$\boxed{23/04/2021}$ Radiation Pattern of Antenna:

Recall: Power pattern = (voltage pattern)$^2$

$$P_n(\theta) = [V_n(\theta)]^2$$

$\left[\because P = \frac{V^2}{R}\right]$

$\downarrow$

$-3\,dB$, ie, 0.5 points = 70.7% or 0.707 points

for beamwidth　　for beamwidth


$$0.5 = (0.707)^2$$

(Beam) solid angle : $d\Omega = \sin\theta\, d\theta\, d\phi$

$$\Rightarrow \Omega = \int d\phi \int \sin\theta\, d\theta$$

## Beam area

$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta,\phi)\, \underbrace{d\Omega}_{d\Omega = \sin\theta\, d\theta\, d\phi} \qquad (sr)}$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta,\theta) \sin\theta \, d\theta \, d\phi$$

where $P_n(\theta,\theta) = V_n^2(\theta,\phi)$

Example Let $E(\theta) = \cos^2\theta$
or
$V(\theta)$ , $0 \le \theta \le 90^\circ$

Beam area

Solution

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta)\,\underbrace{d\Omega}_{\sin\theta\, d\theta\, d\phi}$$

Now, $P_n(\theta) = (V(\theta))^2 = \cos^4\theta$

$$\therefore \Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \cos^4\theta \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow \Omega_A = -2\pi \left[ \frac{1}{25} \cos^5\theta \right]_0^{\pi/2}$$

$$\Omega_A = \frac{2\pi}{5} = \frac{2 \times 3.14}{5} = \underline{1.26 \text{ sr}}$$

Beam area can also be approximated as:-

$$\boxed{\Omega_A \approx \theta_{HP} \phi_{HP}}$$

where $\theta_{HP}$ & $\phi_{HP}$ are HPBW in two principal planes ($\theta$ & $\phi$)

For eg:- If $\theta_{HP} \approx \phi_{HP} \approx 66^\circ$

then $\Omega_A = \theta_{HP}\,\phi_{HP}$

$= 66^\circ \times 66^\circ$

$= 4356^{\square}$ $\longrightarrow$ square degree

But 1 sq radian = 3283 sq degree
(or 1 Sr)

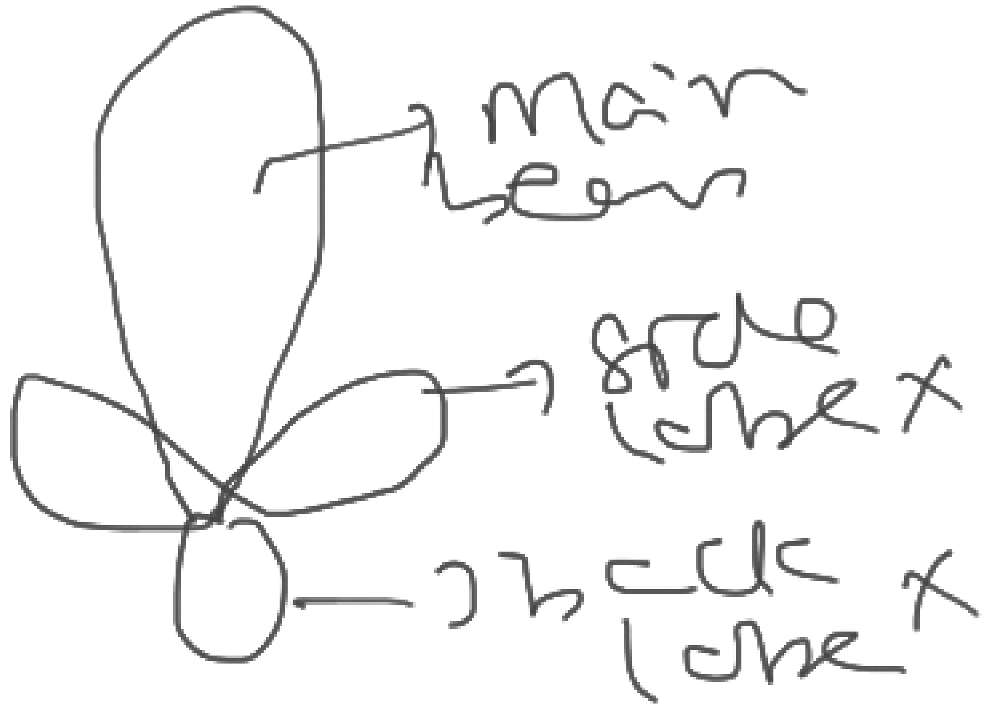
$\therefore \Omega_A = \dfrac{4356}{3836} = 1.33\ \text{Sr}$

# Beam efficiency ($\epsilon$)

$$\epsilon = \frac{\Omega_M}{\Omega_m}$$ (dimensionless) or %age

where $\Omega_M$ = beam area of main beam (or principal) lobe or beam

& $\Omega_m$ = beam area of side lobe & back lobe



# Directivity & Gain of Antenna (G)

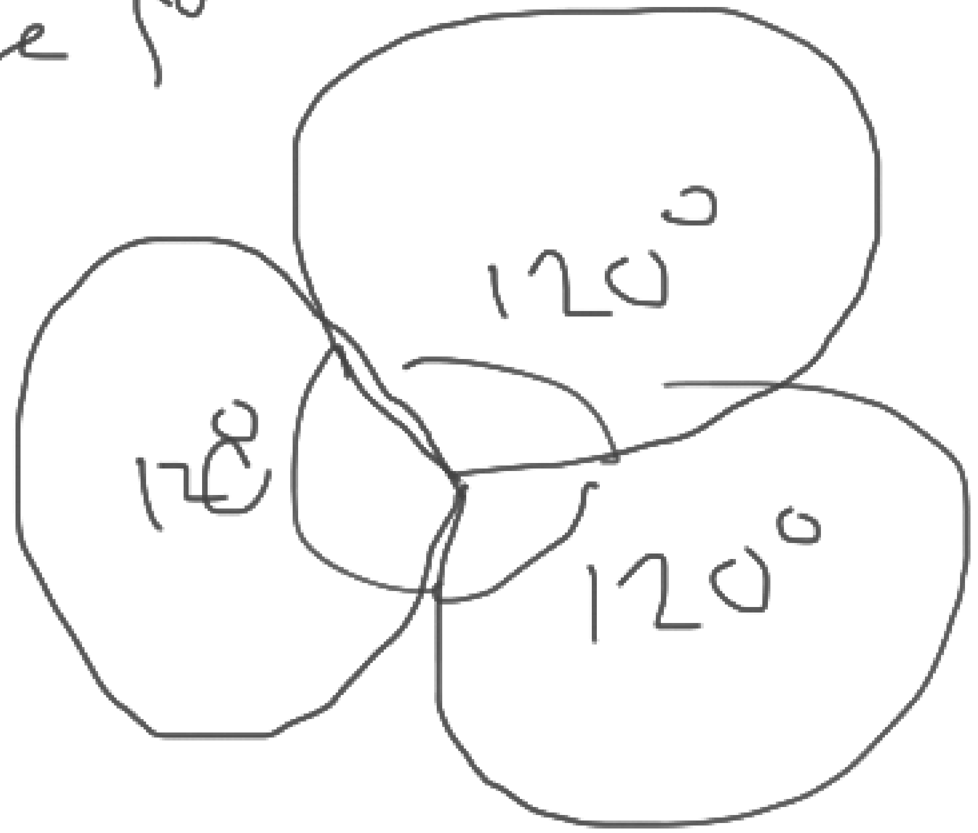
Single antenna : $G = 1$
$= 0\,dB$

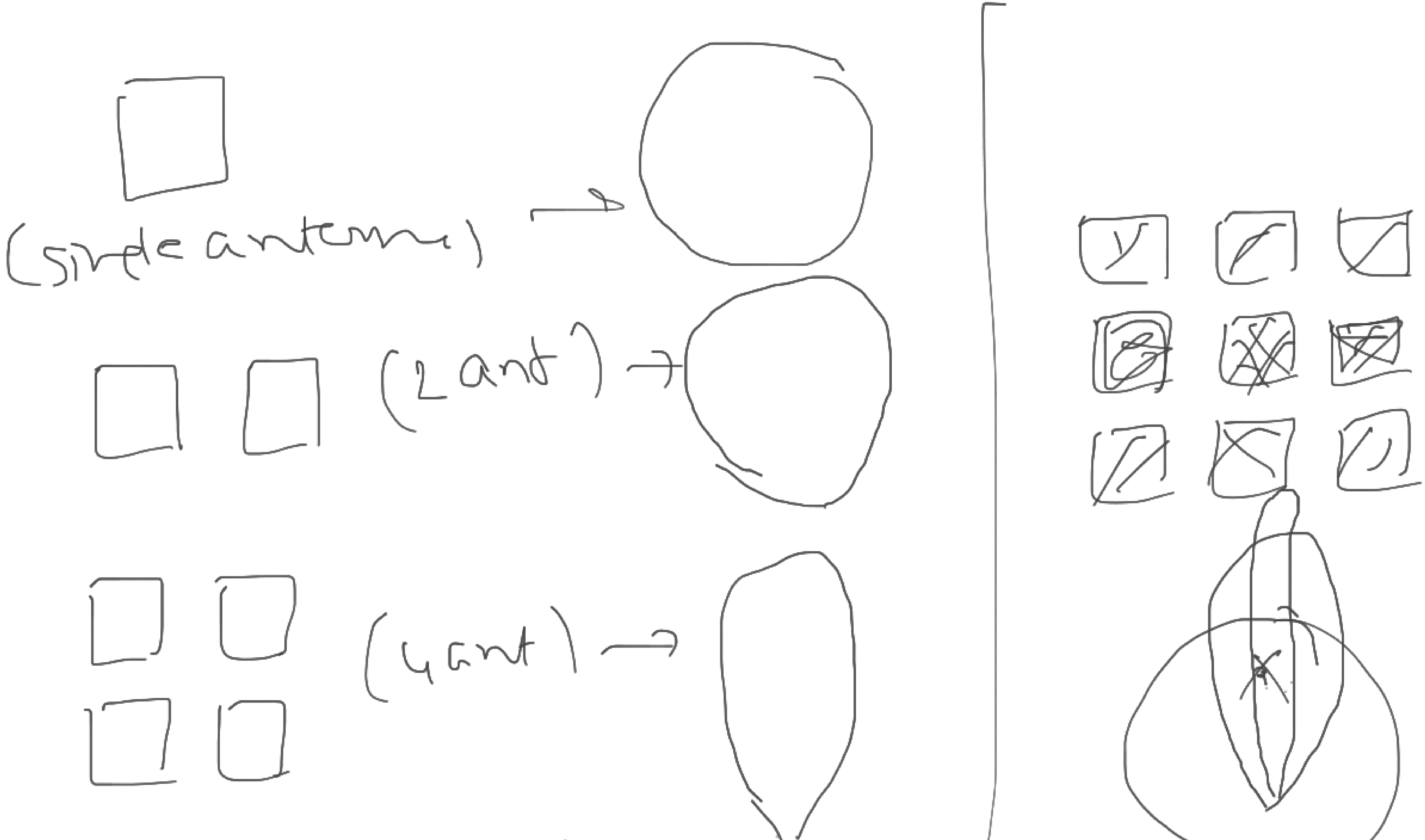
→ omnidirectional pattern

→ dumble shaped or 8 pattern


→ Directional

→ more directional more power




search → detect → track → hit target

Array of antenna → Advantages

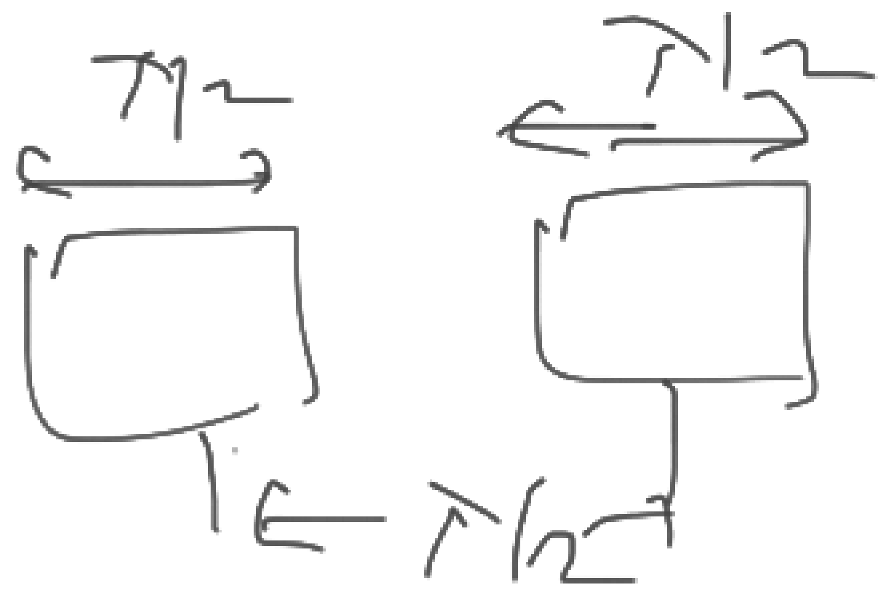
2 or more antennas

→ to increase directivity

ie, beam more directional

→ to increase gain

$L =$



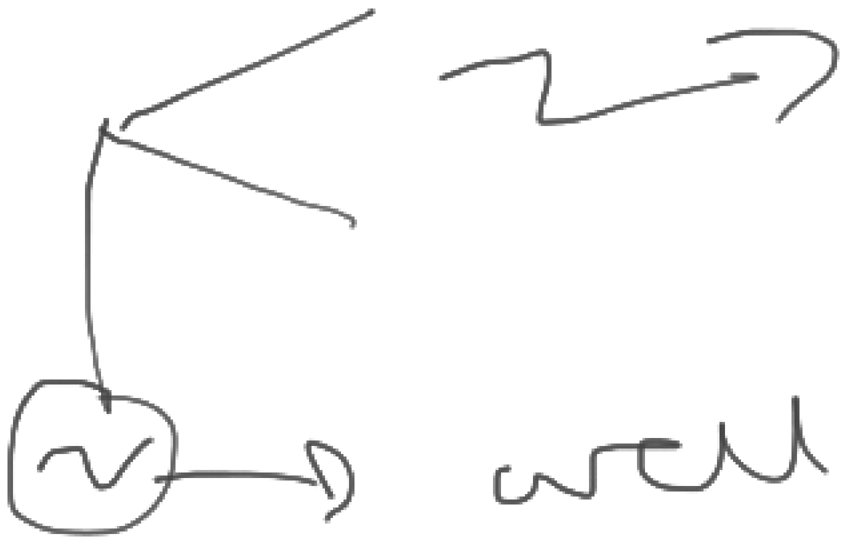

$\lambda = \frac{c}{f}$ → design freq

Gain (G) $\propto$ Directivity (D)

$\Rightarrow \quad G = \underbrace{\epsilon}_{\substack{\text{efficiency} \\ \text{(constant of} \\ \text{proportionality)} \\ k}} D$

$$G\big|_{dB} = \epsilon\big|_{dB} + D\big|_{dB}$$

# How to measure antenna gain (G)



→ no well defined o/p port

→ well defined i/p port

Friss

Read

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

2 identical antenna

then $G_t = G_r = G$

design freq

$\lambda = c/f$

$G = ?$ $\quad P_t , P_r$

G can be determined

$R$ (far field region $> 2D^2/\lambda$)

For lossless antenna, k or $E = 1$

$0 \leq E \leq 1$

(0% to 100%)

$\Rightarrow G = D$

$$\boxed{D = \frac{4\pi}{\Omega_A}},$$

$\Omega_A$ = beam area

$$= \iint_{4\pi} P_n(\theta, \phi)\, d\Omega$$

where $d\Omega = \sin\theta\, d\theta\, d\phi$

& $P_n(\theta) = V_n^2(\theta)$

or

$E_n^2(\theta)$

Approximately

$$\Omega_A \simeq \theta_{HP}\,\phi_{HP}$$

$\Omega_A \downarrow$ sq degree, $\theta_{HP} \downarrow$ degree, $\phi_{HP} \downarrow$ degree

$$D = \frac{4\pi}{\Omega_A}$$

$$= \frac{41{,}253^{\square}}{\theta_{HP}^{\circ}\,\phi_{HP}^{\circ}}$$

where $41{,}253^{\square}$ = no. of square degrees in sphere

$$= 4\pi\left(\frac{180}{\pi}\right)^2 \text{ sq. degree}$$

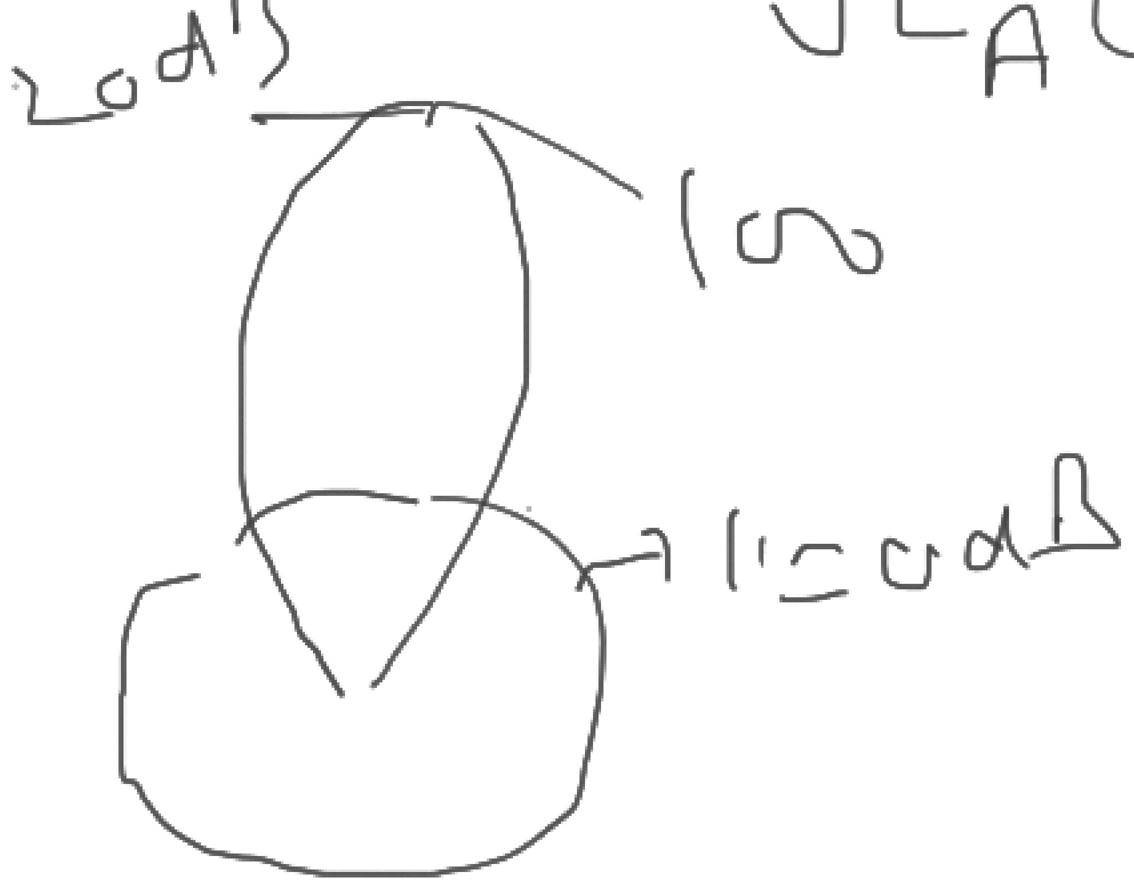
$4\pi$ steradian

$= 4\pi \times 3283 = 41{,}253^{\square}$

Let HPBW in 2 planes is 20° each

ie, $\theta_{HP} = \phi_{HP} = 20°$

$$D = \frac{4\pi (sr)}{\Omega_A (sr)} \simeq \frac{4\pi (sr)}{\theta_{HP} \phi_{HP}}$$




$$= \frac{41,253^{\square}}{20° \times 20°}$$

$$= 100 \text{ (dimensionless)}$$

$$D\Big|_{dB} = 10\log(100)$$

$$= 20 dB$$

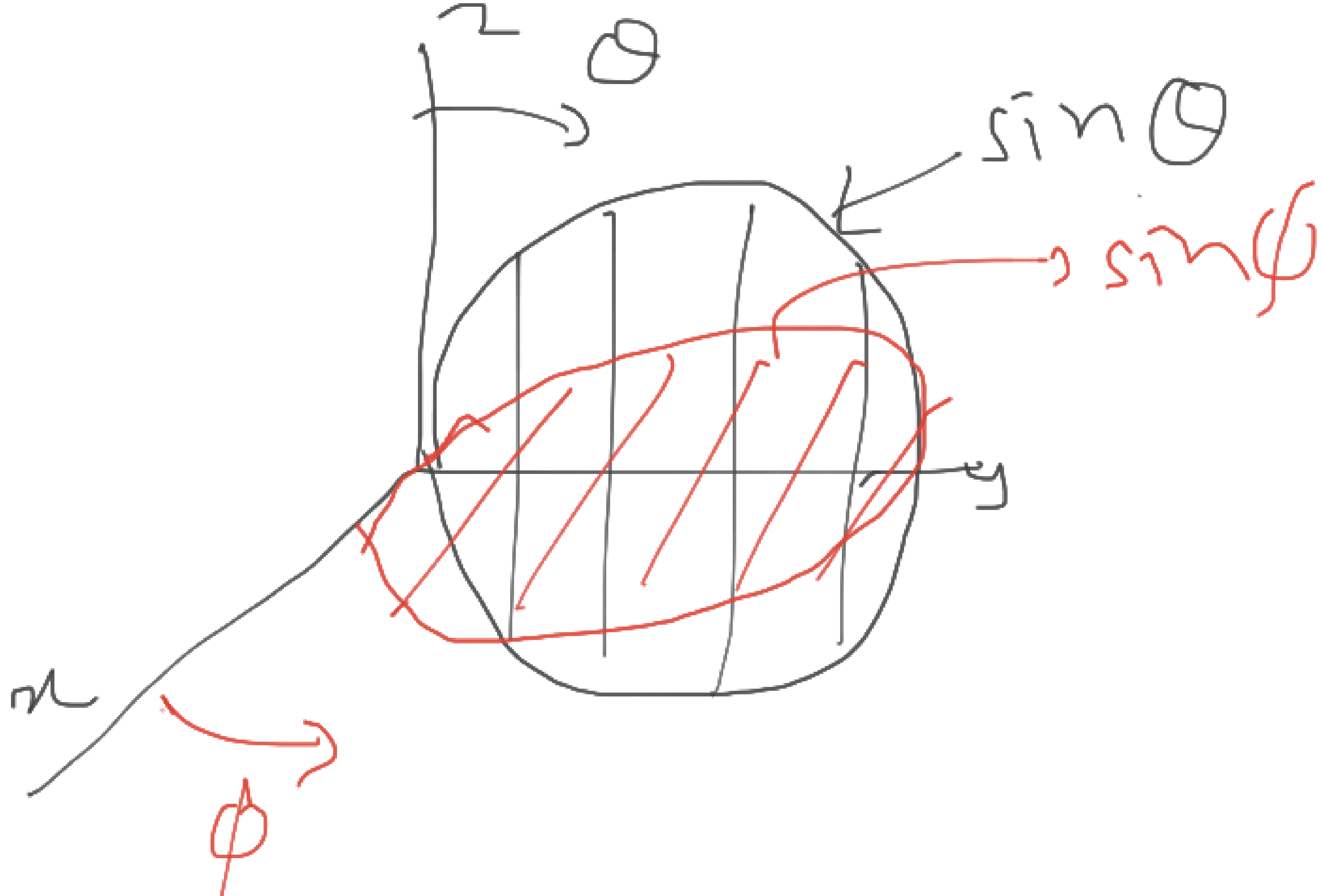
Ex :- Let $E(\theta, \phi) = \sin\theta \sin\phi$

or $V(\theta, \phi)$

$0 \le \theta \le \pi$

$0 \le \phi \le \pi$

Find exact directivity in azimuthal direction




$\Rightarrow \theta_{HPBW} = \theta_{HP}$
$= 90^\circ$

$\Rightarrow \phi_{HPBW} = \phi_{HP}$
$= 90^\circ$

Soln: Approx. Directivity

$$D \approx \frac{41,253^\circ}{\theta_{HP}\,\phi_{HP}} = \frac{41253^\circ}{90^\circ \times 90^\circ} = 5.1$$

$$D|_{dB} = 10\log(5.1)$$

Exact Directivity

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{\pi}\int_0^{\pi} P_n(\theta,\phi)\, \underbrace{d\Omega}_{\sin\theta\, d\theta\, d\phi}}$$

$$P_n(\theta) = V_n^2(\theta)$$

$$\Rightarrow D = \frac{4\pi}{\int_0^\pi \int_0^\pi (\sin\theta \sin\phi)^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\int_0^\pi \int_0^\pi \sin^3\theta \sin^2\phi \, d\theta \, d\phi}$$

$$= \frac{4\pi}{2\pi/3} = 6 \qquad \left(\begin{array}{c}\text{approx} \\ D = 5\end{array}\right)$$

$$D|_{dB} = 10\log(6)$$

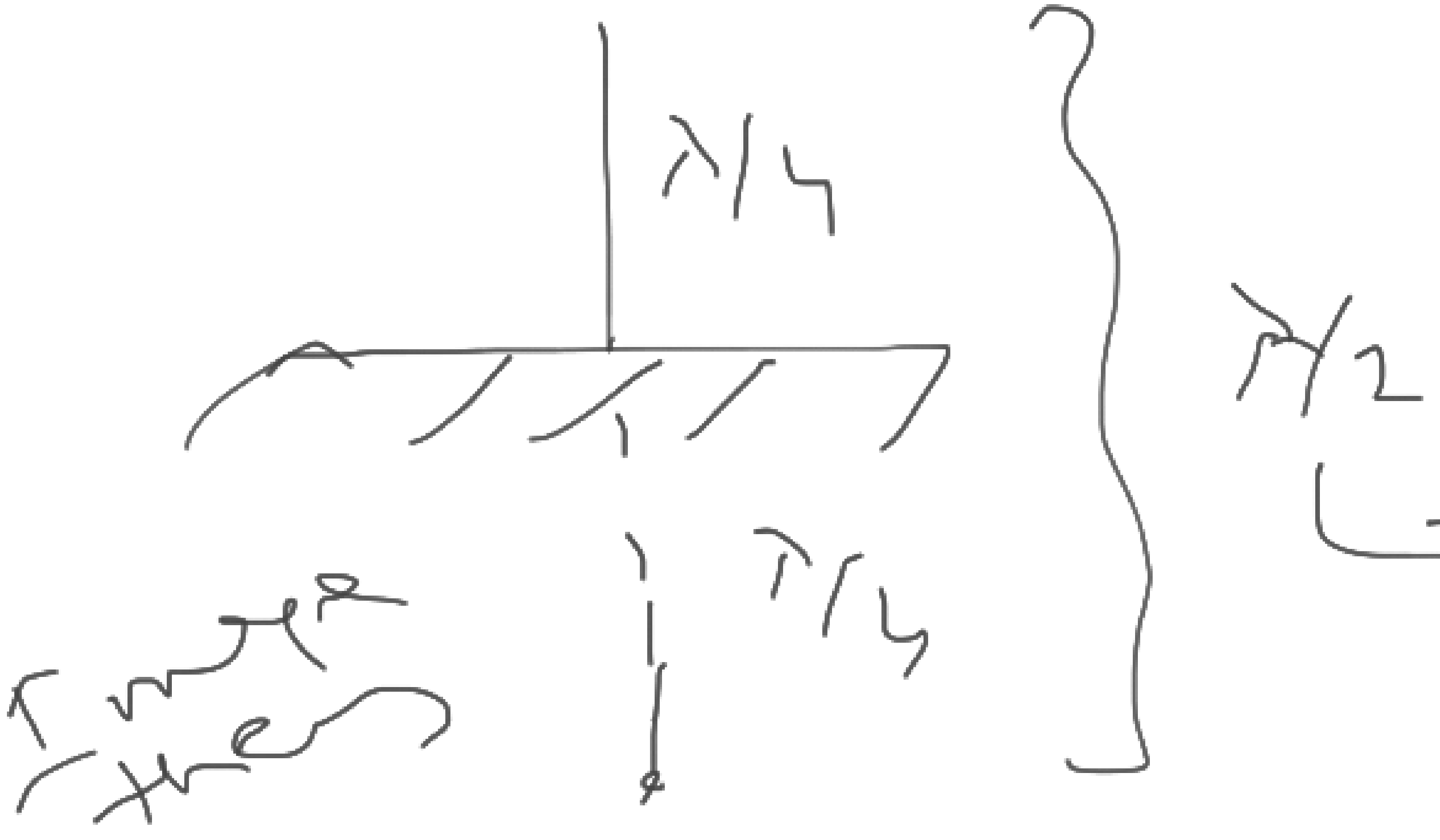
FM antenna (75 cm long)

88 to 108 MHz

Let $f = 100$ MHz, $\lambda = \frac{c}{f} = 300$ cm

$\frac{\lambda}{2} = 150$ cm

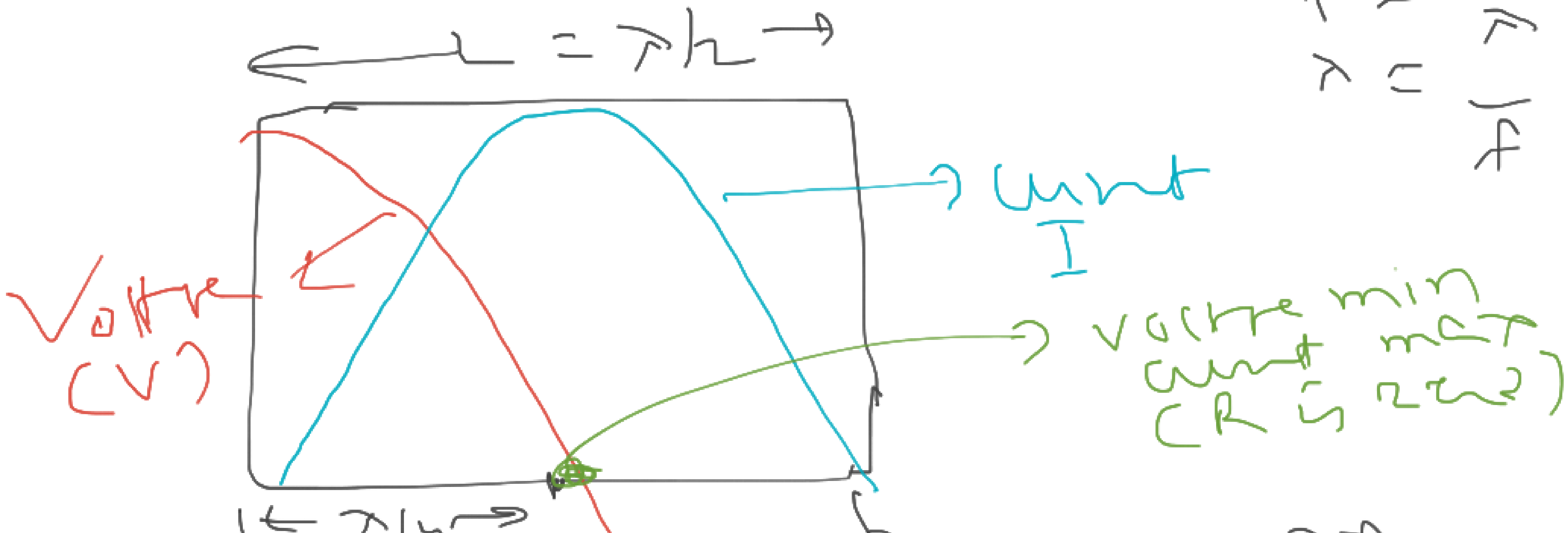
$\frac{\lambda}{4} = 75$ cm




$\lambda/2$ is more efficient than $\lambda/4$ antenna

$f = \frac{c}{\lambda}$

$\lambda = \frac{c}{f}$



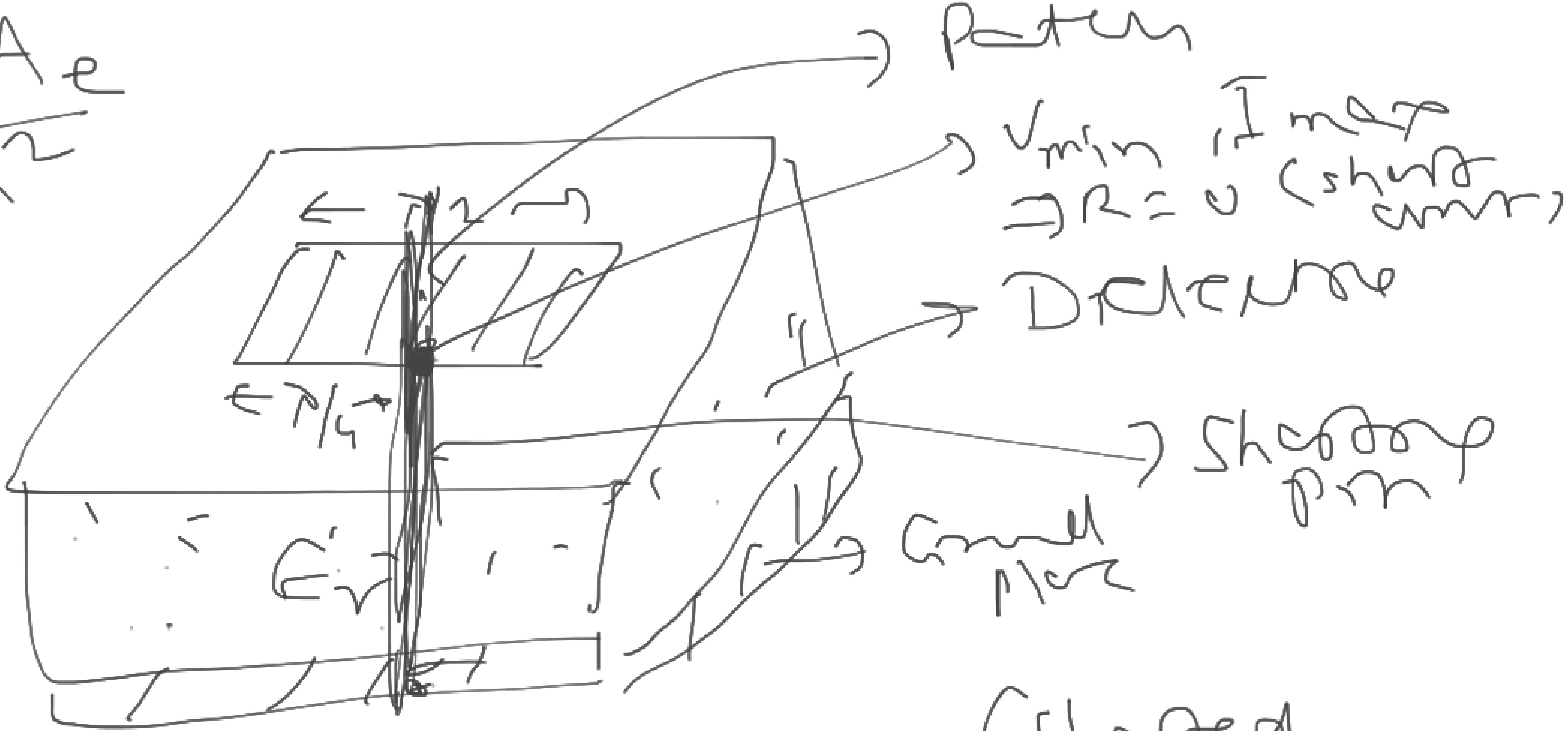

voltage max
current min
⇓
R is max

voltage max
current min
↓
R is max
↳ loss
↳ radiation loss

## Microstrip (or Printed antennas)

$$G = \frac{4\pi A_e}{\lambda^2}$$

$A_e$ = effective aperture area




(Shorted antenna)