Prioritization Of Receivers For Minimum Possible Error Boundary In Time Difference Of Arrival Algorithm

SAURAV KAITY[1] AND NIKHIL KUMAR SINGH[2]

[1] Scientist- E, CDP, DRDO(ITR), CHANDIPUR
[2] Department of Electronics and Electrical Communication Engineering, IIT Kharagpur

Compiled June 27, 2018

ABSTRACT: This paper describes a research and development approach to problem of position estimation from Time Difference Of Arrival (TDOA) measurements that occurs in a range of applications from wireless communication networks to electronic warfare positioning. Correlation analysis of the transmitted signal to two receivers gives rise to one hyperbolic function. With more than two receivers, we can compute more hyperbolic functions, which ideally intersect in one unique point. With TDOA measurement uncertainty, we face a non-linear estimation problem. We here present an approach to prioritize location of receivers in order to minimize error boundary between true and estimated values of transmitter locations.

1. INTRODUCTION

Position information brings tremendous advantages to some genuine applications going from cargo tracking, tourist guiding, emergency evacuation, to endless use situations. As cell phones wind up universal, contextual awareness applications have turned out to be prominent, and the indoor positioning system has gained huge consideration. The time-based localization method, including one- way time of arrival (TOA) and time difference of arrival (TDOA), exploits the fine delay resolution property of wideband signals and has extraordinary potential for giving high precision area estimation. Be that as it may, the two strategies confront a noteworthy test, that is, synchronization is required among the clocks of the included nodes with a timing accuracy corresponding to the desired localization accuracy.

Estimation of the location of the source of a signal (e.g., transmitting radio, acoustic or optic signs, and so on.) has been a subject of research for a considerable length of time and keeps on accepting much enthusiasm for the signal processing research community, including radar (Bahl and Padmanabhan, 2000 and Yan et al., 2007), sonar (Carter, 1981; Leonard and Durrant-Whyte, 1991; and Leonard and Durrant-Whyte, 2012), mobile communications (Caffery, 2000; Gustafsson and Gunnarsson, 2005; and Caffery and Stuber, 1998), multimedia (Brandstein and Silverman, 1997; Wang and Chu,1997; and Akyildiz et al., 2007), animal tracking (Spiesberger and Fristrup, 1990), remote sensor systems (WSNs; Akyildiz et al., 2002; Chen et al.,2002; Patwari et al., 2005; and Mao et al., 2007), and the Global Positioning System (GPS; Hofmann-Wellenhof et al., 2013). Be that as it may, vulnerabilities coming about because of the tweak of a signal propagated through an inhomogeneous medium and in estimations of received signs can rapidly corrupt position estimation precision when utilizing standard techniques to assess transmitter position (Sayed et al., 2005; Güvenç and Chong, 2009; and Tan et al., 2011). Verifiably, estimation of the highlights of received signals required for contribution to position estimation calculations has depended on four strategies: time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and received signal strength (RSS).

In this paper, we analyze various algorithms and attempt at prioritizing location of receivers for minimizing error of boundary between true and estimated coordinates of transmitters.

2. OVERVIEW OF DIFFERENT METHODS

The Taylor-series strategy linearizes the set of equations in (3) by Taylor-series expansion, at that point utilizes an iterative strategy to understand the system of straight equations. The iterative technique starts with an underlying assumption and enhances the estimate at every iteration by deciding thelocal linear least-square solution. The Taylor-series can give exact outcomes and is vigorous. It can likewise make utilization of repetitive estimations to enhance the position localization arrangement. Be that as it may, it requires a good initial guess and can be computationally expansive. For most circumstances, linearization of the nonlinear equations does not present undue blunders in the position location evaluation.
For discretionarily put base stations and a steady systems of equations in which the number of euations measures up to the number of unknown source coordinates to be solved, Fang [1] gives a correct answer for the equations of (3). Be that as it may, his answer does not make utilization of excess estimations made at extra collectors to improve position location accuracy. Moreover, his strategy encounters an uncertainty issue because of the inborn squaring operation. These ambiguities can be settled utilizing from the earlier data or using symmetry properties. Dissimilar to the calculations said beforehand, this strategy gives a closed form and exact solution and it is moreover computationally less intensive than the Taylor-series method.
A non-iterative solution for the hyperbolic position estimation issue which is fit of accomplishing ideal execution for discretionarily set sensors was proposed by Chan [5]. The solution is in closed form and substantial for both distant and close sources. At the point when TDOA estimation mistakes are little, this strategy is an estimation to the maximum likelihood estimator. It gives an explicit solution form that isn't accessible in the Taylor-series method. It is additionally superior to Fang's strategy as it can exploit. repetitive estimations like the Taylor-series method. In any case, it needs from the earlier data to determine an ambiguity in its estimations like the Fang's strategy.

3. DERIVATIONS

When an electromagnetic wave occurs at a transmitter, it arrives at a set of different receivers at different times.By exploiting the differences in the arrival time of the sound to the receivers, TDOA locates the source of the sound. As a function, it takes a set of receivers signals as input and returns the coordinates of the source relative to the receivers array. To elaborate, let us define a set of receivers and signal source, with R being receivers and T being transmitter

$$R_i = [x_i, y_i, z_i]^T fori = 1, 2, \cdots, N \tag{1}$$

where N denotes the number of receivers

$$T = [x_0, y_0, z_0]^T \tag{2}$$

The range distances from each receiver to transmitter is given by

$$r_i = \sqrt[2]{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} \tag{3}$$

Now the time difference of arrival between receiver i and receiver j can be expressed as

$$t_{ij} = \frac{r_j - r_i}{c} \tag{4}$$

$j > i$, and c being velocity of light. Using nonlinear regression, this equation can be converted to the form of a hyperbola. Once enough hyperbolas have been calculated, the position of the transmitter can be calculated by finding the intersection.
Let $x_i(t)$ and $x_j(t)$ be signals received at R_i and R_j respectively. One way to calculate $t_i j$ is given by means of standard cross-correlation function.

$$R_{x_i x_j}(\tau) = E[x_i(t) x_j^*(t - \tau)] \tag{5}$$

We would require four equations to obtain a solution here, which can be obtained by expressions of r_i, r_j, r_k, r_l.

$$r_i - r_j = r_{ij} \tag{6}$$

$$r_i - r_k = r_{ik} \tag{7}$$

$$r_k - r_j = r_{kj} \tag{8}$$

$$r_k - r_l = r_{kl} \tag{9}$$

Where,

$$r_{ij} = \sqrt[2]{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} - \sqrt[2]{(x_0 - x_j)^2 + (y_0 - y_j)^2 + (z_0 - z_j)^2} \tag{10}$$

$$r_{ik} = \sqrt[2]{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} - \sqrt[2]{(x_0 - x_k)^2 + (y_0 - y_k)^2 + +(z_0 - z_k)^2} \tag{11}$$

$$r_{kj} = \sqrt[2]{(x_0 - x_k)^2 + (y_0 - y_k)^2 + (z_0 - z_k)^2} - \sqrt[2]{(x_0 - x_j)^2 + (y_0 - y_j)^2 + (z_0 - z_j)^2} \tag{12}$$

$$r_{kl} = \sqrt[2]{(x_0 - x_k)^2 + (y_0 - y_k)^2 + (z_0 - z_k)^2} - \sqrt[2]{(x_0 - x_l)^2 + (y_0 - y_l)^2} + (z_0 - z_l)^2 \tag{13}$$

The four equations given above can be solved and rearranged to obtain two plane equations as follows:

$$y = Ax + Bz + C \tag{14}$$

$$y = Dx + Ez + F \tag{15}$$

Where,

$$A = \frac{R_{ik}x_{ji} - R_{ij}x_{ki}}{R_{ij}y_{ki} - R_{ik}y_{ji}} \tag{16}$$

$$B = \frac{R_{ik}z_{ji} - R_{ij}z_{ki}}{R_{ij}y_{ki} - R_{ik}y_{ji}} \tag{17}$$

$$C = \frac{R_{ik}[R_{ij}^2 + x_i^2 - x_j^2 + y_i^2 - y_j^2 + z_i^2 - z_j^2] - R_{ij}[R_{ik}^2 + x_i^2 - x_k^2 + y_i^2 - y_k^2 + z_i^2 - z_k^2]}{2[R_{ij}y_{ki} - R_{ik}y_{ji}]} \tag{18}$$

$$D = \frac{R_{kl}x_{jk} - R_{kj}x_{lk}}{R_{kj}y_{lk} - R_{kl}y_{jk}} \tag{19}$$

$$E = \frac{R_{kl}z_{jk} - R_{kj}z_{lk}}{R_{kj}y_{lk} - R_{kl}y_{jk}} \tag{20}$$

$$F = \frac{R_{kl}[R_{kj}^2 + x_k^2 - x_j^2 + y_k^2 - y_j^2 + z_k^2 - z_j^2] - R_{kj}[R_{kl}^2 + x_k^2 - x_l^2 + y_k^2 - y_l^2 + z_k^2 - z_l^2]}{2[R_{kj}y_{lk} - R_{kl}y_{jk}]} \tag{21}$$

Solving equations 14 and 15, gives a linear equation for x in terms of z,

$$Ax + Bz + C = Dx + Ez + F \tag{22}$$

$$x = Gz + H \tag{23}$$

Where,

$$G = \frac{E - B}{A - D} \tag{24}$$

$$H = \frac{F - C}{A - D} \tag{25}$$

Substituting equation 23 back into 14, gives a linear equation for y in terms of z,

$$y = Iz + j \tag{26}$$

Where,

$$I = AG + B \tag{27}$$

$$J = AH + C \tag{28}$$

Equations 23 and 26, if substituted back into equation 11, gives

$$K = R_{ik}^2 + x_i^2 - x_k^2 + y_i^2 - y_k^2 + z_i^2 - z_k^2 + 2x_{ki}H + 2y_{ki}J \tag{29}$$

And,

$$L = 2[x_{ki}G + y_{ki}I + 2z_{ki}] \tag{30}$$

$$M = 4R_{ik}^2[G^2 + I^2 + 1] - L^2 \tag{31}$$

$$N = 8R_{ik}^2[G(x_i - H) + I(y_i - J) + z_i] + 2LK \tag{32}$$

$$O = 4R_{ik}^2[(x_i - H)^2 + (y_i - J)^2 + z_i^2] - K^2 \tag{33}$$

Final solution:

$$z = \frac{N}{2M} + \sqrt[2]{(\frac{N}{2M})^2 - \frac{O}{M}} \tag{34}$$

$$y = Iz + J \tag{35}$$

$$x = Gz + H \tag{36}$$

4. SIMULATIONS

We wrote a small script to generate all possible combinations of receiver locations with each combination consisting of exactly 4 receiver locations. We ran simulations for each combinations and observed results.

Figures and Tables should be labelled and referenced in the standard way using the `\label{}` and `\ref{}` commands.

A. Sample Figure

Figure 1 shows an example figure.

B. Sample Table

Table 1 shows an example table.

Table 1. Shape Functions for Quadratic Line Elements

local node	$\{N\}_m$	$\{\Phi_i\}_m$ $(i = x, y, z)$
$m = 1$	$L_1(2L_1 - 1)$	Φ_{i1}
$m = 2$	$L_2(2L_2 - 1)$	Φ_{i2}
$m = 3$	$L_3 = 4L_1L_2$	Φ_{i3}

5. SAMPLE EQUATION

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i}^{n} X_i \tag{37}$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

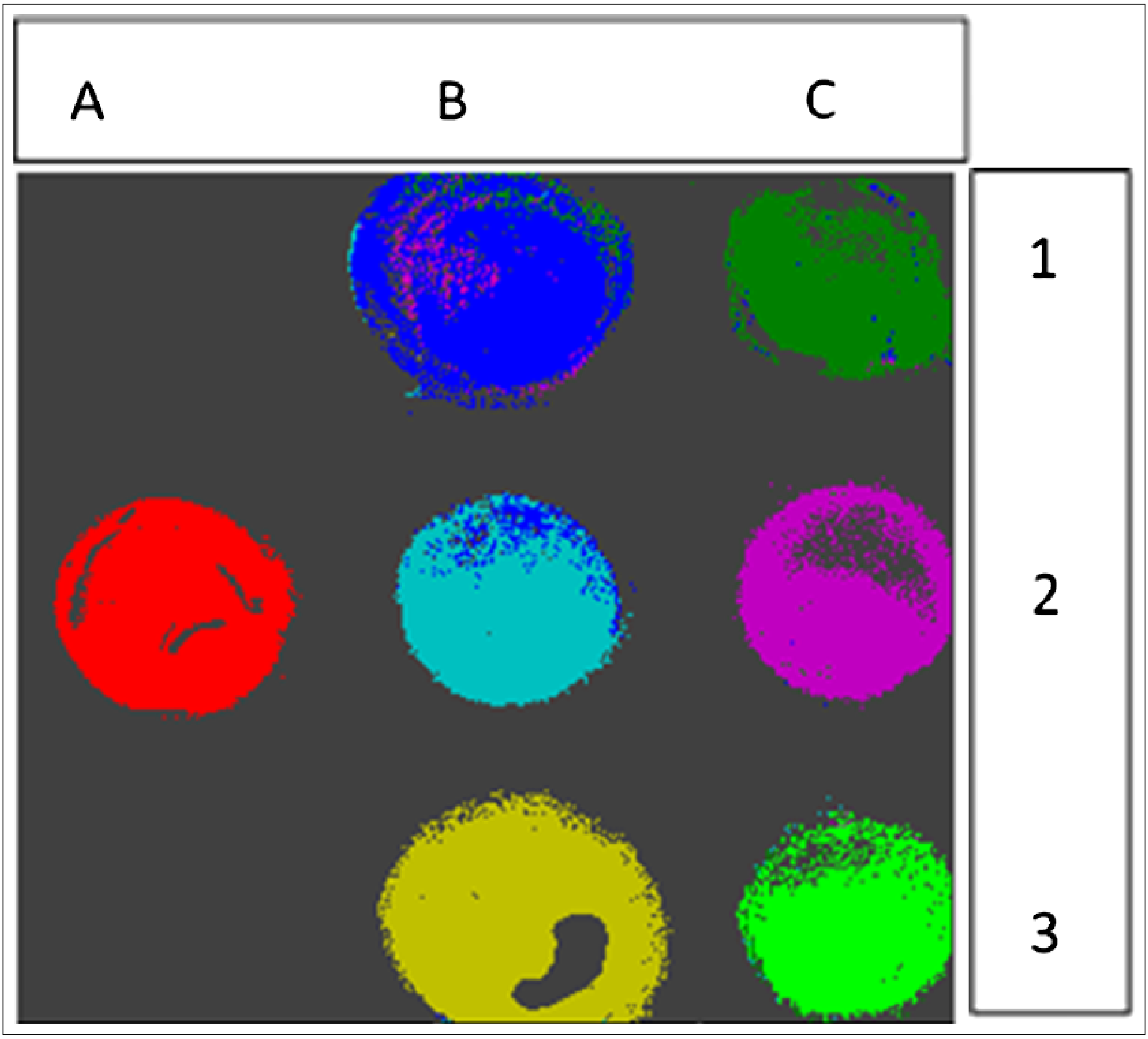

Fig. 1. False-color image, where each pixel is assigned to one of seven reference spectra.

6. SAMPLE ALGORITHM

Algorithms can be included using the commands as shown in algorithm 1.

Algorithm 1. Euclid's algorithm

procedure EUCLID(a, b) ▷ The g.c.d. of a and b
    $r \leftarrow a \bmod b$
    while $r \neq 0$ **do** ▷ We have the answer if r is 0
        $a \leftarrow b$
        $b \leftarrow r$
        $r \leftarrow a \bmod b$
    return b ▷ The gcd is b

REFERENCES