Solution: Tutorial 2

Q:1

(a) $g(t) = Ae^{j(2\pi f_0 t + \theta)}$

Power signal. See Q:3(b)

(b) $g(t) = Ae^{-bt}(t > 0)$

$$E = \int_{-\infty}^{\infty} \left|Ae^{-bt}\right|^2 \, dt$$

$$= A^2 \int_{-\infty}^{\infty} e^{-2bt} \, dt$$

$$= \frac{A^2}{2b}$$

Energy signal. Therefore power is zero.

(c) $g(t) = t(t > 0)$

It is always increasing function, so energy is infinity.

For Power

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |t|^2 \, dt$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{t^3}{3}\right]_{-T}^{T}$$

$$= \infty$$

Neither power nor energy signal.

(d) $g(t) = Kt^{-1/4}(t > 0)$

Neither power nor energy signal as above.

Q:2

Ans:

Since $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental periods T_1 and T_2 respectively, we have

$$x_1(t) = x_1(t+T_1) = x_1(t+mT_1) \qquad m \text{ is positive integer}$$

$$x_2(t) = x_2(t+T_2) = x_2(t+kT_2) \qquad k \text{ is positive integer}$$

Thus,

$$x(t) = x_1(t+mT_1) + x_2(t+kT_2)$$

In order to $x(t)$ be periodic with period T one needs,

$$x(t+T) = x_1(t+T_1) + x_2(t+T_2) = x_1(t+mT_1) + x_2(t+kT_2)$$

Thus we must have

$mT_1 = kT_2 = T$ Or

$$\frac{T_1}{T_2} = \frac{k}{m} = rational\ number\ .$$

In other words, the sum of the two period signals is periodic if the ratio of their respective period's can be expressed as a rational numbers. If the ratio is irrational then x(t) cannot be periodic.

Given the signal, $x(t) = \cos(60\pi t) + \sin(50\pi t)$

From the signal we can find the fundamental time periods,

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{60\pi} = \frac{1}{30}$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{50\pi} = \frac{1}{25}$$

Therefore,

$$\frac{T_1}{T_2} = \frac{25}{30} = \frac{5}{6}$$

Hence the fundamental time period of the x(t) is

$$T = 6T_1 = 5T_2 = 0.2\sec$$

Q:3

(1) $x_1(t) = e^{-5t}u(t)$

$$E_\infty = \int_{-\infty}^{\infty} |x_1(t)|^2 dt$$

$$E_\infty = \int_{-\infty}^{\infty} |e^{-5t}u(t)|^2\, dt$$

$$E_\infty = \int_{0}^{\infty} \left|e^{-5t}\right|^2 dt$$

$$E_\infty = \int_{-\infty}^{\infty} e^{-10t}\, dt$$

$$= \frac{1}{10}$$

It is energy signal. Power of energy signal is zero.

(2) $x_2(t) = e^{j(4t+\pi/4)}$

$$E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} 1\, dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_2(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \times 2T$$

$$= 1$$

(3) $x_1(n) = \left(\frac{1}{3}\right)^n u(n)$

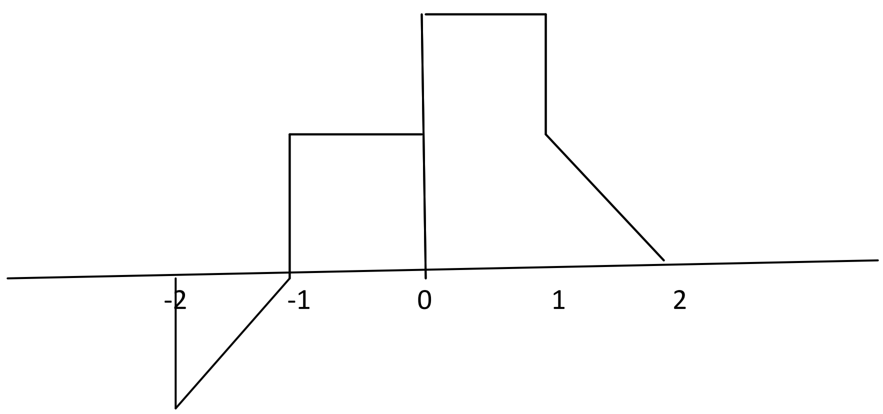
$$E_\infty = \sum_{-\infty}^{\infty} \left| \left(\frac{1}{3}\right)^n u(n) \right|^2$$

$$E_\infty = \sum_{0}^{\infty} \left(\frac{1}{9}\right)^n$$

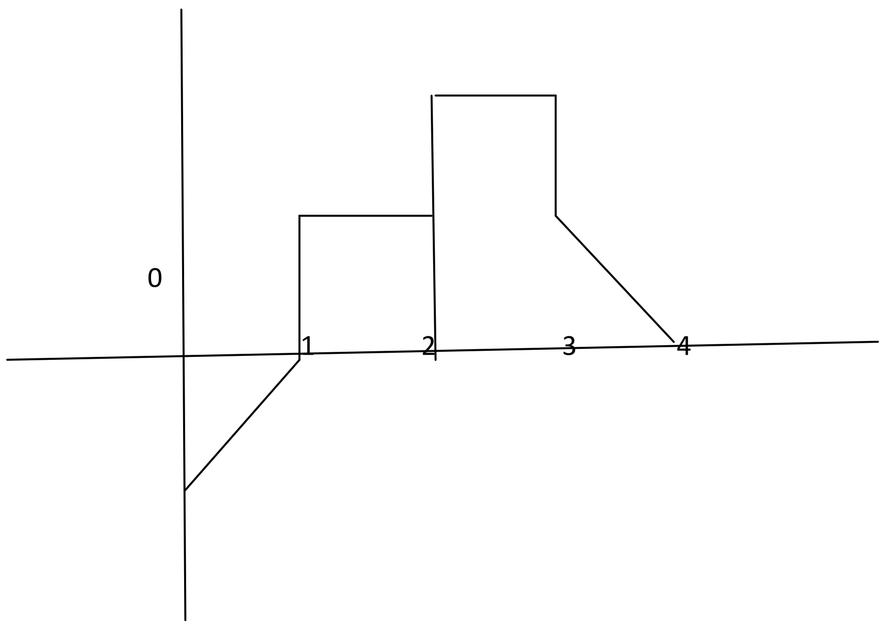
$$= \frac{9}{8}$$

$$P_\infty$$

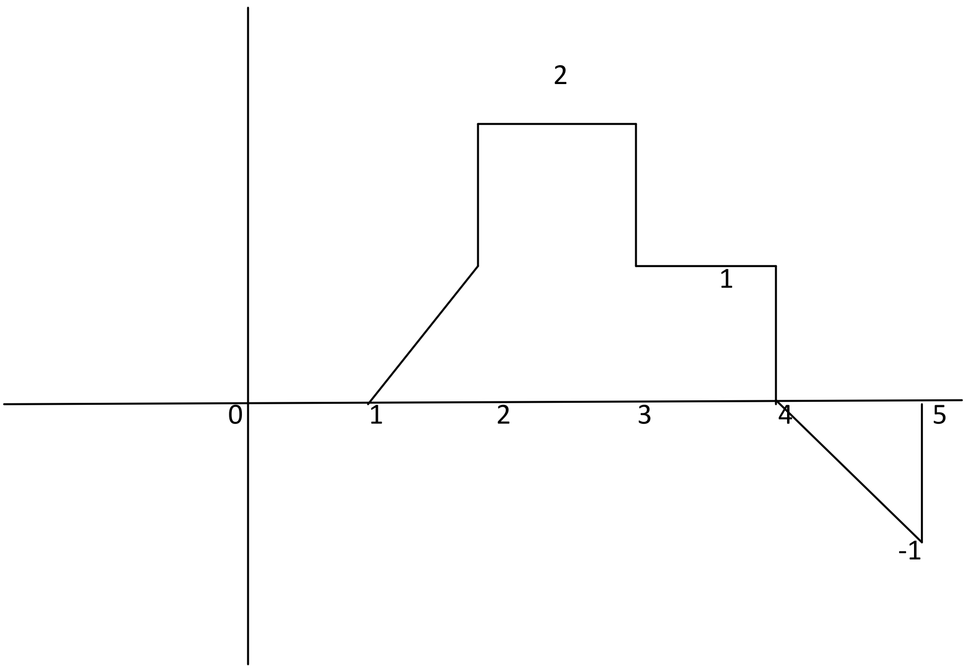
Q 4



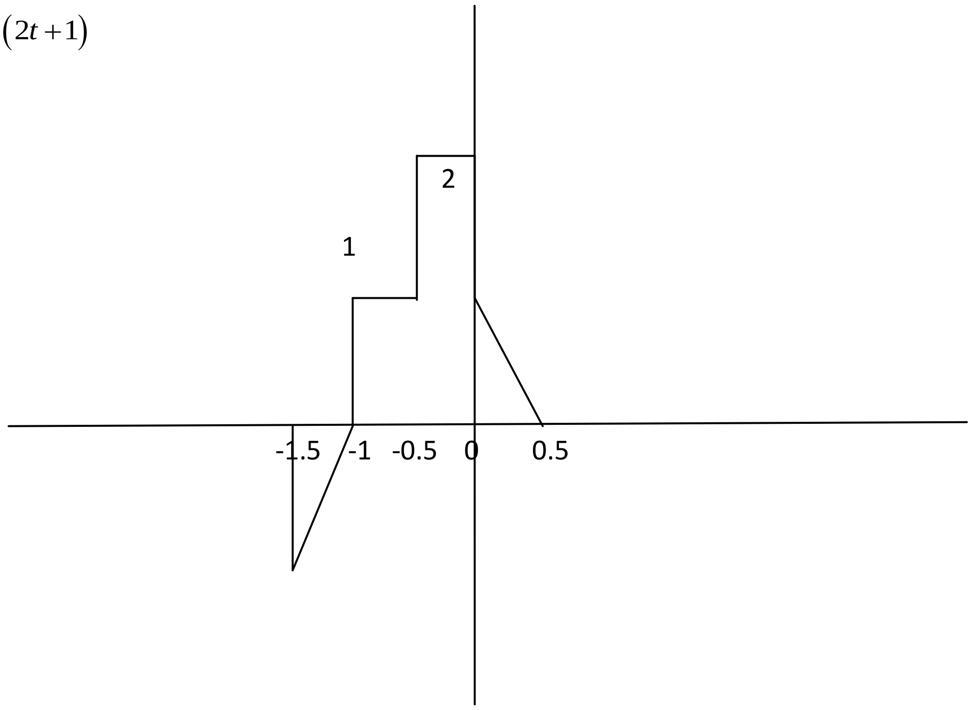
(a) $x(t-2)$



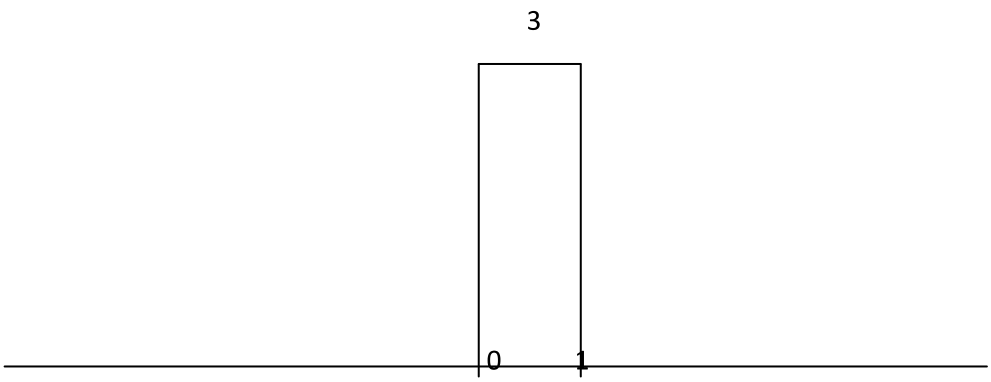
(b) $x(3-t)$



(C) $x(2t+1)$

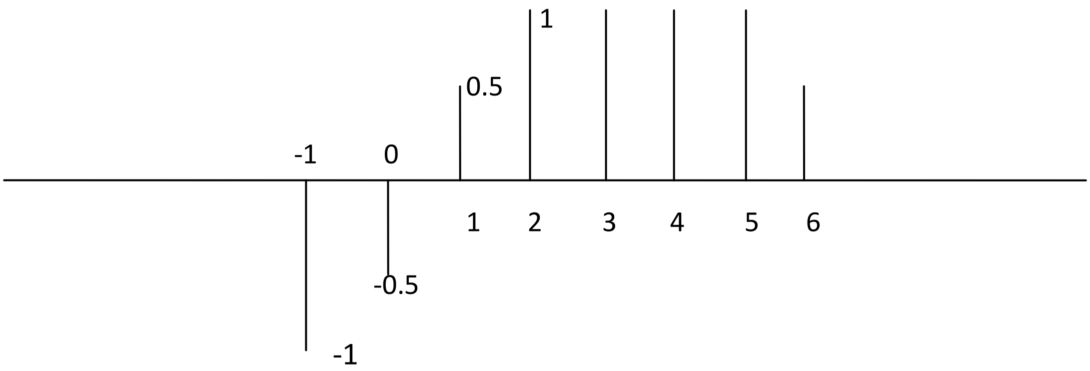


(d)

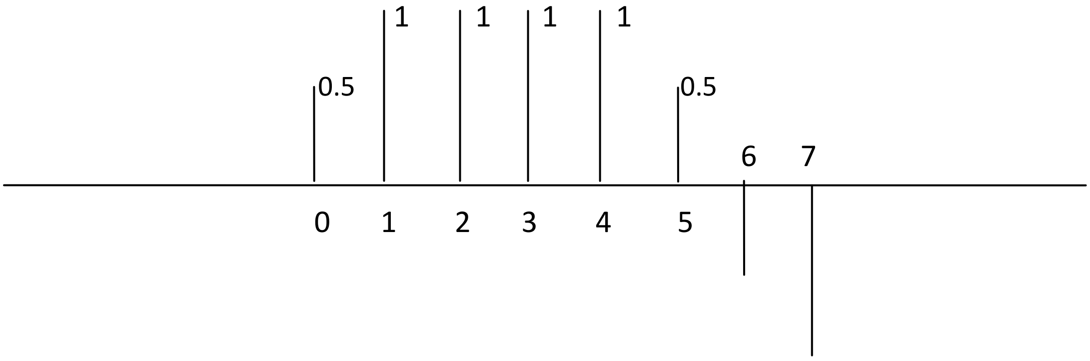


Q5

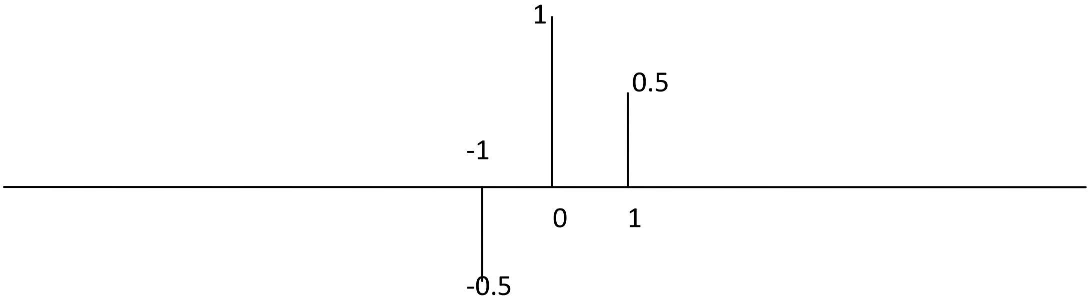
(a) $x(n-3)$



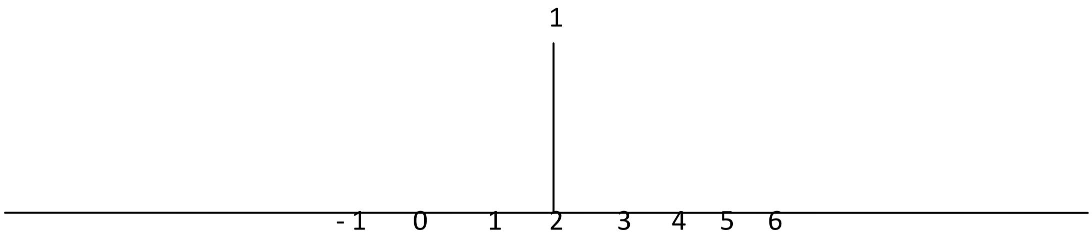
(b) $x(3-n)$



(c) $x(3n)$

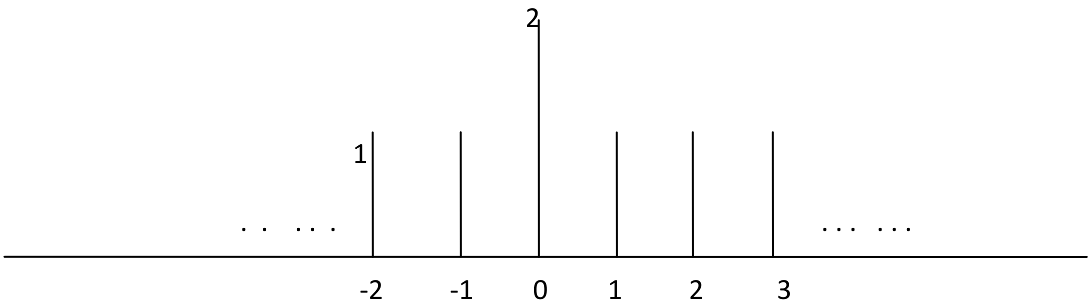


(d) $x(n-2)\delta(n-2)$



Q6

(a) $x_1(n) = u(n) + u(-n)$

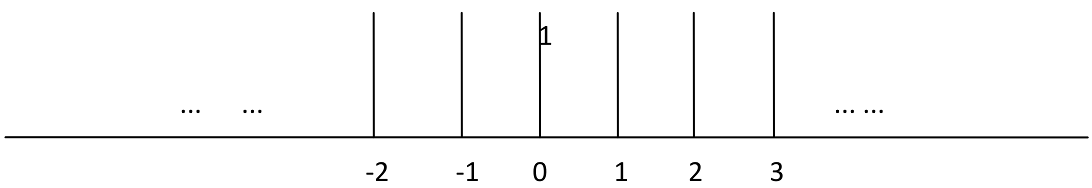


Not periodic

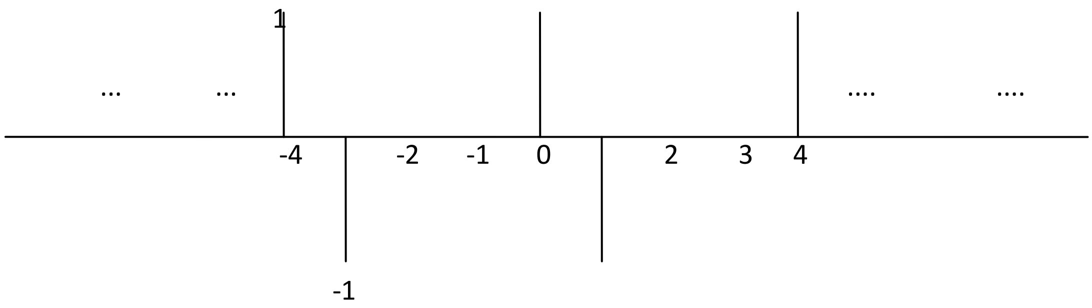
(b) $x_2(t) = 2e^{j(t+\pi/4)}u(t)$

Not periodic because unit step is multiplied to it.

(c)) $x_2(n) = u(n) + u(-n) - \delta(n)$



(d) $x_3(n) = \sum_{k=-\infty}^{+\infty} \{\delta(n-4k) - \delta(n-1-4k)\}$



Q :7

(a) $e^{j(w_0+2\pi)t} \neq e^{jw_0t} \qquad t \in R$

Because $e^{j2\pi t} \neq 1 \qquad for\ all\ t \in R$

(b) $e^{j(w_0+2\pi)n} = e^{jw_0n} \qquad n \in Z$

$Because \quad e^{j2\pi n} = 1 \qquad for\ n \in Z$

Q:8

Energy is infinite and power is 1. See Q:3(b) for solution