Today's Topic

09.02.2021

* Inverse Kinematics (IK)

$v' = T v$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} R(\theta) & t \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\downarrow$

Non-linear function

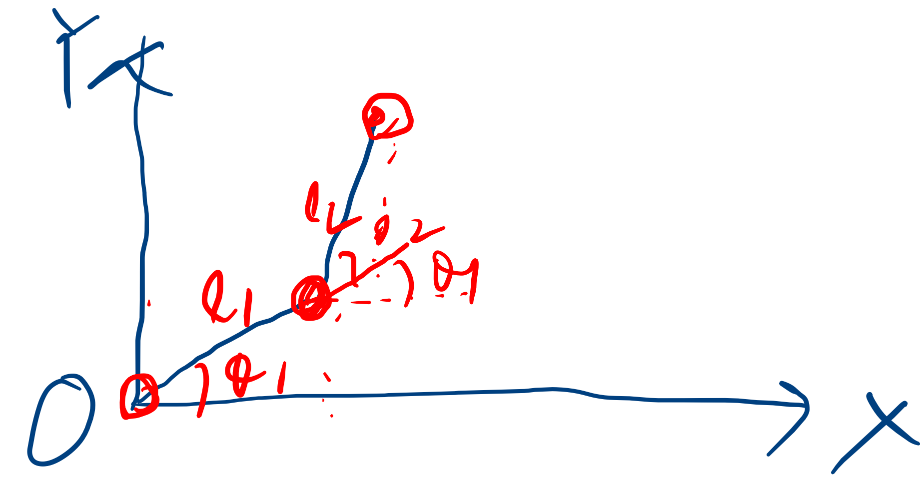
2 x 2 Homogeneous Matrix

2 x 3

3 x 3 - Homogeneous Matrix

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)$$



$$q_1 = \theta_1$$

$$q_2 = \theta_2$$

$$q_3 = \theta_1+\theta_2$$

x

y

ϕ

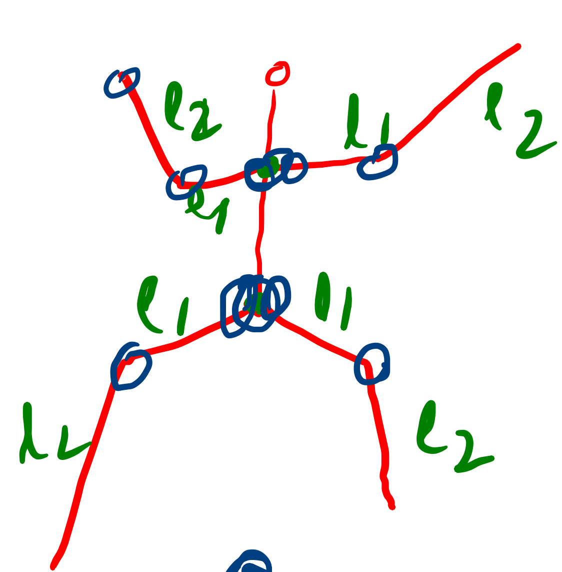
$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = T \begin{pmatrix} x \\ y \\ \phi \end{pmatrix}$$

$\Downarrow$

Robot dynamics

Motors understand θ

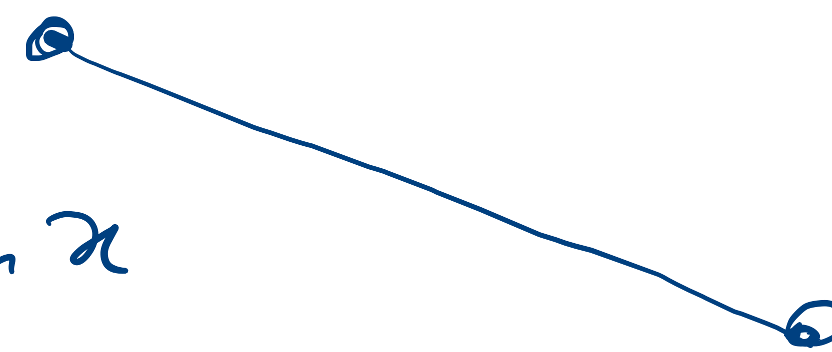
Humanoid Robot



$$x = f(\theta)$$

If we know the θ

we can calculate position x



$$\xrightarrow{\theta} \boxed{FK} \longrightarrow x$$

FK: Forward Kinematics

θ can be defined using our knowledge

$\boxed{\text{Knowledge based AI}} \Rightarrow \theta \Rightarrow$ FK

Forward Kinematics

IK

$x, y \rightarrow \boxed{\text{Inverse Kinematics}} \xrightarrow{\theta}$

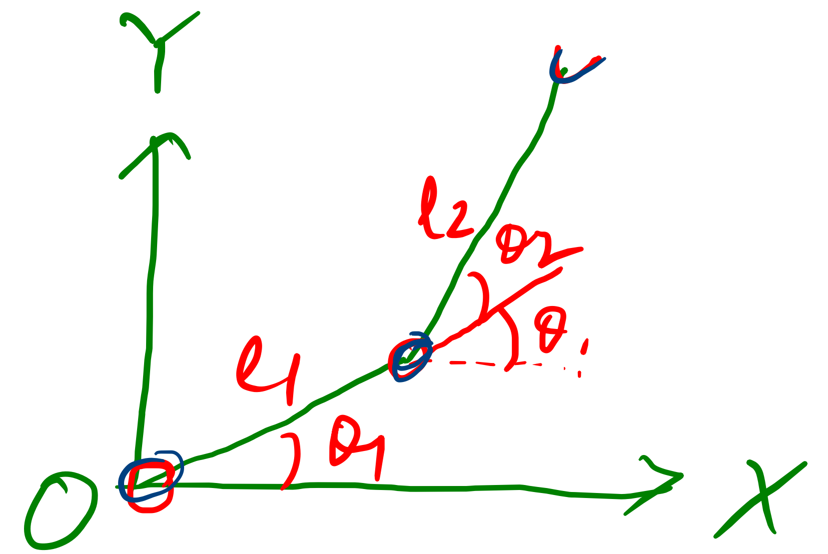
1. Camera
2. Sensors (GPS)

* Forward Kinematics(FK): position (x) of links from angle, θ

* Inverse Kinematics(IK): joints angle (θ) from links position x

FK: $\underline{x} = f(\theta)$

IK: $\theta = f^{-1}(x)$



$\boxed{\text{FK:}}$

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

* Forward Kinematics (FK): position (x) of links from angle, θ

* Inverse Kinematics (IK): joints angle (θ) from links position x

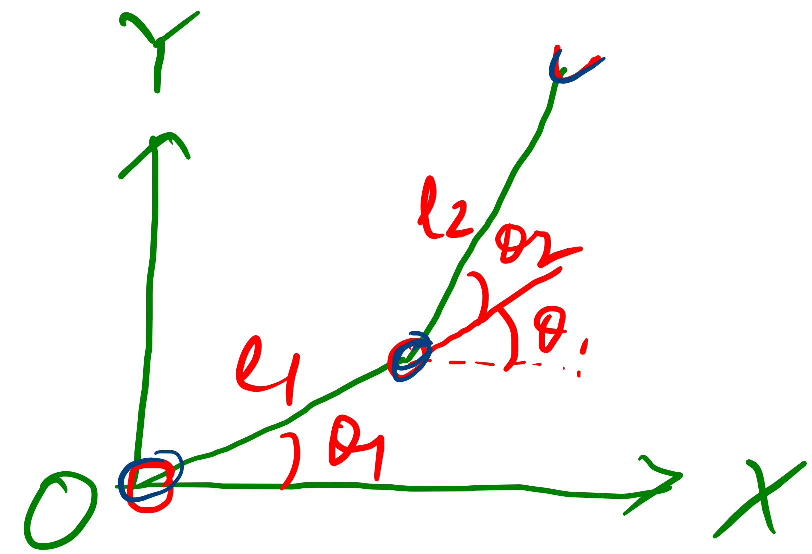
FK: $x = f(\theta)$

IK: $\theta = f^{-1}(x)$

$\boxed{FK:}$

$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$

$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$



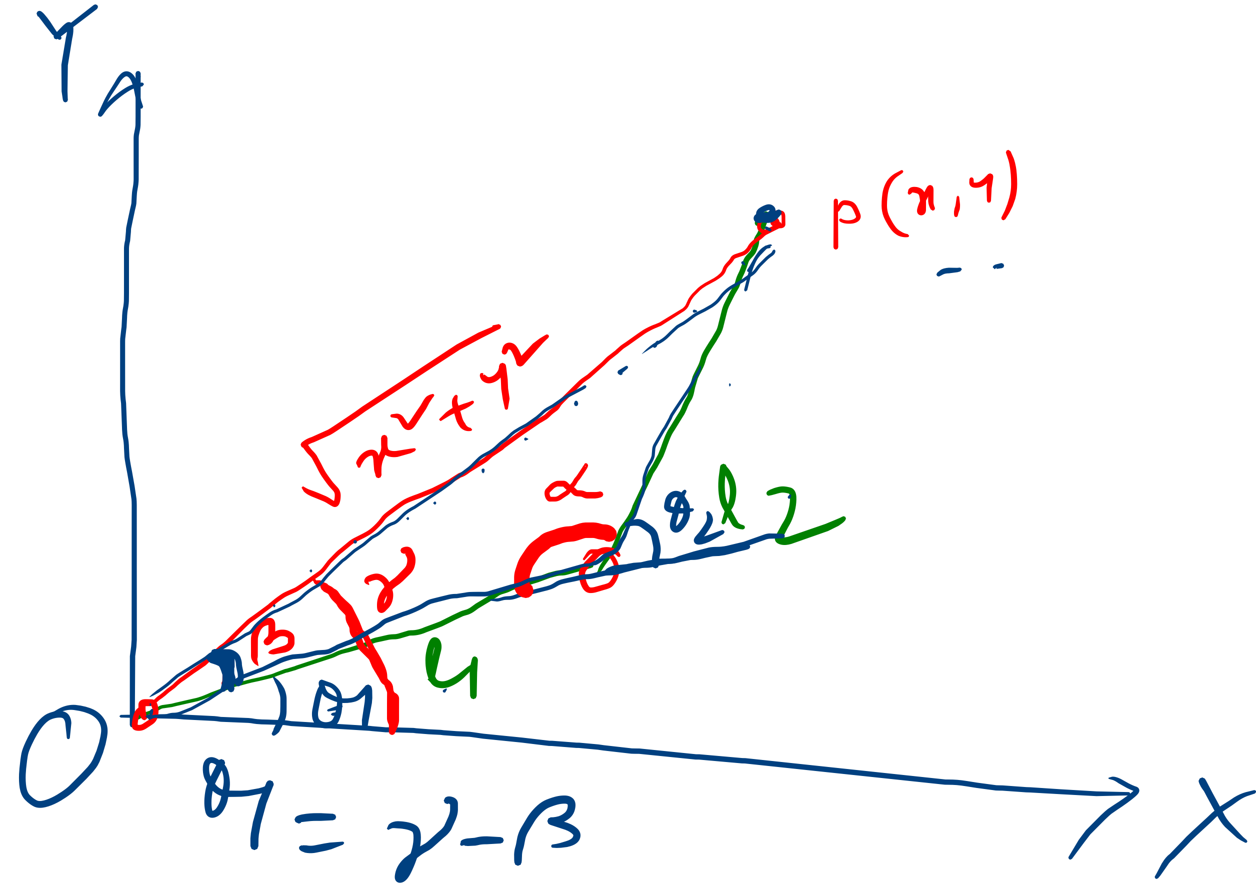
Cosine Theorem

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\underline{\theta_1} \& \underline{\theta_2}$? $\underline{x \& y}$ are given from Camera

$\Downarrow$

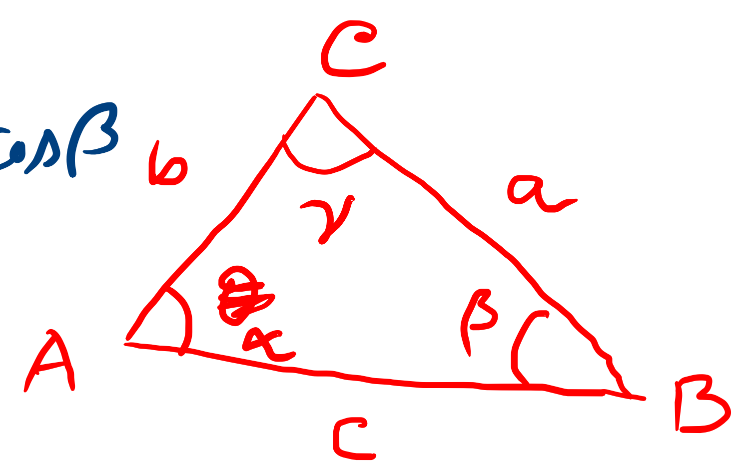
fed to servo-motor



$\theta_1 = \gamma - \beta$

IK

Law of Cosines



① $l_2^2 = l_1^2 + (\sqrt{x^2+y^2}) - 2l_1\sqrt{x^2+y^2}\cos\beta$

② $\tan\gamma = y/x$ or, $\gamma = \operatorname{atan}(y/x)$

③ $\theta_1 = \gamma - \beta$

$$l_2^2 = l_1^2 + \left(\sqrt{x^2+y^2}\right) - 2 l_1 \sqrt{x^2+y^2} \cos\beta$$

or, $$\cos\beta = \frac{l_1^2 - l_2^2 + (x^2+y^2)}{2 l_1 \sqrt{x^2+y^2}}$$

$$\gamma = \tan^{-1}(y/x)$$

$$\theta_1 = \gamma - \beta$$

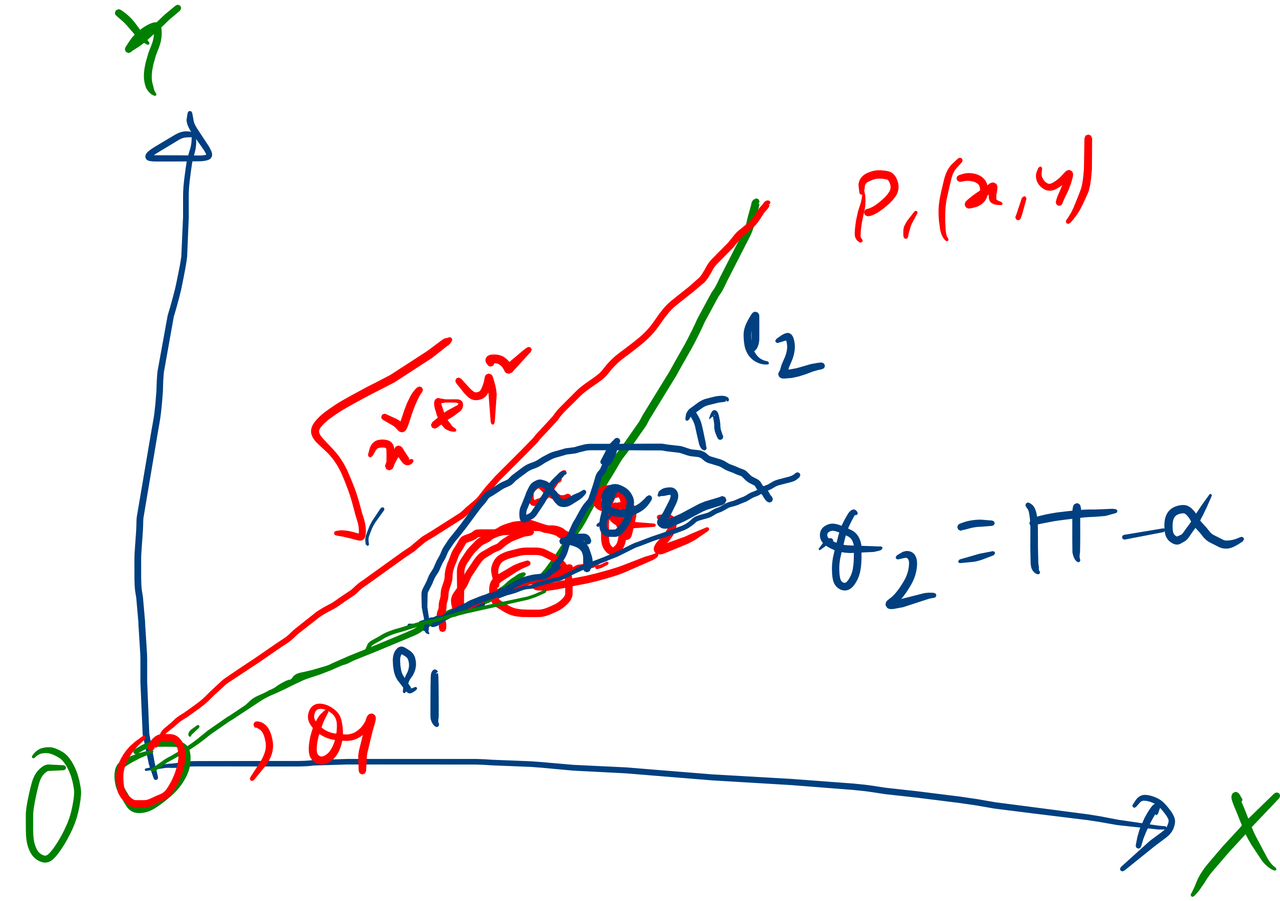
$$\theta_1 = \tan^{-1}(y/x) - \cos^{-1}\left[\frac{l_1^2 - l_2^2 + (x^2+y^2)}{2 l_1 \sqrt{x^2+y^2}}\right]$$

$\theta_2 =$

$\theta_2 = \pi - \alpha$

$$\left(\sqrt{x^2+y^2}\right)^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\alpha$$

or, $$\cos\alpha = \frac{l_1^2 + l_2^2 - (x^2+y^2)}{2 l_1 l_2}$$

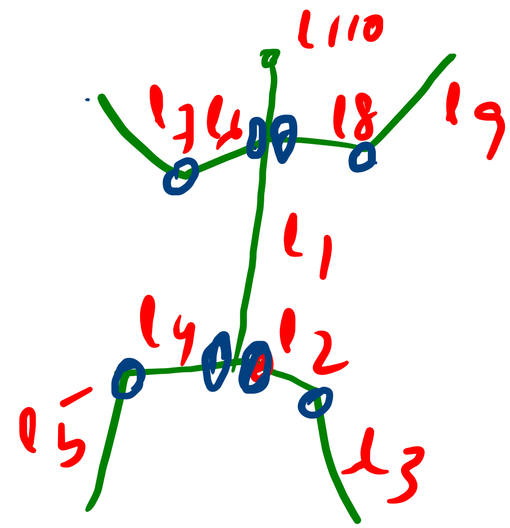


$$\theta_2 = \pi - \cos^{-1}\left[\frac{l_1^2 + l_2^2 - (x^2+y^2)}{2 l_1 l_2}\right]$$

$\boxed{\theta_1 \;\&\; \theta_2 \text{ are calculated}}$

$v' = [T]\,v \Rightarrow$ position

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = J \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$



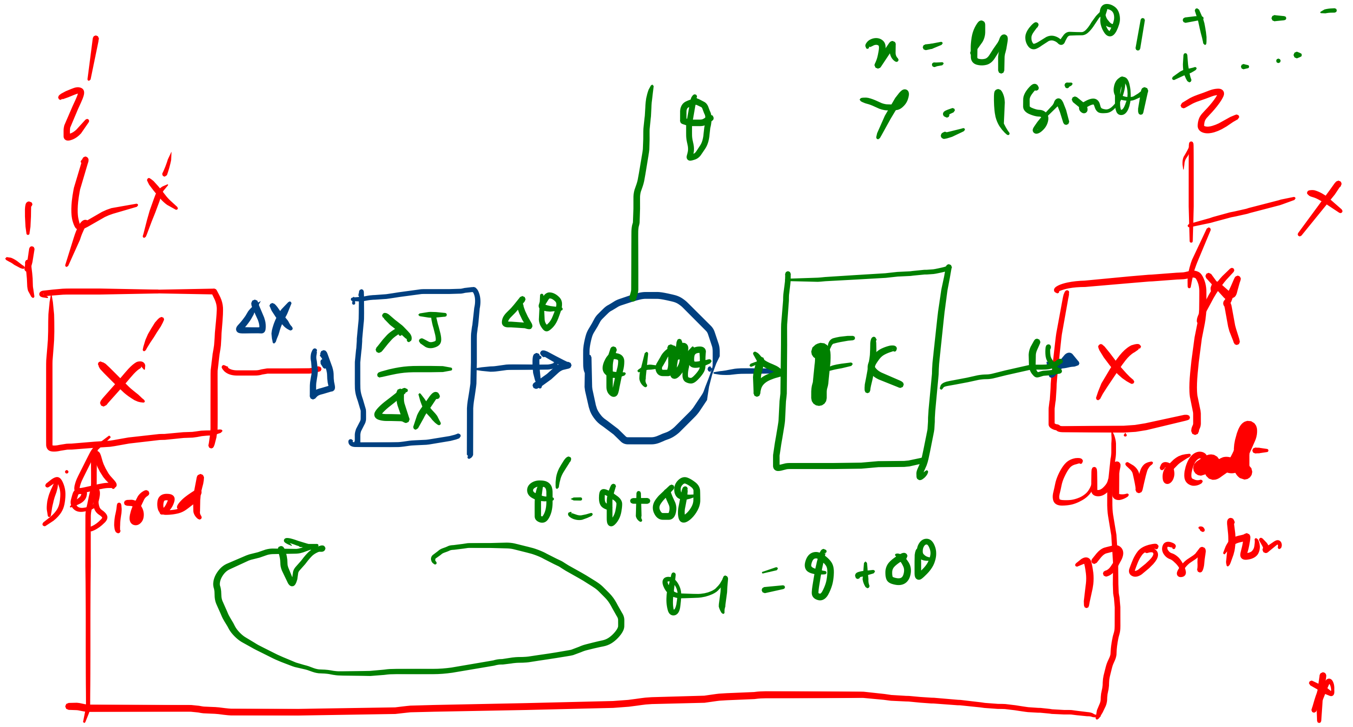
$\theta_i \; [i = 1, 2, 3, \dots, 10]$

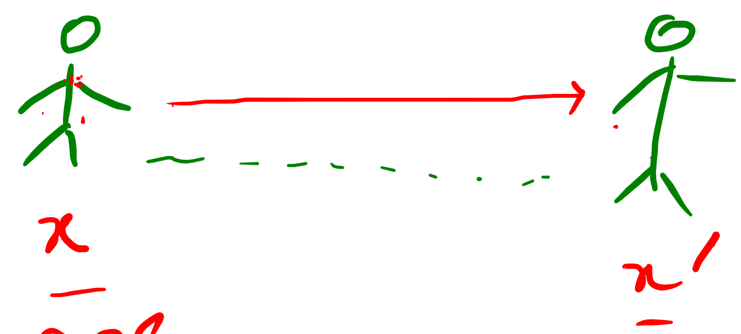
$l_i \; [i = 1, 2, 3, \dots, 10]$

position, velocity, & acceleration can be calculated from co-ordinate Transformation

$$\boxed{x = J\theta} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = J \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

IK : Numerical method





* Reference position
* target position
* Desired position

* GPS
* Camera

* $\Delta x = x' - x$ $\Delta x = 10^{-6}$

* $x = J\theta \Rightarrow \Delta x = J\Delta\theta$ or, $\Delta\theta = J^{-1}/\Delta x \Rightarrow \Delta\theta = \frac{\lambda J}{\Delta x}$

$\boxed{\text{Robot Data Structure \& Programming}}$ ⇒ URDF