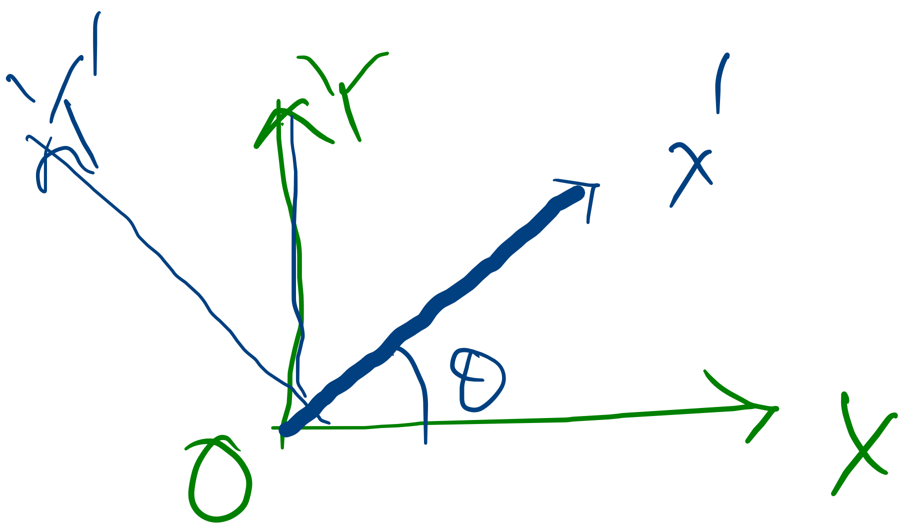
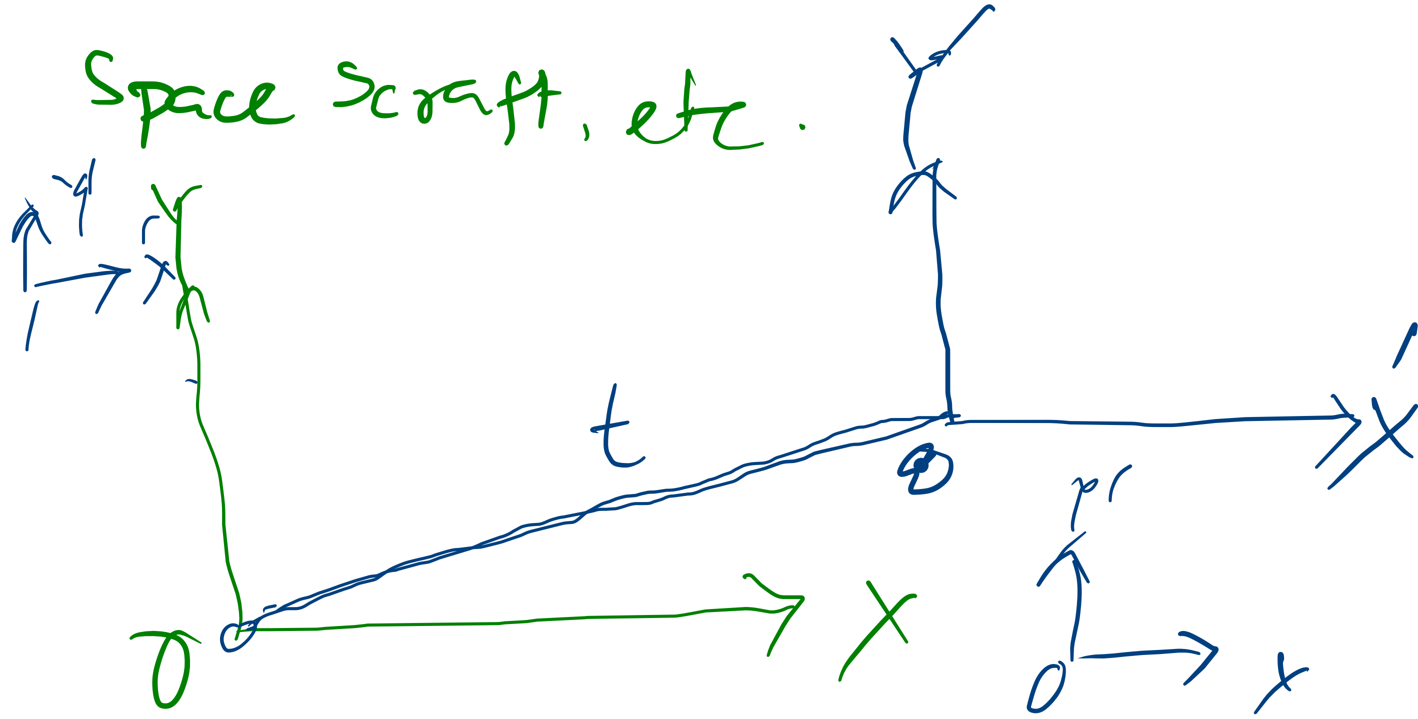
Today's Topic   Date: 12.02.2021

3D-Coordinatesystem & ~~Application~~

1. Rocket, Airplane, Space craft, etc.

2D





General Transformation

: Rotion + Translation

$$p' = T\,p \qquad P \Rightarrow \text{position}$$

$$p' = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} p$$

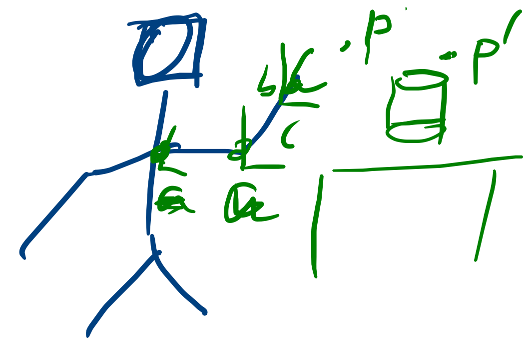
or,
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y' \\ 1 \end{bmatrix}$$

R: rotation

t: translation matrix

$$T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

$$p' = {}^{a}T_{b}\,{}^{b}T_{c}\,p$$

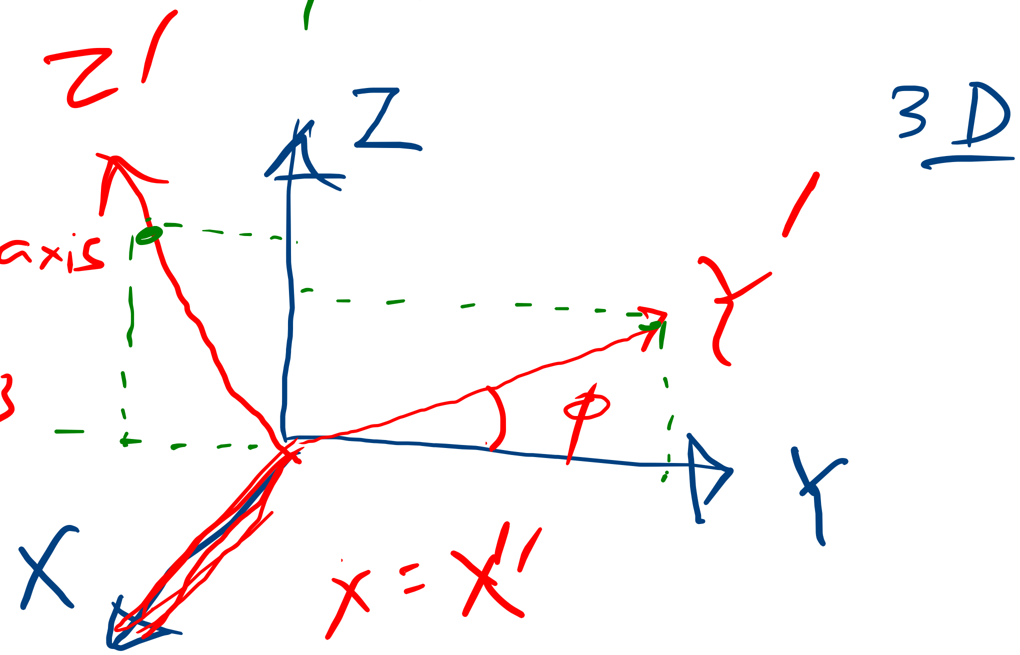


$$\boxed{p' = {}^{a}T_{b}\,{}^{b}T_{c}\cdots\cdots T_{N}\,p}$$

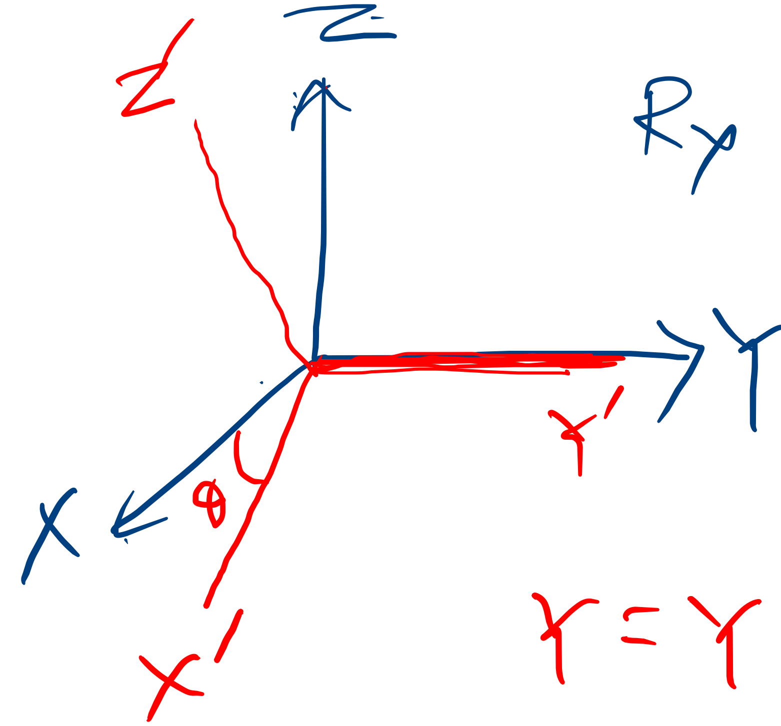
3D - Co-ordinate system

Rotation around x-axis

$$R_x = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad 3\times 3$$



$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

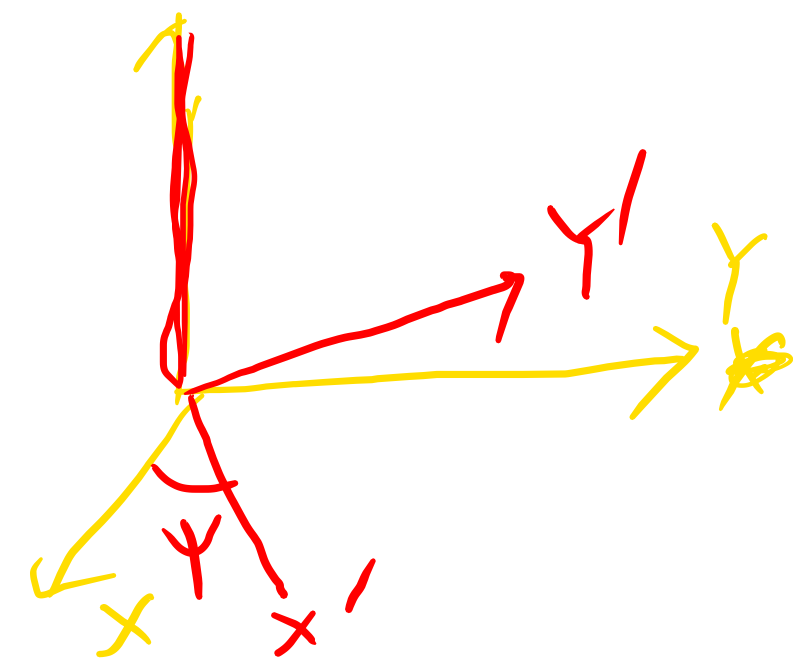


$Y = Y'$

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$Z = Z'$

$$R_Z = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



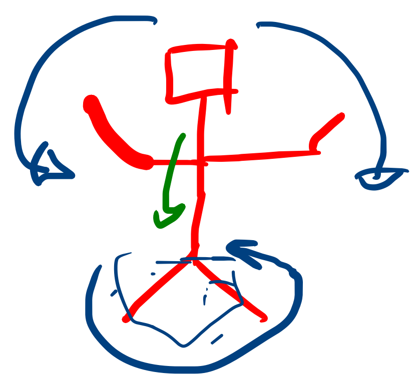
$$R_{xyz} = R_x \cdot R_y \cdot R_z$$

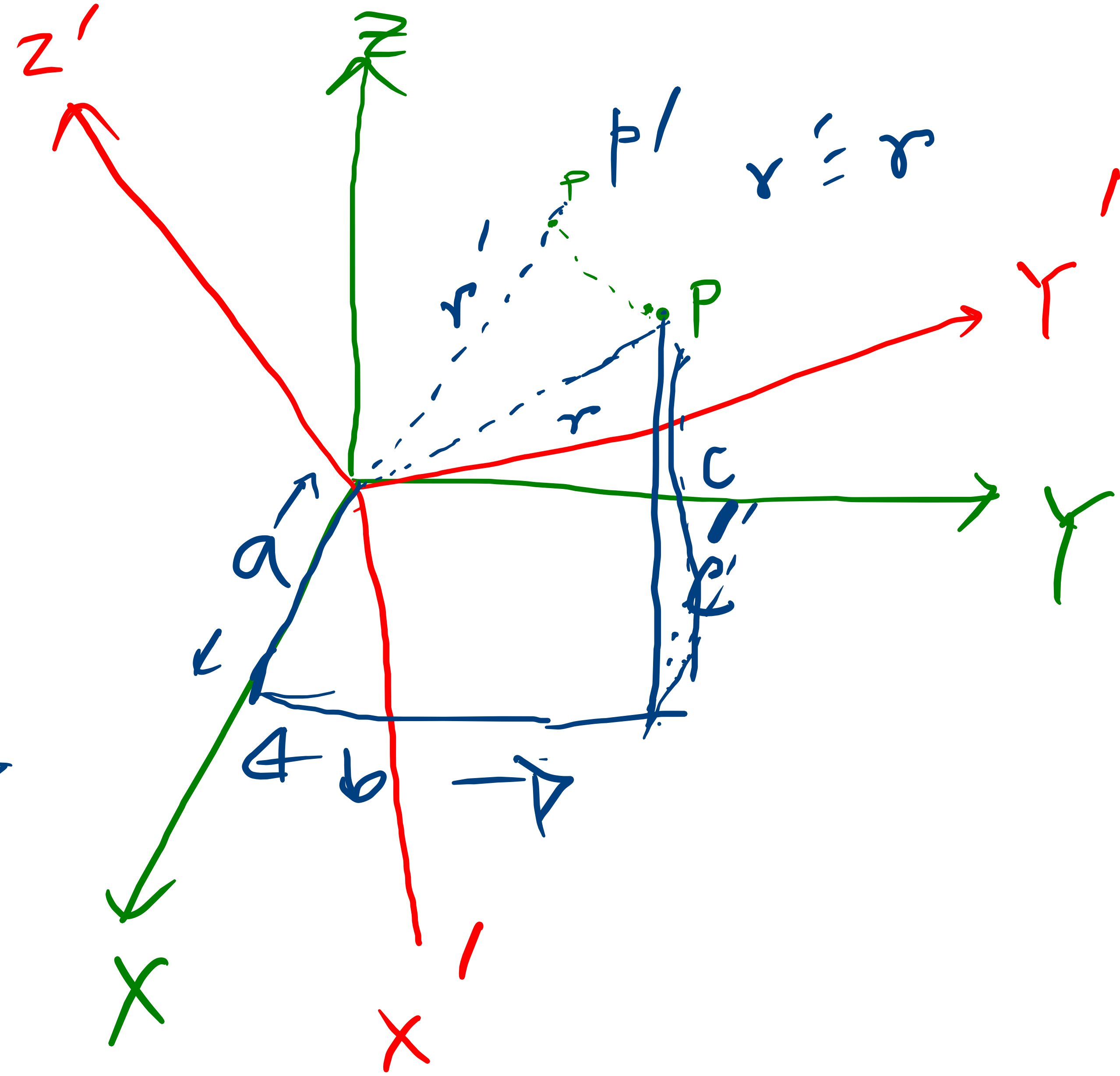
$$p' = R_{xyz} \cdot p$$

$R_x \Rightarrow$ Roll

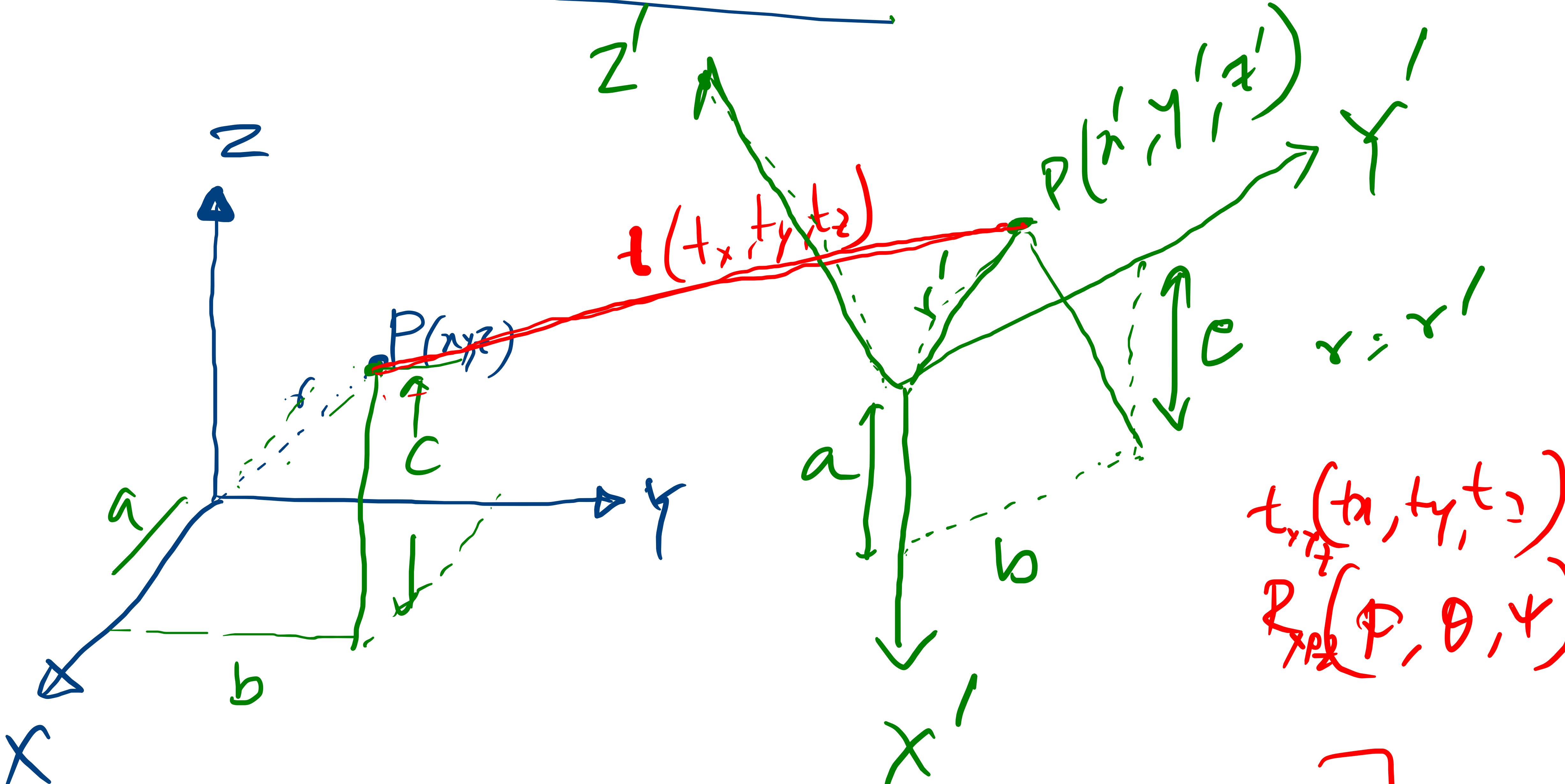
$R_y \Rightarrow$ Pitch

$R_z \Rightarrow$ Yaw





General 3-D Transformation



$$T = R + t = \begin{bmatrix} R_{xyz} & t_{xyz} \\ 0 & 1 \end{bmatrix}$$

$$P' = T\,P'$$

$$T = \begin{bmatrix} R_{xyz} & t_{xyz} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} R_{xyz} & \begin{matrix} t_x \\ t_y \\ t_z \end{matrix} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$R_{xyz} = R_x(\varphi) \cdot R_y(\theta) \cdot R_z(\psi)$$

$$R_{xyz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \times \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{xyz} = \begin{bmatrix} C_\psi C_\theta & -S_\psi C_\varphi + C_\psi S_\theta S_\varphi & S_\psi S_\varphi + C_\psi S_\theta C_\varphi \\ S_\psi C_\theta & C_\psi C_\varphi + S_\psi S_\theta S_\varphi & -C_\psi S_\varphi + S_\psi S_\theta C_\psi \\ -S_\theta & C_\theta S_\varphi & C_\theta C_\varphi \end{bmatrix}$$

$C_\psi = \cos\psi$ $\qquad$ $S_\psi = \sin\psi$

$C_\theta = \cos\theta$ $\qquad$ $S_\theta = \sin\theta$