SINGLE TIME CONSTANT (STC) CIRCUITS

These circuits use atleast one reactive element — L or C which stores energy and one or more resistors which dissipate energy. Together they decide the time-variant (transient) or frequency-variant (steady state) behaviour of circuit to input signals.

The circuit has a time constant τ which is defined as $\tau = RC$ or $\tau = L/R$.

Most STC network can be represented by two types of networks or filters

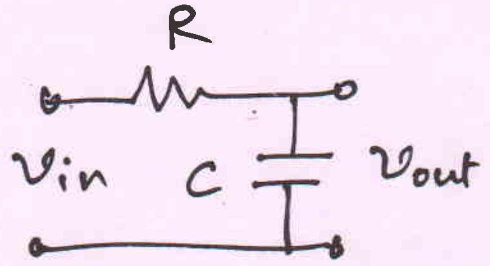
(i) Low-pass or LPF and

(ii) High-pass or HPF.

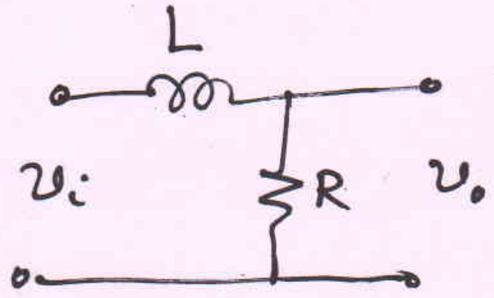
Low pass permits DC and low frequencies to pass through from input to output and block all higher frequencies.

High pass block DC and attenuate lower frequencies and pass higher frequencies.

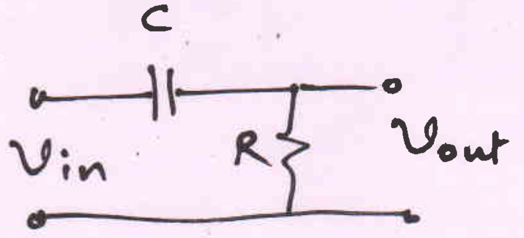
Low-Pass Circuits:

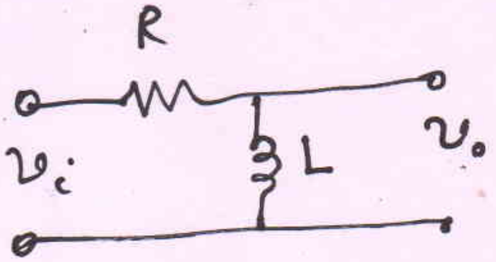


or



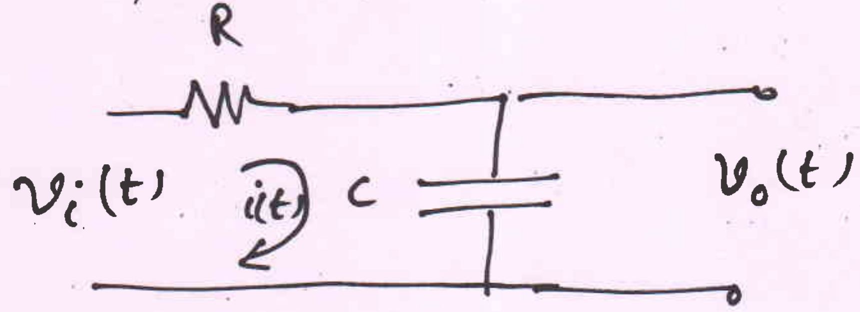
High-Pass Circuits:





We will now focus only on R-C circuits in context of BJT amplifiers.

Analysis of a Low-Pass Circuit

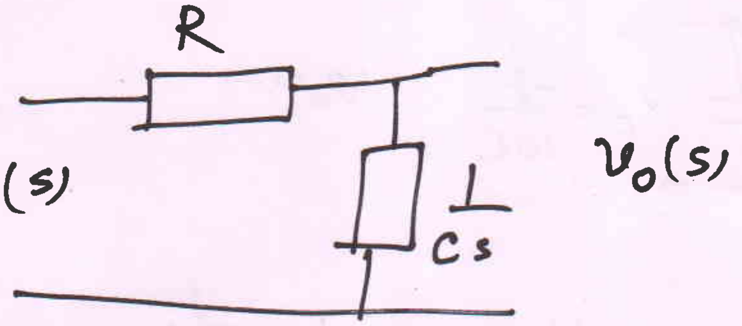


we can represent voltage and current quantities in time domain, or in s (Laplace) domain or in physical frequencies $s = j\omega$

$$v_o(t) = \frac{1}{C}\int i\,dt$$

where $i(t) = \dfrac{v_i(t)}{\text{Total impedance}}$

in s domain reactance of a capacitor is $\frac{1}{Cs}$



The Transfer Function in s domain $T(s)$ is

$$T(s) \equiv \frac{v_o(s)}{v_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1 + CSR}$$

if $RC = \tau$ then

$$\boxed{T(s) = \frac{1}{1+s\tau}}$$

Replacing $s = j\omega$ for physical frequencies

$$T(j\omega) = \frac{1}{1 + j\omega\tau} = \frac{1}{1 + j(\omega/\omega_0)}$$

where $\omega_0 = 1/\tau$ = constant based on circuit component values

~~RC~~

$\omega_0 = 1/RC$ in radians/second.

Note that T is 1 at $\omega = 0$ and 0 at $\omega = \infty$.
This is Low Pass behaviour. DC is allowed & high frequency is attenuated or blocked.

The magnitude of ...

$$|T(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}$$

and phase response is given by

$$\phi(\omega) = -\tan^{-1}(\omega/\omega_0)$$

as ω changes from 0 to ∞ the phase angle changes from 0° to -90°.

at $\omega = \omega_0$ $\quad |T(j\omega)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

The voltage falls to $1/\sqrt{2}$ value or 0.707 times.

at $\omega = \omega_0$ $\quad \phi(\omega) = -45^\circ$

$\omega = \omega_0$ is a very important frequency. It is called as "Corner Frequency", "3-db frequency", "0.707 times gain frequency" or "half-power frequency".

We form an expression of $T(j\omega)$ in db form

T.F. in db form = $20 \log(|T(j\omega)|)$

so if $T(j\omega)$ gets doubled in log it is 6 db.

" " " 10 times " " " 20 db.

If original TF has a multiplier coeff. of say K then in db form the log is taken of $(|T(j\omega)|/K)$ so that max. value we get is not K but 1.0

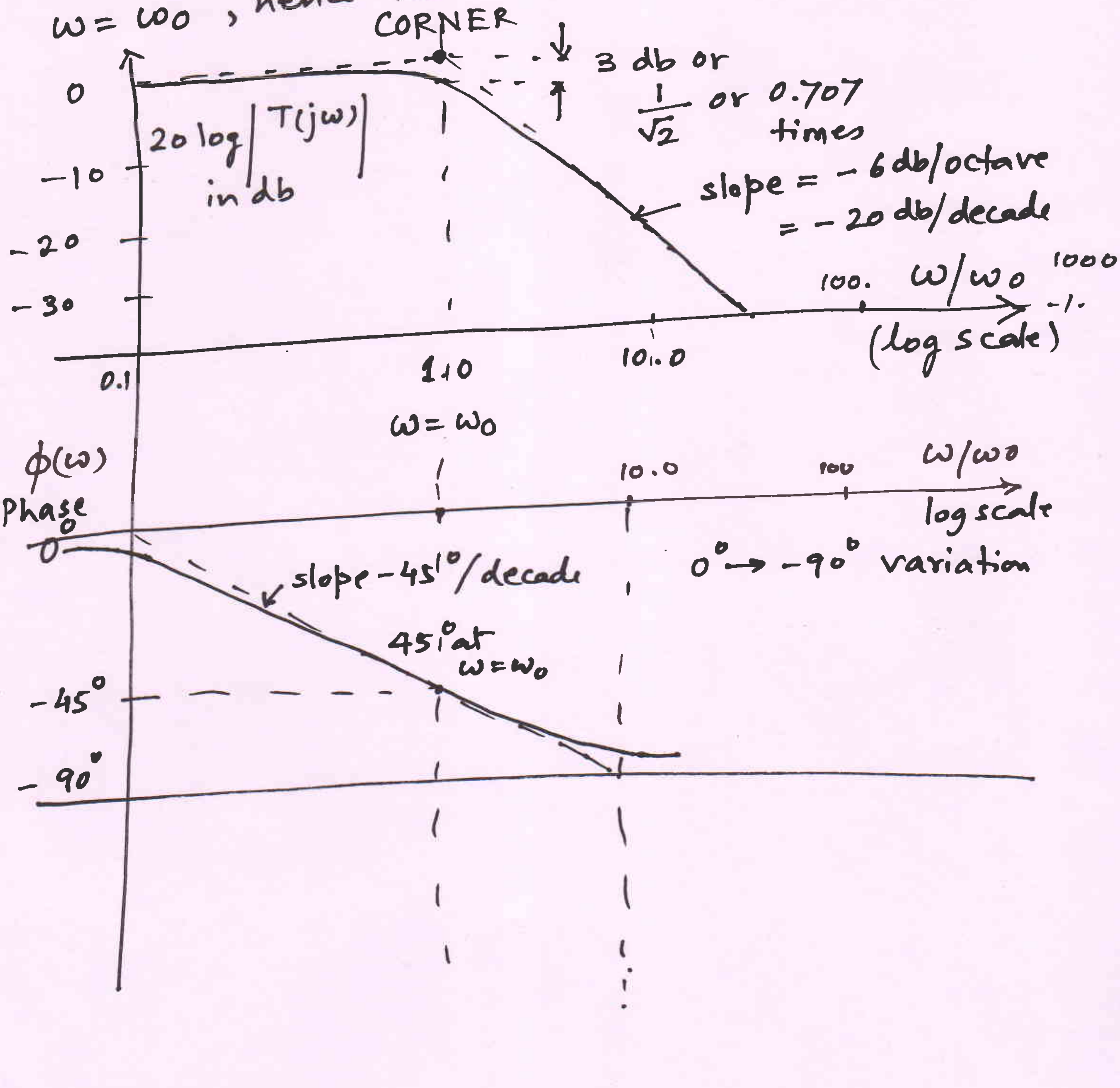
This NORMALISES ALL $T(j\omega)$ PLOTS.

BODE PLOT

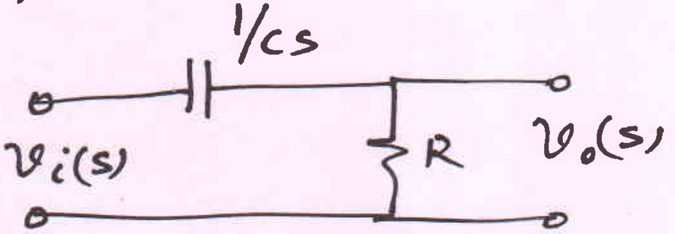
This was devised by H. Bode. It is a plot of a function expressed in decibel (which is log) with frequency in log, normalised to (ω/ω_0).

For $|T(j\omega)| = \dfrac{1}{\sqrt{1+(\omega_0/\omega)^2}}$

the magnitude curve is closely defined by two asymptotic straight lines which meet at $\omega = \omega_0$, hence the name corner frequency.



Analysis of High-Pass Circuit



Transfer Function $T(s) = \dfrac{v_o(s)}{v_i(s)}$

$$T(s) = \frac{R}{R + \frac{1}{cs}} = \frac{csR}{1 + csR} = \frac{s\tau}{1+s\tau}$$

where $\tau = RC = 1/\omega_0$

dividing $T(s)$ by τ in numerator & denominator

$$T(s) = \frac{s}{\frac{1}{\tau} + s} = \boxed{\frac{s}{s + \omega_0}}$$

Calculating magnitude at ~~real~~ physical frequencies

$$T(j\omega) = \frac{j\omega}{j\omega + \omega_0} = \frac{1}{1 - j(\omega_0/\omega)}$$

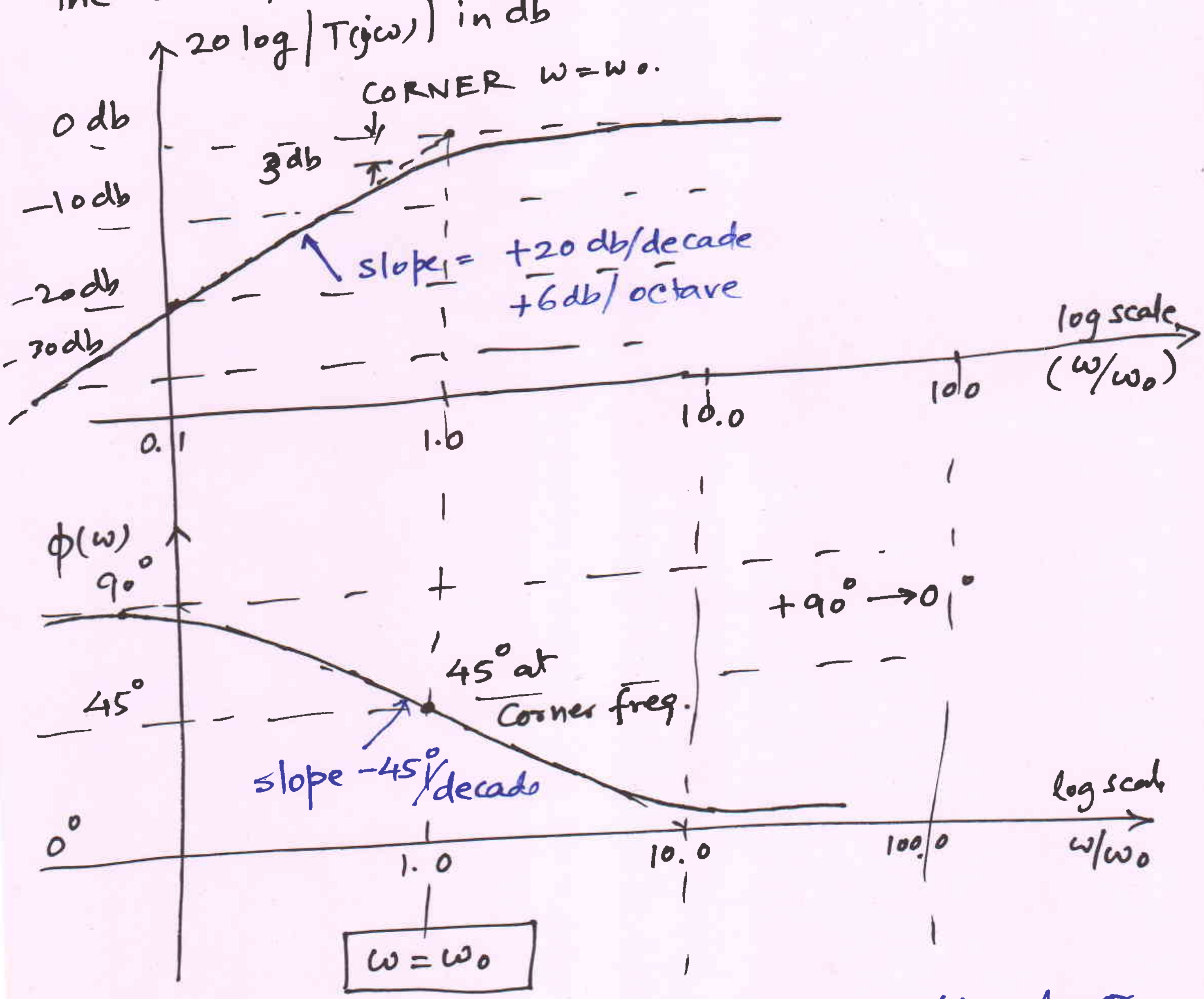
$$|T(j\omega)| = \frac{1}{\sqrt{1 + (\omega_0/\omega)^2}}$$ and phase is

$$\phi(\omega) = \tan^{-1}(\omega_0/\omega).$$

Note that as ω increases $|T|$ becomes ≈ 1. At $\omega = \infty$, $|T| = 0$. At small value of ω, $|T|$ is small.

This is HIGH-PASS behaviour.

The Bode plot of high-pass circuit is :



To summarise, A LPF gives a -20 db/decade or downward slope while a HPF gives $+20$ db/decade upward slope. A LPF phase changes from 0° to -90° with a slope of -45°/decade. A HPF phase falls from $+90^\circ$ to 0° with a slope of -45°/decade.

BODE PLOT OF MORE COMPLEX FUNCTIONS

A more complex function can be described as a polynomial in numerator and denominator

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s^1 + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

where coefficients a and b are "real numbers" and the order m of numerator is smaller than or equal to order n of denominator. For a stable circuit i.e. which ~~go~~ do not generate any signal of its (it does not have any internal oscillators or signal sources) "THE ROOTS OF THE DENOMINATOR POLYNOMIAL ALL HAVE NEGATIVE REAL PARTS".

second form of representing T(s) is ...

$$T(s) = a_m \frac{(s-z_1)(s-z_2) \dots (s-z_m)}{(s-P_1)(s-P_2) \dots (s-P_n)}$$

where a_m = multiplicative constant or DC Gain

$z_1, z_2 \dots z_m$ are roots of numerator polynomial Called as "TRANSFER FUNCTION ZEROES" or simply ZEROES

and

$P_1, P_2 \dots P_n$ are roots of denominator polynomial called as "TRANSFER FUNCTION POLES" or simply POLES.

when the frequency $s =$ any zero freq. the
T. F. value is 0 or no output, due to that factor.

when the frequency $s =$ any pole frequency
then T.F. value is ∞, due to that factor 0 in denominator.

Poles and Zeroes can be either real or complex numbers. Since a and b are real nos., it is necessary that complex poles and zeros must occur in conjugate pairs such as $x + jy$ and $x - jy$.

How to plot Complex Functions in Bode Plot ?

For constructing Bode Plots, it is most convenient to express transfer function factors in the form $\left(1 + \frac{s}{a}\right)$. This readily gives us a corner frequency at $\omega = a$.

Note that a zero tends to increase the value of Tr. function at a slope of +20 db/decade.
A pole, on the other hand, tends to decrease the value of Transfer function at a slope of −20 db/decade.
Since these are factors in $T(s)$ function, in log they become additive of log quantities (if multiplied) or Subtractive (if divided by).

How to plot a function which has corner freq. of zeroes at $\omega = 10, 20$ rad./sec.
corner freq. of poles at $\omega = 100, 1000$ rad./sec.

The expression can be given as

$$T(s) = \frac{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)}{\left(1+\frac{s}{100}\right)\left(1+\frac{s}{1000}\right)}$$

(i) Plot +20 slope lines at $s = 10$ & $s = 20$

(ii) " −20 " " " $s = 100$ & $s = 1000$.

The above 2 lines will add while low 2 lines will have a subtractive effect.

In all, add 4 lines to generate overall line or Plot which is SUM of all 4 parts