FORMULAE — SPECIAL RELATIVITY

Classical Velocity Addition: $u = u' + v$

Galilean Transformation: $x' = x - vt$,

$y' = y$, $z' = z$, $t' = t$.

(Exchange primes, $v \longrightarrow -v$ for inverse transform)

Time Dilation: $\Delta t = \gamma \Delta t_0$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = v/c$

$\Delta t_0 \rightarrow$ Proper time interval.

Length Contraction: $l = \frac{l_0}{\gamma} = \sqrt{1-(v/c)^2}\, l_0$

Lorentz Transformation: $x' = \gamma(x - vt)$,

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$, $y = y'$, $z = z'$.

Doppler Shift:

$\nu_{obs} = \sqrt{\frac{1-\beta}{1+\beta}}\, \nu_s$ (receding)

$\nu_{obs} = \sqrt{\frac{1+\beta}{1-\beta}}\, \nu_s$ (approaching)

$\nu = c/\lambda$.

Relativistic Velocity Addition:

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2} u_x'\right)}, \quad u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2} u_x'\right)}$$

Inertial and Gravitational Mass: $F = ma$, $W = m'g$, $g = \frac{GM_e}{R_e^2}$.

$m = m'$, $\vec{F}' = \vec{F} - m\vec{A}$

Momentum in Relativity: $p = m_0 \dfrac{dx}{dt_0}$

$m_0 \longrightarrow$ mass in rest frame, $t_0 \longrightarrow$ proper time

$$p = (m_0 \gamma)\frac{dx}{dt} = m\frac{dx}{dt} = \gamma m_0 v$$

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-(v/c)^2}}$$

Schwarzschild Radius: $R_s = \dfrac{2GM}{c^2}$

Gravitational Redshift: $\dfrac{\Delta\nu}{\nu} = -\dfrac{GM}{Rc^2}$ & $\dfrac{\Delta\lambda}{\lambda} = \dfrac{GM}{Rc^2}$

Force in Relativity: $F = \dfrac{d(p)}{dt} = \dfrac{d(mv)}{dt}$

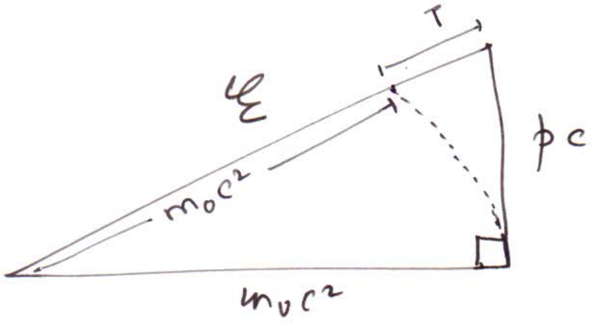
Kinetic Energy: $T = \dfrac{m_0c^2}{\sqrt{1-(v/c)^2}} - m_0c^2 = (\gamma - 1)m_0c^2$

Total Energy: $\mathcal{E} = mc^2 = T + m_0c^2 = \gamma m_0 c^2$

Rest Energy: m_0c^2

$$\mathcal{E}^2 = (pc)^2 + (m_0c^2)^2$$

Pythagorean relation



$\beta = \dfrac{v}{c} = \dfrac{pc}{\mathcal{E}}$ (when $m_0 = 0$, $\mathcal{E} = pc$).

$\therefore$ $\beta = 1$ when $m_0 = 0 \Rightarrow v = c$