Tutorial 2

SC-220 Groups and linear algebra Autumn 2019

(Dihedreal group, Subgroups, Cyclic groups, Permutation groups)

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(1) Show that the following subsets of D_4 are actually subgroups.
i) $\{1, r^2, s, r^2 s\}$ ii) $\{1, r^2, rs, r^3 s\}$

(2) Determine if the following set of matrices are subgroups of $GL_n(\mathbb{R})$.
i)The diagonal $n \times n$ matrices with no zeros on diagonal.
ii)The $n \times n$ matrices with determinant -1.
iii) The set of $n \times n$ matrices such that $A^T A = \mathrm{I}$.

(3) Find the order of
i) 2,6,10 in the additive group $\mathbb{Z}_{36}$.
ii)2 in the multiplicative group $\mathbb{Z}_{13}^*$.

(4) What are the generators of $\mathbb{Z}_5$? What about $\mathbb{Z}_9$ and $\mathbb{Z}_{12}$? Do you notice a pattern?

(5) Show that D_n is generated by by two elements rs and $r^2 s$.

(6) Let x and g be elements of a group G. Show that x and gxg^{-1} have the same order. Now show that xy and yx have the same order for any two elements x, y in G.

(7) Consider the group of invertible 2×2 matrices with entries in real numbers under matrix multiplication $GL_2(\mathbb{R})$. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ belong to $GL_2(\mathbb{R})$. Compute the order of A, B, AB, BA

(8) Let σ be the permutation $1 \mapsto 3$, $2 \mapsto 4$,$3 \mapsto 5$,$4 \mapsto 2$,$5 \mapsto 1$
and let τ be the permutation $1 \mapsto 5$, $2 \mapsto 3$,$3 \mapsto 2$,$4 \mapsto 4$,$5 \mapsto 1$
Find the cycle decompositions of σ^2, $\sigma\tau$ and $\tau^2\sigma$

(9) Show that if σ is the m-cycle $(a_1 a_2 ... a_m)$ then $|\sigma| = m$.

(10) Compute the order of the element (13)(246) in S_6.

(11) Show that if $n \geq m$ then the number of m-cycles in S_n is given by

$$\frac{n(n-1)(n-2)\cdots(n-m+1)}{m}$$
