Quantisation of Radiation

The Compton Effect

Arthur Holly Compton

Planck: Emission and absorption of electromagnetic radiation is quantised: Quantum of $\boxed{\mathcal{E} = h\nu}$

Einstein: Transmission is also quantised (unlike a wave).

$\boxed{h\nu = W_0 + \frac{1}{2} mv^2}$ $\boxed{h\nu = W_0 + e\phi_s \quad (e\phi_s = \frac{1}{2}mv^2)}$

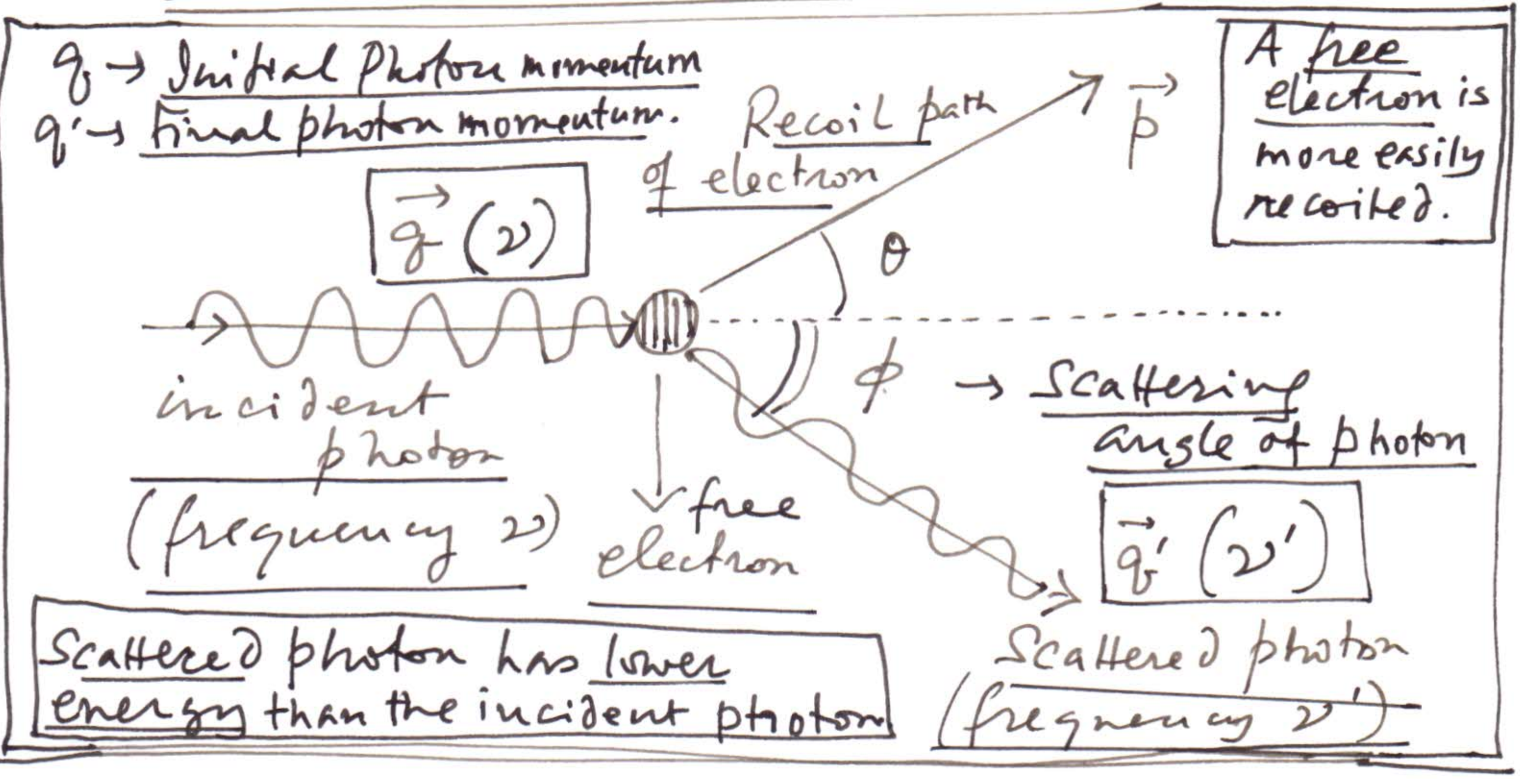
Not bound strongly to atoms

Compton: Photons collide with → FREE electrons like particles Can collide (eg. billiards balls).

Photons with NO rest mass, but energy $\propto h\nu$.

Compton combined the concept of a photon with Einstein's Special Theory of Relativity.

Photon-Electron Collision



1/. Electron is at rest initially. (zero momentum).

2/. Final electron momentum is $\vec{p}$.

3/. Initial electron energy is $\boxed{m_e c^2}$.

4/. Final electron energy is

$$\boxed{\sqrt{p^2c^2 + m_e^2c^4}}$$

$\to$ Special Theory of Relativity

5/. Initial photon energy is $\boxed{h\nu}$.

6/. Final photon energy is $\boxed{h\nu'}$.

(The final photon energy is lower than the initial photon energy). $(\nu' < \nu)$

(P.T.O.)

(vector)

7/. Initial photon momentum is $\vec{q}$.

Magnitude is $\boxed{|\vec{q}| = \frac{h\nu}{c}}$ $\boxed{\begin{array}{l} \mathcal{E} = pc \\ p = \frac{\mathcal{E}}{c} = \frac{h\nu}{c} \end{array}}$

8/. Final photon momentum is $\vec{q}'$

Magnitude is $\boxed{|\vec{q}'| = \frac{h\nu'}{c}}$ (vector)

Energy Conservation:

$$\boxed{h\nu + m_e c^2 = h\nu' + \sqrt{p^2c^2 + m_e^2 c^4}}$$

Momentum Conservation:

$$\boxed{\vec{q} = \vec{p} + \vec{q}'}$$

Initial ~~photo~~ electron momentum is zero.

$$\Rightarrow \boxed{\vec{p} = \vec{q} - \vec{q}'}$$

$$\Rightarrow \boxed{\vec{p}\cdot\vec{p} = p^2 = (\vec{q} - \vec{q}')\cdot(\vec{q} - \vec{q}')}.$$

Writing the vector momentum conservation equation ~~as~~ in a scalar form.

(P.T.O.)

$$\Rightarrow p^2 = \vec{q}\cdot\vec{q} + \vec{q}'\cdot\vec{q}' - \vec{q}\cdot\vec{q}' - \vec{q}'\cdot\vec{q}$$

$$\Rightarrow \boxed{p^2 = q^2 + q'^2 - 2\,\vec{q}\cdot\vec{q}'}$$

But $\boxed{\vec{q}\cdot\vec{q}' = q q' \cos\phi}$

$$\Rightarrow \boxed{p^2 = q^2 + q'^2 - 2 q q' \cos\phi}$$

$\boxed{q = \frac{h\nu}{c}}$ and $\boxed{q' = \frac{h\nu'}{c}}$

← Derived from the momentum conservation condition.

$$\boxed{p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\phi}$$

→ (A)

From the energy conservation equation

on squaring →

$$\boxed{p^2c^2 + m_e^2c^4 = \left(h\nu - h\nu' + m_ec^2\right)^2}$$

$$\Rightarrow \boxed{\begin{aligned} p^2c^2 = {} & (h\nu)^2 + (h\nu')^2 + (m_ec^2)^2 \\ & + 2(h\nu)(m_ec^2) - 2(h\nu)(h\nu') \\ & - 2(h\nu')(m_ec^2) - m_e^2c^4 \end{aligned}}$$

Now divide throughout by c^2. The above equation will have only p^2 on the left hand side, like Eqn. (A).

(P.T.O.)

Derived from the energy conservation condition.

$$p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2h\frac{(m_e \cancel{c^2})}{\cancel{c^2}}(\nu - \nu') - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)$$

$$\Rightarrow \boxed{p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2m_e h(\nu - \nu') - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)} \longrightarrow (B)$$

With (A) − (B), we get,

$$\boxed{2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)(1 - \cos\phi) - 2m_e h(\nu - \nu') = 0}$$

$$\boxed{\lambda = \frac{c}{\nu}} \quad \text{and} \quad \boxed{\lambda' = \frac{c}{\nu'}}$$

(Transforming to the wavelength).

$$\Rightarrow \boxed{2\frac{h^2}{\lambda\lambda'}(1 - \cos\phi) = 2m_e h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)}.$$

$$\Rightarrow \frac{\cancel{2}h}{\cancel{\lambda\lambda'}}(1 - \cos\phi) = \cancel{2}m_e c\left(\frac{\lambda' - \lambda}{\cancel{\lambda\lambda'}}\right)$$

$$\Rightarrow h(1 - \cos\phi) = m_e c(\lambda' - \lambda).$$

(P.T.O.)

Compton Scattering

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\phi\right)}$$

Formula

$\boxed{\lambda' > \lambda}$ because $\boxed{\nu' < \nu}$ $\boxed{h\nu' < h\nu \quad \nu \propto 1/\lambda}$

$\Rightarrow \boxed{(\lambda' - \lambda) > 0}$ $\boxed{\Delta\lambda = \lambda' - \lambda}$

$$\Rightarrow \boxed{\Delta\lambda = \lambda_c \left(1 - \cos\phi\right)}$$

Compare with $\boxed{\lambda = \frac{h}{mv}}$ (de Broglie)

where $\boxed{\lambda_c = \frac{h}{m_e c}}$ → Compton wavelength of the free electron.

ϕ → Scattering angle.

$m_e = 9.11 \times 10^{-31}$ kg.

$h = 6.626 \times 10^{-34}$ J s.

$c = 3 \times 10^{8}$ m s^{-1}.

For the electron

$\lambda_c = 2.4 \times 10^{-12}$ m $= \boxed{2.4 \text{ pm}}$.

Fractional change in wavelength,

$$\boxed{\frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda}\left(1 - \cos\phi\right)}$$

A large $\boxed{\Delta\lambda/\lambda}$ is required for appreciable Compton scattering.

(P. T. O.)

Conditions for large $\frac{\Delta\lambda}{\lambda}$

1/. λ_C has to be large for significant fractional change. $\left(\because \boxed{\lambda_C \propto 1/m}\right)$

$\Rightarrow$ Small mass. Smallest mass obtainable : electron. (me)

2/. λ has to be small for large $\boxed{\Delta\lambda/\lambda}$. X-rays are more effective in this case. ($\lambda \sim 1 Å$)

3/. i) when $\boxed{\phi = 0}$, $\cos\phi = 1$.

$\Rightarrow \boxed{1 - \cos\phi = 0}$. $\rightarrow$ No scattering.

ii) when $\boxed{\phi = 90^\circ}$, $\cos\phi = 0$

$\Rightarrow \boxed{1 - \cos\phi = 1}$.

iii) when $\boxed{\phi = 180^\circ}$, $\cos\phi = -1$

$\Rightarrow \boxed{1 - \cos\phi = 2}$.

Maximum Energy Transfer.

$\therefore$ High scattering angle ϕ will increase $\boxed{(1-\cos\phi)}$, which will raise the value of $\boxed{\Delta\lambda/\lambda}$. $\boxed{\phi = 180^\circ}$ is the highest scattering angle.

High frequency photons on small targets (electrons) will cause inelastic scattering.