Tutorial 5

SC-220 Groups and Linear algebra Autumn 2019
(Normal Subgroups, Quotient groups)

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(1) Which are the subsets of $\mathbb{R} \times \mathbb{R}$ are equivalence relations
i) $\{(x, y)|x - y \text{ is and even integer}\}$
ii) $\{(x, y)|x - y \text{ is rational}\}$
i) $\{(x, y)|x + y \text{ is rational}\}$
i) $\{(x, y)|x - y \geq 0\}$

(2) Find all the cosets of
i) The subgroup $4\mathbb{Z}$ of $\mathbb{Z}$. What is the quotient group $\mathbb{Z}/4\mathbb{Z}$.
ii)The subgroup $< 2 >$ of $\mathbb{Z}_{12}$. What is the quotient group $\mathbb{Z}_{12}/ < 2 >$.

(3) Let G be a group and H be a subgroup of G. Show that the following three definitions of being a Normal subgroup are equivalent
i)$gH = Hg\, \forall g \in G$
ii)$gHg^{-1} = H\, \forall g \in G$
iii)$ghg^{-1} \in H\, \forall\, g \in G$ and $h \in H$

(4) Let G a group then $Z(G) = \{x \in G|xg = gx\, \forall g \in G\}$ is the set all elements of G that commute with every other element of G. It is called the center of the group G. Show that $Z(G)$ is a normal subgroup of G.

(5) Let G be a group and let $H = \{xyx^{-1}y^{-1}|x, y \in G\}$. Show that H is a Normal Subgroup of G

(6) Find all the normal subgroups of D_4
