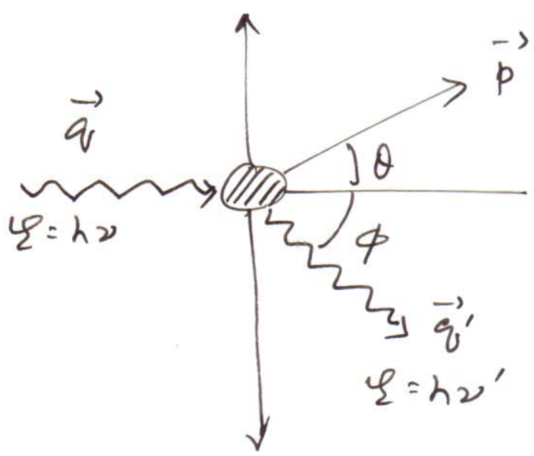
FORMULAE

COMPTON EFFECT



$$|\vec{q}| = \frac{h\nu}{c}, \quad |\vec{q}\,'| = \frac{h\nu'}{c}$$

$$c = \nu\lambda \; ; \quad p\sin\theta = \frac{h\nu'}{c}\sin\phi$$

$$p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right) \times \cos\phi$$

or $p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2 m_e h(\nu - \nu') - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)$

Wave length shift : $\lambda' - \lambda = \Delta\lambda = \lambda_c (1 - \cos\phi)$.

$\lambda_c = \frac{h}{m_e c} \rightarrow$ Compton wavelength of electrons

$m_e \rightarrow$ mass of an electron.

For maximum energy loss, $\Delta\lambda = 2\lambda_c$
because $\phi = 180°$, and $\cos\phi = -1$.

Fractional shift : $\frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda}(1 - \cos\phi)$.

$\cos\phi = 1 - \frac{hc}{\lambda_c}\left(\frac{1}{\xi'} - \frac{1}{\xi}\right)$ where $\xi' = \frac{hc}{\lambda'}$
and $\xi = hc/\lambda$

$\Delta\xi$

$= \xi - \xi' = hc \cdot \frac{\Delta\lambda}{\lambda(\lambda + \Delta\lambda)}, \quad \lambda' = \lambda + \Delta\lambda$.

$$\lambda^2 + (\Delta\lambda)\lambda - \Delta\lambda \frac{hc}{\Delta\xi} = 0 \Rightarrow \lambda = \frac{-\Delta\lambda + \sqrt{(\Delta\lambda)^2 + 4\Delta\lambda(hc/\Delta\xi)}}{2}$$

(Taking only positive root)