Tutorial 13

SC-220 Groups and linear algebra Autumn 2019

(Diagonalization,eigen values and eigen vectors)

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(1) Can these matrices be diagonalized? If so find an invertible matrix P such that $P^{-1}AP$ is diagonal

a)$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ over $\mathbb{R}$ b)$A = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ over $\mathbb{R}$

c)$A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$ over $\mathbb{C}$ d)$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}$ over $\mathbb{C}$

(2) Let D denote the derivative map which is a linear map on the space of differentiable functions. Show that the functions $\sin kx$ and $\cos kx$ $(k \neq 0)$ are eigenvectors of D^2. What are the eigenvalues ?

(3) Let $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ be such that $T(A) = A^T$. Is T diagonalizable? If so find a a diagonal representation and change of basis matrix in which T can be diagonalized.

(4) Let A, B be linear maps from a vector space V onto itself. If $AB = BA$ then show that if v is an eigenvector of A with eigenvalue λ then Bv is also an eigen vector of A with eigenvalue λ. (assume Bv not equal to zero).

(5) Show that the eigen values of a Unitary operator are of the form $e^{i\theta}$ for some θ

(6) Solve the following system of differential equations

$$\begin{aligned} \dot{x}_1(t) &= 3x_1 + x_2 + x_3 \\ \dot{x}_2(t) &= 2x_1 + 4x_2 + 2x_3 \\ \dot{x}_3(t) &= -x_1 - x_2 + x_3 \end{aligned}$$