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Transmission line

TM mode - transverse electromag wave

Types of T lines
Open 2 wire
Coaxial
Rectangular
Circular

V & I change with length

Loss less propagation

$$V(z,t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^-$$

Loss less

$$\gamma = j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$

$$v_p = \frac{1}{\sqrt{LC}}$$

Lossy

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

attenuation / phase of wave

Low loss

$$\gamma = \alpha + j\beta$$

$$Z_0 = \sqrt{L/C}$$

$$\beta = j\omega\sqrt{LC}$$

$\frac{V}{I}$ of incident wave
reflected wave

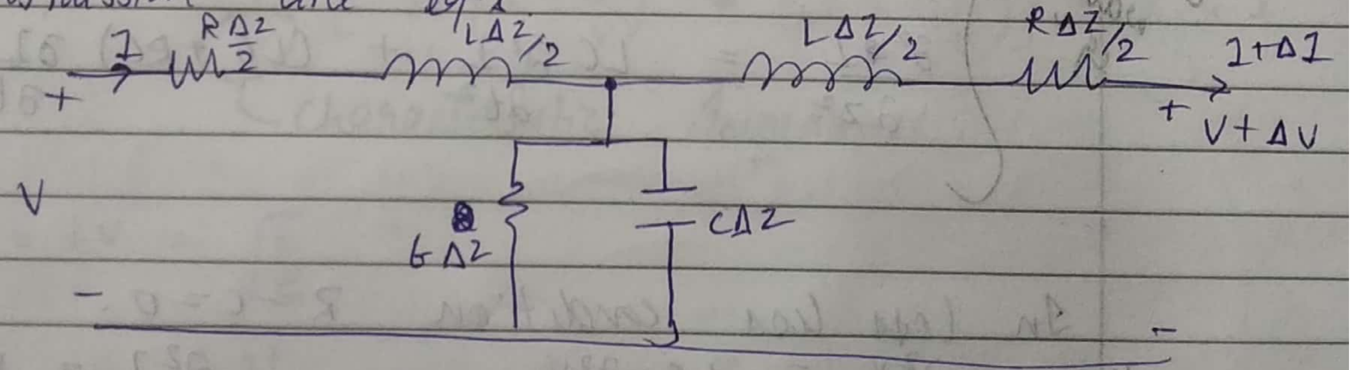
$$\alpha = \left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)\frac{1}{2}$$

Notes

lumped elements: time delay to cross $\simeq 0$
distributed : if inductor, cap., r large the time delay ↑
wave faster if L, C ↓
$\therefore v = \frac{1}{\sqrt{LC}}$ lossless transmission

Series of lumped L & C are connected in ladder network form. charging of L & C continues down the network lane. The position of wavefront can be identified as the pt. btw. 2 adjacent capacitors that exhibit most difference btw their charged levels.
when R is connected to such network the discharge of C starts from $3 \to 2 \to 1$ ⇒ when network is completely discharged a voltage pulse is obtained at the R. So this is called pulse forming network.

⇒ Transmission line eqn's



KVL $\quad \frac{\Delta V}{\Delta z} = -\left(RI + L\frac{\partial I}{\partial t} + \frac{1}{2}L\frac{\partial \Delta I}{\partial t} + \frac{1}{2}R\Delta I\right)$

$$\frac{\partial V}{\partial z} = -\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z}\right)\left(RI + L\frac{\partial I}{\partial t}\right)$$

$\Delta z \to 0$

$$\boxed{\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right)}$$

KCL

$$I = I_g + I_c + (I + \Delta I) = G\Delta z\left(V + \frac{\Delta V}{2}\right) + C\Delta z \frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2}\right) + (I + \Delta I)$$

$$\therefore \frac{\partial I}{\partial z} = -\left(1 + \frac{\Delta z \partial}{2\partial z}\right)\left(GV + C\frac{\partial V}{\partial t}\right)$$

$\Delta z \to 0$

$$\boxed{\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right)}$$

☐ – Telegraphist's Eqⁿ.

Differentiating $\partial V/\partial z$ by dz & $\partial I/\partial z$ by ∂t & rearranging

General wave eqⁿ

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV$$

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + (LG + RC)\frac{\partial I}{\partial t} + RGI$$

In Loss less condition $R = C = 0$.

★ $\therefore \dfrac{\partial^2 V}{\partial z^2} = LC\dfrac{\partial^2 V}{\partial t^2}$, $\dfrac{\partial^2 I}{\partial z^2} = LC\dfrac{\partial^2 I}{\partial t^2}$

$$V(z,t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^-$$

wave velocity = const.

f_1 +ve z ‖ $t\uparrow \Rightarrow z\uparrow$ in the dirⁿ to keep track of

f_2 −ve z ‖ $t\uparrow \Rightarrow z\downarrow$ $f_1(0)$ & $f_2(0)$.

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_1}{\partial(t - z/v)} \cdot \frac{\partial(t - z/v)}{\partial z} = f_1' \cdot \left(-\frac{1}{v}\right)$$

$$\frac{\partial f_1}{\partial t} = f_1' \cdot (1)$$

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{v^2} f_1'' \qquad \frac{\partial^2 f_1}{\partial t^2} = f_1''$$

$$= LC f_1'' \quad (\because \star)$$

$$\therefore v = \frac{1}{\sqrt{LC}}$$

$$\rightarrow \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \qquad \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial t} = \frac{1}{Lv} (f_1' - f_2')$$

$$I(z,t) = \frac{1}{Lv} \left[f_1\left(t - \frac{z}{v}\right) - f_2\left(t + \frac{z}{v}\right) \right] = I^+ + I^-$$

characteristic Impedance.

$$Z_0 = Lv = \sqrt{\frac{L}{C}}$$

$$V^+ = Z_0 I^+ \qquad V^- = -Z_0 I^-$$

- Let f_1, f_2 be cosine func'n.

$$V(z,t) = |V_0| \left(\underbrace{\cos(\omega t - \beta z)}_{\text{forward}} + \underbrace{\cos(\omega t + \beta z)}_{\text{backward}} \right)$$

$$\beta = \frac{\omega}{v_p} \Rightarrow \leftarrow \text{spatial freq.}$$

phase const.

phase velocity

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β = phase shift per unit distance

$\therefore$ For distance λ & $t=0$

$V_f = V_{f0} = |V_0| \cos \beta z$

$\beta \lambda = 2\pi$

$\therefore \lambda = \frac{2\pi}{\beta} = \frac{V_p}{f}$

* Complex Analysis of sinusoidal waves

$V(z,t) = \frac{1}{2} V_0 \, e^{j\omega t} \cdot e^{\pm j\beta z}$ + conjugate

Complex instantaneous voltage $= V_0 \, e^{\pm j\beta z} \cdot e^{j\omega t} = V_c$

Phase voltage $= V_s = V_0 \, e^{\pm j\beta z}$

defined when we have sinusoidal steady state condition

$\Rightarrow V_0$ is independent of time

like observing standing wave in space at $t=0$.

Real inst. Volt. $= V(z,t) = \frac{1}{2} V_c + cc. = Re(V_c)$

$= \frac{1}{2} V_s \cdot e^{j\omega t} + cc.$

$V(z,t) = Re\{$ Phasor voltage $\times e^{j\omega t})$

Double derivative + rearrange

* TL & sol^n in phasor form

$$\frac{\partial^2 V_s}{\partial z^2} = -\omega^2 LC \, V_s + j\omega (LG + RC) V_s + RG V_s$$

$$= \underset{Z}{(R + j\omega L)} \underset{Y}{(G + j\omega C)} V_s = \gamma^2 V_s$$

$Z = R + j\omega L$ = Net series impedance

$Y = G + j\omega C$ = " shunt admittance

γ = propagation const. $= \sqrt{(R+j\omega L)(G+j\omega C)}$

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$\gamma = \alpha + j\beta = \sqrt{Z.Y}$

β ← phase const.

α ← attenuation const.

$\therefore \frac{\partial^2 V_s}{\partial z^2} = \gamma^2 V_s$

$\therefore V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$

$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$

Relⁿ btw I & V by Telegraphists' eqⁿs.

$\frac{dV_s}{dz} = -\left(R + L\frac{dI}{dt}\right) = -(R + j\omega L) I_s = -Z I_s$

$\frac{dI_s}{dz} = -\left(G + C\frac{dV_s}{dt}\right) = -(G + j\omega C) V_s = -Y V_s$

$-\gamma\left(V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}\right) = -Z\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$

equating coeffs.

$\therefore Z_0 = \frac{Z}{\gamma}$

$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}}$

$\therefore Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = |Z_0| e^{j\theta} = |Z_0| e^{j(\theta - \xi)}$

Characteristic Impedence

* LOW LOSS propagation

$V_s(z) = V_0^+ e^{-\alpha z} . e^{-j\beta z} + V_0^- e^{\alpha z} . e^{j\beta z}$

$\Rightarrow V_s$ attenuates with propagation dist. at a rate of $\underline{\underline{\alpha}}$ Nepers/m

$\lambda = \frac{2\pi}{\beta}$ & $\beta = \frac{\omega}{V_p}$

Low loss when $\alpha \to 0$

Scanned by CamScanner

In Low loss $R << \omega L$ & $G << C\omega$

$\therefore \gamma = \alpha + j\beta = [(R+j\omega L)(G+j\omega C)]^{1/2}$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}$$

$$\alpha + j\beta = j\omega\sqrt{LC}\left(1+\frac{R}{2j\omega L}+\frac{1}{8}\frac{R^2}{\omega^2 L^2}\right)\left(1+\frac{G}{2j\omega C}+\frac{G^2}{8\omega^2 C^2}\right)$$

$$= j\omega\sqrt{LC}\left(1+\frac{1}{2j\omega}\left(\frac{R}{L}+\frac{G}{C}\right)+\frac{1}{8\omega^2}\left(\frac{R^2}{L^2}+\frac{G^2}{C^2}-\frac{2RG}{LC}\right)\right)$$

$$\boxed{\begin{aligned}\alpha &= \frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)\\ \beta &= \omega\sqrt{LC}\left[1+\frac{1}{8}\left(\frac{G}{\omega C}-\frac{R}{\omega L}\right)^2\right]\end{aligned}}$$

$v_p = \frac{\omega}{\beta}$ is freq. dependent

v_g = group vel. $= \frac{d\omega}{d\beta}$ = depends on freq. & leads to sig. distortion

Heaviside's Condition

With R & G non zero, phase & group velocities that are const. with freq. can be obtained when

$R/L = G/C$

$\rightarrow \beta = \omega\sqrt{LC} \Rightarrow$ sig. is distortionless.

Generally loss increases with increasing freq. ($\because \uparrow\omega \Rightarrow \uparrow R$)

This effect is known as skin effect loss.

$$\therefore Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}\left[\frac{1+\frac{R}{2j\omega L}+\frac{R^2}{8\omega^2L^2}}{1+\frac{G}{2j\omega C}+\frac{G^2}{8\omega^2C^2}}\right]$$

$\times$ & $\div$ by cc. of D^n & simplify by neglecting higher terms of R^2G & G^2R etc.

$$Z_0 = \sqrt{\frac{L}{C}}\left(1+\frac{1}{2\omega^2}\left(\frac{1}{4}\left(\frac{R}{L}+\frac{G}{C}\right)^2-\frac{G^2}{C^2}\right)+\frac{j}{2\omega}\left(\frac{G}{C}-\frac{R}{L}\right.\right.$$

when low loss $R/L = G/C \Rightarrow Z_0 = \sqrt{L/C}$

POWER

$$P = VI$$
$$= |V_0||I_0|\, e^{-2\alpha z}\cos(\omega t-\beta z)\; \therefore \cos(\omega t-\beta z+\theta)$$

$$\langle P\rangle = \frac{1}{T}\int_0^T P\,dt$$

$$= \frac{1}{2}|V_0||I_0|\,.\,e^{-2\alpha z}\cos\theta \quad = \frac{1}{2}\frac{|V_0|^2}{Z_0}\cdot e^{-2\alpha z}\cos\theta$$

$$= \frac{1}{2}\,Re\,\{V_s I_s^*\}$$

$$= \langle P(0)\rangle e^{-2\alpha z}$$

$$\text{Power loss dB} = 10\log_{10}\left[\frac{\langle P(0)\rangle}{\langle P(z)\rangle}\right] = 8.69\,\alpha z$$

* Wave Reflection

$$V_i(z) = V_{0i}\, e^{-\alpha z} \cdot e^{-j\beta z}$$
$$V_r(z) = V_{0R}\, e^{+\alpha z} \cdot e^{+j\beta z}$$

$$\Gamma(x) = \frac{V_r}{V_i} = |\Gamma|\, e^{j(\phi + 2\beta x)}$$

$$V_L = V_{0i} + V_{0R}$$
$$I_L = I_{0i} + I_{0R}$$
$$= \frac{1}{Z_0}\left(V_{0i} - V_{0r}\right) = \frac{V_L}{Z_L} = \frac{(V_{0i} + V_{0R})}{Z_L}$$

$$\Gamma(x=0) = \frac{V_{0r}}{V_{0i}} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma|\, e^{j\phi_r}$$

For lossless line Γ is purely real

$$\therefore V_L = V_{0i} + \Gamma V_{0i} = (1+\Gamma) V_{0i}$$
$$= V_{0i} + V_{0R}$$

$\therefore \tau$ = transmission coeff. $= \frac{V_L}{V_{0i}} = 1 + \Gamma$

$$\tau = \frac{2 Z_L}{Z_L + Z_0}$$

$\tau > 1$ but avg. P at load $\leq$ that in incident w.

For no reflection $\Gamma = 0$ $\Rightarrow$ Load impedence = line "

if so then load is said to be matched with line.

$$\frac{\langle P_r \rangle}{\langle P_i \rangle} = |\Gamma|^2$$

$$\langle P_i \rangle = \frac{1}{2} Re\left\{ \frac{V_0 V_0^*}{|Z_0|} e^{-2\alpha L} \cdot e^{j\theta} \right\} = \frac{|V_0|^2}{2|Z_0|} \cdot e^{-2\alpha L} \cos\theta$$

$$\langle P_r \rangle = \frac{\Gamma^2 |V_0|^2}{2|Z_0|} e^{-2\alpha L} \cos\theta$$

Fraction of incident power that is transmitted into the load is $\frac{\langle P_T \rangle}{\langle P_i \rangle} = 1 - |\Gamma|^2$ = P dissipated

* Standing Wave.

$\left.\begin{array}{l} Z_L = 0 \\ Z_L \text{ real } \& < Z_0 \end{array}\right\} \phi = \pi$

$$V(x) = V_i e^{-j\beta x} + V_r e^{j\beta x}$$

$$= V_i e^{j\beta x} \left(1 + \frac{V_r}{V_i} e^{2j\beta x}\right)$$

Standing wave.

$$V(x) = \underbrace{V_i e^{j\beta x}}_{} \left(1 + |\Gamma| e^{j(\phi + 2\beta x)}\right)$$

For a particular x is const.

$$V(x)_{max} = V_i e^{j\beta x} (1 + |\Gamma|) \quad ; 2\beta x + \phi = \pm 2n\pi$$

$$V(x)_{min} = V_i e^{j\beta x} (1 - |\Gamma|) \quad ; 2\beta x + \phi = \pm (2n+1)\pi$$

$V(x)_{max}$ when $x = -\frac{1}{2\beta}(2n\pi + \phi) = -\left(\frac{\phi}{2\beta} + \frac{n\pi}{\beta}\right)$

$$x_{max} = -\left(\frac{\phi}{2\beta} + \frac{n\lambda}{2}\right)$$

For real & +ve impedance $\phi = 0$ $(\because Z_L > Z_0 \Rightarrow \phi = 0)$

$\therefore x_{max} = \frac{-n\lambda}{2}$; $0, \frac{-\lambda}{2}, -\lambda, \frac{-3\lambda}{2} \dots$

when maxima at load or

$V(x)_{min} \Rightarrow x_{min} = -\left(\frac{\phi}{2\beta} + \frac{(2n+1)\pi}{2\beta}\right)$

$$= -\left(\frac{\phi}{2\beta} + \frac{(2n+1)\lambda}{4}\right)$$

$-\lambda/4, -3\lambda/4, -5\lambda/4$

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s = voltage standing wave ratio

$$= \frac{1+|\Gamma|}{1-|\Gamma|} \qquad 0 \le |\Gamma| \le 1$$

$$1 \le S < \infty$$

Book:

⇒ 2 special cases:

1. Slotted line terminated by matched impedance ~~has~~ then no reflected wave. ⇒ probe gives same V at every pt.
Equal amplitude voltages are characteristic of an unattenuated traveling wave.

2. If terminated by open or short circuit. (⇒ Z_L is purely imaginary) ⇒ V_L is standing wave.
No O/P at nodes
For short circuit $\phi = \pi$ voltage varies as $|\sin \beta z|$

Perfectly matched load ⇒ VSWR = 1
Total reflecting load " = ∞

$$V_{ST} = V_0 (e^{-j\beta z} + |\Gamma| e^{j(\beta z + \phi)}) = V_0 e^{+j\phi/2} (e^{-j\beta z} e^{-j\phi/2} + |\Gamma| e^{j\beta z} e^{j\phi/2})$$

$$= V_0 (1 - |\Gamma|) e^{-j\beta z} + 2V_0 |\Gamma| e^{j\phi/2} \cos(\beta z + \phi/2)$$

$$v(z,t) = V_0 (1 - |\Gamma|) \cos(\omega t - \beta z) \leftarrow \text{Travelling wave}$$
$$+ 2|\Gamma| V_0 \cos(\beta z + \phi/2) \cos(\omega t + \phi/2)$$

← Standing wave

$Re(V_{ST}(z) \cdot e^{j\omega t})$

Min. when S wave achieves null.
Max. " Total = $V_0(1+|\Gamma|)$

* Noise.

1 Thermal Noise / Johnson Noise

~~The free e's within~~

e^- have KE ∵ energy exchange b/w conductor & the surroundings. Motion randomised through collisions with imperfections in structure of conductor. This is conductors resistance. ∵ temp. ∴ Thermal Noise

Avg. or mean noise voltage across conductor = 0.
But RMS is finite & can be measured
MS ∝ R of conductor
∝ Absolute temp
∝ Freq. bandwidth of device measuring the noise

$$E_n^2 = 4RkTB_n \leftarrow \text{MS voltage}$$

B_n ← Noise Bandwidth

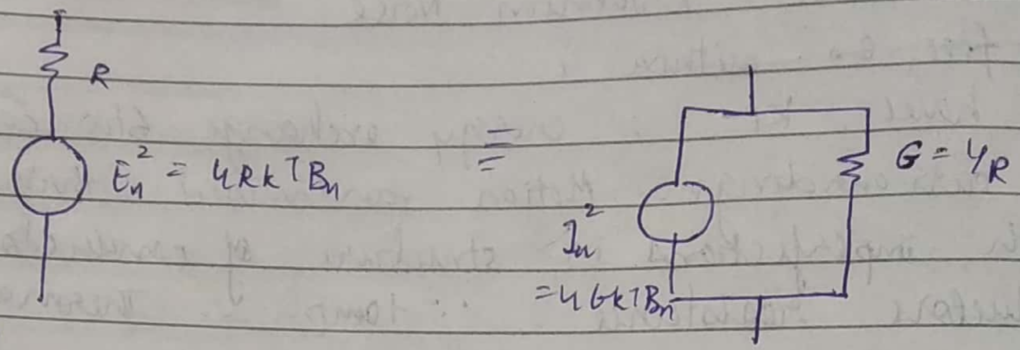
$k = 1.38 \times 10^{-23}$ J/K.

$$\text{RMS} = \sqrt{4RkTB_n}$$

Presence of RMS V at terminals of R suggest that it ~~can~~ can be considered as a source of electrical noise power.
To abstract noise power R is connected to resistive load & in thermal equi. the load would supply as much energy to R as it receives.

Available avg. power = Max. avg. power the source can deliver.

$$P_n = kTB_n$$



Thermal noise generator

$$E_n = \sqrt{E_{n1}^2 + E_{n2}^2 \ldots}$$

If resistance left open circuited $\Rightarrow R \to \infty$
$\Rightarrow E \to \infty$

2 things prevent this →

1. Derivation of noise is based on classical thermodynamics & not quantum mechanical effects.
$\because$ QME $\Rightarrow$ Energy drops with inc. freq. which limits the noise power available.
But QM imp. when freq e Infrared.

2. All circuits contain reactance $\Rightarrow$ finite limit on bandwidth. In open circuited resistor, the self capacitance sets limit on bandwidth.

R in series

$$E_n^2 = 4R_{ser} \cdot kTB_n$$

$$= E_1^2 + E_2^2 - - -$$

R in parallel

$$I_n^2 = 4G_{par} kTB_n$$

$$= I_1^2 + I_2^2 \cdots$$

$\Sigma M Sq. I$ = M Sq. noise current.

But better to work with ~~potential~~ noise voltage.

$$E_n^2 = 4R_{par} kTB_n$$

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2} \cdots$$

* Reactance.
  doesn't generate thermal noise. ∵ can't dissipate power.
  In thermal equilibrium, equal amt. of power must be exchanged. ⇒ If R supplies P_1 to reactance the reactance must supply thermal noise power $P_2 = P_1$

But reactance doesn't dissipate power ⇒ $P_2 = 0$
∴ P_1 has to be zero.

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* Spectral Densities

Bandwidth B_n is property of external measuring or receiving sys. & is assumed flat so that available spectral density

$$G_a(f) = \frac{P_n}{B_n} = kT$$

Spectral D. of MSq. Voltage

$$G_v(f) = \frac{E_n^2}{B_n} = 4RkT$$

SD are flat => independent of freq. so thermal noise is sometimes referred to as white noise.

When white noise is passed through a network the SD varies by the shape of network freq. response.