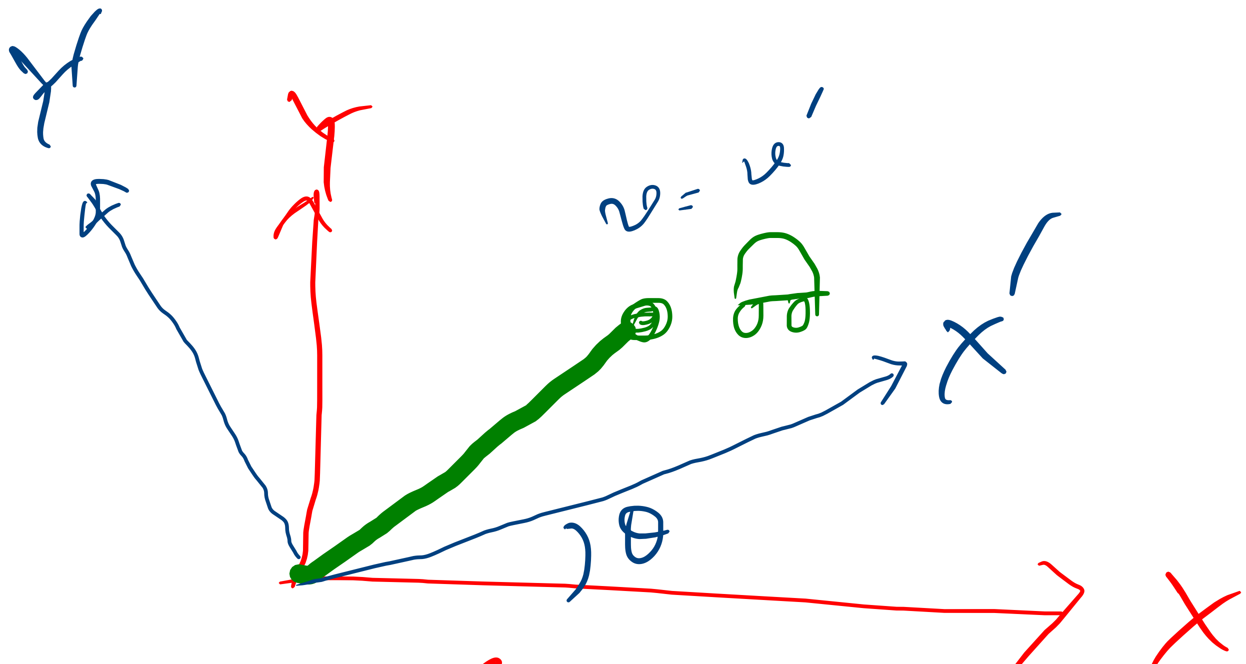
IE 410 Lecture 6 Feb 02, 2021

1. Passive Transformation
2. Active Tranformation

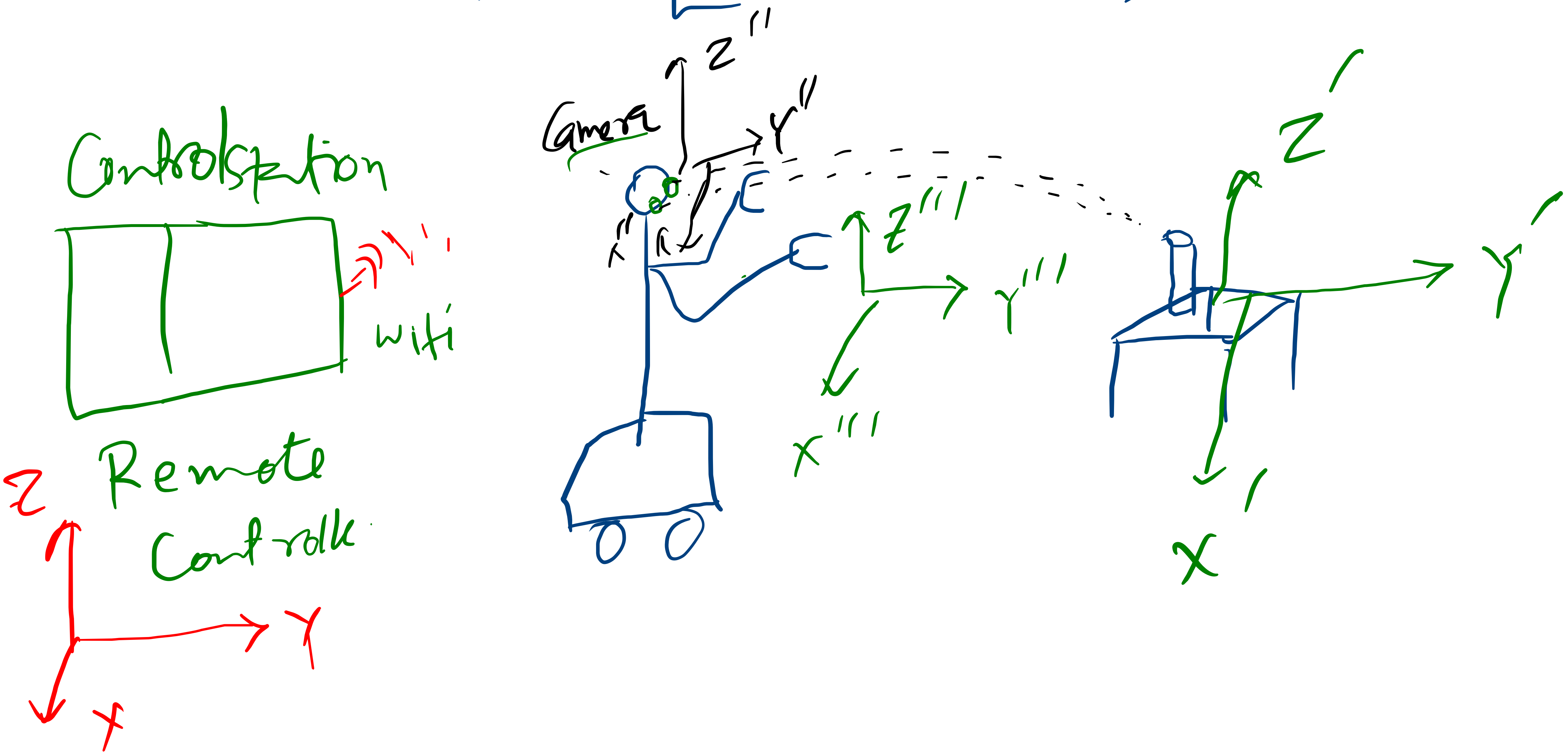


fixed frame of Reference

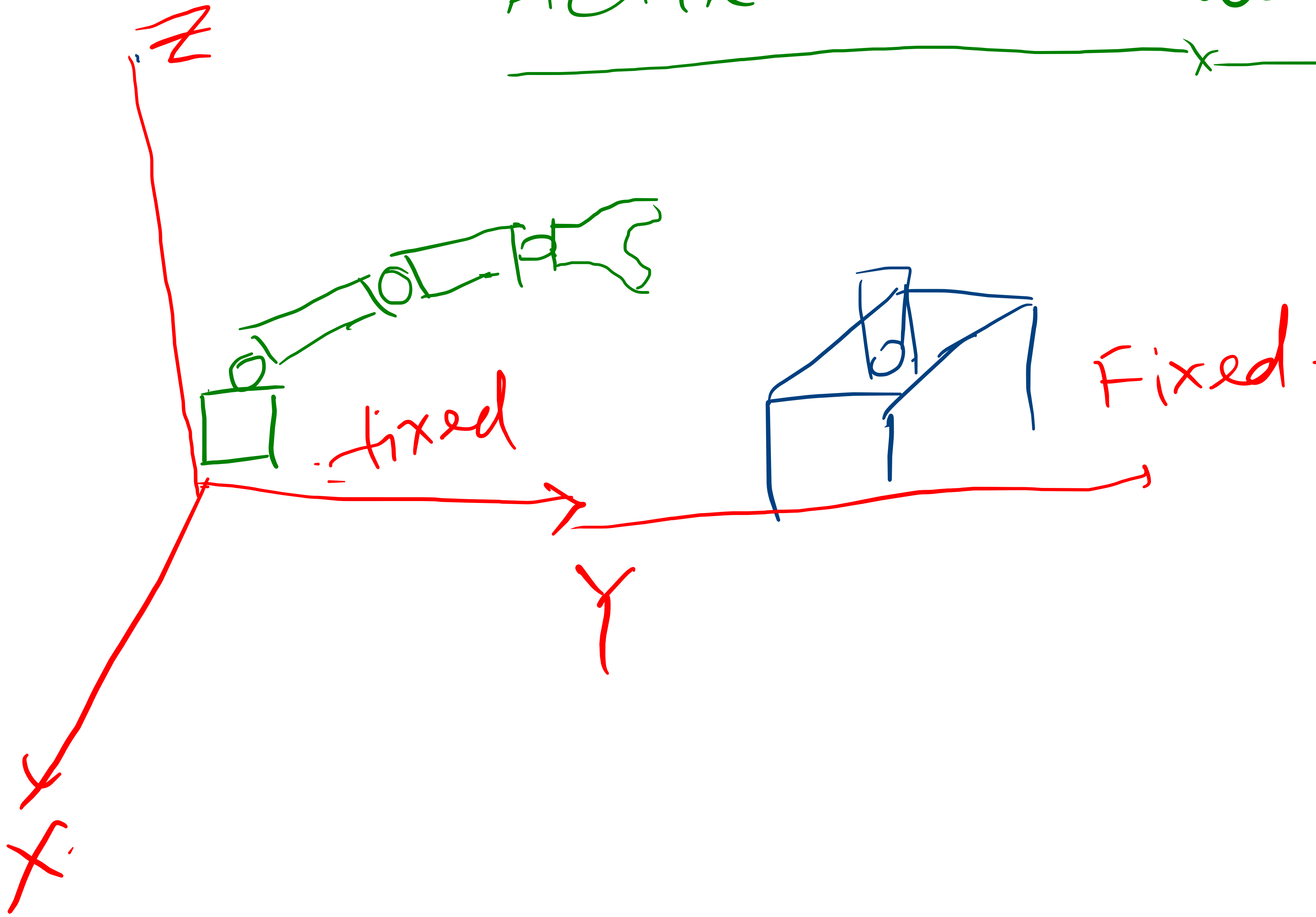
Embedded coordinate system

$$x' = R(\theta)\, x$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



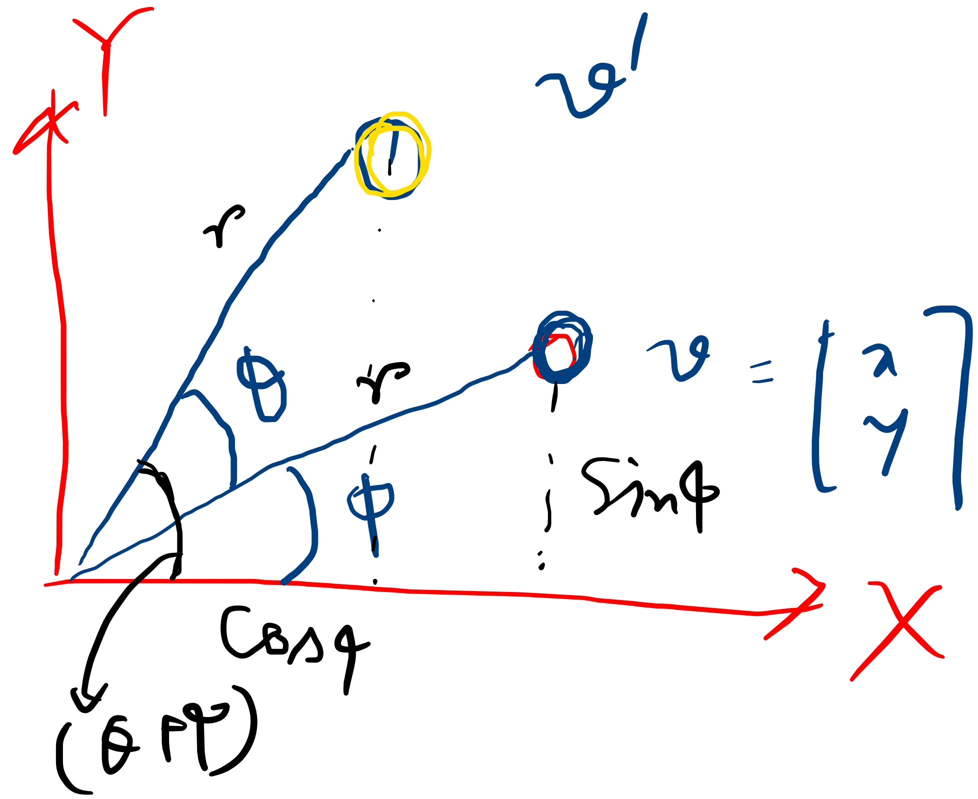
Active Co-ordinate Transformation



①

$$v = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$x = r\cos\phi$$
$$y = r\sin\phi$$



②

$$v' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r\cos(\theta+\phi) \\ r\sin(\theta+\phi) \end{bmatrix}$$

$$x' = r\cos(\theta+\phi)$$
$$y' = r\sin(\theta+\phi)$$

4

$$\boxed{\begin{aligned} \cos(A+B) &= \cos A\cos B - \sin A\sin B \\ \sin(A+B) &= \sin A\cos B + \cos A\sin\phi \end{aligned}}$$

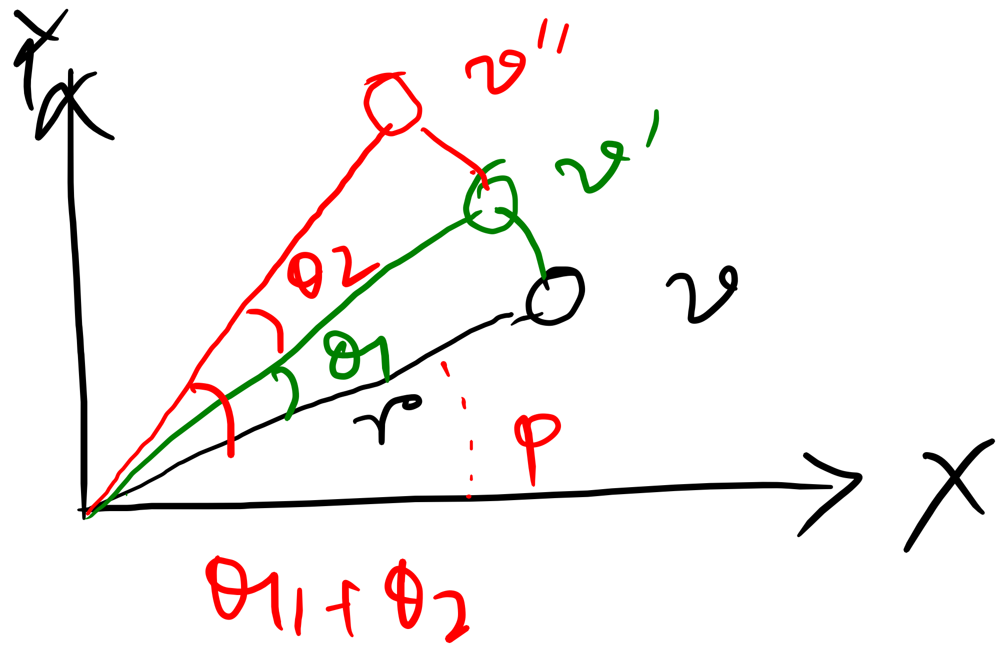
$$v' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r\cos\theta\cos\phi - r\sin\theta\sin\phi \\ r\sin\theta\cos\phi + \cos\theta\sin\phi \end{bmatrix}$$

$$x = r\cos\phi$$
$$y = r\sin\phi$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$v' = R(\theta)\, v' \qquad R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



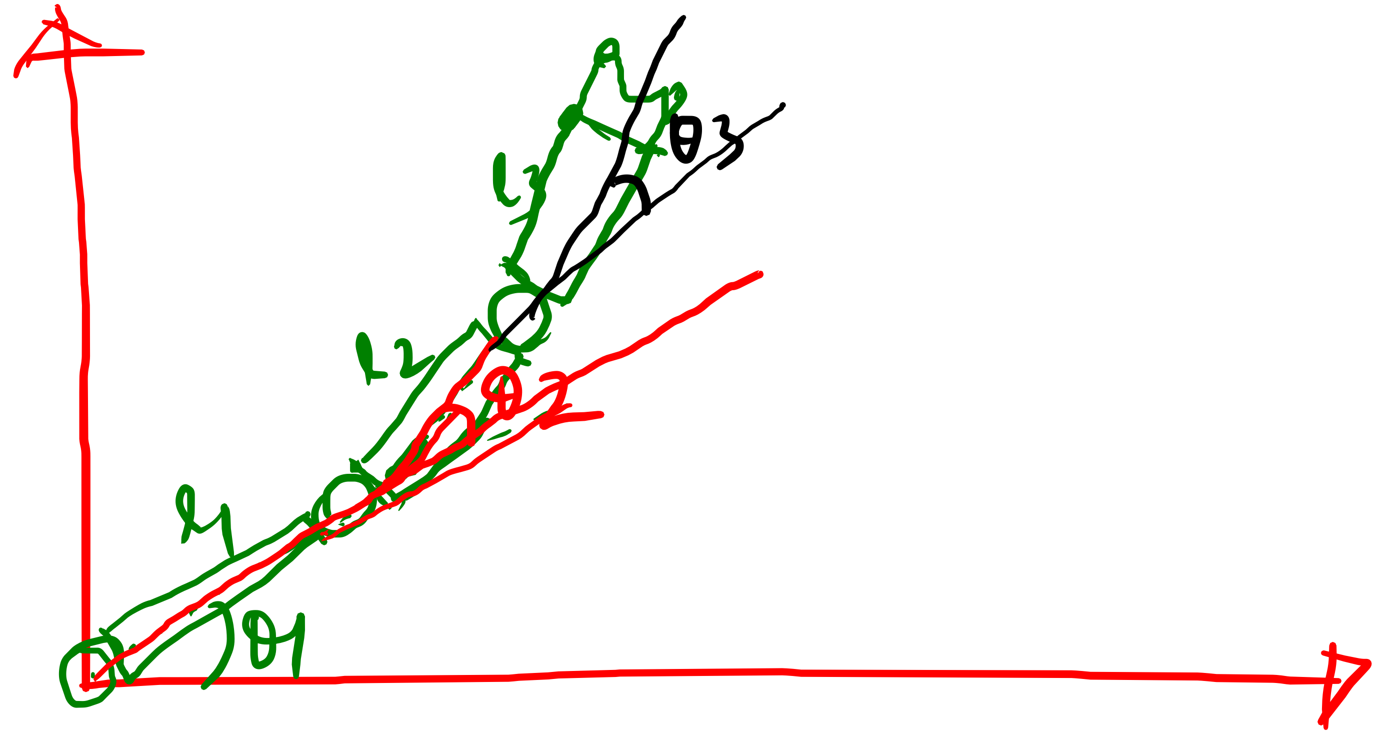
$$R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$$

$$R(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \qquad R(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

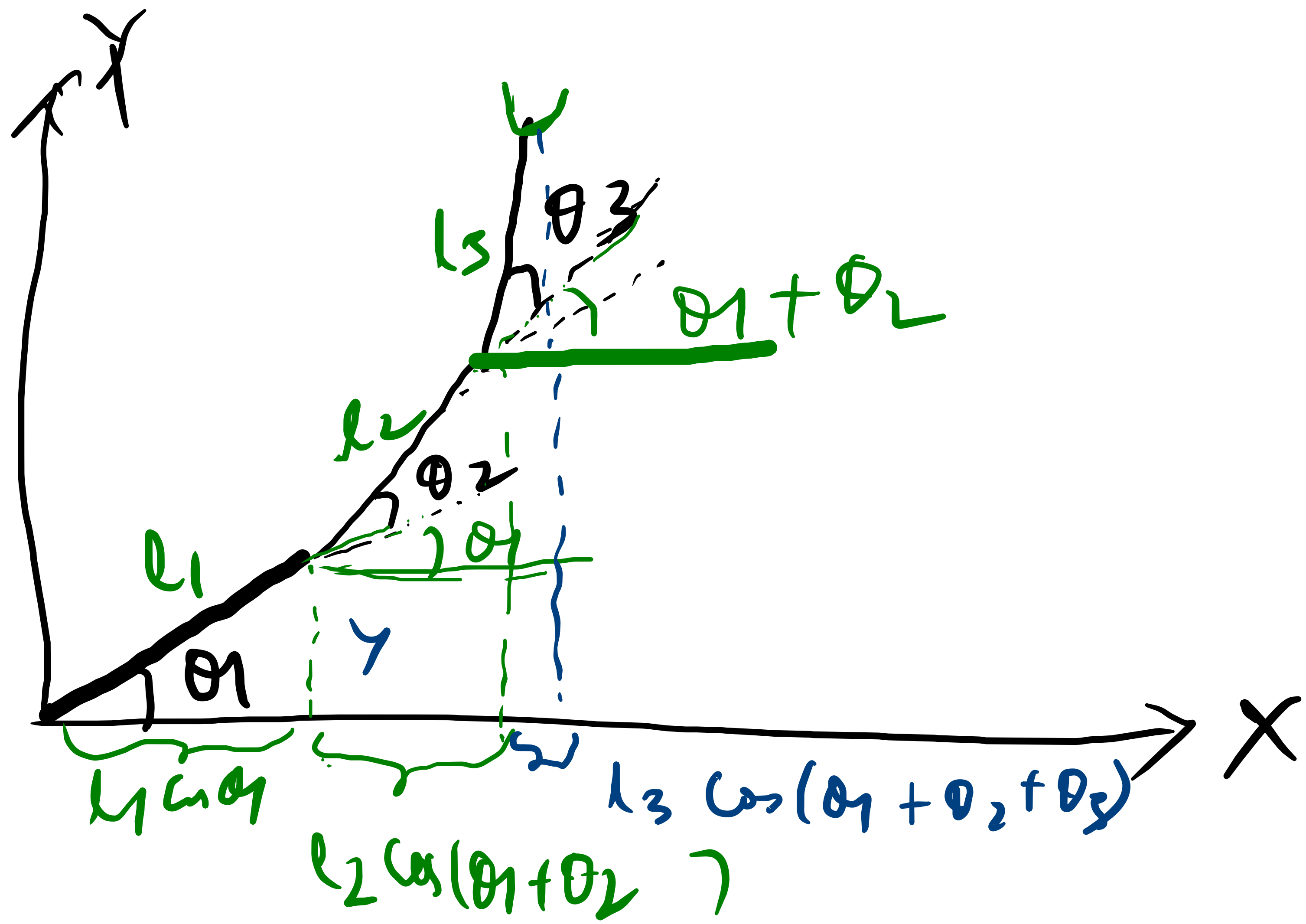
$$R(\theta_1) * R(\theta_2)$$

$$= \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{bmatrix}$$

$$= R(\theta_1 + \theta_2)$$



l1
l2
l3
θ1
θ2
θ3



Y
X
l1
l2
l3
θ1
θ2
θ3
θ1 + θ2
y
l1 cos θ1
l2 cos(θ1 + θ2)
l3 cos(θ1 + θ2 + θ3)

Position

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$$

Servomotor

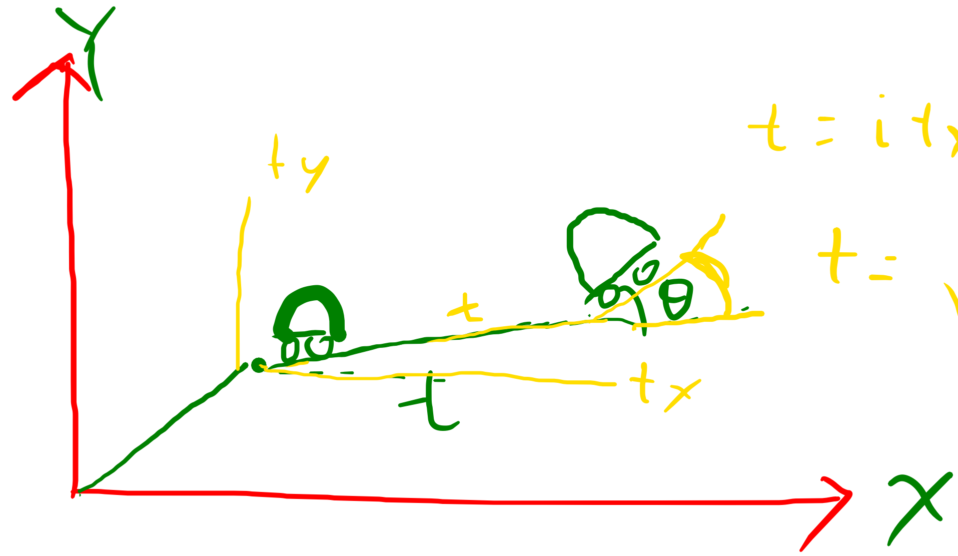
Velocity, acceleration Calculation

$$v_x = \frac{dx}{dt} = v(\theta, \dot{\theta})$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt} = a_x(\theta, \dot{\theta}, \ddot{\theta})$$

$$v' = R(\theta)v + t$$

$$\boxed{v' = R(\theta)v + t}$$



$t = i t_x + j t_y$

$t = \sqrt{t_x^2 + t_y^2}$

$v = v_x i + j v_y$

$v = \sqrt{v_x^2 + v_y^2}$

Note

$$v' = R(\theta)v$$

$R(\theta) = 2 \times 2$ matrix

$$= \begin{array}{c c} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \end{array}$$

$$v' = R(\theta)\begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} v_x' \\ v_y' \end{bmatrix} = R(\theta)\begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} v_x' \\ v_y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$2\times 1 = 2\times 3$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$v_x' = x'$

$v_y' = y'$

$v_x = x$

$v_y = y$

$$\cancel{\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

$$\cancel{2\times 1 \qquad 2\times 3 \Rightarrow 2\times 2}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$R(\theta, t) = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{Homogeneous Matrix}}$ used for General Transformation

$v_x' = x'$, $v_y' = y'$, $v_x = x$, $v_y = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

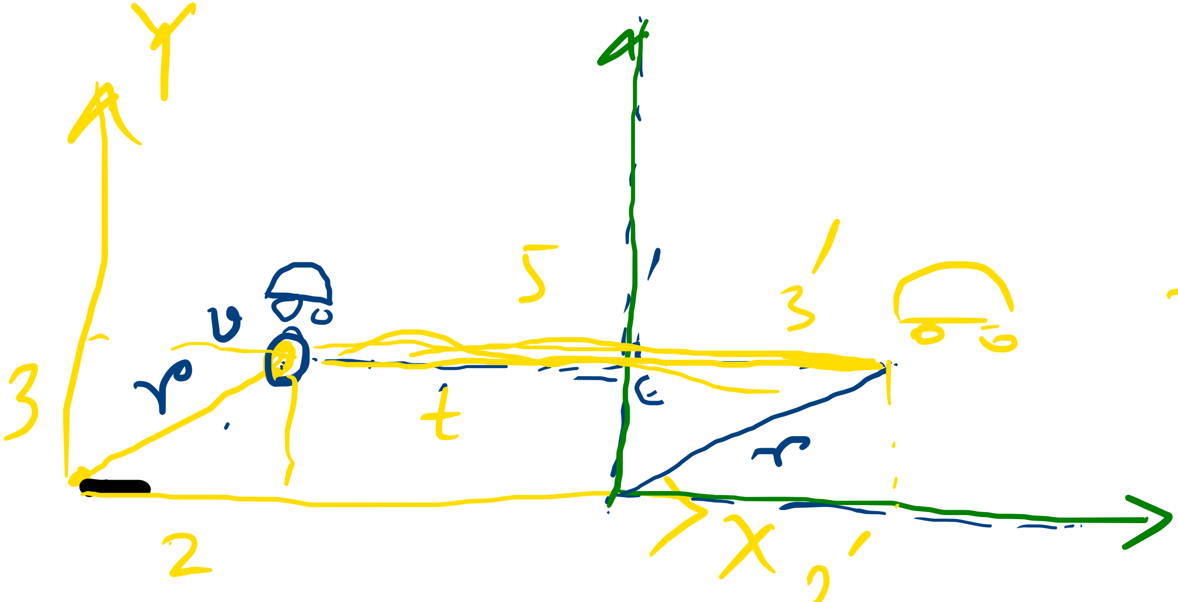
$$= \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \end{bmatrix}}_{2 \times 3} \begin{bmatrix} x \\ y \end{bmatrix}$$

- in-homogeneous matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Matrix

Co-ordinate transformation

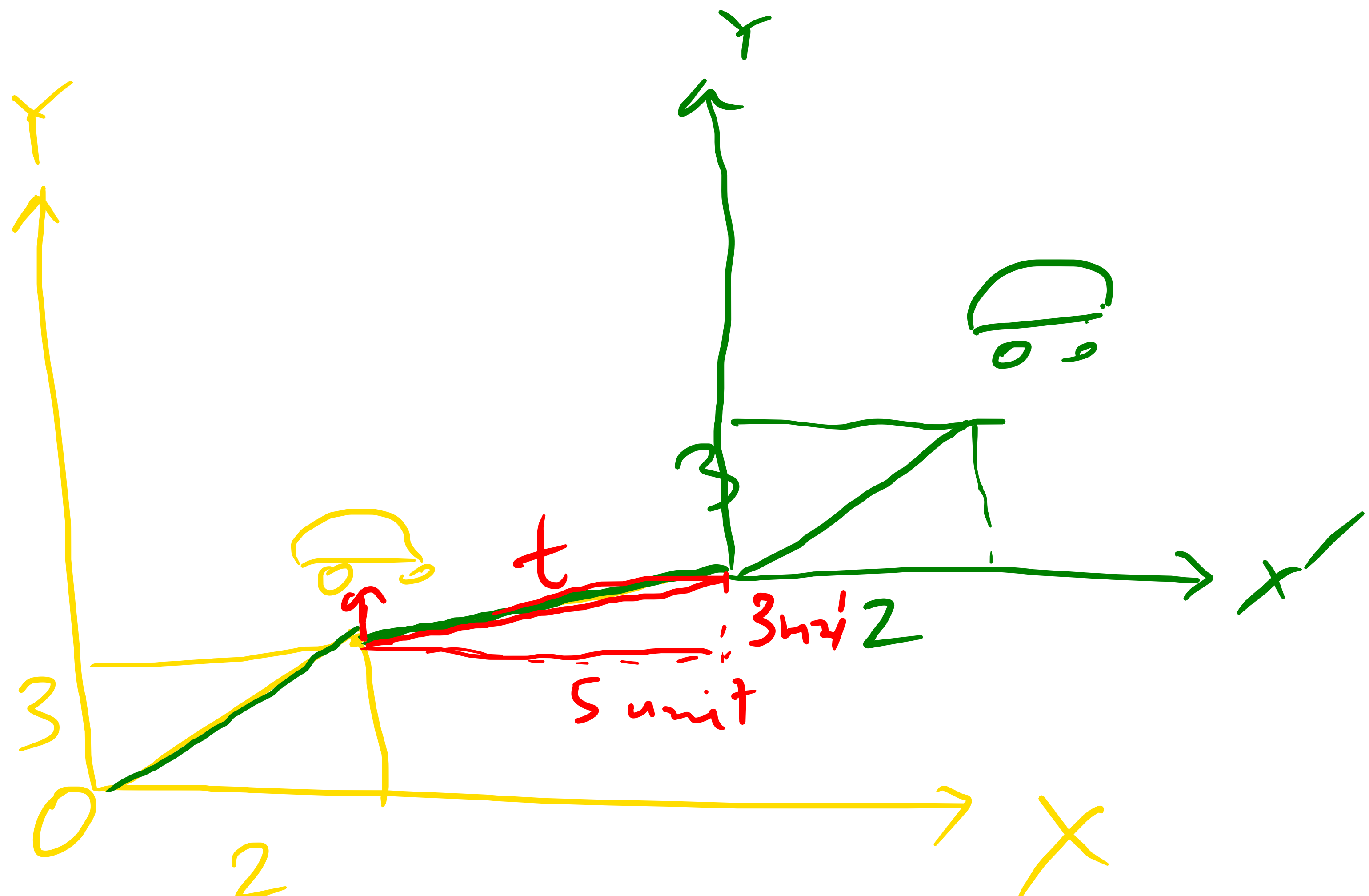


$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

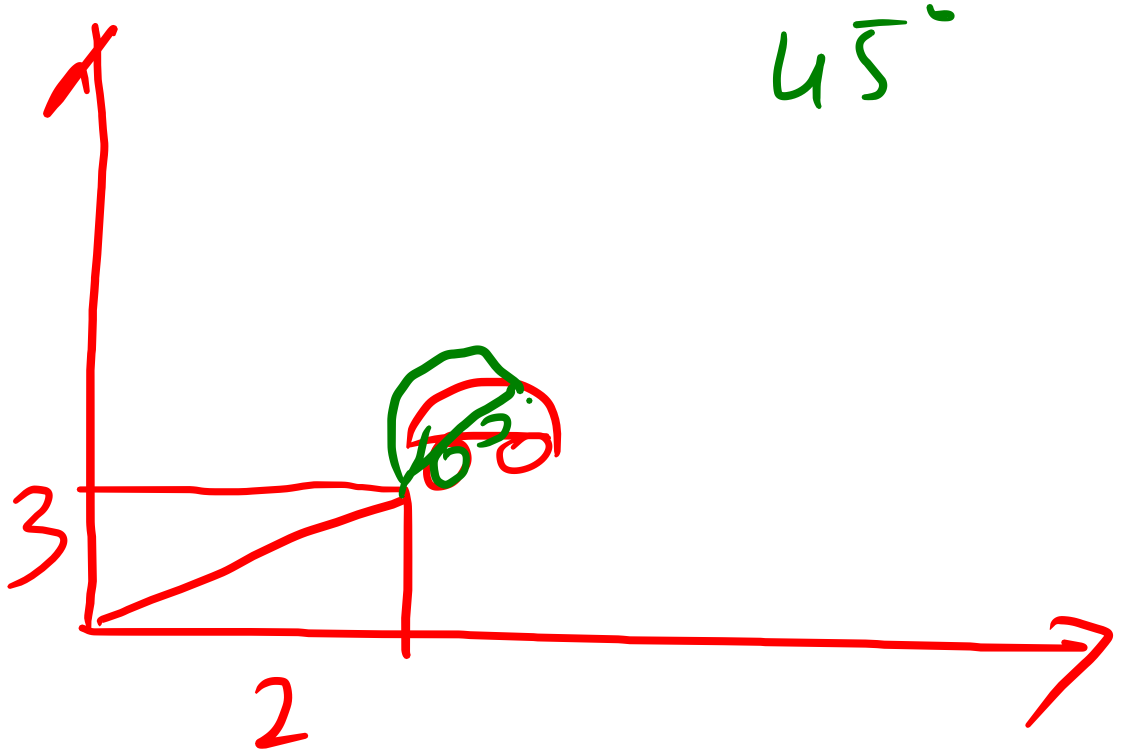
$v' =$

$$v' = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

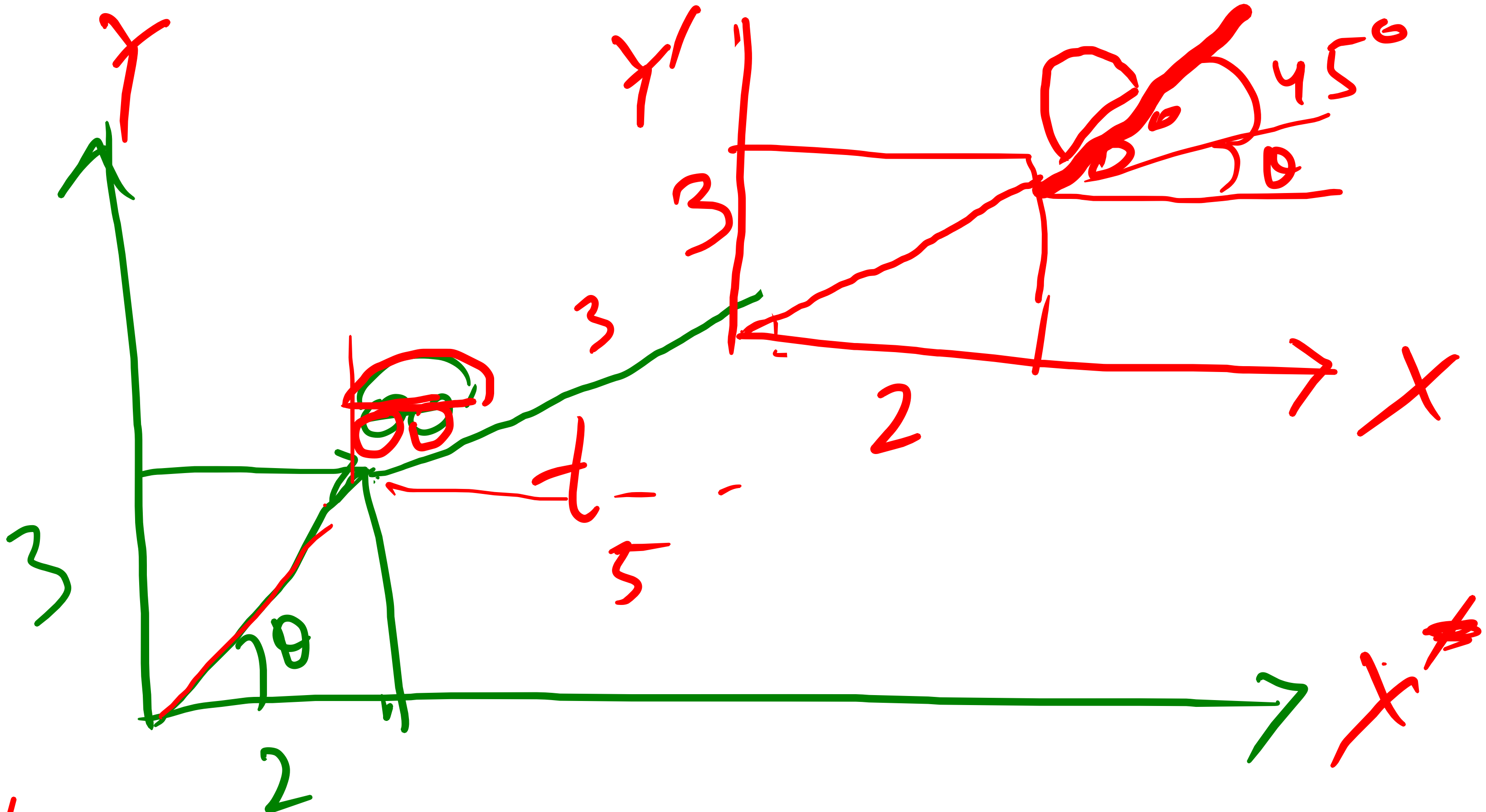
$$= \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



$$t\begin{bmatrix} x \\ y' \end{bmatrix} = t\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \cos 45^\circ & \sin 45^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



v'

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \overset{t}{\begin{bmatrix} 5 \\ 3 \end{bmatrix}} + \overset{R(\theta)}{\begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$