Groups and linear algebra (SC220) Autumn 2019
In Sem -II Time: 1hr 30 min

Name: ______________________________

Student I.D.: ______________________________

Section 1. True/False (2 pts. each)

Print "T" if the statement is true, otherwise print "F". In either case give a justification or a counter example. No points will be awarded for just writing T or F.

______ The matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ is invertible

______ Let V be a vector space and $V = U_1 \oplus W$ and $V = U_2 \oplus W$, where U_1, U_2, W are subspaces of V, then it is necessarily true that $U_1 = U_2$

______ A and B are $n \times n$ matrices. If A is similar to B then A^2 is similar to B^2

______ In $C[-\pi, \pi]$ the set of vectors $\{\cos^2 x, \sin^2 x, \cos 2x\}$ are linearly independent

______ In the vector space $C[-1, 1]$ the subset $W = \{f(x) \in C[-1, 1] : f(-1) = -f(1)\}$ is a subspace.

______ There equation $Ax = b$ where $A = \begin{pmatrix} 1 & 3 & -2 & 0 \\ 2 & 6 & -2 & 4 \\ 0 & 1 & 1 & 3 \end{pmatrix}$ $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $b = \begin{pmatrix} 3 \\ 18 \\ 10 \end{pmatrix}$ has infinitely many solutions.

______ The set of vectors $\{(1, -1, 1, 0), (0, 1, 1, 1), (3, -1, 5, 2)\}$ in $\mathbb{R}^4$. is linearly independent.

______ In $Q_2[x]$ with basis $\{1, x, x^2\}$, the linear transformation that takes T that $p(x)$ to $p(x+1)$ has matrix representation $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

______ In $M_2(\mathbb{R})$ let $U = \left\{ \begin{pmatrix} a & a \\ b & 0 \end{pmatrix} | a, b \in \mathbb{R} \right\}$ and $V = \left\{ \begin{pmatrix} 0 & x \\ y & x \end{pmatrix} | x, y \in \mathbb{R} \right\}$ then $\dim(U + V) = 4$

______ The two matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$ are similar

Section 2. Short Answer (10 pts each)

Answer all problems in as thorough detail as possible.

1. In $\mathbb{R}^2$ let L be a line that makes an angle θ with the x-axis. Show that the matrix representation of the operator $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ which is reflection about line L in the standard basis $\{e_1, e_2\}$ is given by $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

2. Let $\{e_1, e_2, \ldots, e_n\}$ be a basis of a vector space V then show that $\{e_1, e_1 + e_2, e_1 + e_2 + e_3, \ldots, e_1 + e_2 + \cdots + e_n\}$ is also a basis of V

3. Use Kirchoff's current and voltage laws to find the currents I_1, I_2 and I_3

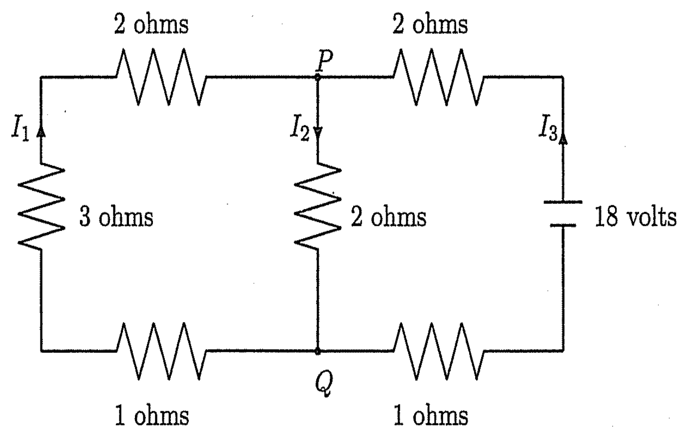

Figure 1: Electrical Circuit