DESIGN AND ANALYSIS OF ALGORITHMS

IT - 216

(2)

STABLE MATCHING PROBLEM (1962)

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STATEMENT :

$M = \{m_1, m_2, \ldots, m_n\}$

$W = \{w_1, w_2, \ldots, w_n\}$

M is a set of n men.

W is a set of n women.

$M = \{ m_1, m_2, \ldots, m_n \}$

$W = \{ w_1, w_2, \ldots, w_n \}$

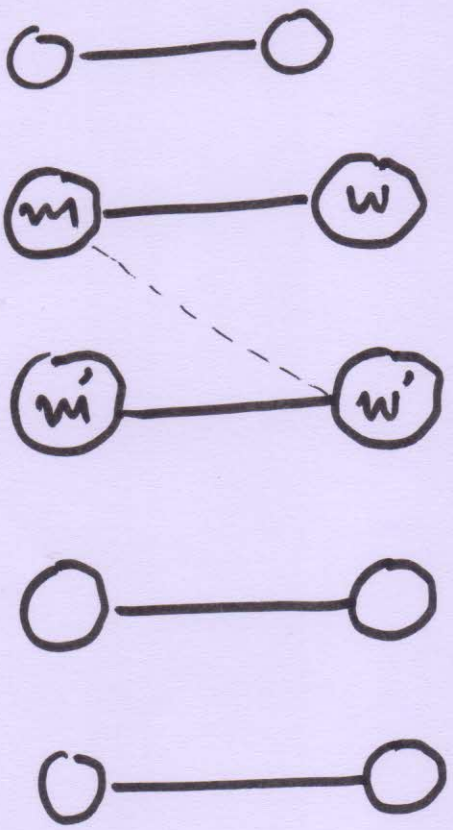
$S \subseteq M \times W$ is **MATCHING**
if each $m \in M$ and each $w \in W$
appears in at most one pair.

$S \subseteq M \times W$ is **PERFECT MATCHING**
if each $m \in M$ and each $w \in W$
appears in precisely one
pair.

PREFERENCE

- Each man $m \in M$ ranks all the women.
- Each woman $w \in W$ ranks all the men.
- No ties

Given the following S



But:

- m prefers w' to w
- w' prefers m to m'

(m, w') is an **instability** w.r.t S

$M = \{m_1, m_2, \ldots, m_n\}$

$W = \{w_1, w_2, \ldots, w_n\}$

$S \subseteq M \times W$ is **STABLE** MATCHING/MARRIAGE if:

(1) S is a perfect matching

(2) There is no instability pair wrt S.

PROBLEM STATEMENT (8)

Given $M = \{m_1, m_2, \ldots, m_n\}$

$W = \{w_1, w_2, \ldots, w_n\}$

- Each $m \in M$ has a preference list
- Each $w \in W$ has a preference list.

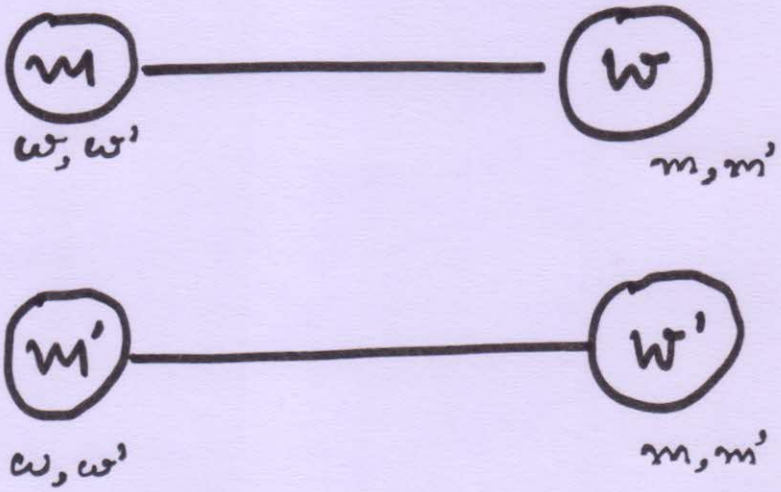
FIND A STABLE MARRIAGE

FOLLOWING QUESTIONS ARISES :

(1) Does there exist a SM for every set of preference list

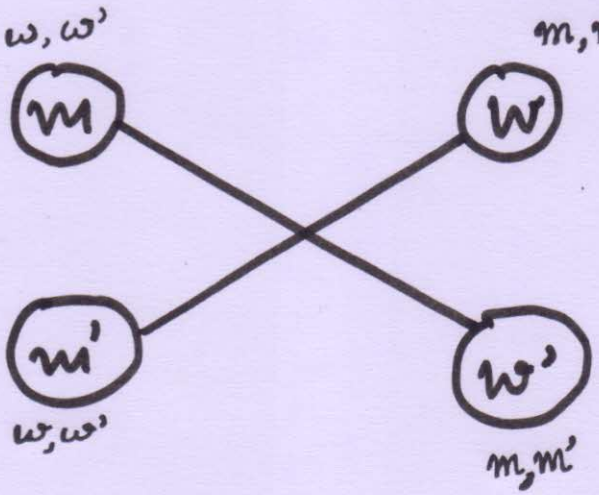
(2) Given a set of preference lists, how to construct a SM algorithmically

EXAMPLE

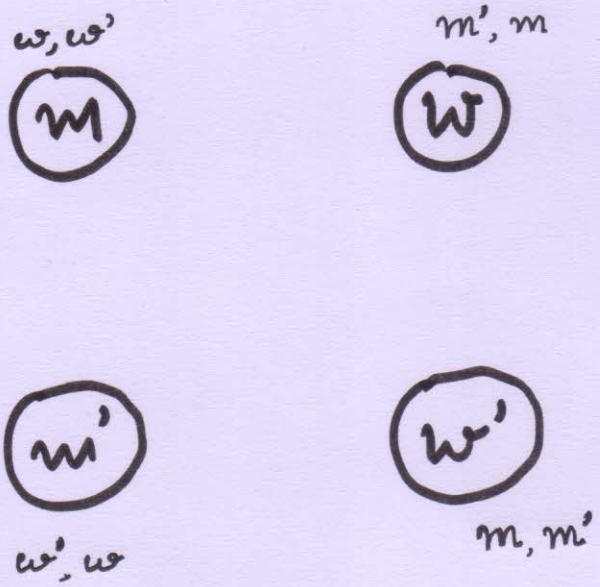


Is the marriage stable?

ANOTHER EXAMPLE :

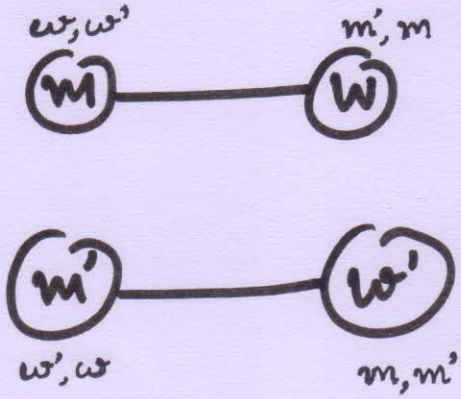


Is the marriage stable now?

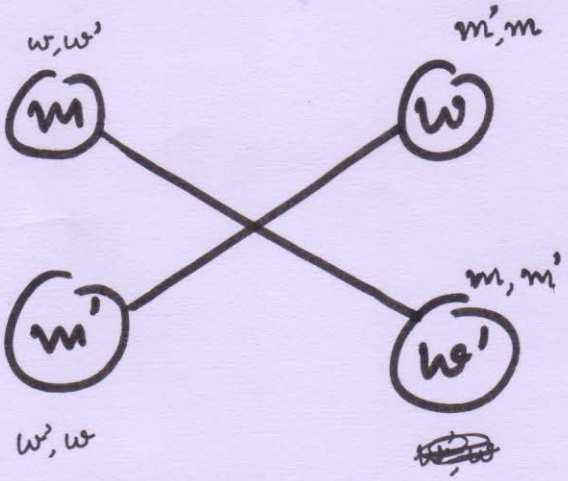


FIND a SM, plz.

THERE COULD BE MULTIPLE SMs.



1st



2nd

ALGO (PLs)

INITIALIZE $S = \phi$

WHILE ($\exists m \in M$ s.t

(a) m is single

(b) $\exists$ Woman not proposed by m)

w ← TOP OF PL of m s.t m has not proposed w yet.

If (w is single), then $S = S + (m, w)$

Else

If (w prefers m to its current partner m'), then $S = S + (m,w) - (m', w)$

Else w rejects m.

End If.

Endif

End While

RETURN S

- Is the ALGO correct?
- what is the Complexity?

WE NEED TO ESTABLISH THE ABOVE TWO POINTS.

OBSERVATION 1

THE ALGORITHM TERMINATES.

Proof :

OBSERVATION 2

THE SET S ~~ALY~~ ALWAYS REMAINS A MATCHING DURING THE ENTIRE ALGORITHM.

Proof :

OBSERVATION 3

EACH $w \in W$ gets engaged when she receives the first proposal, and never gets disengaged there after. Her parter gets better with every new marriage.

. Proof:

LEMMA

Set S returned after the end of the algorithm is a PM (perfect matching)

Proof:

THEOREM

THE SET S RETURNED AT THE END OF THE ALGO IS A SM.

Proof :