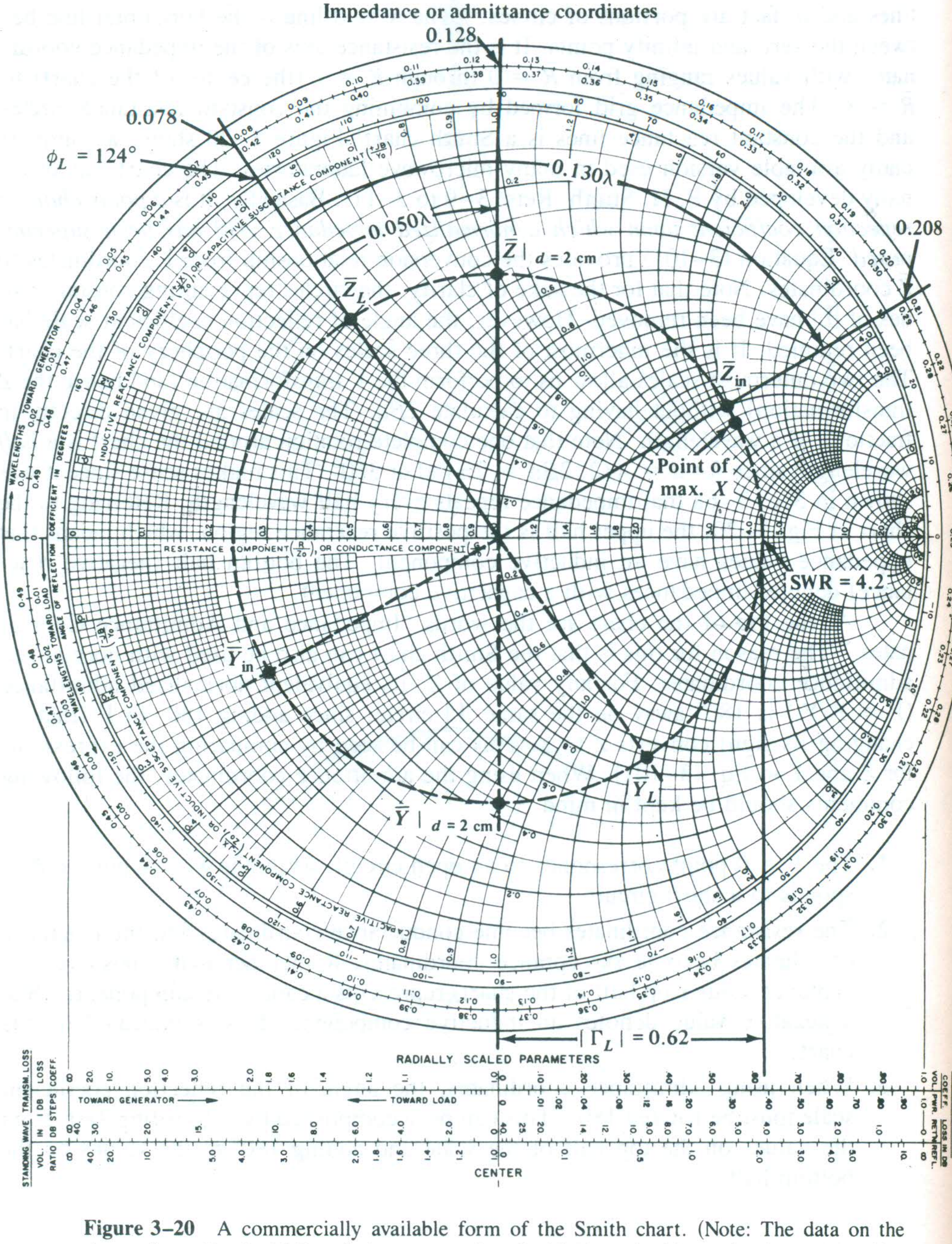

Figure 3–20 A commercially available form of the Smith chart. (Note: The data on the chart refer to Ex. 3–6.) Permission to reproduce Smith charts in this text has been granted by Phillip H. Smith, Murray Hill, N.J., under his renewal copyright issued in 1976.

Note that there are two wavelength scales on the periphery of the chart. One is labeled *Wavelengths toward Generator* and the other *Wavelengths toward Load*. The first one is used when determining the impedance at a point *nearer the input* than the known impedance. This clockwise rotation is referred to as *moving toward the generator*, the assumption being that a generator is connected to the input. The second scale is used in determining the impedance at a point *nearer the load* than the known impedance. This counterclockwise rotation is referred to as *moving toward the load*.

The following example illustrates the graphical procedure for determining the impedance and admittance transformation due to a length of transmission, line as well as other useful properties of the Smith chart.

Example 3–6:

A 5.2 cm length of lossless 100 ohm line is terminated in a load impedance $Z_L = 30 + j50$ ohms.

(a) Calculate $|\Gamma_L|$, ϕ_L, and the SWR along the line.

(b) Determine the impedance and admittance at the input and at a point 2.0 cm from the load end. The signal frequency is 750 MHz and $\lambda = \lambda_0$.

Solution:

(a) Plot the normalized load impedance $\bar{Z} \equiv Z_L/Z_0 = 0.30 + j0.50$ on the Smith chart in Fig. 3–20. This is done by starting from the 0.30 point on the resistance axis and moving up 0.50 reactance units along the constant resistance circle. Next, draw a circle with its center at $\bar{Z} = 1$ (the center of the chart) and a radius equal to the distance between $\bar{Z} = 1$ and $\bar{Z}_L$. This is shown in the figure and will hereafter be referred to as the SWR circle. It is, in fact, the constant $|\Gamma|$ circle for the given value of load impedance. The $\bar{Z}_L$ point on the Smith chart corresponds to its Γ_L value on the polar chart. Since the angle of the reflection coefficient scale has been retained, the value of ϕ_L (124° in this case) can be obtained from the chart as shown in the figure. The value of $|\Gamma_L|$ is obtained by measuring the radius of the SWR circle on the Reflection Coefficient-Vol. scale located at the bottom of the chart. In this example problem, $|\Gamma_L| = 0.62$.

One reason that the constant $|\Gamma|$ circle is called the *SWR circle* is that its intersection with the right half of the resistance axis (that is, between 1 and ∞) yields the SWR due to $\bar{Z}_L$. For this case, the SWR is 4.2. Since SWR is so easily obtained, many engineers prefer to calculate $|\Gamma|$ from Eq. (3–53) rather than using the Reflection Coefficient-Vol. scale. The reason that the resistance axis between 1 and ∞ serves as a SWR scale is that it corresponds to positive real values of Γ. As such it represents the magnitude of Γ for all normalized impedances on the SWR circle. Equation (3–101) transforms these values of Γ into $\bar{R} = (1 + |\Gamma|)/(1 - |\Gamma|)$, which is exactly the equation for SWR (Eq. 3–52). The unity SWR circle is simply a point at the center of the chart, while the infinite SWR circle is the periphery of the chart and is equivalent to $|\Gamma| = 1.00$.

(b) A graphical method of obtaining Γ_{in} when Γ_L and βl are known has been described. For a lossless line, it consists of rotating clockwise $2\beta l$ on the constant $|\Gamma|$ circle. The procedure for obtaining $\bar{Z}_{in}$ from $\bar{Z}_L$ is *exactly the same* except that the impedance coordinates of the Smith chart are used. The steps are as follows:

1. Plot $\bar{Z}_L$ (0.30 + j0.50 in this case) and draw its SWR circle (4.20 in this case).
2. Draw a radial line from the center of the chart through $\bar{Z}_L$ to the periphery. Read the value on the *Wavelengths toward Generator* scale (0.078 in this case).

This value in itself has no physical meaning. It is merely the starting point of the clockwise rotation in the next step.

3. Since $\lambda_0 = 40$ cm at 750 MHz and the input is 5.2 cm from the load, rotate clockwise *from* 0.078 a distance $l/\lambda = 5.2/40 = 0.130$. Draw a radial line from the center of the chart through the 0.208 point on the outer scale as shown in Fig. 3–20. The intersection of the radial line with the SWR circle represents $\bar{Z}_{\text{in}}$ since it corresponds to the Γ_{in} point on the polar reflection coefficient chart. In this case, $\bar{Z}_{\text{in}} = 2 + j2$ or $Z_{\text{in}} = 200 + j200$ ohms.

To obtain the impedance at $d = 2$ cm, start from $\bar{Z}_L$ and rotate clockwise $2/40 = 0.050$ and draw a radial line through the 0.128 point as shown. Its intersection with the SWR circle yields $\bar{Z} = 0.47 + j0.93$ or $Z = 47 + j93$ ohms.

This simple graphical method of determining the impedance transformation due to a length of lossless line is the most useful characteristic of the Smith chart. The procedure for determining the admittance transformation is *exactly the same* since all admittance points are directly opposite their corresponding impedance points on the SWR circle. Thus from the figure, $\bar{Y}_L = 0.88 - j1.47$, $\bar{Y}_{\text{in}} = 0.25 - j0.25$, and at $d = 2.0$ cm, $\bar{Y} = 0.43 - j0.87$. Multiplying these values by $Y_0 = 0.01$ mho gives the unnormalized admittance values.

Part *a* of this example problem illustrates how to determine $|\Gamma_L|$, ϕ_L, and SWR when Z_L and Z_0 are known. Since the line is lossless, the SWR and $|\Gamma|$ are the same at all other points on the line. The angle of the reflection coefficient, however, is a function of position and can be read on the periphery of the chart. In this example, $\phi_{\text{in}} = 30°$ while at $d = 2$ cm, ϕ is equal to 88°.

Part *b* describes the graphical solution to the impedance/admittance transformation equation for lossless lines.[16] Given $\bar{Z}_L$, the normalized impedance at any other point on the line is obtained by rotating clockwise on a fixed SWR circle the appropriate distance d/λ. Thus for a given load impedance, the SWR circle represents the locus of all possible impedance and admittance values available on the lossless line. Stated another way, given $\bar{Z}_L$, it is the locus of all possible values of $\bar{Z}_{\text{in}}$ and $\bar{Y}_{\text{in}}$ obtainable by varying the line length l. This is an example of how a good graphical procedure can show the effect of a variable (l) on the desired result (Z_{in}). For instance, it is obvious from Fig. 3–20 that if $\bar{Z}_L = 0.30 + j0.50$, varying the line length will never result in $\bar{Z}_{\text{in}} = 0.70 + j0.40$ since it is not on the SWR circle containing $\bar{Z}_L$. If the reader is ambitious, try proving this analytically. To further emphasize the chart's usefulness, consider the ease with which the following problem is solved. Given the impedance values in Ex. 3–6, what value of line length maximizes the reactive portion of the input impedance? With $\bar{Z}_L = 0.30 + j0.50$ plotted in Fig. 3–20, the SWR circle represents all possible values of $\bar{Z}_{\text{in}}$ that can be obtained by varying l. A brief look at the chart shows that the reactive portion is maximized at the point where the SWR circle is tangent to the reactance lines. A positive reactive maximum occurs when $\bar{Z}_{\text{in}} \approx 2.3 + j2.0$. A radial line through this point intersects the *Wavelengths toward Generator* scale at 0.216. Therefore, the 100 ohm line must be $(0.216 - 0.078)$ or 0.138λ long.

[16] For a lossy line, a second SWR circle is required. It is obtained by multiplying $|\Gamma_L|$ by $e^{-2\alpha d}$ and converting the resulting $|\Gamma|$ to SWR. The intersection of the radial line with the circle defined by the new SWR value yields $\bar{Z}$ at d units from the load.