Tutorial 1

SC-220 Groups and linear algebra Autumn 2019

(Groups)

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(1) Find all the rotational symmetries of the cube.

(2) If G is a group then show the following
i) The identity element of G is unique
ii)For $x \in G$ then x has a unique inverse
iii)For $a, b \in G$ there is a unique x such that $a * x = b$

(3) Determine whether the binary operation $*$ gives a group structure
i)Let $*$ be defined on $\mathbb{Z}$ by $a * b = ab$
ii)Let $*$ be defined on $\mathbb{R}^+$ by $a * b = \sqrt{ab}$
iii)Let $*$ be defined on $\mathbb{R} - \{0\}$ by $a * b = \frac{a}{b}$

(4) Let $G = \{a + \sqrt{2}b \in \mathbb{R} | a, b \in \mathbb{Q}\}$
i) Prove that G is a group under addition.
ii)Prove that the non-zero elements of G are a group under multiplication.

(5) Let S be the set of all real numbers except -1. Define an operation $*$ on S by

$$a * b = a + b + ab$$

i) Show that $< S, * >$ is a group.
ii)Find the solution to the following equation in S.

$$2 * x * 3 = 7$$

(6) Show that a group of three elements is commutative

(7) If x and y are elements of a group show that $(x * y)^{-1} = y^{-1} * x^{-1}$.

(8) Prove that if $x^2 = 1$ for all $x \in G$ then G is a commutative (abelian) group.
