SPECIAL TOPIC : DOPPLER EFFECT OF LIGHT.

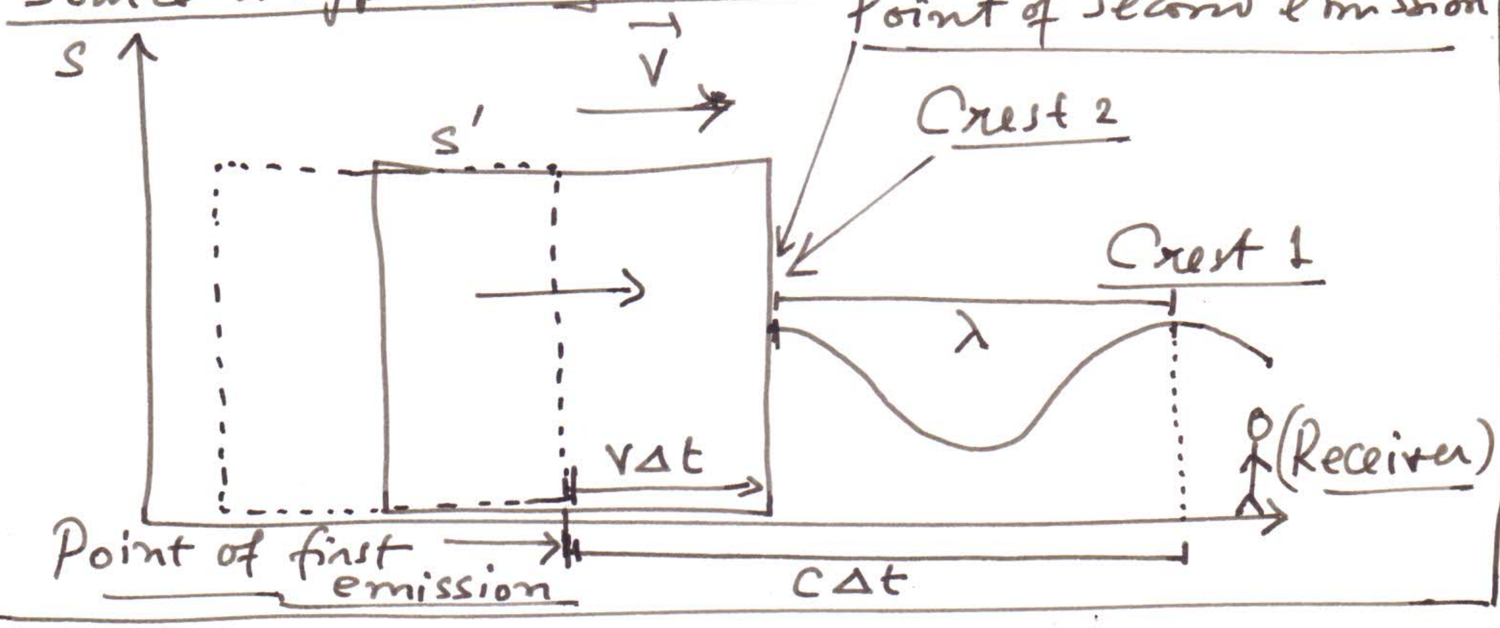
DOPPLER EFFECT: The frequency of a wave changes (sound waves) if either the source or the receiver is in motion. (Christian Doppler)

For light, we have to consider:

1/. Relativistic effects and time dilation.

2/. Relative motion ONLY between source and receiver because c is universally fixed.

Source is approaching receiver



Δt is the time between two successive emissions as seen from the static frame S.

$$\boxed{\lambda = c\Delta t - v\Delta t}$$

If $v \neq 0$, λ is reduced $\Rightarrow$ The waves are squeezed up.

The waves propagate with a speed c (always) $\rightarrow$ Einstein's postulate.

$\Rightarrow$ Observed frequency in frame S,

$$\boxed{\nu_{obs} = \frac{c}{\lambda} = \frac{c}{(c-v)\Delta t}}$$

$$\Rightarrow \boxed{\nu_{obs} = \frac{1}{(1-\beta)\Delta t}} \quad \text{where } \boxed{\beta = v/c}.$$

In frame S', the time measured between two successive emissions is $\Delta t'$. This is the proper time. (Two events occurred at the same ~~place~~ position)

Time dilation $\boxed{\Delta t = \gamma \Delta t'}$

$$\Rightarrow \boxed{\nu_{obs} = \frac{1}{(1-\beta)\gamma\Delta t'}}$$

[The front part of S'.]

(time between successive crests)

$\underline{\Delta t'}$ is also the time period of the emission from the source.

$\therefore$ $\boxed{\Delta t' = \frac{1}{\nu_s}}$ and $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$.

$$\Rightarrow \nu_{obs} = \frac{1}{(1-\beta)\gamma\Delta t'} = \frac{1}{(1-\beta)}\sqrt{1-\beta^2}\cdot\nu_s$$

$\Rightarrow$ $\boxed{\nu_{obs} = \sqrt{\frac{1+\beta}{1-\beta}}\,\nu_s}$ $\boxed{\because\ 1-\beta^2 = (1+\beta)(1-\beta)}$

Frequency increases — for source approaching receiver.

For source receding from receiver,
$\boxed{V \rightarrow -V}$ and $\boxed{\beta \rightarrow -\beta}$.

$\therefore$ $\boxed{\nu_{obs} = \sqrt{\frac{1-\beta}{1+\beta}}\,\nu_s}$ Frequency decreases

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EDWIN HUBBLE

Application: Hubble's law (Expanding Universe)

$$\frac{\nu_s}{\nu_{obs}} = \sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{(1+\beta)(1-\beta)^{-1}} \simeq \sqrt{(1+\beta)^2}$$

$[\beta \ll 1\ (V \ll c)]$

Also $\frac{\nu_s}{\nu_{obs}} = \frac{\lambda_{obs}}{\lambda_s} \simeq 1+\beta$ $\Rightarrow$ $\boxed{\frac{\lambda_{obs}-\lambda_s}{\lambda_s} \simeq \frac{V}{c}}$

DOPPLER

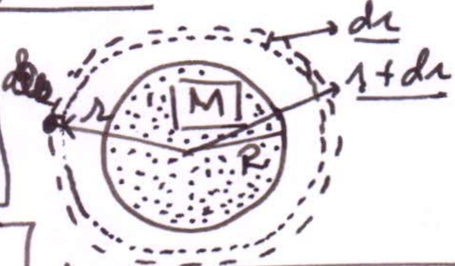
$\Rightarrow$ Redshift $\boxed{\frac{\Delta\lambda}{\lambda_s} \simeq \frac{V}{c}}$

Distance

$\boxed{V = H_0 d}$ → Hubble's law of recession of galaxies.

Hubble constant

Gravitational Redshift : Alternative Derivation



Work done to travel, against gravity, by a displacement dr

$$\boxed{dW = -\frac{GMm}{r^2}\,dr}$$

$$\Rightarrow \boxed{-\frac{GM}{r^2}\left(\frac{h\nu}{c^2}\right)dr = h(\nu + d\nu) - h\nu}$$

$\because\ m = \frac{h\nu}{c^2}$ photon "effective" mass.

$$\Rightarrow \boxed{-\frac{GM}{c^2}\frac{dr}{r^2} = \frac{d\nu}{\nu}}$$

Not a quantum effect (h cancels)

Now $\boxed{\nu = c\lambda^{-1}} \Rightarrow d\nu = -c\lambda^{-2}\,d\lambda$

$$\therefore\ \frac{d\nu}{\nu} = -\frac{c\,d\lambda}{\lambda(\lambda\nu)} \Rightarrow \boxed{\frac{d\nu}{\nu} = -\frac{d\lambda}{\lambda}}$$

Hence, $+\frac{GM}{c^2}\int_R^r \frac{dr}{r^2} = +\int_\lambda^{\lambda+\Delta\lambda}\frac{d\lambda}{\lambda}$ | Wavelength increases

$$\Rightarrow \boxed{\frac{GM}{c^2}\left.\left(-\frac{1}{r}\right)\right|_R^r = \ln\lambda\Big|_\lambda^{\lambda+\Delta\lambda}}$$

$$\Rightarrow \boxed{\frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{r}\right) = \ln\left(\frac{\lambda+\Delta\lambda}{\lambda}\right) = \ln\left(1+\frac{\Delta\lambda}{\lambda}\right)}$$

When $\boxed{r \gg R}$ and $\boxed{\Delta\lambda \ll \lambda}$ (small shift)

Far away when gravity is weak.

$$\boxed{\frac{GM}{Rc^2} \simeq \frac{\Delta\lambda}{\lambda}}$$

$\because\ \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots \simeq x$ when $x \ll 1$

Gravitational Red Shift.