Signals and Systems (CT 203)

Solutions of Tutorial Sheet-03 DA-IICT, Gandhinagar.

1. Determine and sketch the even and odd parts of the signals shown below in Fig.1 and 2.

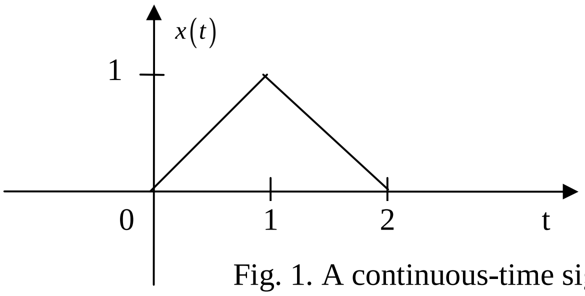

Fig. 1. A continuous-time signal

Ans:-

$x_{even}(t) = \frac{1}{2}\{x(t) + x(-t)\}$

$x_{odd}(t) = \frac{1}{2}\{x(t) - x(-t)\}$

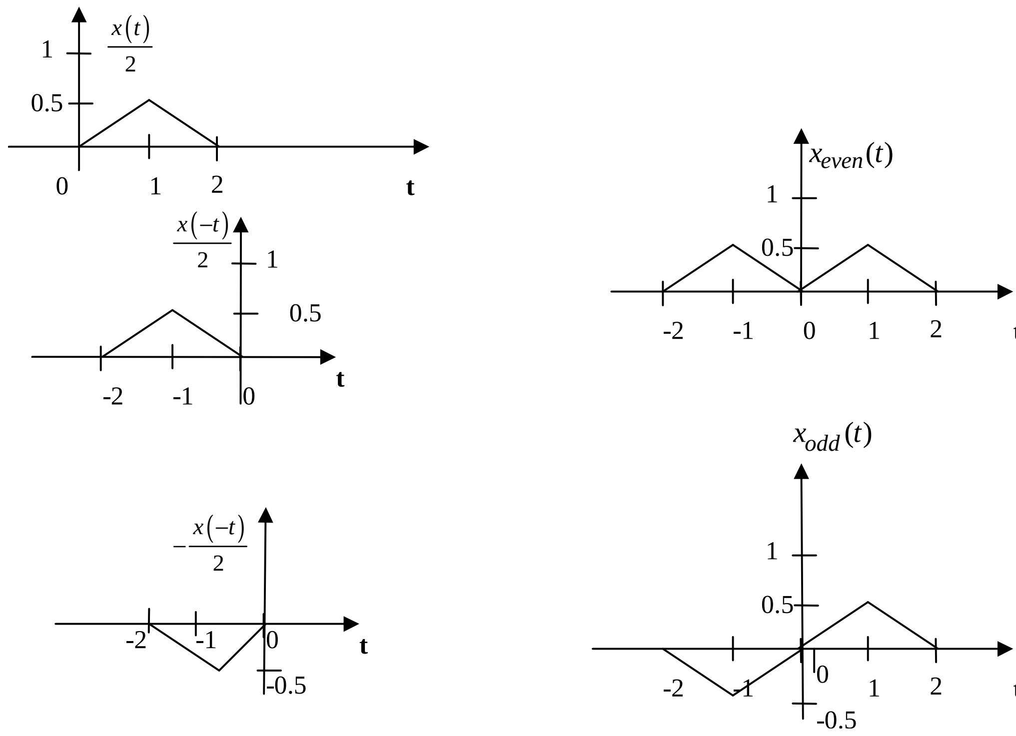

Fig. 1.1. Answer for Fig.1

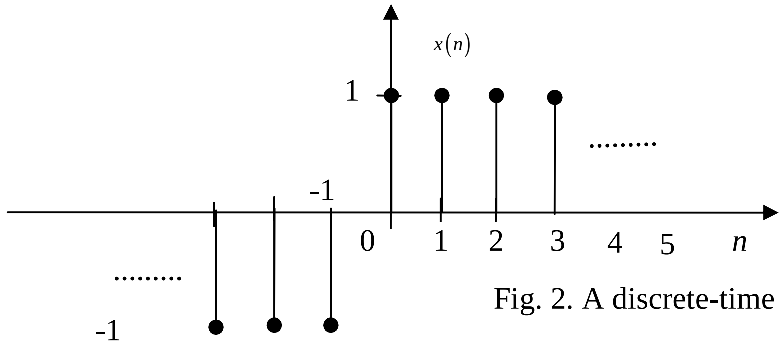

Fig. 2. A discrete-time signal

$$x_{even}[n] = \tfrac{1}{2}\{x[n] + x[-n]\}$$
$$x_{odd}[n] = \tfrac{1}{2}\{x[n] - x[-n]\}$$

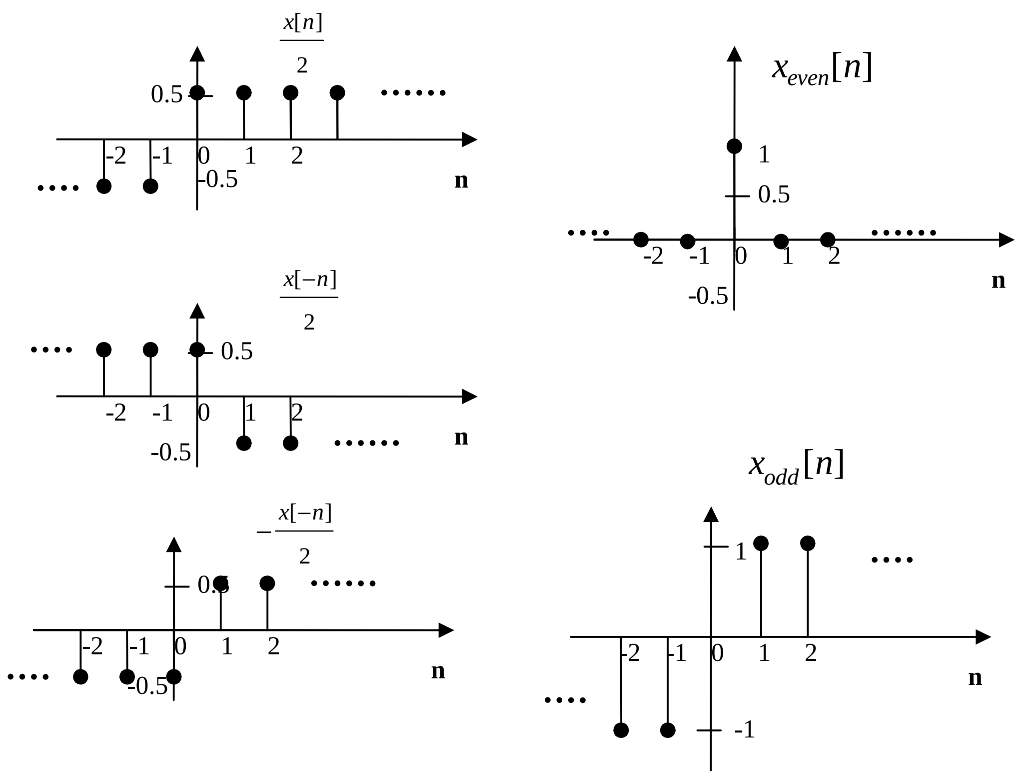

Fig. 2.1. Answer for Fig.2

2. Determine the fundamental period of the discrete-time signal given by

$$x(n) = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$$

Ans:-

$$x(n) = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$$

The first exponential in eq. has fundamental period *3*. It can be easily verified. In particular, note that the angle $(2\pi/3)n$ of the first term must be incremented by multiple of 2π for the values of this exponential begin repeating.
We then immediately see that if n is incremented by 3, the angle part will be incremented by a single multiple of 2π.

With regard to the second term, we see that incrementing the angle $(3\pi/4)n$ by 2π would require n to be incremented by $8/3$. That is not possible because n is restricted to be integer only. Similarly, incrementing angle by 4π would require a noninteger increment of $16/3$ to n. That is also not possible. However, the incrementing angle by 6π requires increment of 8 to n. Thus, $n = 8$ is fundamental period of second term.

So, Fundamental period of $x(n)$ =LCM {3,8} =24. This again can be verified by substituting $n = 24$, the angles in both exponentials are integer multiple of 2π.
$\boxed{\text{ANS=24}}$.

3. Develop mathematical model for series RC circuit

Ans:-

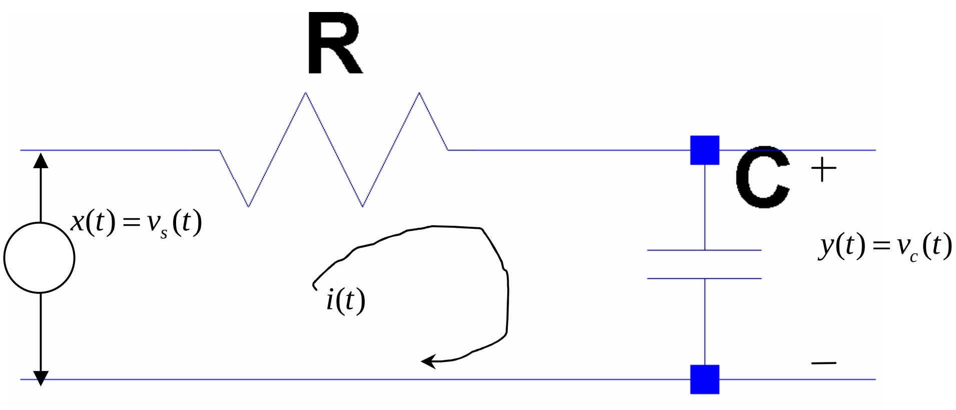

Fig. 3. A Series RC Circuit

Input- $x(t) = v_s(t)$ is supplied voltage.

Output- $y(t) = v_c(t)$ is voltage across the capacitor.

Here, Voltage across resistor= $v_r(t) = v_s(t) - v_c(t) = x(t) - y(t)$

$$i(t) = \frac{v_r(t)}{R} = \frac{v_s(t) - v_c(t)}{R} = \frac{x(t) - y(t)}{R}$$ (R and C are in series so current is same.) --(1)

Also, current through capacitor is proportional to rate of change of voltage across it.

$$i(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt} \qquad \text{--(2)}$$

From (1) and (2), we can say that,

$$\boxed{\begin{aligned} &\frac{x(t) - y(t)}{R} = C\frac{dy(t)}{dt} \\ &\therefore \frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t) \end{aligned}}$$

ANS

4. Develop mathematical model for automobile systems

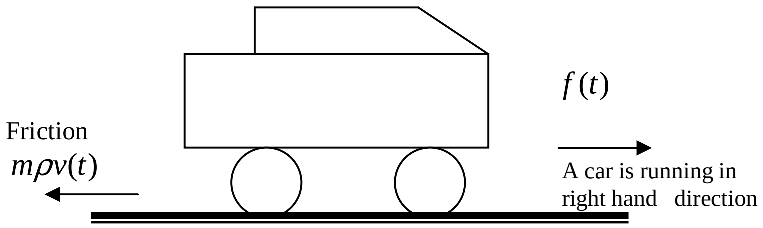

Fig. 4. An Automobile System

Considering Fig. 4, we regard the force $f(t)$ as the input and the velocity $v(t)$ as the output. Let m be the mass of automobile then $mv\rho$ is the resistance due to friction. Now, equating acceleration, i.e., the time derivative of velocity- with net force divided by mass,

Net Force $= f(t) - m\rho v(t)$

Acceleration= $\dfrac{dv(t)}{dt} = \dfrac{\text{Net Force}}{\text{Mass}} = \dfrac{f(t) - m\rho v(t)}{m}$

$$\boxed{\frac{dv(t)}{dt} + \left(\frac{\rho}{m}\right)v(t) = \left(\frac{1}{m}\right)f(t)}$$

Take $x(t) = f(t)$ and $y(t) = v(t)$

$$\boxed{\frac{dy(t)}{dt}+\left(\frac{\rho}{m}\right)y(t)=\left(\frac{1}{m}\right)x(t)}$$

5. Develop mathematical model for series RLC circuit as approximation to a physical system

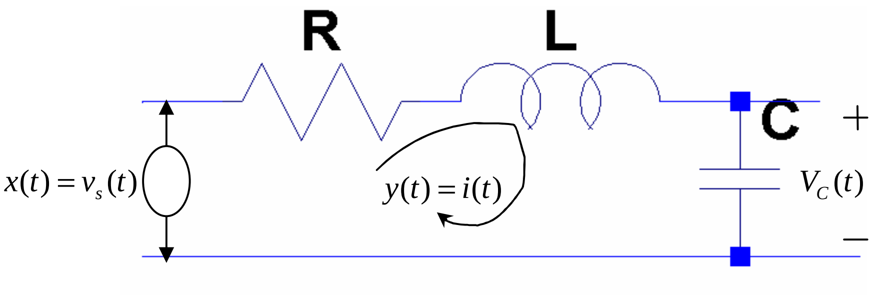

Fig. 5. A Series RLC Circuit

Input signal $=x(t)=v_s(t)$ is supplied voltage.

Output signal $y(t)=v_C(t)$ is current through capacitor.

$x(t)=$ Voltage across R + Voltage across L +Voltage across C

$$\therefore x(t)=V_R(t)+V_L(t)+V_C(t)$$

$$\therefore x(t)=Ri(t)+L\frac{di(t)}{dt}+\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$\therefore x(t)=Ry(t)+L\frac{dy(t)}{dt}+\frac{1}{C}\int_{-\infty}^{t} y(\tau)d\tau$$

Taking differentiation

$$\therefore \frac{dx(t)}{dt}=R\frac{dy(t)}{dt}+L\frac{d^2y(t)}{dt^2}+\frac{1}{C}y(t)$$

$$\boxed{\therefore \frac{d^2y(t)}{dt^2}+\left(\frac{R}{L}\right)\frac{dy(t)}{dt}+\left(\frac{1}{LC}\right)y(t)=\left(\frac{1}{L}\right)\frac{dx(t)}{dt}}$$ **ANS**

6. Develop mathematical model for mass-spring and damper system.

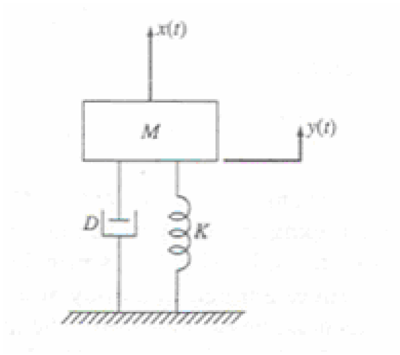

Fig. 6. A Schematic Diagram of Mass-spring-damper System

Consider Fig. 6. the input $x(t)$ to the system is the external force applied to the mass, which causes mass to move up or down. The output of the system is the displacement $y(t)$ of the mass, which is measured with respect to an equilibrium position. When the mass is moved upward by the external force $x(t)$ from its equilibrium position, the displacement $y(t)$ will be positive. In this case, the spring is expanded and thus will resist the upward motion, resulting in a negative force applied to the mass. In addition, the inertia force and damping force will also resist the upward motion and therefore result in negative forces applied to the mass.

Let's count all forces.

F_1 = Force due to spring = $Ky(t)$

F_2 = Damping force = $D\dfrac{dy(t)}{dt}$

F_3 = Inertia force = $M\dfrac{d^2y(t)}{dt^2}$

In equilibrium condition,

$$x(t) = F_1(t) + F_2(t) + F_3(t)$$

$$\boxed{\therefore x(t) = Ky(t) + D\frac{dy(t)}{dt} + M\frac{d^2y(t)}{dt^2}}$$ **ANS**

7. Develop mathematical model for system as electric motor with load

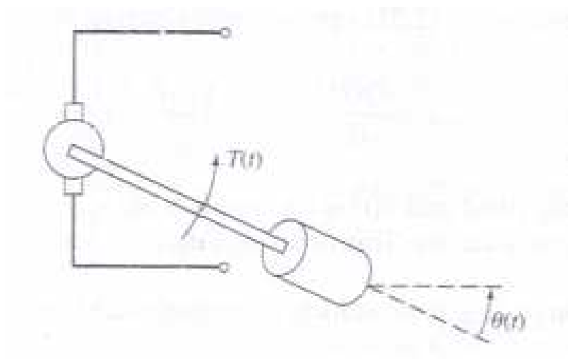

Fig. 7. Motor with load

Consider a motor with load as shown in schematically in Figure 7. The load indicated in Figure is some structure, such as a valve or plate, to which the motor shaft is connected. The input in this example is the Torque $T(t)$ applied to the motor shaft that is generated by the motor. The torque $T(t)$ is generated by the motor via a process that depends on the type of motor being used. The output of the motor with load is the angular position $\theta(t)$ of motor shaft relative to a reference position. The torque is resisted by the inertia torque and damping torque.

$$T(t) - I\frac{d^2\theta(t)}{dt^2} - k_d\frac{d\theta(t)}{dt} = 0$$

Where, I is the moment of inertia of the motor and load and k_d is the viscous friction coefficient of the motor and load. Rearranging the terms results in the following second order differential equation for the motor with load,

$$\boxed{I\frac{d^2\theta(t)}{dt^2} + k_d\frac{d\theta(t)}{dt} = T(t)}$$ **ANS**