Groups and linear algebra (SC220) Autumn 2018
In Sem -II Time: 1hr 30 min

Name: ______________________________

Student I.D.: ______________________________

Section 1. True/False (2 pts. each)

Print "T" if the statement is true, otherwise print "F". In either case give a justification or a counter example.

_______ Let $D = \frac{d}{dt}$. Consider the set $\mathcal{B} = \{e^t, te^t\}$ be a linearly independent set of functions. Let $\mathcal{B}$ generate a vector space V, then D is a linear map from V into itself. The matrix of D relative to basis $\mathcal{B}$ is $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

_______ Every vector space has a finite basis

_______ The dimension of the set of complex numbers with the scalar field being the real numbers is 2

_______ The vectors $\{1, 1+x, 1+x^2\}$ are linearly independent set in the space of polynomials with degree less than or equal to 2

______ Any subset of a linearly dependent set of vectors is linearly dependent.

______ In $M_2(\mathbb{R})$ let $U = \left\{ \begin{pmatrix} a & a \\ b & 0 \end{pmatrix} | a, b \in \mathbb{R} \right\}$ and $V = \left\{ \begin{pmatrix} 0 & x \\ y & x \end{pmatrix} | x, y \in \mathbb{R} \right\}$ then $\dim(U + V) = 4$

______ If $A : V \to V$ is a linear operator on a vector space V then $\text{range}(A) \cap \ker(A^T) = 0$

______ The set of vectors $\{(1, -1, 1, 0), (0, 1, 1, 1), (3, -1, 5, 2)\}$ in $\mathbb{R}^4$. is linearly independent.

______ There equation $Ax = b$ where $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ 1 & 3 & 4 \end{pmatrix}$ $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $b = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ has infinitely many solutions.

______ Let V be a vector space and $V = U_1 \oplus W$ and $V = U_2 \oplus W$, where U_1, U_2, W are subspaces of V, then it is necessarily true that $U_1 = U_2$

Section 2. Short Answer (10 pts each)

Answer all problems in as thorough detail as possible.

1. Let $T : V \to V$ be a linear operator on a finite dimensional vector space V of dimension n and let U be a subspace of V with dimension $p < n$. Also, U is an invariant subspace of T, that is, $T(U) \subseteq U$. Show that the matrix representation of T in an appropriate basis is of the form $T = \begin{pmatrix} A & B \\ O & C \end{pmatrix}$ where A is $p \times p$ matrix, B is $p \times n - p$ matrix, C is $n - p \times n - p$ matrix and O is all zero $n - p \times p$ matrix.

2. Consider $P : \mathbb{R}^2 \to \mathbb{R}^2$ that projects a vector onto the line $y = 3x$ and $R : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects about the line $y = 3x$. Find the matrix representation of P and R in the standard basis $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.

3. A hypothesis is that change in the price of bread is a linear combination of wheat and change in price of the minimum wage, that is

$$B = \alpha W + \beta M$$

The following is change (In Rupees) in price of the bread, wheat and minimum wages for three consecutive years. Estimate the change in price of bread in Year 4 if wheat prices and minimum wage each fall by Rs 1.

	Year 1	Year 2	Year 3
B	+1	+1	+1
W	+1	+2	+0
M	+1	+0	-1