Tutorial 7

SC-220 Groups and Linear Algebra Autumn 2019

(Linear independence, basis)

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(1) Find three vectors in $\mathbb{R}^3$ that are linearly dependent and such that any two of them are linearly independent.

(2) Consider the complex vector space $V = \mathbb{C}^3$ and the list (v_1, v_2, v_3) of vectors in V , where $v_1 = (i, 0, 0)$, $v_2 = (i, 1, 0)$, $v_3 = (i, i, -1)$:
(a) Prove that $\text{span}(v_1, v_2, v_3) = V$.
(b) Prove or disprove: (v_1, v_2, v_3) is a basis for V .

(3) Are the vectors $x_1 = (1, 1, 2, 4), x_2 = (2, -1, -5, 2), x_3 = (1, -1, -4, 0), x_4 = (2, 1, 1, 6)$ linearly independent in $\mathbb{R}^4$? If not find a basis for the subspace of $\mathbb{R}^4$ spanned by the four vectors.

(4) Let V be the vector space of 2×2 matrices over the complex numbers. As seen this has dimension 4. Find a basis of V consisting of matrices A_1, A_2, A_3 and A_4 such that $A_j^2 = A_j$ for each j.

(5) Determine the dimension of each of the following subspaces of $\mathbb{R}^4$.
(a) $\{(x_1, x_2, x_3, x_4) | x_4 = 0\}$.
(b) $\{(x_1, x_2, x_3, x_4) | x_4 = x_1 + x_2\}$.
(c) $\{(x_1, x_2, x_3, x_4) | x_4 = x_1 + x_2, x_3 = x_1 - x_2\}$.

(6) Let V be the vector space of 2×2 matrices over the complex numbers. Let W_1 be the matrices of the form $\begin{pmatrix} x & -x \\ y & z \end{pmatrix}$ and let W_2 be the set of matrices of the form $\begin{pmatrix} a & b \\ -a & c \end{pmatrix}$.
i) Show that W_1 and W_2 are subspaces of V.
ii)Find the dimensions of $W_1, W_2, W_1 + W_2$ and $W_1 \cap W_2$.
