Physical Systems

Physical examples of dynamical systems

Radioactivity: Unstable nucleus changing spontaneously into another nucleus by the process of radioactive decay.

$$\boxed{\frac{dx}{dt} = -\lambda x}$$

(Rate $\propto$ State) Decay Equation.

$(\lambda > 0) \rightarrow$ Decay constant

Integral solution is $\int \frac{dx}{x} = -\lambda \int dt$

$\Rightarrow \ln x = \ln x_0 - \lambda t \Rightarrow \boxed{x = x_0 e^{-\lambda t}}$.

$\Rightarrow -\lambda t = \ln\left(\frac{x}{x_0}\right) \Rightarrow \boxed{\lambda t = \ln\left(\frac{x_0}{x}\right)}$, x_0 is the integration constant.

Half-life is where $\boxed{x = x_0/2}$, where x_0 is the initial amount. If $T_{1/2}$ is the half life,

$\lambda T_{1/2} = \ln\left(\frac{x_0}{x_0/2}\right) \Rightarrow \boxed{T_{1/2} = \frac{1}{\lambda}\ln 2} \Rightarrow \boxed{T_{1/2} \simeq \frac{0.693}{\lambda}}$

Half life examples: U-238, $T_{1/2} = 4.5 \times 10^9$ yrs

Ra-226, $T_{1/2} = 1600$ yrs, Pb-210, $T_{1/2} = 22$ yrs.

i.) α-decay: $\boxed{{}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He}$ α-particles are ${}^4He$ nuclei.

α-decay causes a shift by two columns to the left in the chemical periodic table. Occurs in the case of unstable large nuclei.

(P.T.O.)

In the nucleus protons repel one another due to the Coulomb potential $\boxed{\sim \frac{1}{r}}$.

The nucleus, consisting of protons and neutrons is held together due to an attractive strong force potential $\boxed{\sim -\frac{1}{(r/r_0)} e^{-(r/r_0)}}$ (Yukawa) among neutrons and protons. In this potential function, r_0 is a characteristic length of the nuclear dimensions. If $r > r_0$, the strong attractive force decays rapidly, much more than the Coulomb repulsion.

When the proton number is large, a disproportionately large number of neutrons is needed to apply the strong attraction. But with this the nucleus also becomes large in size, and thus unstable, because the strong force does not have a long effect. The instability results in the α- decay.

ii) β- decay: $\boxed{{}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e}$ β- particles are electrons.

Or $\boxed{{}^{1}_{0}n \longrightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \bar{\nu}}$ $\bar{\nu}$ is an anti-neutrino.

(P.T.O.)

β- decay causes a shift by one column to the right in the chemical periodic table.

Occurs in the case of large neutron number.

iii) Positron emission : $\boxed{{}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}e + \nu}$ $\nu \rightarrow$ neutrino

Occurs in the case of large proton number.

Nuclear Reactions

Fission : Example → $\boxed{{}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n}$

i.) The 3 neutrons released cause chain reaction.

ii.) The fission is achieved by shooting a slow-moving neutron at U-235 nuclei. By the de Broglie relation $\boxed{\lambda = h/mv}$, a slow-moving neutron will have a large wave length and, hence, a greater cross-section of reaction.

iii.) The difference of the mass between the reactants and products is mass defect, Δm. Energy produced $= (\Delta m)c^2 \sim 10^2$ MeV

Fusion: Smaller ~~nuclei~~ nuclei fuse to form large nuclei.

i) Energy is extracted out of fusion instars till iron (Fe)

ii) Elements beyond iron are fused under extreme conditions of a supernova burst.

Radiometric Dating

Radiocarbon and geological dating.

Radiocarbon $\boxed{^{14}_{6}C}$: Cosmic rays enter the Earth's atmosphere and collide with the nuclei of atoms to produce neutrons. The neutrons react with $^{14}_{7}N$ as follows:

$$\boxed{^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}p}$$ to produce radiocarbon $\boxed{^{14}_{6}C}$.

β-decay of $^{14}_{6}C$: $\boxed{^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e}$

For $^{14}_{6}C$, $\boxed{T_{1/2} \simeq 5600 \text{ years}}$. The $^{14}_{6}C$ atoms are ingested in plants during photosynthesis and enter wood and the animal food chain. The decay of ^{14}C in dead wood (from charcoal or wood structures of ancient civilisations) helps in dating of the wood. The decay equation is $\boxed{\dot{x} = -\lambda x}$. At $t = t_0$, $x = x_0$

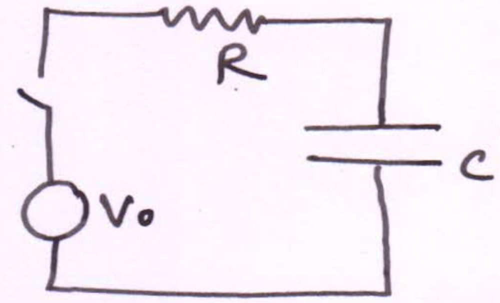
$$\therefore \boxed{x = x_0 e^{-\lambda(t-t_0)}} \Rightarrow \boxed{t - t_0 = \frac{1}{\lambda}\ln\left(\frac{x_0}{x}\right)}$$

$$\Rightarrow \boxed{t - t_0 = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{x_0}{x}\right) = \frac{T_{1/2}}{\ln 2}\ln\left[\frac{\dot{x}(t_0)}{\dot{x}(t)}\right]}$$

Since x_0 is difficult to estimate, it is easier to find the decay rates $\dot{x}(t_0)$ and $\dot{x}(t)$ for dating.

For geological dating nuclei with longer $T_{1/2}$ are used.
Example: U-238, $T_{1/2} = 4.5 \times 10^9$ yrs.

Q-R-C Circuit



i) Potential drop at the resistor, $\boxed{V = IR}$.

ii) At the capacitor, $\boxed{Q = VC} \Rightarrow \boxed{V = Q/C}$

Over the full circuit: $\boxed{V_0 = IR + Q/C}$

The current, $\boxed{I = \frac{dQ}{dt}} \Rightarrow R\frac{dQ}{dt} = V_0 - \frac{Q}{C}$

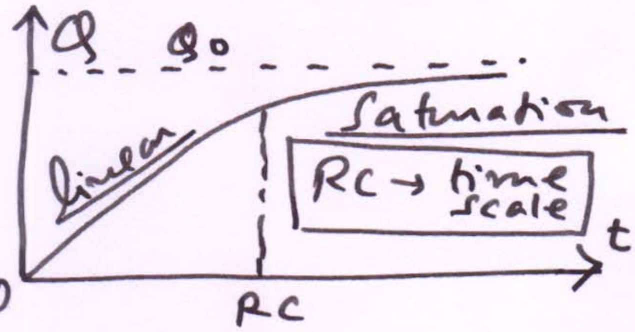
$\Rightarrow \boxed{\frac{dQ}{dt} = \frac{V_0}{R} - \frac{Q}{RC}}$ in the form $\boxed{\frac{dx}{dt} = a - bx}$

$a \to V_0/R$, $b \to 1/RC$.

$Q = \frac{V_0/R}{1/RC}\left(1 - e^{-t/RC}\right)$

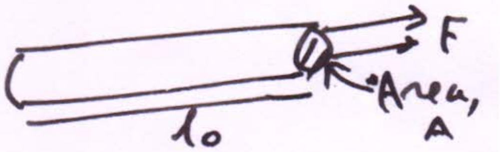
$\Rightarrow \boxed{Q = Q_0\left(1 - e^{-t/RC}\right)}$

$\boxed{Q_0 = V_0 C} \to$ limiting value

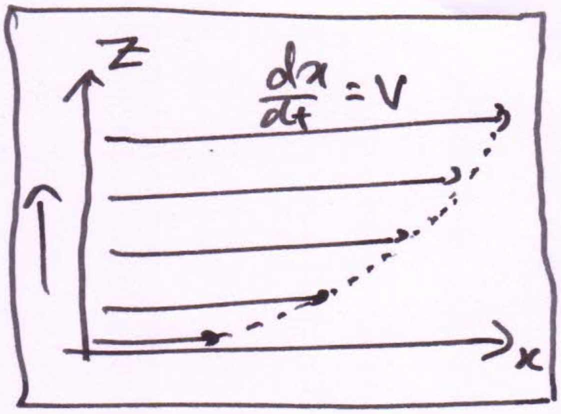


Viscoelasticity (Viscosity + Elasticity)

Elastic Property: Stress = $\frac{\text{Force}}{\text{Area}}$.



» $\boxed{\sigma\,(\text{stress}) = \frac{F}{A} \propto \epsilon}$, where ϵ is elongation (strain) over the original length l_0. $\boxed{\sigma \propto \epsilon}$ i.e. $\boxed{\text{Stress} \propto \text{Strain}}$. $\Rightarrow \boxed{\sigma = Y\epsilon}$, where Y is Young's modulus, an elastic property of a solid (with a defined structure). P.T.O.



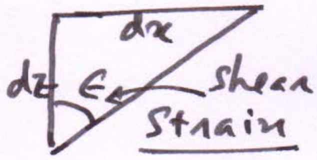
Viscous property: There is a viscous drag (fluid friction) between adjacent layers of a flowing liquid.

The drag force F, over an area A, is

$\boxed{F \propto A \frac{dv}{dz}}$ $\Rightarrow$ $\boxed{\sigma \text{ (viscous stress)} = \frac{F}{A} \propto \frac{dv}{dz}}$

$\therefore \boxed{\sigma = \eta \frac{dv}{dz}}$ where η is the coefficient of viscosity.

Now $\boxed{\sigma = \eta \frac{d}{dz}\left(\frac{dx}{dt}\right) = \eta \frac{d}{dt}\left(\frac{dx}{dz}\right)}$



With ϵ being the strain, we write

$\boxed{\tan \epsilon = \frac{dx}{dz}}$. For small ϵ, $\boxed{\tan \epsilon \simeq \epsilon \simeq \frac{dx}{dz}}$

$\therefore \boxed{\sigma = \eta \frac{d\epsilon}{dt}}$. Combining with $\boxed{\sigma = Y\epsilon}$ to account for both elasticity and viscosity we get, $\boxed{\sigma = Y\epsilon + \eta \frac{d\epsilon}{dt}}$, which is the equation of viscoelasticity, for a fixed stress σ.

$\Rightarrow \boxed{\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} - \frac{Y}{\eta}\epsilon}$ in the form $\boxed{\frac{dx}{dt} = a - bx}$.

$a \rightarrow \sigma/\eta$ and $b = Y/\eta$.

$\boxed{\epsilon = \frac{\sigma}{Y}\left(1 - e^{-\frac{Yt}{\eta}}\right)}$. The natural time scale in this system is η/Y.

Solid rocks flow out under the weight of Earth matter above it (Kelvin). Viscosity is like fugitive elasticity (Maxwell)