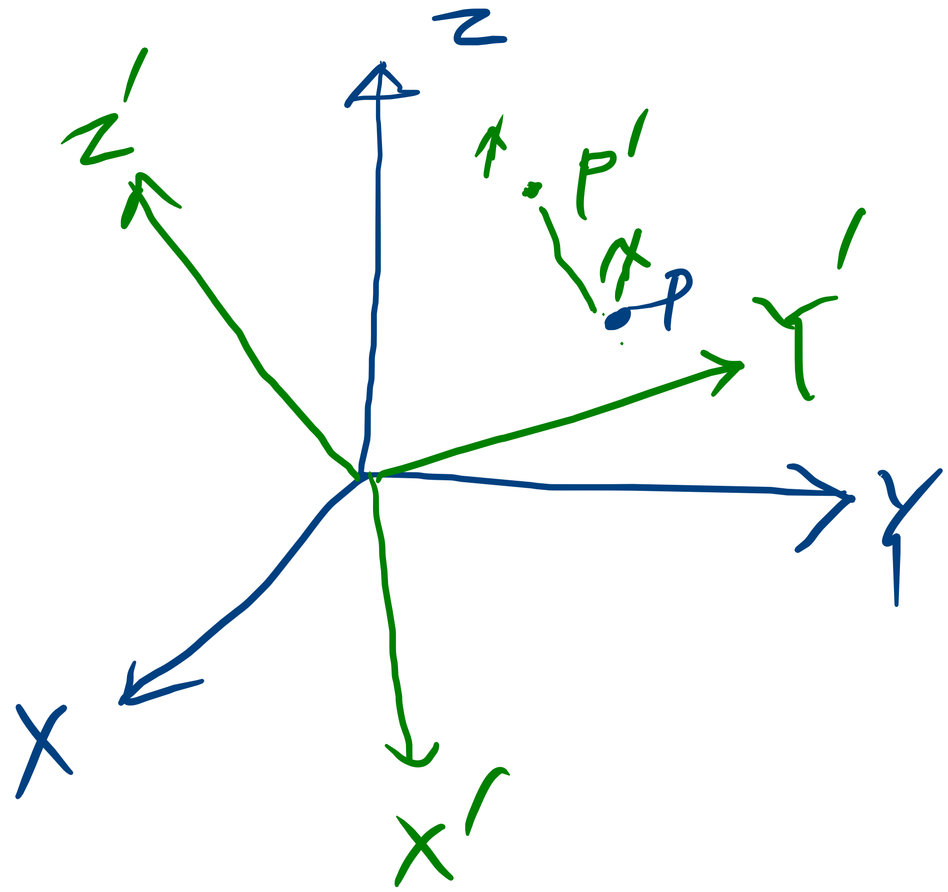
Date : 16.02.2021

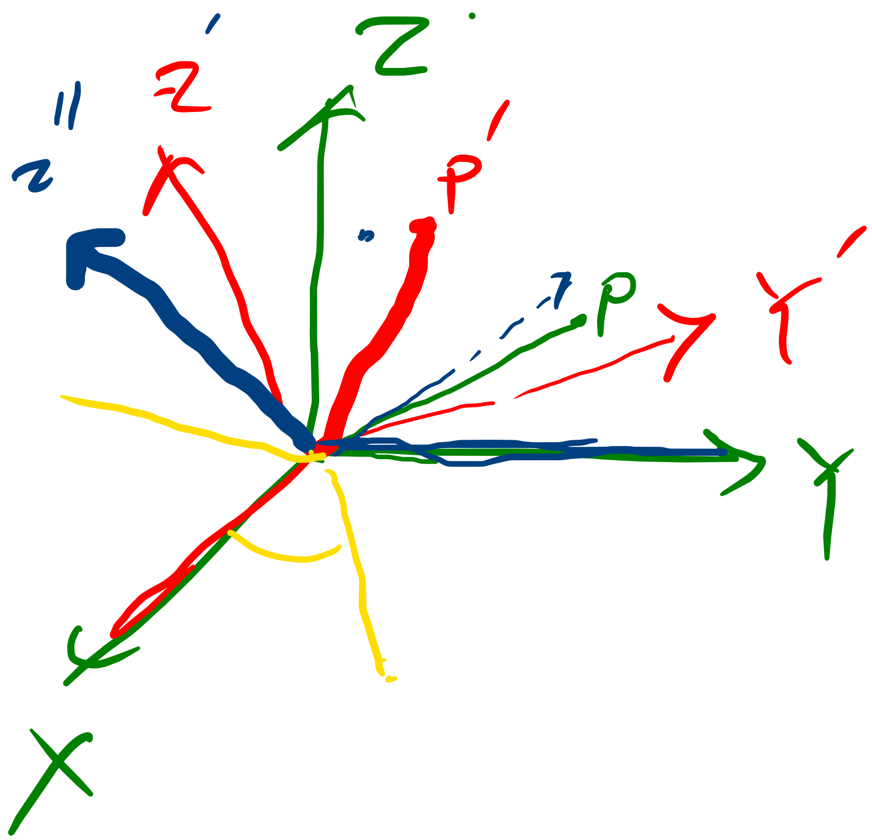


X - roll ⇒ ϕ

Y → pitch ⇒ θ

Z → Yaw ⇒ ψ

$$R = R(\phi)\, R(\theta)\, R(\psi)$$



Successive Rotation

$$\begin{array}{ccccc} x & & Y & & \\ \downarrow & & \downarrow & & \downarrow \\ \phi & \longrightarrow & \theta, & \longrightarrow & \psi \end{array}$$

$$R = R_x R_y R_z = XYZ$$

$$R = R_x R_y R_z$$

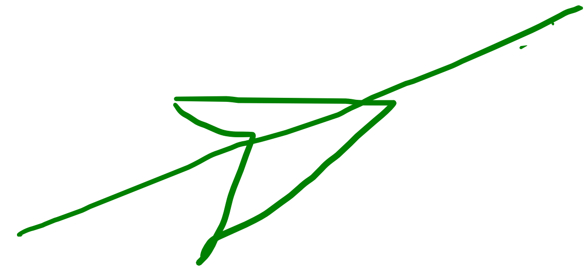
$$= XYZ$$

$X \rightarrow R_x$

$Y \rightarrow R_y$

$Z \rightarrow R_z$

$$R = XYZ$$



$\boxed{\text{Optimization}}$ in Computing

For single task may take

10^{-6} s

or, $10\,\mu s$

1. Hardware level
2. Math Formulation
3. Algo

Euler Angles :

$X \to \phi$
$Y \to \theta$
$Z \to \psi$

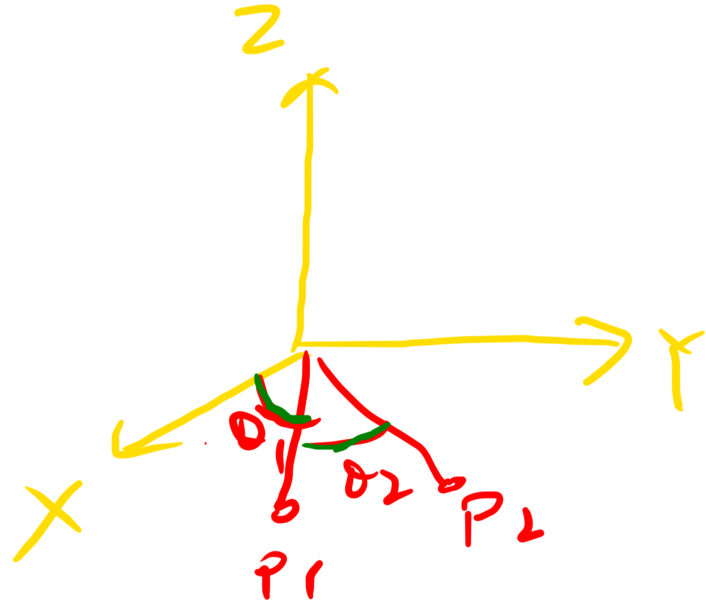
(I) Rotation can not be more than 3
XYZ

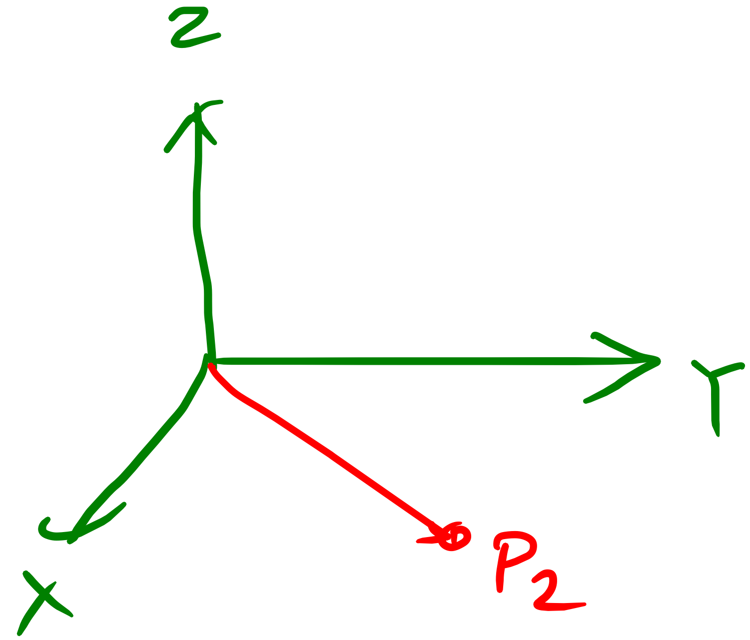
(ii) No two successive Rotation

(X, Y, Z), XXZ ∃ XZX
XXX :

XXX, XXY, XXZ, XYX, XYY, XYZ, XZX, XZY, XZZ
YXX, YXY, YXZ, YYX, YYY, YYZ, YZX, YZY, YZZ
ZXY, ZXY, ZXZ, ZYX, ZYY, ZYZ, ZZX, ZZY, ZZZ

27 Sequences





12 ⇒ Sequences 'Cardan' Angles → roll, pitch, yaw

$$XYX = R_x R_y R_x = R$$

$$XZX = R_x R_z R_x = R$$

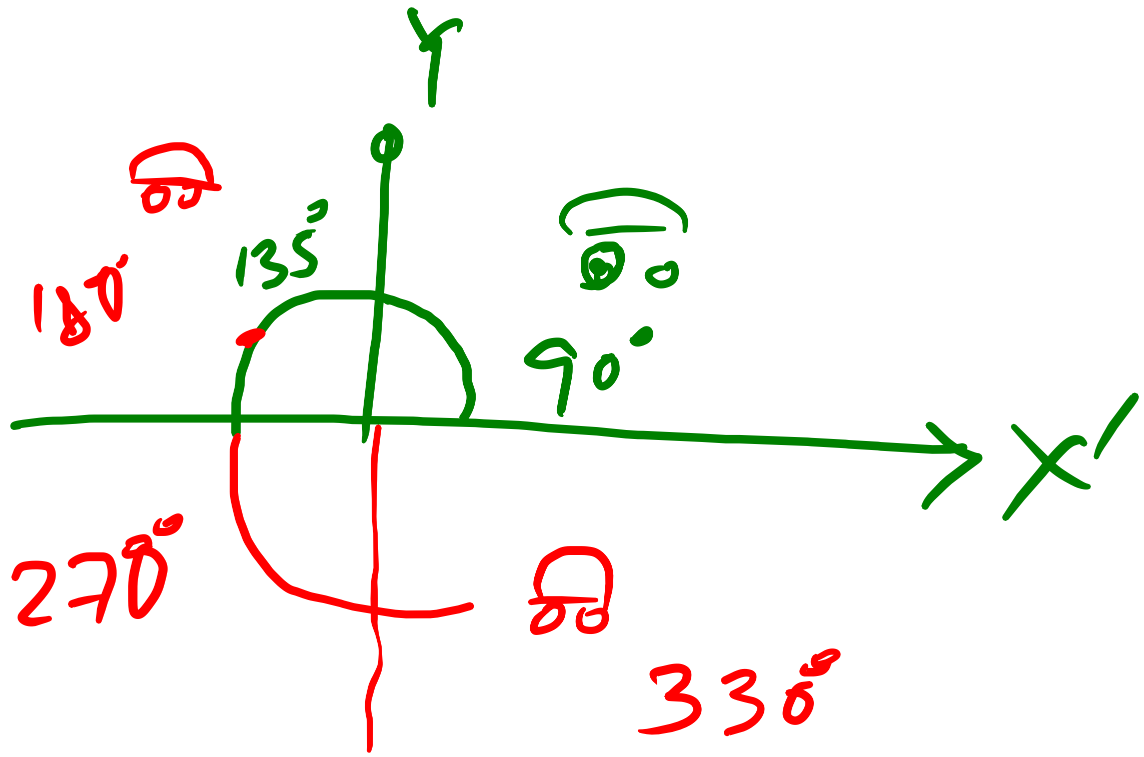
$$12 \begin{bmatrix} XYX & XYZ & XZX & XZY \\ YXY & YXZ & YZX & YZY \\ ZXY & ZXZ & ZYX & ZYZ \end{bmatrix}$$

$$R = R_1 R_2 R_3 \cdots R_N$$

$$\left.\begin{matrix} XYX, XZX \\ YXY, YZY \\ ZXZ, ZYZ \end{matrix}\right\} \text{Accurat Euler Angle}$$

Accuracy in the form of Computing task 10^{-6}s

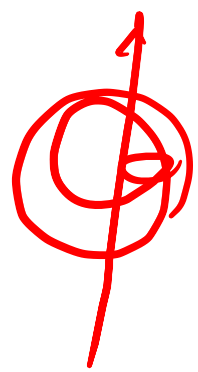
$\approx 10\,\mu s$



Sin function

$-\sin$

$+\ -$



$\left.\begin{array}{l} \theta \to + \\ \theta \to - \end{array}\right\}$ Discontinuity

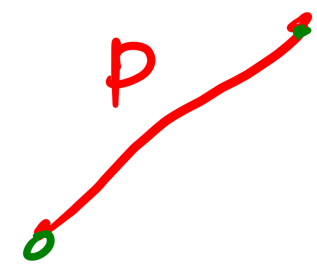
1. Homogeneous Transformation in 3D (Matrix)

$$P' = R\,P$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$R \Rightarrow$ maybe anything

$$R = R_1 \cdot R_2 \cdot R_3 \cdots R_N$$



2. Vector + Rotation

3. Vector + Euler angle

: Quaternion: a hyper complex number

$\underline{a + ib}$

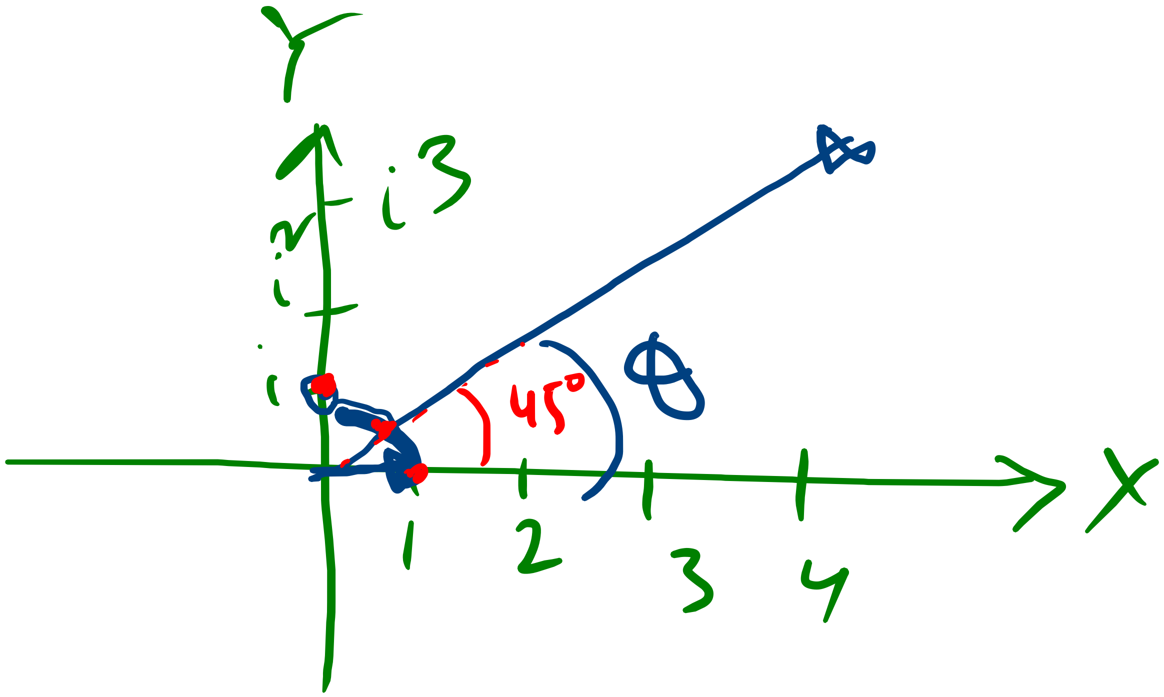
$q = q_0 + iq_1 + jq_2 + kq_3$

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = +1$



$1 \cdot i = i$

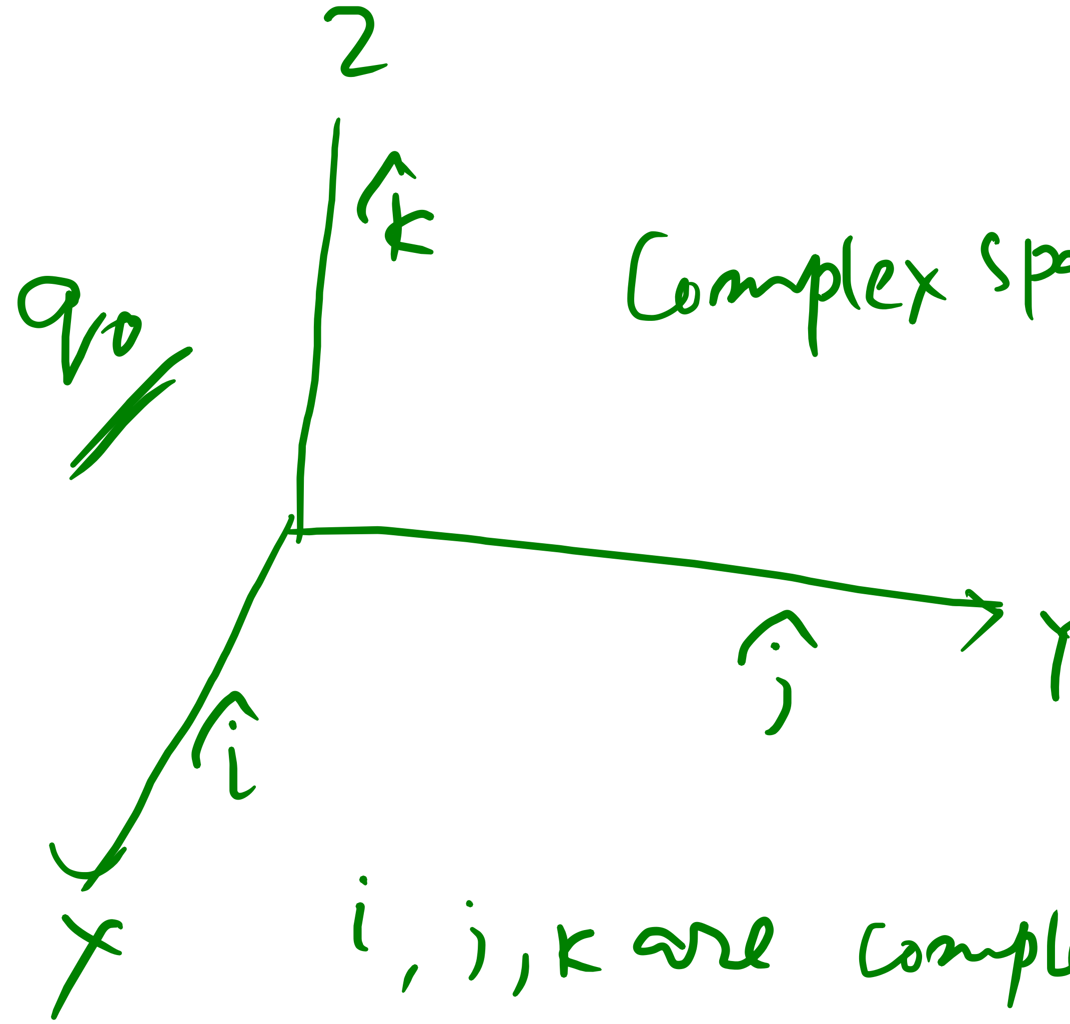
$1 \cdot i^{(1/2)} = 0.7 + 0.7i$

$$i^{(\theta/90^\circ)} = \theta$$

$$|q| = \sqrt{q_0^2 + q_1^2}$$

$$q = q_0 + i\,q_1$$

$$|q| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}$$



Complex space

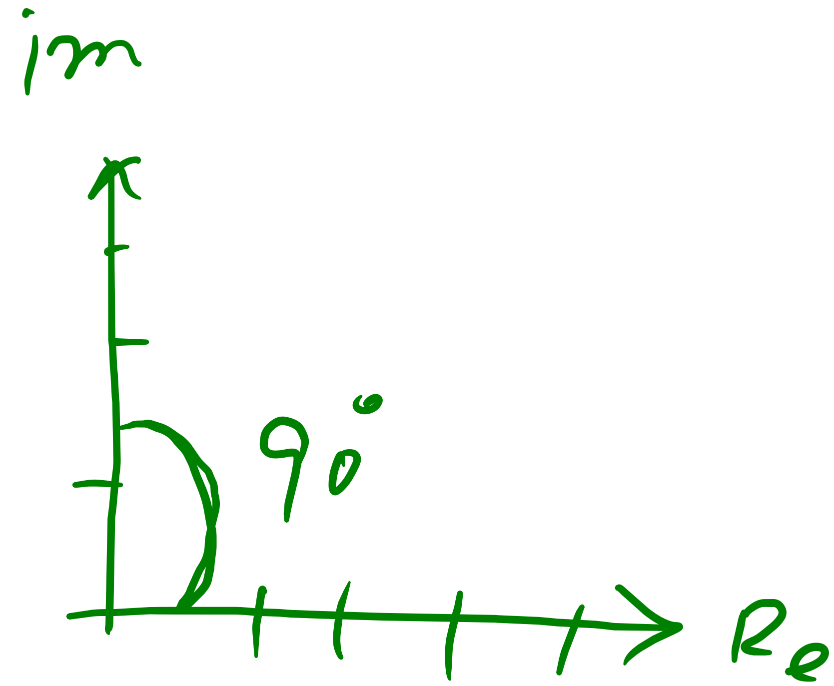
i, j, k are complex quantity

$q_0 \rightarrow$ Real number

$q_1 \rightarrow \underline{x}$

$q_2 \rightarrow \underline{y}$

$q_3 \rightarrow \underline{z}$



$\underline{\phi}, \underline{\theta}, \underline{\psi}$

$=$

ta (y/x

$$
\begin{bmatrix} \Phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \operatorname{atan2}\left(2(q_0 q_1 + q_1 q_3),\ 1-2(q_1^2+q_2^2)\right) \\ \operatorname{asin}\left(2(q_0 q_2 - q_3 q_1)\right) \\ \operatorname{atan2}\left(2(q_0 q_3 + q_1 q_2),\ 1-2(q_2^2+q_3^2)\right) \end{bmatrix}
$$

General Transformation

1. Vector + Roll-Pitch-Yaw
2. Vector + Euler angles
3. Vector + Quaternions
4. **Homogeneous** Transformation.

q_0, q_1, q_2, q_3

$\Downarrow$

ϕ, θ, ψ

$\Longleftrightarrow$ IMU Sensor $\Rightarrow$

* Acc.
* Roll-Pitch-Yaw
* magnetic

Ref. on Quaternions

* "Quaternion proposed standard reference"

NASA JET propulsion Lab, 1979

CalTech

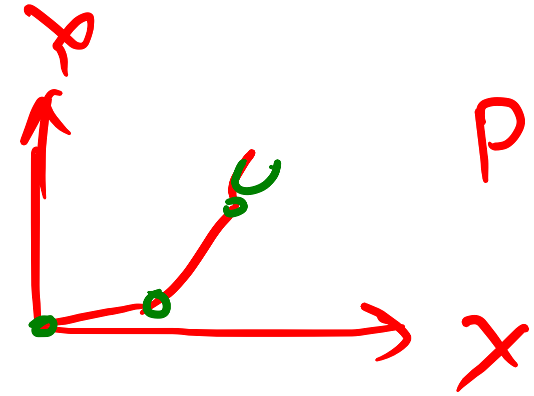
* Drone programming : Local drone

Rs 20,000/-

* Co-ordinate system (2D, 3D)

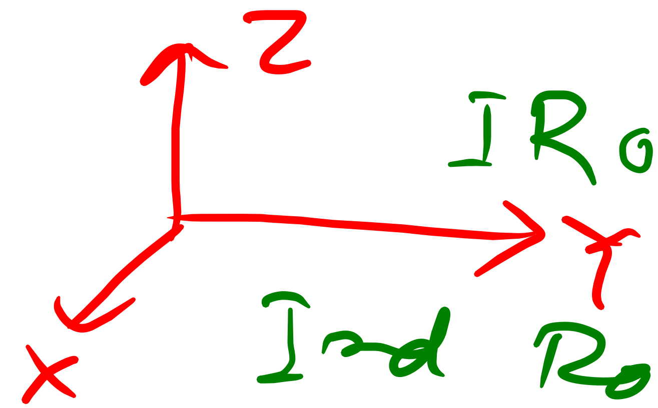
* Co-ordinate transformation (2D, 3D)

* Example : 2D :



Planar Robot

3D :



I Robot

2nd Robot

* Kinetics : $\longrightarrow$ FK, IK $>$ 2D

* 3D Example: Ind Robot
  * Humanoid Robot
    ↓

Dynamics & Control

ROS ⇒ PID control