Physics Theory

Physical constants: $c = 3 \times 10^8\,\mathrm{ms^{-1}}$, $h = 6.626 \times 10^{-34}\,\mathrm{Js}$, $G = 6.67 \times 10^{-11}\,\mathrm{kg^{-1}m^3s^{-2}}$, $k_\mathrm{B} = 1.38 \times 10^{-23}\,\mathrm{JK^{-1}}$, $N_\mathrm{A} = 6.022 \times 10^{23}\,\mathrm{mol^{-1}}$, $e = 1.6 \times 10^{-19}\,\mathrm{C}$, $m_\mathrm{e} = 9.11 \times 10^{-31}\,\mathrm{kg}$, $m_\mathrm{n} \simeq m_\mathrm{p} = 1836 m_e$.

- **Basic concepts of relativity, wave-particle duality and gravity (10-01-2020):**

1. A rocket ship is $100\,\mathrm{m}$ long on the ground. When in flight its length appears $99\,\mathrm{m}$ to an observer on the ground. What is the speed of the rocket in terms of c? [$0.14c$]
2. A metre stick moving with respect to a stationary observer appears only $500\,\mathrm{mm}$ long. What is its relative speed? [$0.87c$]
3. A meson has a proper lifetime of $2 \times 10^{-6}\,\mathrm{s}$. What is its lifetime to a stationary observer who sees it travel at a speed of $0.998c$? How far does the observer see it travel? What distance would it travel without relativistic effects? [$31.6\,\mu\mathrm{s}$, $9.5\,\mathrm{km}$, $0.6\,\mathrm{km}$]
4. A certain particle has a proper lifetime of $1 \times 10^{-7}\,\mathrm{s}$. How far does a stationary observer see it to travel if its speed is $0.99c$? [$211\,\mathrm{m}$]
5. Deduce the fractional increase of the mass of a particle for velocity $0.1c$. [5×10^{-3}]
6. Two particles travel in the same direction. Particle A has a speed of $0.9c$ with respect to the fixed ground frame. Particle B has a relative speed of $0.9c$ with respect to particle A. Using the relativistic addition formula, find the speed of particle B with respect to the fixed ground. [$0.99c$]
7. Two particles are ejected in opposite directions from a radioactive atom at rest in the laboratory. Each particle has a speed of $0.8c$ as measured in the fixed laboratory frame. Use the relativistic velocity addition formula to know the speed of one particle with respect to the other. [$0.98c$]
8. Compute the effective mass of a 5000Å photon. [$4.4 \times 10^{-36}\,\mathrm{kg}$]
9. Estimate the energy of γ-rays with wavelength $10^{-13}\,\mathrm{m}$ and radio waves with wavelength $10\,\mathrm{m}$. [$2 \times 10^{-12}\,\mathrm{J}$, $2 \times 10^{-26}\,\mathrm{J}$]
10. Estimate the de Broglie wavelength of a particle of $1\,\mathrm{g}$ mass moving with a velocity of $1\,\mathrm{ms^{-1}}$, and a free electron with a kinetic energy of $200\,\mathrm{eV}$. [$6.6 \times 10^{-31}\,\mathrm{m}$, $8.7 \times 10^{-11}\,\mathrm{m}$]
11. Discuss wave-particle duality in the two previous questions, pointing out a major difference.
12. To collapse the Sun as a black hole, estimate the Schwarzschild radius within which all of the solar mass should be concentrated. Mass of the Sun, $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$. Also determine the density of this compact spherical object. [$3\,\mathrm{km}$, $1.8 \times 10^{19}\,\mathrm{kg\,m^{-3}}$]
13. If the Schwarzschild radius of a black hole were to be of the order of the Planck length, then estimate the corresponding mass of the black hole. Compare this mass with the Planck mass that you can derive dimensionally in terms of c, h, G. [$10^{-8}\,\mathrm{kg}$]
14. Discuss the conceptual difference between the random velocity of molecules in a gas and the escape velocity of a particle from a gravitational field. In this connection, suggest a reason as to why the moon does not have an atmosphere.
15. Make an order-of-magnitude estimation of the average kinetic energy of helium molecules in a gas at room temperature ($300\,\mathrm{K}$). [$10^{-21}\,\mathrm{J}$]

- **Radioactive dating (24-01-2020):**

  1. $1\,\mathrm{gm}$ of living wood gives an average of 6.68 disintegrations in $1\,\mathrm{min}$. Estimate the age of two separate samples of charcoal, with average disintegration of 0.97 and 4.09 in the same unit. Living wood contains radioactive C-14, with $T_{1/2} \simeq 5600\,\mathrm{yrs}$. [$15600\,\mathrm{yrs}$, $3960\,\mathrm{yrs}$]
  2. How long does it take for 60% of a sample of radon-222 to decay? ($T_{1/2} = 3.82\,\mathrm{days}$). [$5.05\,\mathrm{days}$]
  3. The activity of a radioactive sample decreases to 15% of its original value in $10\,\mathrm{days}$. Estimate its half life. [$3.7\,\mathrm{days}$]
  4. Tritium (H-3) has a half-life of $12.5\,\mathrm{yrs}$. What fraction of a sample of tritium will remain undecayed after $25\,\mathrm{yrs}$? [$1/4$]

- **Fluid systems – liquid and air (13-02-2020):**

  1. The viscosity of castor oil is $2.42\,\mathrm{N\,s\,m^{-2}}$, and its density is $940\,\mathrm{kg\,m^{-3}}$. A steel ball of radius $2\,\mathrm{mm}$ falls through the oil. The density of steel is $7800\,\mathrm{kg\,m^{-3}}$, and $g = 10\,\mathrm{m\,s^{-2}}$. Calculate the terminal velocity and the time for the velocity to become terminal. [$2.5\,\mathrm{m\,s^{-1}}$, $2.9 \times 10^{-3}\,\mathrm{s}$]
  2. Spherical powder particles are stirred up in water $10\,\mathrm{cm}$ deep. Find the radius of the largest particle in suspension after $24\,\mathrm{hours}$. The density of the powder is $4\,\mathrm{g\,cm^{-3}}$, and the viscosity of water is 0.010 cgs unit. ($g = 981\,\mathrm{cm\,s^{-2}}$). [$4.2 \times 10^{-5}\,\mathrm{cm}$]
  3. A skydiver falls through air without opening the parachute. If the numerical value of the air drag coefficient is $0.25\,\mathrm{kg\,m^{-1}}$, find the terminal speed for a $50\,\mathrm{kg}$ skydiver. [$44\,\mathrm{m\,s^{-1}}$]
  4. Two spherical rain drops of equal size fall through air with a steady velocity of $0.15\,\mathrm{m\,s^{-1}}$. The drops coalesce to form a larger spherical drop. Estimate its terminal velocity. [$0.21\,\mathrm{m\,s^{-1}}$]
  5. A horizontal slab of air has a thickness $\mathrm{d}z$. The pressure from below, holding up the slab, balances both the pressure from above and the weight of the slab. With this fact find an expression for $\mathrm{d}P/\mathrm{d}z$ in terms of the density of air. Then use the ideal gas equation to show that

     $$\frac{\mathrm{d}P}{\mathrm{d}z} = -\frac{mg}{k_\mathrm{B}T}P\,,$$

     where m is the average mass of an air molecule. Assuming that T is constant, derive the pressure-height equation $P(z) = P(0)\exp(-mgz/k_\mathrm{B}T)$. If T varies as $T = T_0 - \lambda z$, where $T_0 \simeq 300\,\mathrm{K}$ is the temperature at the sea level, then argue that for $(T_0/\lambda) \gg z$, the approximation of constant T is valid. Using $m = 29\,\mathrm{amu}$, $g = 9.8\,\mathrm{m\,s^{-2}}$, $P(0) = 1\,\mathrm{atm}$ and $\lambda = 10^{-2}\,\mathrm{K\,m^{-1}}$, estimate the air pressure at the peak of Mt. Everest ($8895\,\mathrm{m}$). [$0.38\,\mathrm{atm}$]
  6. Air flows over the top of an airplane wing with speed v_1, and past the underside of the wing with speed v_2. Both sides of the wing have area A, and the air density is ρ. Using Bernoulli's equation show that the upward lift force on the wing is approximately $\rho A(v_1^2 - v_2^2)/2$. If $v_2 = 110\,\mathrm{m\,s^{-1}}$, and the pressure difference between the top and the underside of the wing is $900\,\mathrm{Pa}$, estimate v_1. Density of air $\rho = 1.3 \times 10^{-3}\,\mathrm{g\,cm^{-3}}$. [$116\,\mathrm{m\,s^{-1}}$]

- **Oscillators (27-02-2020):**

  1. The components of an electric circuit are an inductor with $L = 10^{-3}\,\mathrm{H}$, a capacitor with $C = 2 \times 10^{-5}\,\mathrm{F}$ and a resistor, all connected in series. Determine the critical resistance for which the current will just be oscillatory. [$14.14\,\Omega$]

2. A voltage of $5\,\mu$V is applied across a superconducting Josephson junction. What is the frequency of the resulting oscillating current? [2.4×10^9 Hz]

3. For each of the following damped oscillator equations, determine the eigenvalue(s), and write the general solution in terms of two arbitrary constants. Also state the type of damping:

   (a) $\ddot{x} + 5\dot{x} + 6x = 0$ [$\lambda_{1,2} = -3, -2$, overdamped]

   (b) $\ddot{x} + 6\dot{x} + 9x = 0$ [$\lambda_{1,2} = -3, -3$, critically damped]

   (c) $\ddot{x} + 2\dot{x} + 5x = 0$ [$\lambda_{1,2} = -1 \pm 2i$, underdamped]
