observed in free state below Hagedorn temperature – Color Confinement

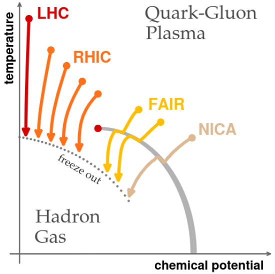
Open questions in QCD

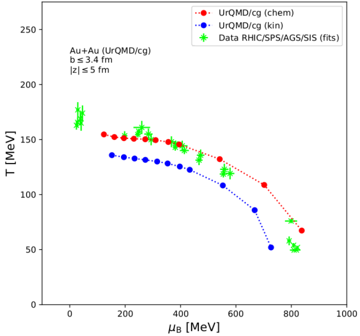
Mapping out QCD phase diagram:
What are the phases of strongly interacting matter?
Location of critical point ?

What are the equilibrium and transport properties of quark-gluon plasma ?

Onset of deconfinement:
as a function of temperature and chemical potentials?
as a function of relativistic heavy-ion collision energy and system size?

Eur. Phys. J. A 56:267 (2020)
Rep. Prog. Phys. 84 056301 (2021)





Müller, Israel and Stewart (MIS) 2^{nd} order Hydrodynamics (1/2)

- Expt data provide strong support for hydrodynamics as the appropriate effective theory for RHIC.
- Relativistic conservation laws: , $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu N^\mu = 0$.
- The local values of temperature, $\mathcal{T}(x)$, fluid velocity, $u_\mu(x)$ and chemical potential, $\mu(x)$ are the chosen hydrodynamic variables.

Energy-momentum tensor decomposition

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + (w^\mu u^\nu + w^\nu u^\mu) + \Pi^{\mu\nu} \qquad (2)$$

w^μ – Transverse vector coefficient
$\Delta^{\mu\nu} \equiv (g^{\mu\nu} + u^\mu u^\nu)$ – Projector operator orthogonal to the fluid velocity, u^μ
ϵ – Energy density
P – Pressure
$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$ – Tensor introduced to account for the dissipative effects
Π – Bulk pressure
$\pi^{\mu\nu}$ – Shear stress tensor

Inter. Journal of Modern Physics, 19 1 pp.1-53 (2010)

Müller, Israel and Stewart (MIS) 2^{nd} order Hydrodynamics (2/2)

Shear stress tensor for MIS hydro

$$\pi^{\mu\nu} = -\eta\Big(2\sigma^{\mu\nu} + \frac{4}{3}\frac{\tau_\pi}{\eta} d_\mu u^\mu \pi^{\mu\nu} + \frac{\tau_\pi}{\eta}\Delta^\mu_\alpha \Delta^\nu_\beta D\pi^{\alpha\beta} + \frac{\lambda_0}{\eta}\tau_\pi(\pi^{\mu\lambda}\Omega^\nu_\lambda + \pi^{\nu\lambda}\Omega^\mu_\lambda)\Big)$$

Here, λ_0 is a scalar coefficient and Ω is a traceless, anti-symmetric, transverse vorticity tensor. d_μ is the covariant derivative, $D = u^\mu d_\mu$, is the comoving time derivative.

Bulk pressure for MIS hydro

$$\Pi = -\zeta\left(d_\mu u^\mu + \frac{\tau_\Pi}{\zeta} u^\alpha d_\alpha \Pi + \frac{4}{3}\frac{\tau_\Pi}{\zeta}\Pi \; d_\mu u^\mu\right) \tag{3}$$

The value of τ_Π, λ_0, τ_π, η, ζ has to be provided for solving the above two equations.

τ_Π and τ_π are the bulk viscosity and shear relaxation times, representing how quickly the above 2^{nd} order form of Π and $\pi^{\mu\nu}$ relaxes to its leading-order forms.

Eur. Phys. J. C (2013) 73:2524

Non-hydro modes

If the dynamics show quasi-universality at large time scale, one can derive hydrodynamics from it.

Energy momentum tensor for microscopic theory in a non-equilibrium state

$$T^{\mu\nu} = \langle \hat{T^{\mu\nu}} \rangle_{eq} + \delta \langle \hat{T^{\mu\nu}} \rangle \quad (4)$$

where,

first term – equilibrium values of $T^{\mu\nu}$
second term – deviations from that equilibrium value.

This second term is given under linear response theory in terms of the retarded $2-$point correlator of $T^{\mu\nu}$.

This correlator when expressed in the Fourier space, has singularities in ω-plane. This is the **quasinormal mode**.

2018 Rep. Prog. Phys. 81 046001

Quasinormal modes

Quasinormal frequency

$$\omega = \omega_h + i\,\omega_{nh} \tag{5}$$

where,

ω_h – excitation frequency for equilibrium plasma and is called a hydrodynamic mode.

ω_{nh} – transient mode or non-hydrodynamic mode frequency and is associated with the dissipative effects.

Transient mode disrupts hydrodynamization and is controlled with the relaxation time parameter.

Initial conditions for hydrodynamics

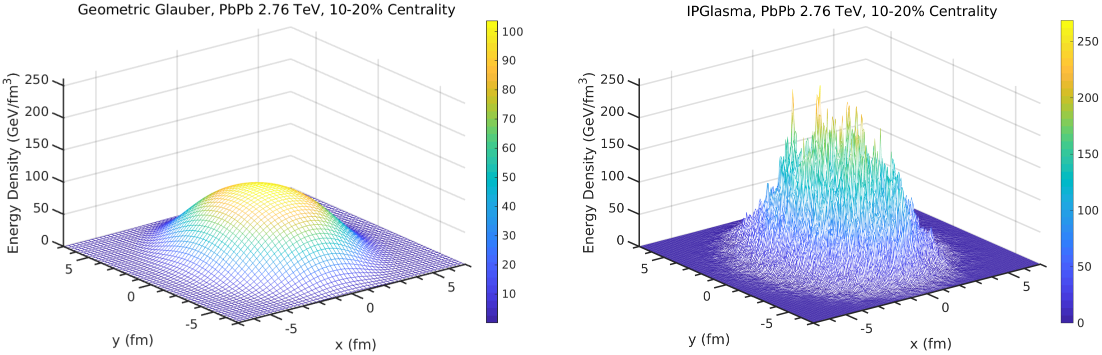
IPGlasma

- Effective theory wavefunction of the high energy nuclei/hadron – **Color Glass Condensate**.
- The distribution of nucleons acts as sources of initial state fluctuations.
- Gaussian sampled color charges act as a source for gluon fields which are evolved using classical Yang-Mills equations.
  PRC 86, 034908 (2012), PRL 108, 252301 (2012)

Glauber

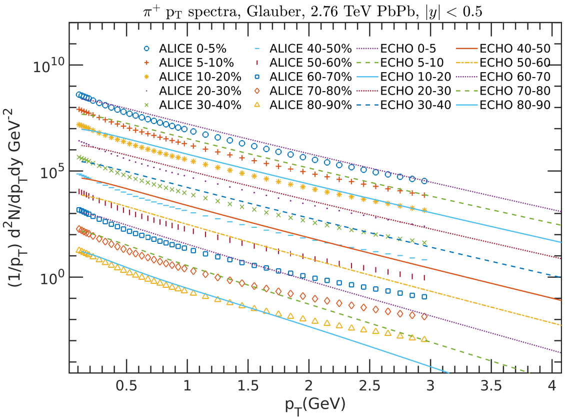
- Glauber assumes nucleons being distributed with Wood-Saxon profile and having independent linear trajectories.
- Wood-Saxon distribution has a smooth plateau for nucleus which decays softly towards the edges.
  Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243

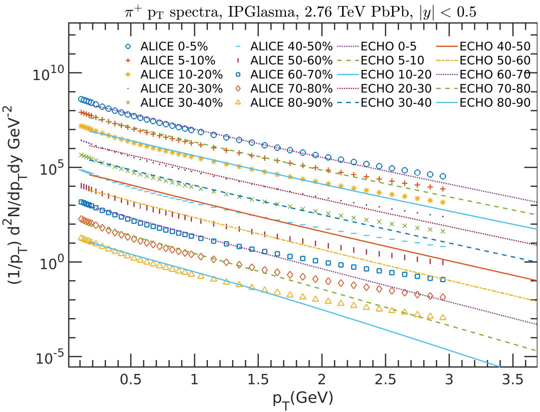
Initial energy density plots



- Notice IPGlasma profile being extremely jagged and spiky, whereas Glauber initial condition is quite smooth.
- Two extreme limits of the initial state fluctuation viz. very high and very low spatial eccentricities.

Pion p_T spectra plots





- Slightly better match for IPGlasma than Glauber.
- Spectra comply to experimental values only in low p_T regime, where the hydrodynamic mode operates.
  Phys. Rev. C 93, 034913 (2016)

Thermodynamic eccentricities

Spacial eccentricity:

$$e_c = \frac{\int dx\ dy\ [(x - x_0)^2 - (y - y_0)^2]}{\int dx\ dy\ [(x - x_0)^2 + (y - y_0)^2]} \tag{6}$$

Momentum eccentricity:

$$e_p \equiv \frac{\int d^2x_\perp (T^{xx} - T^{yy})}{\int d^2x_\perp (T^{xx} + T^{yy})} \tag{7}$$

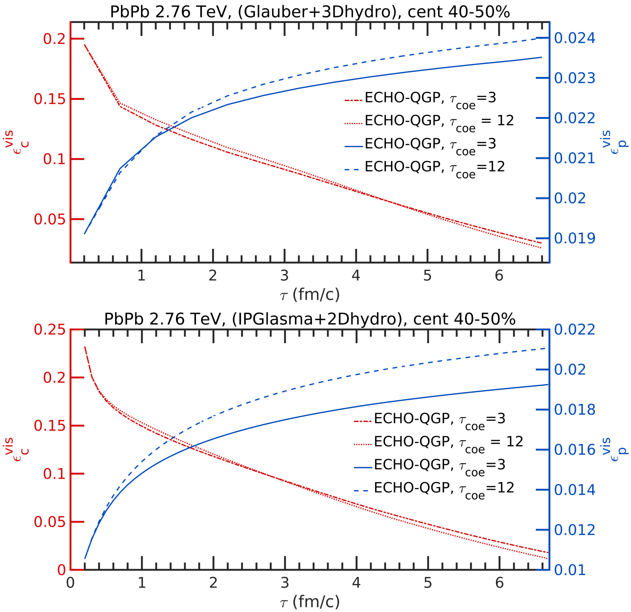
$$e_p \equiv \frac{\int d^2x_\perp (\epsilon + P)\,(u^x u^x - u^y u^y) + \pi^{xx} + \pi^{yy}}{\int d^2x_\perp [(\epsilon + P)\,(u^x u^x + u^y u^y) + 2P + \pi^{xx} + \pi^{yy}]} \tag{8}$$

$(\pi^{xx} + \pi^{yy})$ term has been added as viscous corrections to ideal hydrodynamic momentum eccentricity.

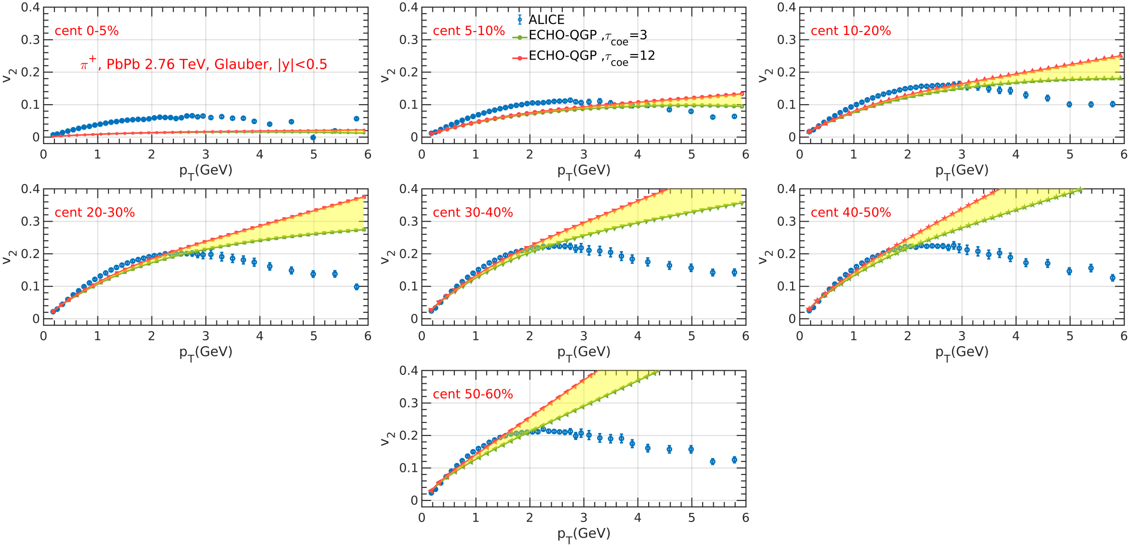
Eur. Phys. J. C (2015) 75 429

Eccentricity during hydrodynamic evolution

- e_c vaguely represents initial state fluctuations.
- Initial state fluctuations leads to momentum anisotropy in the system, which translates to larger collective flow.
- Notice the larger e_c for IPGlasma initial condition.
- Also notice larger change in e_p due to increase in relaxation time.

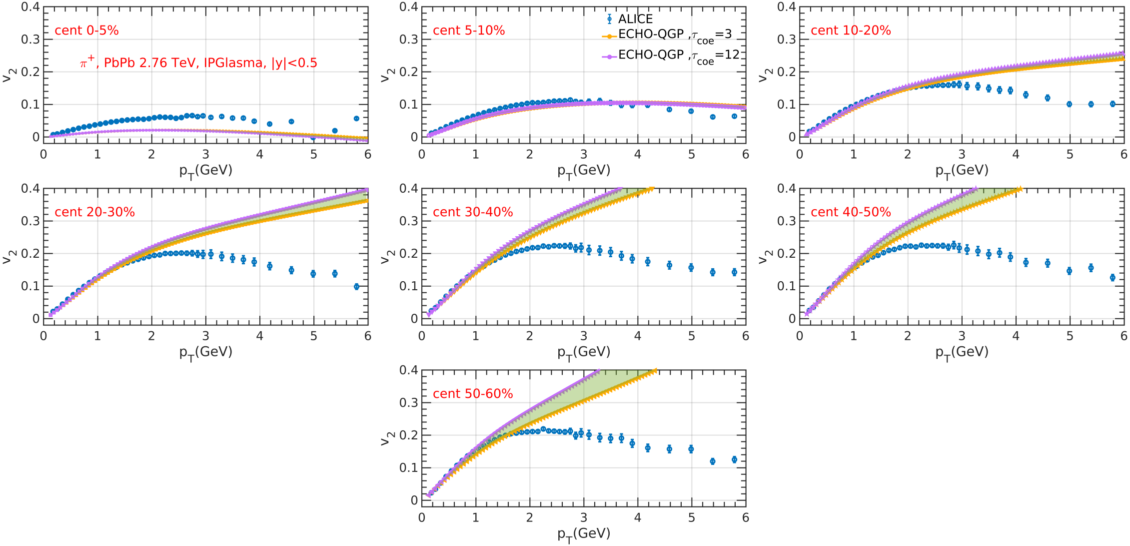


Effect of non-hydro mode in Glauber elliptic flow



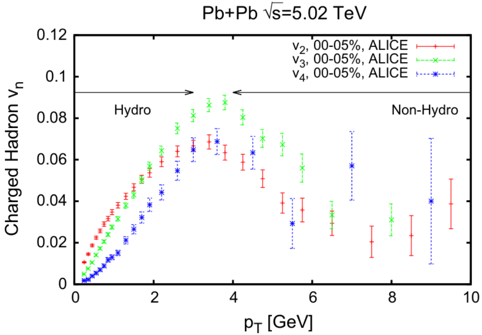
Shaded area represents increase in flow due to the use of the larger relaxation time.

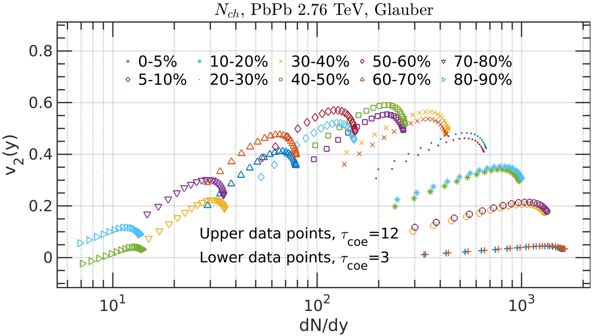
Effect of non-hydro mode in IPGlasma elliptic flow



J. High Energ. Phys. 2015, 190 (2015)

Elliptic flow vs $dN/d\eta$ plot





- P. Romatschke suggested that abrupt change in elliptic flow(v_2) for variation in relaxation times at low multiplicity could potentially indicate breakdown of the hydrodynamics.
- Low p_T hydro regime where hydrodynamic results show agreement with data.

P. Romatschke, Eur. Phys. J C 77, 1 (2017)

Conclusion

- Even the most peripheral PbPb, 2.76 TeV collisions are within the hydrodynamic regime.
- Large value of relaxation time leads to increase in elliptic flow for MIS hydrodynamic theory especially at high p_T.
- We see a larger flow increment in the Glauber model than in IPGlasma model.
- Elliptic flow in the most central collisions is not affected by the non-hydro mode structure of the theory. For increasing impact parameter, the non-hydro mode participation also increases. Could we compare flow from different models at the most central collisions without worrying about non-hydro mode structure of the theory? caveat!
- Initial state fluctuations have a noticeable effect on the elliptic flow.

Thank you