Newton's laws of motion

First law: Every object maintains its state of rest, or uniform motion along a straight line, unless impressed upon by an external force.

1/. Force does not maintain motion, but only causes change in motion.

2/. Absence of forces (not just a complete cancellation of forces) implies the need to define force as something that changes motion.

3/. Law of inertia implies a natural tendency to resist change in motion.

Second law: $\boxed{\vec{F} \propto m\vec{a}}$ $\vec{a} = \frac{d\vec{v}}{dt}$

$\therefore \boxed{\vec{F} = km\vec{a}}$ $k = 1$ in suitably chosen units

1/ For the same force acting on two objects of different masses $\boxed{|\vec{F}| = m_1|\vec{a}_1| = m_2|\vec{a}_2|} \Rightarrow \boxed{|\vec{a}| \propto \frac{1}{m}}$

Hence, with greater mass, the tendency to resist change in motion would be greater.

$\therefore$ Mass is a measure of inertia.

2/. $\boxed{\vec{F} = m\frac{d\vec{v}}{dt}}$ $\quad \vec{F}\cdot\vec{v} = m\,\vec{v}\cdot\frac{d\vec{v}}{dt}$

Now $\boxed{\vec{v} = \frac{d\vec{l}}{dt}}$ $\Rightarrow$ $\vec{F}\cdot\frac{d\vec{l}}{dt} = m\,\vec{v}\cdot\frac{d\vec{v}}{dt}$.

$\Rightarrow \vec{F}\cdot\frac{d\vec{l}}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) \Rightarrow \boxed{\vec{F}\cdot d\vec{l} = d\left(\frac{mv^2}{2}\right)}$

$\therefore$ Work done $= \vec{F}\cdot d\vec{l} =$ Change in kinetic energy.

In one-dimension, $\boxed{d\vec{l} \equiv dx\,\hat{x}}$ $\boxed{|\vec{F}(\vec{x})| \equiv F(x)}$

$\therefore \vec{F}\cdot d\vec{l} = F(x)\,dx = d\left(\frac{mv^2}{2}\right)$

$$\Rightarrow \boxed{\int_{v_1}^{v_2} d\left(\frac{mv^2}{2}\right) = \frac{m}{2}v_2^2 - \frac{mv_1^2}{2} = \int_{x_1}^{x_2} F(x)\,dx}$$

changes the speed

Work done is difference of kinetic energy

3/. Further, $\boxed{d\left(\frac{mv^2}{2}\right) - F(x)\,dx = 0}$

Write $\boxed{F(x) = -\frac{dU(x)}{dx}}$ $\quad$ $U(x) \rightarrow$ Potential function

$\therefore d\left(\frac{mv^2}{2}\right) + \frac{dU}{\cancel{dx}}\cancel{dx} = 0 \Rightarrow \int d\left(\frac{mv^2}{2}\right) + \int dU$ ~~= Constant~~ = Constant

$\Rightarrow \boxed{\frac{mv^2}{2} + U(x) = \mathcal{E}}$ Conservation of Energy.

P.T.O.

$\Rightarrow \boxed{\frac{1}{2}mv_1^2 + U(x_1) = \frac{1}{2}mv_2^2 + U(x_2)}$ When.

$\boxed{U(x_1) = U(x_2)}$, no change in kinetic energy occurs when there is no potential difference.

41. $\boxed{F = m\frac{d^2x}{dt^2}} \Rightarrow \boxed{\frac{dx}{dt} = v}$ and $\boxed{\frac{dv}{dt} = \frac{F}{m}}$

The integrals give $\boxed{x \equiv x(t)}$ and $\boxed{v \equiv v(t)}$.

At any initial time, two initial conditions are required, x (an initial position) and v (an initial velocity). The former specifies the state, and the latter the rate at which the state is changing. Hence, for a deterministic system, a second-order differential equation is required (with two conditions)

Third law: $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$. Particle 1 acts on Particle 2 (Action). Particle 2 reacts on Particle 2 (Reaction). Action and reaction are on different objects.

At the time of collision between two particles in an isolated system, $\boxed{m_1\frac{d\vec{v}_1}{dt} = -m_2\frac{d\vec{v}_2}{dt}}$.
(by the action-reaction principle)

P.T.O.

$$\Rightarrow \frac{d}{dt}(m_1\vec{v_1}) + \frac{d(m_2\vec{v_2})}{dt} = \frac{d}{dt}\left[m_1\vec{v_1} + m_2\vec{v_2}\right] = \vec{0}.$$

$$\therefore \boxed{m_1\vec{v_1} + m_2\vec{v_2} = \vec{P}} \rightarrow \underline{\text{Conservation of Momentum}}$$

Mass, ~~a~~ Energy and Momentum

i/. <u>Mass</u> → Emerges from the <u>first law</u> as a <u>measure of inertia</u>.

ii/. <u>Energy</u> → Emerges from the <u>second law</u> as a <u>conserved quantity</u>.

iii/. <u>Momentum</u> → Emerges from the <u>third law</u> as a <u>conserved quantity</u>.

Physical ~~[illegible]~~ Points of Newton's laws

1/. <u>Objects have inertia</u>. <u>Mass</u> is a <u>measure of the inertia</u>. <u>Force</u> is needed to <u>overcome inertia</u>.
(change in motion) ← inertia

2/. The <u>second law</u> quantifies <u>force</u>, and <u>makes it measurable in physical ways</u>.

3/. The <u>third law</u>, (~~[illegible]~~ following the <u>knowledge of force</u>), <u>relates force</u> to <u>interactions among objects</u>.