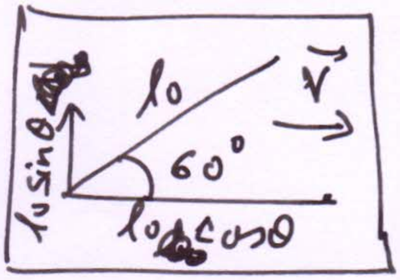
Relativity : Solutions of Selected Problems

Relativity - 1

Q2/. Length contraction is applied only along the direction of the velocity.



$\therefore$ $\boxed{l' = \frac{l_0 \cos\theta}{\gamma}}$ $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$ $\Rightarrow$ $\boxed{\frac{1}{\gamma^2} = 1-\beta^2}$

$\boxed{\beta = 0.8}$

Total contracted length is,

$$\sqrt{l_0^2 \sin^2\theta + \frac{l_0^2 \cos^2\theta}{\gamma^2}}$$

Length contraction is applied only on the horizontal component.

$$= \sqrt{l_0^2 \sin^2\theta + l_0^2 \cos^2\theta - l_0^2 \beta^2 \cos^2\theta}$$

$$= \sqrt{l_0^2 (1 - \beta^2 \cos^2\theta)} = l_0 \sqrt{1 - \cos^2 60° \times 0.8^2}$$

$$= l_0 \sqrt{1 - \frac{0.64}{4}} = l_0 \sqrt{1 - 0.16} = l_0 \sqrt{0.84}$$

Hence contracted length is $\simeq 0.92 l_0$.

Percentage contraction is $\frac{l_0 - 0.92 l_0}{l_0} = 0.08$

$0.08 \times 100\%$ $\longrightarrow$ $\boxed{= 8\%}$ $\longleftrightarrow$ Answer

Q4/. $\boxed{t = \gamma t_0}$. $t_0 = 60$ min , $t = 61$ min

(An hour takes longer by 1 minute)

$\therefore \gamma = \frac{t}{t_0} = \frac{61}{60} = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta^2 = 1 - \left(\frac{60}{61}\right)^2$

$\Rightarrow \boxed{\beta = 0.18} \Rightarrow \boxed{V = 0.18c} \longleftarrow$ Answer

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Q 7/. $\boxed{\gamma t_0 = t = (t_0 + 1)} \Rightarrow \boxed{\gamma = 1 + \frac{1}{t_0}}$

$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = (1-\beta^2)^{-1/2}}$

t, t_0 are in seconds.

$\beta = \frac{V}{c} = \frac{3 \times 10^2}{3 \times 10^8}$

$\Rightarrow \beta = 10^{-6} \Rightarrow \beta^2 = 10^{-12}$.

$\Rightarrow \boxed{\gamma = (1-\beta^2)^{-1/2} \simeq 1 + \frac{1}{2}\beta^2}$ ($\because \beta^2 \ll 1$, by the binomial expansion)

$\Rightarrow \gamma \simeq 1 + \frac{1}{2}\beta^2 \simeq 1 + \frac{1}{t_0} \Rightarrow t_0 = \frac{2}{\beta^2}$ (in seconds)

$\Rightarrow \boxed{t_0 \simeq 2 \times 10^{12} \text{ sec}}$.

1 year $= 3.2 \times 10^7$ sec. $\Rightarrow t_0 = \frac{2 \times 10^{12}}{3.2 \times 10^7}$ years.

$\Rightarrow \boxed{t_0 \simeq 63{,}000 \text{ years}} \longleftarrow$ Answer

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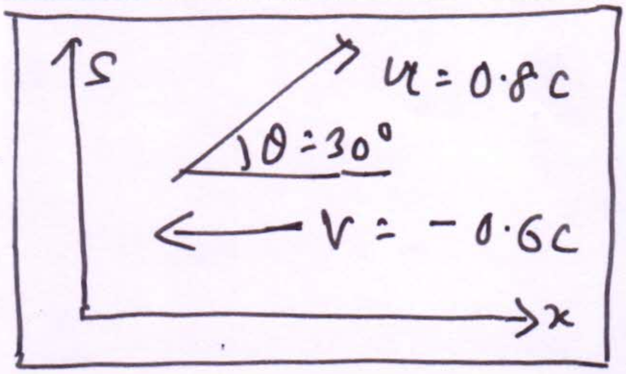
Q 13/. $\boxed{u_x = u \cos\theta}$

$\boxed{u_y = u \sin\theta}$

In the Static frame



(P.T.O.)

(Continued)

Q13/.

$$\boxed{u_x = \frac{u_x' + v}{1 + u_x' v / c^2}} \Rightarrow \boxed{u_x' = \frac{u_x - v}{1 - u_x v / c^2}}$$

u_x' is the x-component of the particle velocity, seen from the moving frame.

$$\boxed{u_y = \frac{u_y'}{\gamma\left(1 + \frac{v u_x'}{c^2}\right)}}, \quad \boxed{u_y' = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2} u_x\right)}}$$

u_y' is the y-component of the particle velocity, seen from the moving frame.

$$\boxed{u_x = 0.8c \cos 30^\circ} \Rightarrow \boxed{u_x = 0.8 \times \frac{\sqrt{3}}{2} c = \boxed{0.693c}}$$

$$\boxed{u_y = 0.8c \sin 30^\circ} \Rightarrow \boxed{u_y = 0.8 \times \frac{1}{2} c = \boxed{0.4c}}$$

$$u_x' = \frac{0.693c + 0.6c}{1 + 0.693 \times 0.6}$$

$$\Rightarrow \boxed{u_x' = 0.913c}$$

$\boxed{v = -0.6c}$, $\boxed{\beta = -0.6}$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.6^2}}$$

$$\Rightarrow \boxed{\gamma = 1.25}$$

$$\boxed{u_y' = \frac{0.4c}{1.25(1 + 0.6 \times 0.693)} = 0.226c}.$$

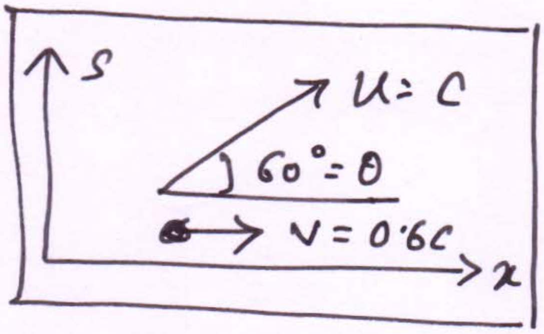
Speed as seen from the moving frame is

$$\sqrt{u_x'^2 + u_y'^2} = c\sqrt{0.913^2 + 0.226^2} = \boxed{0.94c}$$

Answer ↑

Q 14/. $\boxed{u_x = c\cos 60^\circ = c/2}$

$\boxed{u_y = c \sin 60^\circ = c\sqrt{3}/2}$



$\boxed{u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}}$, $\boxed{u_y' = \dfrac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}}$ $\boxed{\beta = 0.6}$

$\boxed{\gamma = \dfrac{1}{\sqrt{1-\beta^2}}} \rightarrow \boxed{\gamma = \dfrac{5}{4}}$

$u_x' = \dfrac{c/2 - 0.6c}{1 - 0.5 \times 0.6} = \dfrac{-0.1c}{1-0.3} \Rightarrow \boxed{u_x' = -c/7}$

$\Rightarrow u_y' = \dfrac{c\sqrt{3}/2}{(5/4)(1-0.5\times 0.6)} = \dfrac{2\sqrt{3}\,c}{5 \times 0.7} = \boxed{\dfrac{2\sqrt{3}}{3.5}c = u_y'}$

$\Rightarrow \boxed{u_y' = \dfrac{4\sqrt{3}}{7}c}$ Angle $\boxed{\tan\theta' = \dfrac{u_y'}{u_x'}}$,

with respect to the x-x' axis (the direction of the moving frame)

$\Rightarrow \tan\theta' = \dfrac{4\sqrt{3}\cancel{c}}{\cancel{7}} \times \dfrac{\cancel{7}}{-\cancel{c}} = -4\sqrt{3} = -6.928$

$\boxed{\theta' = -81.79^\circ}$ Since the x'-component of the velocity is negative and the y'-component of the velocity is positive, the angle should lie in the second quadrant. Hence $\theta' = 180^\circ - 81.79^\circ$.

$\Rightarrow \boxed{\theta' = 98.2^\circ}$

Answer

$\boxed{\text{Speed } u' = \sqrt{u_x'^2 + u_y'^2} = c\sqrt{\dfrac{1}{7^2} + \dfrac{16 \times 3}{7^2}}}$

$\Rightarrow \boxed{u' = c}$ (Same value in all frames)

Q 15/. By Lorentz transformation $\boxed{\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)}$

$$\boxed{\Delta x' = \gamma(\Delta x - v\Delta t)} \Rightarrow \boxed{\Delta x = \gamma(\Delta x' + v\Delta t')}$$

$$\Rightarrow \Delta t' = \gamma\Delta t - \gamma\frac{v}{c^2}\cdot\gamma(\Delta x' + v\Delta t')$$

$\boxed{\Delta x' = 800\,m}$ (measured in moving frame)

$\boxed{\Delta t = 0}$ (simultaneous events in static frame)

$$\therefore \boxed{\Delta t' = -\frac{v\gamma^2}{c^2}(\Delta x') - \gamma^2\frac{v^2}{c^2}\Delta t'}.$$

$$\Rightarrow \Delta t'\left[1 + \gamma^2\frac{v^2}{c^2}\right] = -\frac{v\gamma^2}{c^2}\Delta x' = -\frac{v^2\gamma^2}{c^2}\frac{\Delta x'}{v}$$

$$\Rightarrow \boxed{\Delta t' = -\frac{\Delta x'}{v}\cdot\frac{1}{1+\frac{c^2}{\gamma^2 v^2}}}$$

$$\boxed{\frac{c^2}{\gamma^2 c^2} = \frac{c^2(1-\beta^2)}{v^2} = \frac{1}{\beta^2} - 1}$$

$$\therefore \boxed{\Delta t' = -\frac{\Delta x'}{v}\beta^2} \qquad \because \boxed{1+\frac{c^2}{\gamma^2 v^2} = \frac{1}{\beta^2}}$$

$$\boxed{\beta = \frac{v}{c} = \frac{160\times10^3}{3600\times3\times10^8} = \frac{4}{27}\times10^{-6}}$$

only Take the absolute value

$$\Rightarrow |\Delta t'| = \frac{800\,m}{160\times10^3}\times\frac{16}{27^2}\times10^{-12}\times3600 = \frac{8\times\cancel{16}\times36}{\cancel{16}\times27^2}\times10^{-12}$$

$$\Rightarrow |\Delta t'| = \frac{32}{81}\times10^{-12} \simeq 0.4\times10^{-12}\,s$$

$$\simeq \boxed{0.4\,ps} \longleftarrow \text{Answer}$$