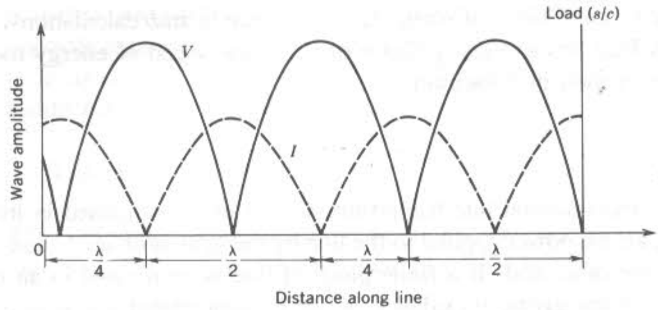


**FIGURE 7-6 Lossless line terminated in a short circuit.**

Consider only the forward traveling voltage and current waves for the moment. At the load, the voltage will be zero and the current a maximum because the load is a short circuit. Note that the current has a finite value since the line has an impedance. At that instant of time, the same conditions also apply at a point exactly one wavelength on the generator side of the load, and so on. The current at the load is always a maximum, although the size of this maximum varies periodically with time, since the applied wave is sinusoidal.

The reflection that takes place at the short circuit affects both voltage and current. The current now starts traveling back to the generator, unchanged in phase (series circuit theory), but *the voltage is reflected with a* 180° *phase reversal*. At a point exactly a quarter-wavelength from the load, the current is *permanently* zero (as shown in Figure 7-6). This is because the forward and reflected current waves are exactly 180° out of phase, as the reflected wave has had to travel a distance of $\lambda/4 + \lambda/4 = \lambda/2$ farther than the forward wave. The two cancel, and a current node is established. The voltage wave has also had to travel an extra distance of $\lambda/2$, but since it underwent a 180° phase reversal on reflection, its total phase change is 360°. Reinforcement will take place, resulting in a voltage antinode at precisely the same point as the current node.

A half-wavelength from the load is a point at which there will be a voltage zero and a current maximum. This arises because the forward and reverse current waves are now in phase (current has had to travel a total distance of one wavelength to return to this point). Simultaneously the voltage waves will cancel, because the 180° phase reversal on reflection must be added to the extra distance the reflected wave has to travel. All these conditions will repeat at half-wavelength distances, as shown in Figure 7-6. Every time a point is considered that is $\lambda/2$ farther from the load than some previously considered point, the reflected wave has had to travel one whole wavelength farther. Therefore it has the same relation to the forward wave as it had at the first point.

It must be emphasized that this situation is permanent for any given load and is determined by it; such waves are truly *standing* waves. All the nodes are permanently fixed, and the positions of all antinodes are constant. Many of the same conditions apply if the load is an open circuit, except that the first current minimum (and voltage

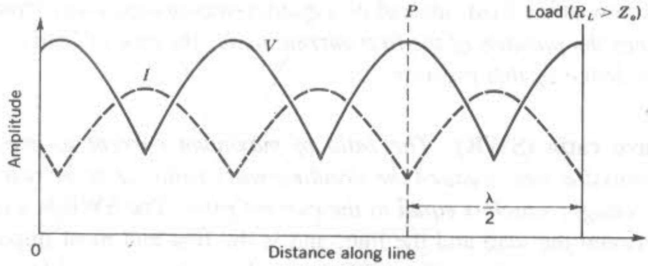


**FIGURE 7-7 Lossless line terminated in a pure resistance greater than $Z_0$ (note that voltage SWR equals current SWR).**

Consider a pure resistance connected to a transmission line, such that $R_L \neq Z_0$. Since the voltage and current vary along the line, as shown in Figure 7-7, so will the resistance or impedance. However, conditions do repeat every half-wavelength, as already outlined. The impedance at *P* will be equal to that of the load, if *P* is a half-wavelength away from the load and the line is lossless.

### 7-1.5 Quarter- and Half-Wavelength Lines

Sections of transmission lines that are exactly a quarter-wavelength or a half-wavelength long have important impedance-transforming properties, and are often used for this purpose at radio frequencies. Such lines will now be discussed.

**Impedance inversion by quarter-wavelength lines** Consider Figure 7-8, which shows a load of impedance $Z_L$ connected to a piece of transmission line of length *s* and having $Z_0$ as its characteristic impedance. When the length *s* is exactly a quarter-wavelength line (or an odd number of quarter-wavelengths) and the line is lossless, then the impedance $Z_s$, seen when looking toward the load, is given by

$$Z_s = \frac{Z_0^2}{Z_L} \tag{7-10}$$

This relationship is sometimes called *reflective impedance;* i.e., the quarter-wavelength reflects the opposite of its load impedance. Equation (7-10) represents a very important and fundamental relation, which is somewhat too complex to derive here, but whose truth may be indicated as follows. Unless a load is resistive and equal to the characteristic impedance of the line to which it is connected, standing waves of voltage and current are set up along the line, with a node (and antinode) repetition rate

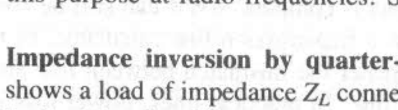


**FIGURE 7-8 Loaded line.**

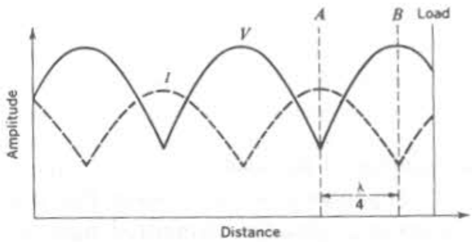


**FIGURE 7-9 Standing waves along a mismatched transmission line; impedance inversion.**

of $\lambda/2$. This has already been shown and is indicated again in Figure 7-9. Note that here the voltage and current minima are not zero; the load is not a short circuit, and therefore the standing-wave ratio is not infinite. Note also that the current nodes are separated from the voltage nodes by a distance of $\lambda/4$, as before.

It is obvious that at the point $A$ (voltage node, current antinode) the line impedance is low, and at the point $B$ (voltage antinode, current node) it is the reverse, i.e., high. In order to change the impedance at $A$, it would be necessary to change the SWR on the line. If the SWR were increased, the voltage minimum at $A$ would be lower, and so would be the impedance at $A$. The size of the voltage maximum at $B$ would be increased, and so would the impedance at $B$. Thus an increase in $Z_B$ is accompanied by a decrease in $Z_A$ (if $A$ and $B$ are $\lambda/4$ apart). This amounts to saying that *the impedance at $A$ is inversely proportional to the impedance at $B$*. Equation (7-10) states this relation mathematically and also supplies the proportionality constant; this happens to be the square of the characteristic impedance of the transmission line. The relation holds just as well when the two points are not voltage nodes and antinodes, and a glance at Figure 7-9 shows that it also applies when the distance separating the points is three, five, seven and so on, quarter-wavelengths.

Another interesting property of the quarter-wave line is seen if, in Equation (7-10), the impedances are normalized with respect to $Z_0$. Dividing both sides by $Z_0$, we have

$$\frac{Z_s}{Z_0} = \frac{Z_0}{Z_L} \tag{7-11}$$

but

$$\frac{Z_s}{Z_0} = z_s$$

and

$$\frac{Z_L}{Z_0} = z_L$$

whence $Z_0/Z_L = 1/z_L$.

Substituting these results into Equation (8-11) gives