 (Satyendranath Bose)

Bose-Einstein Statistics and Condensates

1/. Pertains to indistinguishable particles that do not follow Pauli's Exclusion Principle.

2/. Have integral spins (0, 1, 2, ... in the unit $\hbar$).

3/. Collectively named BOSONS. $\left[\hbar = \frac{h}{2\pi}\right]$

Indistinguishable Particles

1/. Wave-Particle Duality: $\boxed{\lambda = h/p}$.

2/. A free particle is under No force.

Since $\boxed{F = -\frac{dU}{dx}}$ ($F \to$ Force, $U \to$ Potential),

if $\boxed{F = 0}$ $\Rightarrow$ $\boxed{U = \text{Constant} = 0}$ (No potential difference)

Hence, $\boxed{\mathcal{E} = \frac{1}{2}mv^2 = \mathcal{E}_K}$ (Only kinetic energy).

(Total energy)

3/. $\boxed{\mathcal{E}_K = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}}$ $\Rightarrow$ $\boxed{\lambda = \frac{h}{\sqrt{2m\mathcal{E}_K}}}$

4/. In a gas of such particles (large aggregate). $\boxed{\mathcal{E}_K \sim k_B T}$ $\Rightarrow$ $\boxed{\lambda \sim \frac{h}{\sqrt{m k_B T}}}$

$\lambda \to$ Thermal de Broglie Wavelength

(P.T.O.)

5/. When T is large (hot conditions), λ is small. Since λ represents the spatial extent over which a particle can be located, a small value of λ implies prominent particle-like properties (like gas molecules at ordinary room temperature) [Distinguish-able]

6/. When T is small (cold conditions), λ is large. Hence, wave-like properties become prominent. The length of the waves becomes large enough to be comparable to the inter-particle spacing (overlapping occurs).

7/. The overlapping of waves causes indistinguishability (the particles cannot be distinguished from one another).

8/. Hence, cold gases have quantum features, based on the wave-particle duality (not applicable to dilute gases).

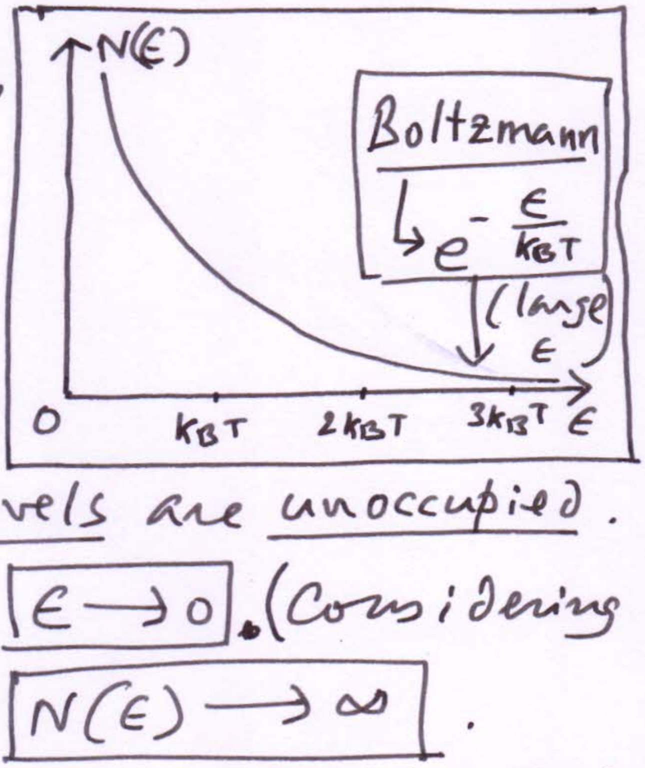
The Energy Distribution function

1/. For particles that do not obey Pauli's Exclusion Principle, the number of particles with energy ϵ is

$$\boxed{N(\epsilon) \sim \frac{1}{A e^{\epsilon/k_B T} - 1}}$$ → The Bose-Einstein Distribution function.

2/. i) When T is small (cold), for $\boxed{\epsilon \gg k_B T}$, $\boxed{N(\epsilon) \to 0}$.

$\boxed{N(\epsilon) \sim e^{-\epsilon/k_B T}}$ (like the Boltzmann function).



Hence, high-energy levels are unoccupied.

ii) For small T, when $\boxed{\epsilon \to 0}$. (Considering $\boxed{A=1}$), $\boxed{e^{\epsilon/k_B T} \to 1}$ ⇒ $\boxed{N(\epsilon) \to \infty}$.

Hence, under ultra cold conditions (10^{-7} K) low-energy levels become overpopulated to a great extent → CONDENSATION.

Forms a Bose-Einstein Condensate.

3/. Experimentally verified for Rb^{87} atom vapour which were LASER Cooled. (Cornell, Wieman, Ketterle).

(Also in superconductivity and superfluidity).