16/04/2021

Antenna Parameters (contd.)

Basic Radiation Equation

$$\boxed{\dot{I} L = Q \dot{v}} \quad (\text{A m s}^{-1})$$

→ Ampere

$\dot{I}$ = time changing current (A s^{-1})

L = length of antenna (m)

Q = charge (coulomb, C)

$\dot{v}$ = ~~time~~ change of velocity or acceleration of charges (m s^{-2})

⇒ Time changing current radiates & accelerated charge radiates

Field Zones (Regions) of an Antenna

D = largest dimension of antenna

$\lambda = c/f$ → freq of antenna

→ **Reactive** or Near field region $\left(r < 0.62\sqrt{\frac{D^3}{\lambda}} \right)$ → or Fresnel region

→ **Radiative** or Far field region $\left(R > \frac{2D^2}{\lambda} \right)$ → Fraunhofer region (desired)

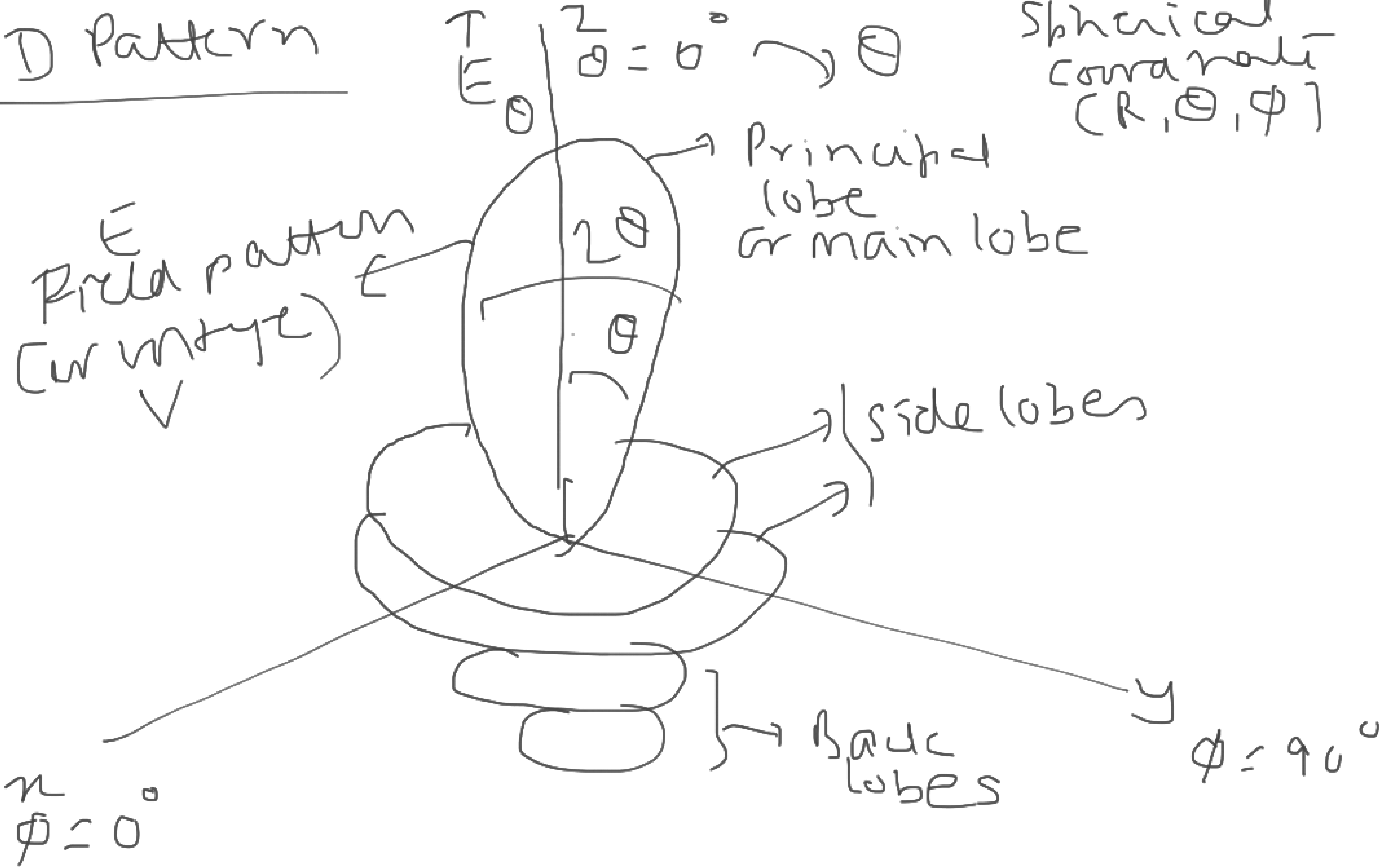
Radiative near field region

$0.62\sqrt{\frac{D^3}{\lambda}} \le r \le \frac{2D^2}{\lambda}$ → NFC (near field communication)

Radiation Pattern of an Antenna

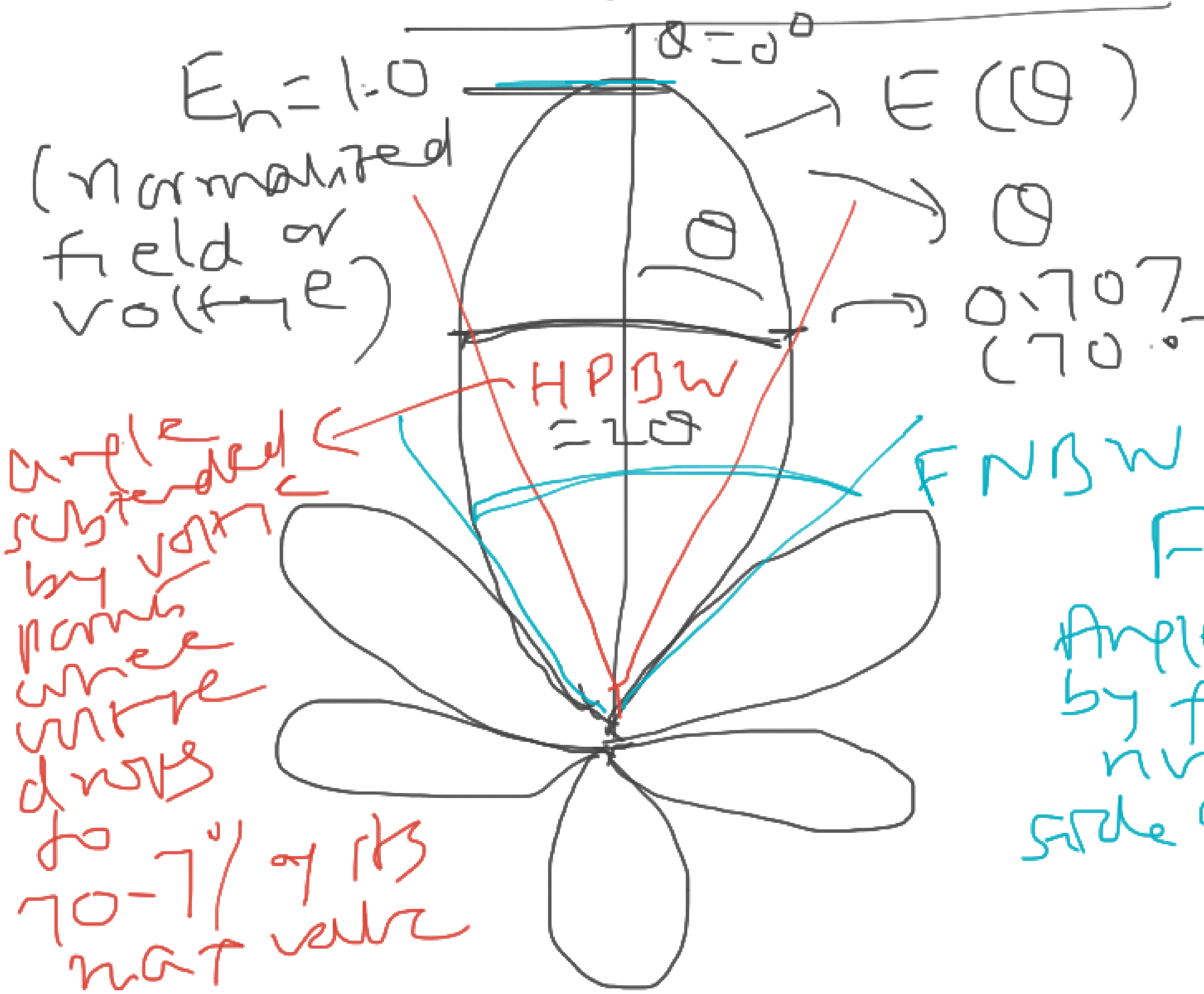
3D Pattern

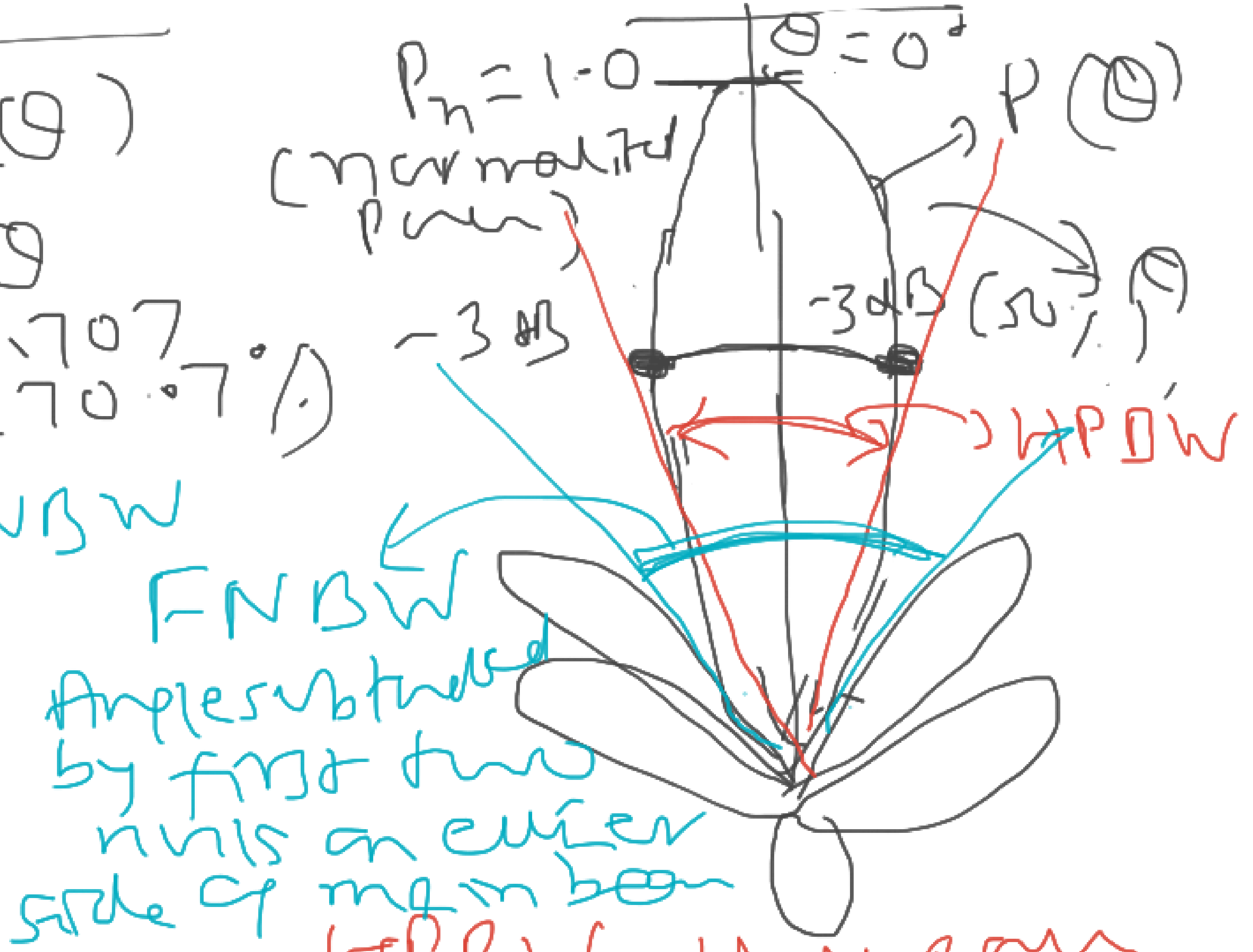
Spherical coordinate (R, θ, ϕ)



Field (or voltage) E V Pattern

Power Pattern





$FNBW \approx 2HPBW$

HPBW = Half power beam width (degrees)

angle subtended by half power points

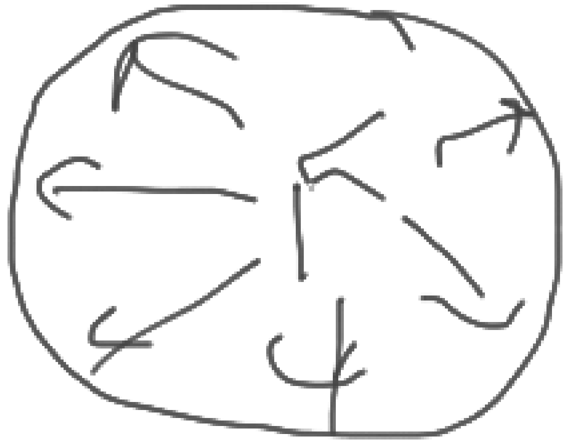
Power pattern = Square of voltage pattern

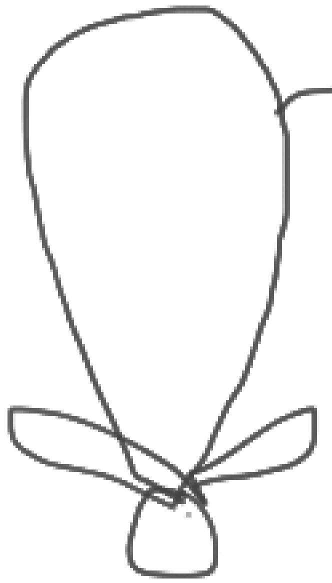
[i.e. $P = \frac{V^2}{R}$]

$$P_n(\theta) = E_n^2(\theta)$$

or

$$V_n^2(\theta)$$





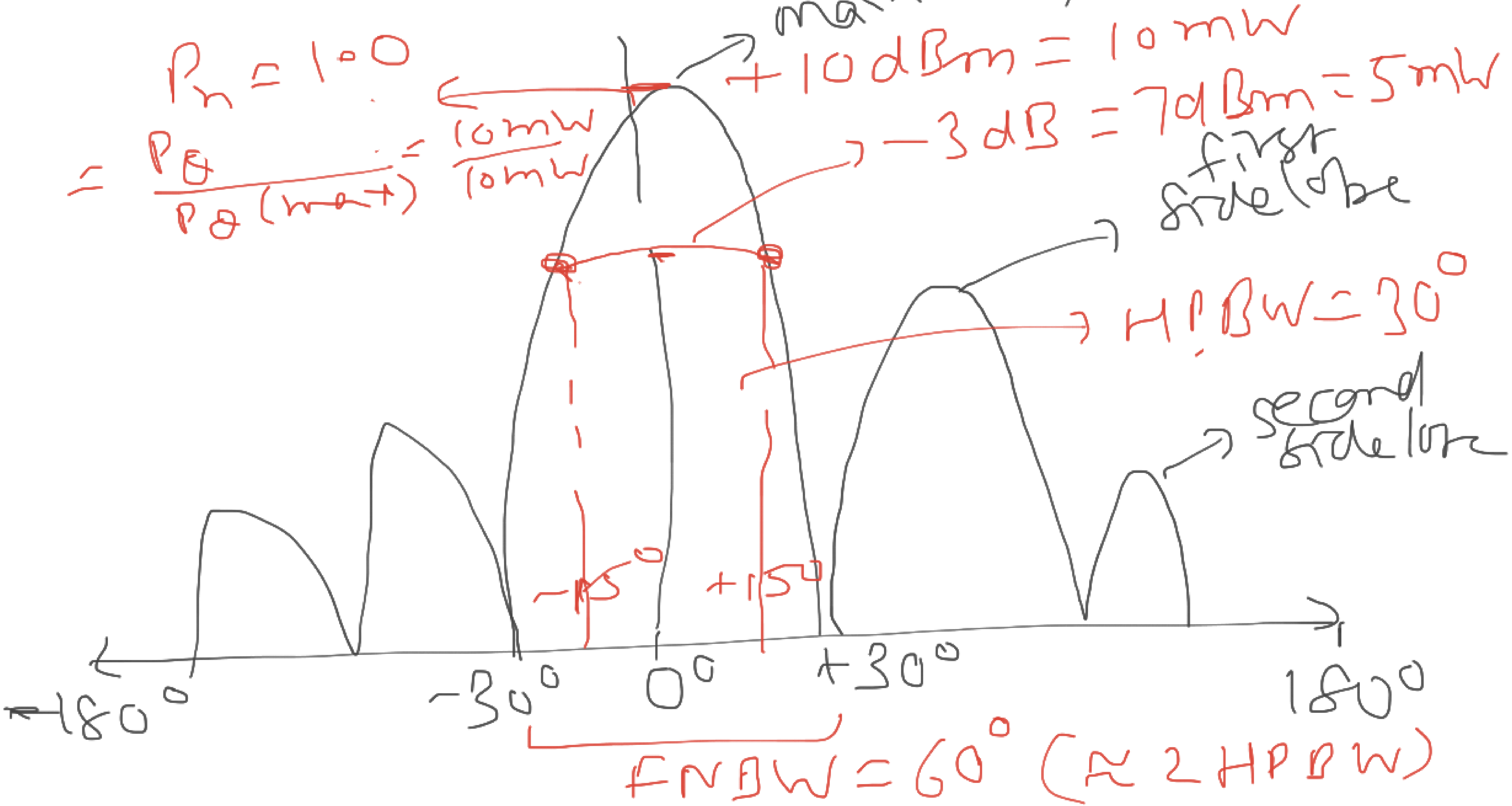
→ Gain increase

Directional antenna

search → detect → track → hit

(omni-directional)

2D Pattern (let's draw mainlobe)



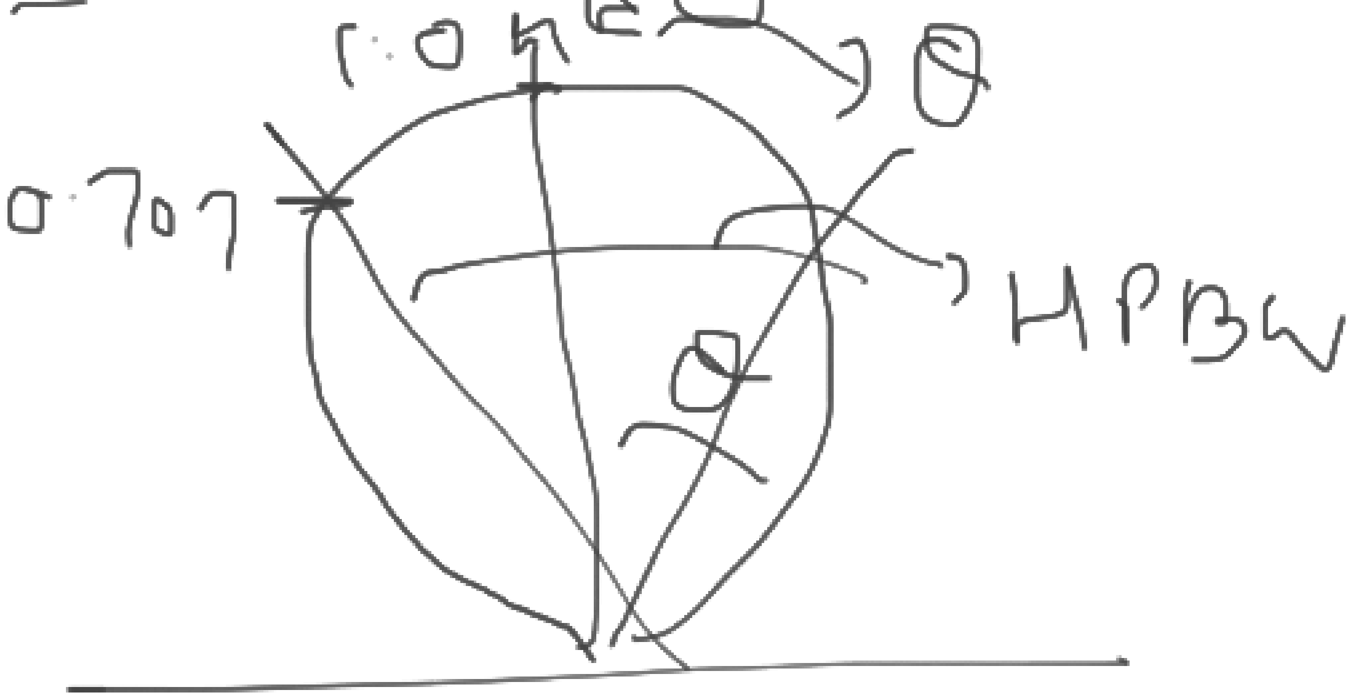
Problem: An antenna has a field pattern (or voltage) given by

$$E(\theta) = \cos^2\theta \ , \quad 0^\circ \le \theta^\circ \le 90^\circ$$

Find HPBW

Solution: For voltage pattern (field),

$E(\theta)$ at half power $= 0.707$ (i.e. $0.707\,E(\theta)$)



$\Rightarrow \cos^2\theta = 0.707$

$\Rightarrow \cos\theta = \sqrt{0.707}$

$\Rightarrow \theta = 33^\circ$

$\therefore HPBW = 2\theta = 66^\circ$

same problem, but pam pattern
HPBW = ?

For pam pattern,
P(θ) at hay pam = 0.5

$\Rightarrow \cos^2\theta = 0.5$

$\Rightarrow \cos\theta$

$\Rightarrow \theta =$ ______

HPBW = 2θ

Problem

$E(\theta) = \cos\theta \cos 2\theta, \quad 0^\circ \leq \theta \leq 90^\circ$

HPBW & FNBW = ?
= ?

Soln:-

$E(\theta)$ at hay pan $= 0.707$

$\Rightarrow \cos\theta \cos 2\theta = 0.707 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \frac{1}{\sqrt{2}\cos\theta} \Rightarrow 2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$

$\Rightarrow \theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$

$$\theta = \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$

Iterating until $\theta' = \theta$ as a first guess,

$\Rightarrow \theta = 22.5^\circ$

Setting $\theta' = 22.5^\circ$, $\theta = 20.03^\circ$, etc ...

until after next iteration $\theta = \theta' = 20.5^\circ$

$$HPBW = 2\theta = 41^\circ$$

For $\overline{FNBW}$ $\Rightarrow \cos\theta \cos 2\theta = 0$

$\Rightarrow \theta = 45^\circ$

$$FNBW = 2\theta = 90^\circ$$

$\therefore FNBW \approx 2\, HPBW$

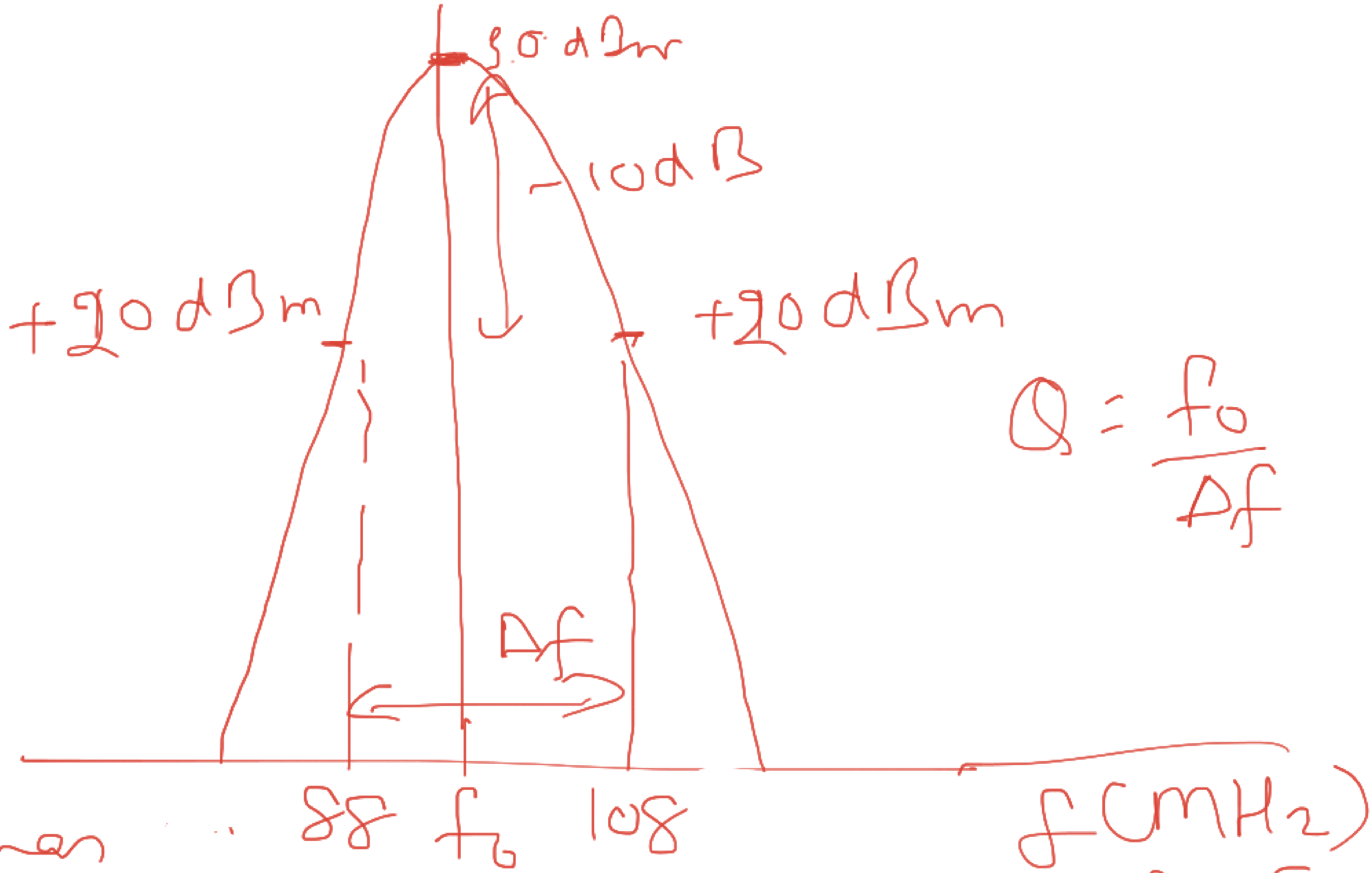
Bandwidth (Hz) → frequencies at
BW which an antenna radiates

Ex: FM radio antenna — 88 to 108 MHz
↳ BW – 20 MHz

Bluetooth system
→ 2400 to 2480 MHz
↳ BW = 80 MHz

Antennas are high Q or very narrow band devices

$$Q = \frac{f_0}{\Delta f}, \quad \Delta f \to 0, \quad Q \to \infty$$



$$Q = \frac{f_0}{\Delta f}$$

Antenna

Bandwidth → w.r.t. -10dB points for FM

Beam Area (or Beam Solid Angle) Ω_A

Area of sphere $= 4\pi R^2$,

where 4π = solid angle subtended by a sphere, sr

solid angle $\rightarrow$ steradian (sr)

$= \text{rad}^2$

$= \left(\frac{180}{\pi}\right)^2 (\text{deg}^2)$

angle $\rightarrow$ radians or degrees

≈ 3282.8064 sq degree or deg^2 or

$1 \text{ sr} \approx 3283 \square$

$\therefore\ 4\pi$ steradians $= 3283 \times 4\pi$

$\approx 41{,}253^{\square}$ → square degrees

= solid angle in a sphere

Beam area or Beam solid angle, Ω_A of antenna is given by

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta,\phi) \underline{\sin\theta \, d\theta \, d\phi}$$

Beamarea

or

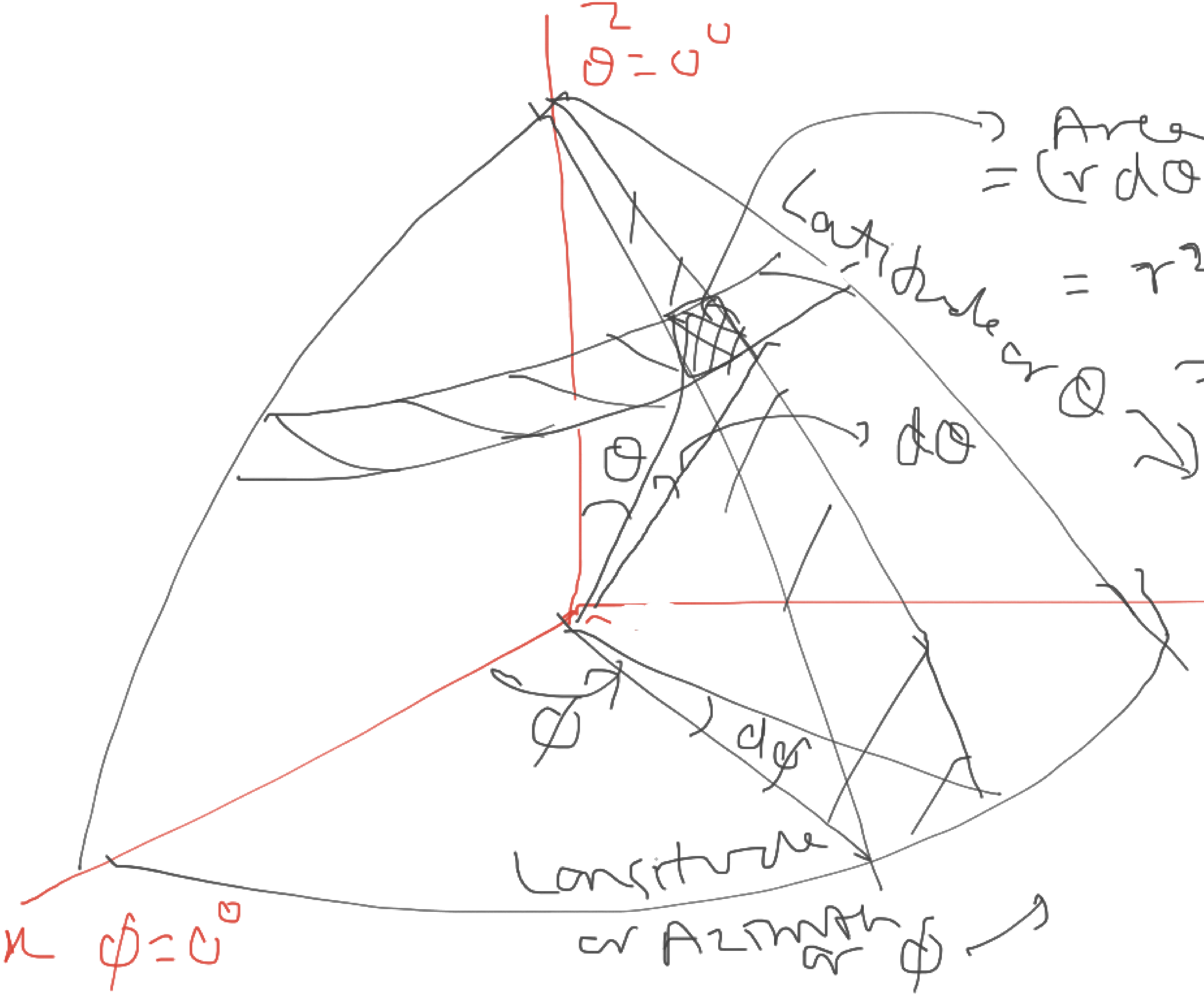
$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta,\phi) \, \underline{d\Omega}}$$

where

$$\boxed{d\Omega = \sin\theta \, d\theta \, d\phi}$$

Rough Approximately $\approx$ $\boxed{\Omega_A = \theta_{HP} \, \phi_{HP}}$ (square degrees)

where θ_{HP} & ϕ_{HP} are HPBW in two planes θ & ϕ (1st line)



Area dA

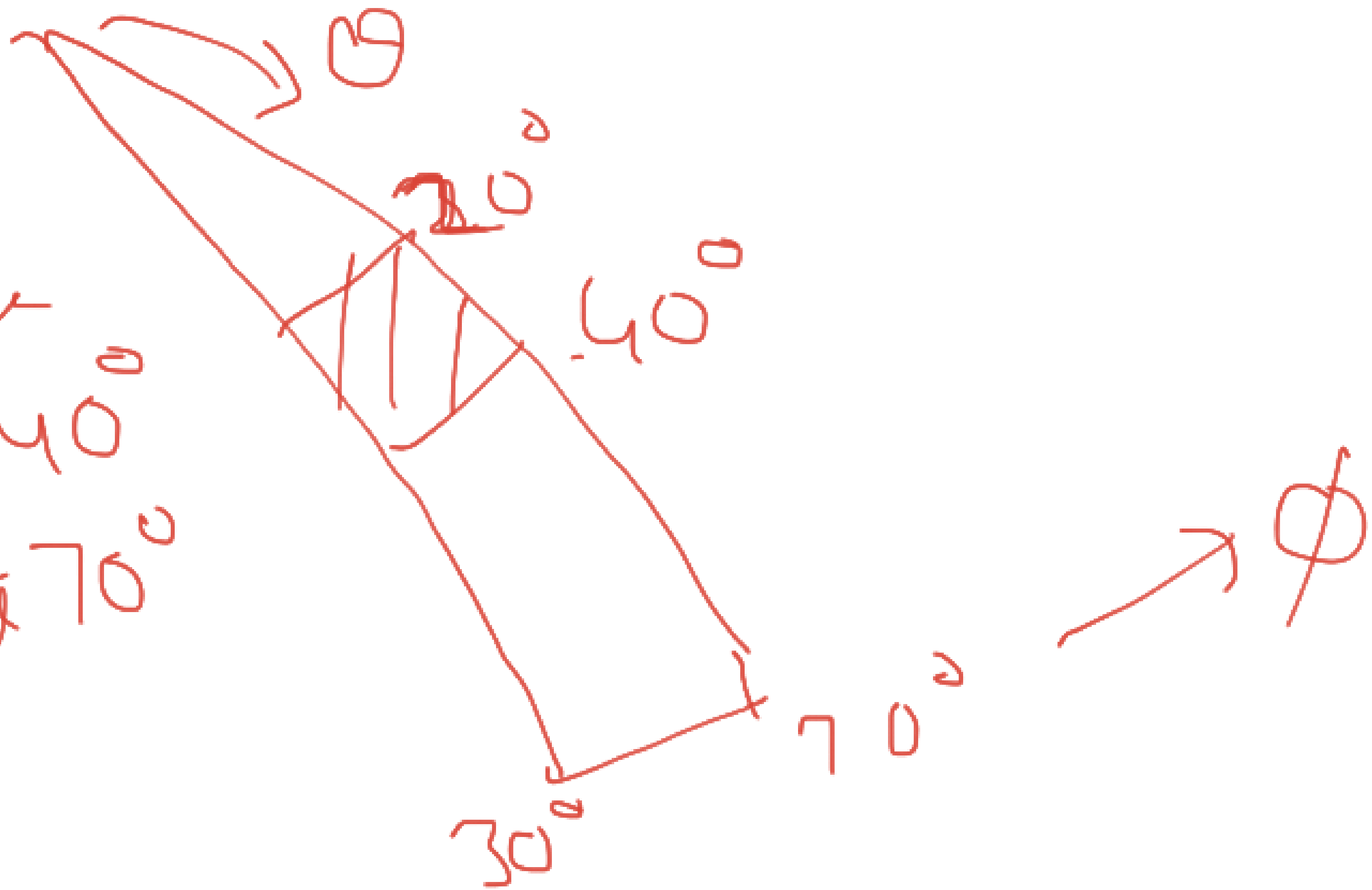
$$= (r\,d\theta)\,(r\sin\theta\;d\theta\,d\phi)$$

$$= r^2\sin\theta\,d\theta\,d\phi$$

$$= r^2\,\underbrace{d\Omega}_{\text{Solid angle}}$$

Example

Find solid angle Ω on a spherical surface between $\theta = 20^\circ$ & 40° & between $\phi = 30^\circ$ & 70°



Soln

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \Omega = \int_{\phi=30^\circ}^{70^\circ} d\phi \int_{\theta=20^\circ}^{40^\circ} \sin\theta \, d\theta$$

$$\Rightarrow \Omega = \underbrace{\frac{40}{360} \cdot 2\pi}\left[-\cos\theta\right]_{\theta=20^\circ}^{40^\circ}$$

$\because 360^\circ = 2\pi$ radians

$\Rightarrow 1^\circ = \frac{2\pi}{360}$ radian

$\Rightarrow 40^\circ = \frac{40}{360} \cdot 2\pi$ radian

$$\Rightarrow \Omega = 0.222\pi \times 0.173$$

$$= 0.121 \text{ sr (steradian)}$$

$$= 0.121 \times 3283 \text{ Square degree}$$

$$\boxed{\Omega = 397^\circ}$$

Last lecture on Friday, 23rd April

- Remaining part of antenna

→ End-sem exam syllabus & pattern