Velocity: Robot Kinematics in 2D

Date: 19.02.2021

$$\Rightarrow \boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$$

if there are no external force

$\Downarrow$

* Control theory

$\Downarrow$

* ML / AI

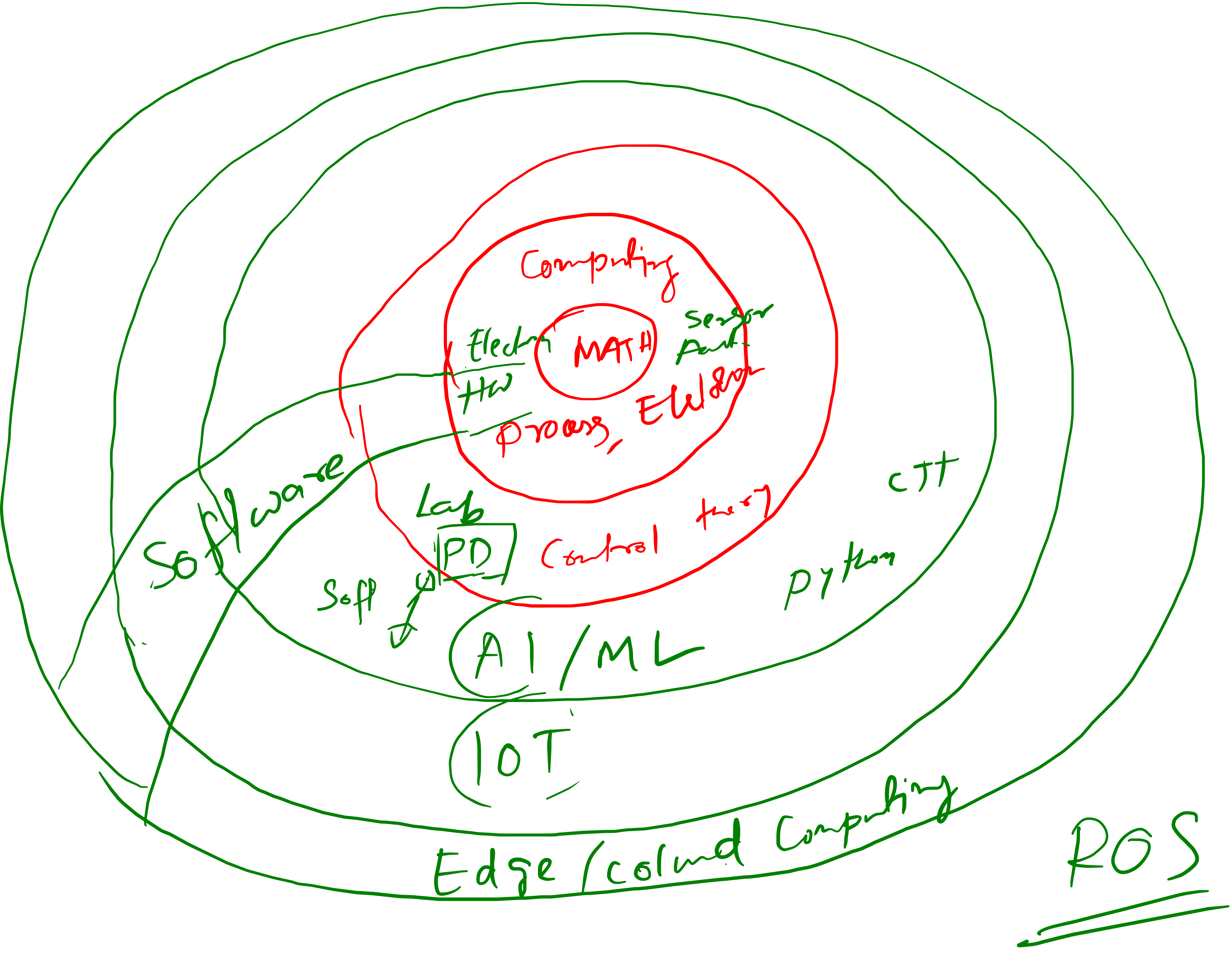
$\downarrow$

* Robotics IOT

$\Downarrow$

* Edge / cloud Computing

Position



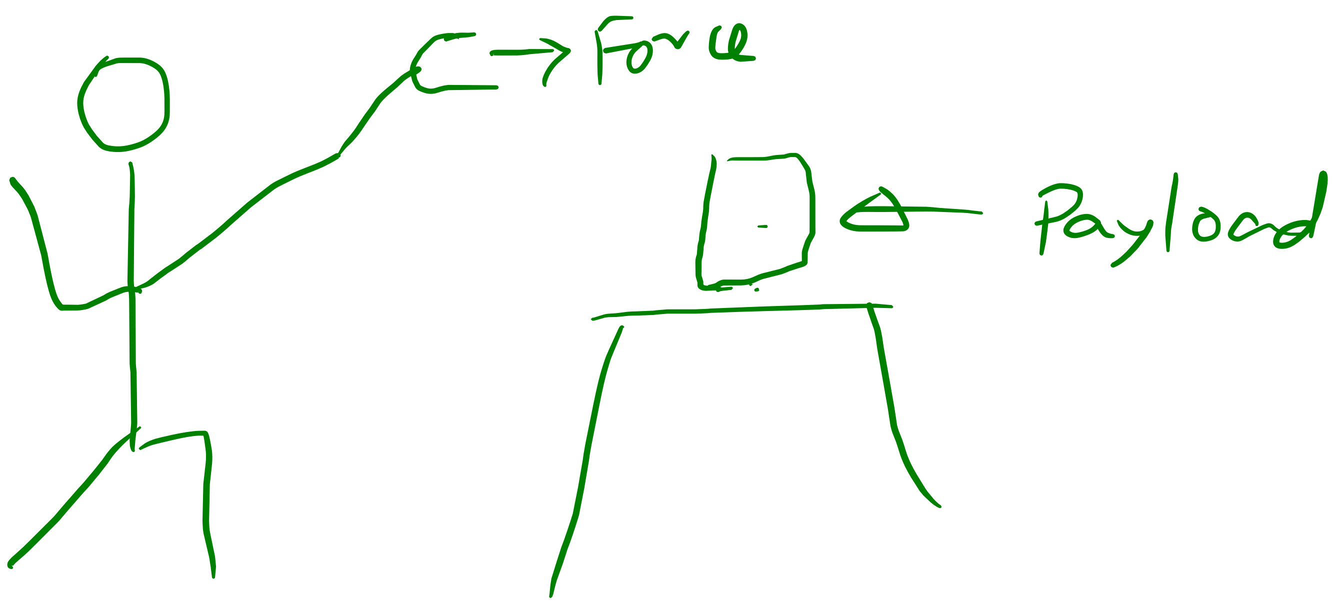
Computing
MATH
Electr
HW
Sensor
Process, EW
Lab
PD
Control theory
Soft
Software
C++
Python
AI/ML
IOT
Edge/cloud Computing
ROS

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$$

no external force

$\neq 0$ if there are external force

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext}}$$



FK: $p' = \psi p$

FK: Given $\theta \longrightarrow$ Get x

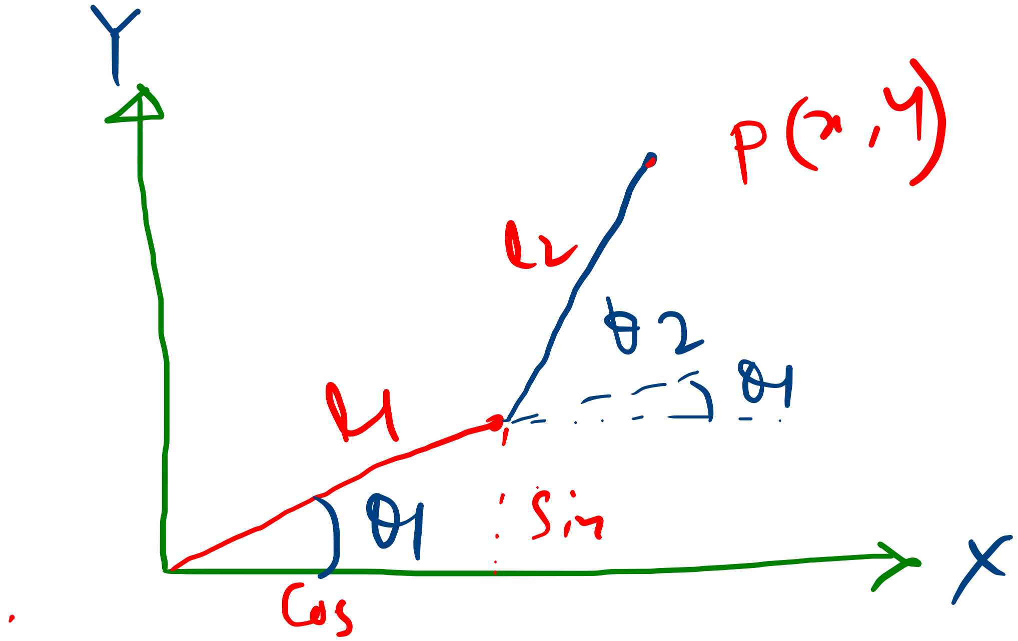
$\theta \to m$

$\to P$

IK: Given $x \longrightarrow \theta$

(1) $\Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \Rightarrow \underline{F_{ext}}$: Force

(2) $\Rightarrow I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + k\theta = 0 \Rightarrow \underline{\tau}_{ext} \Rightarrow$ Torque



$$\frac{d}{dt} = \frac{d}{d\theta}\frac{d\theta}{dt}$$

$$x = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2)$$

$$y = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2)$$

$$\frac{d\cos\theta}{dt} = \frac{d}{d\theta}\cos\theta\frac{d\theta}{dt}$$

$$= -\sin\theta\,\dot{\theta}$$

$$\dot{x} = \frac{dx}{dt} = -l_1\sin\theta_1\dot{\theta}_1 - l_2\sin(\theta_1+\theta_2)*(\dot{\theta}_1+\dot{\theta}_2)$$

$$\dot{y} = \frac{dy}{dt} = l_1\cos\theta_1\dot{\theta}_1 + l_2*\cos(\theta_1+\theta_2)*(\dot{\theta}_1+\dot{\theta}_2)$$

$$\frac{d\theta}{dt}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix}}_{\Psi(\theta)} \underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{\dot{\theta}}$$

$$v = \underline{J(\theta)}\, \dot{\theta}$$

$$\Rightarrow \quad v = \cancel{\Psi}(\theta)\, \dot{\theta} \quad \Rightarrow \text{FK} \rightarrow \text{in velocits}$$

$$\Downarrow J(\theta)$$

$$\underline{\underline{v}} = \boxed{J(\theta)}\,\underline{\underline{\dot{\phi}}} \Rightarrow \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = J(\theta) \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$\Downarrow$

Jacobian $\Rightarrow$ ~~Jacob~~ Jacobi

$\Downarrow$

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix} \quad \text{if } y = f(x)$$

$$\dot{\theta} = J^{-1} v$$

$$J^{-1} = \frac{1}{l_1 l_2 \sin\theta_2} \begin{bmatrix} l_2 \cos(\theta_1+\theta_2) & l_2 \sin(\theta_1+\theta_2) \\ -l_1 \cos\theta_1 - l_2 \cos(\theta_1+\theta_2) & -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$

$\sin\theta_2$

if $\theta_2 = 0$, $\sin\theta_2 = 0$, $J^{-1} = \text{infinity}$

$\boxed{\text{Singularity}}$

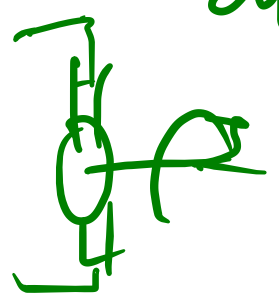
$$\Rightarrow \quad q_{k+1} = q_k + \Delta t \times \dot{\theta} \qquad = \frac{\Delta\theta}{\Delta t}$$

$\Downarrow$

Position.

$$\theta \longrightarrow x : \boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext} \text{ (Force)}}$$

$$x \Rightarrow \theta \Rightarrow m\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + k\theta = \tau_{ext} \text{ (Torque)}$$

 = Torque, rotation speed

Generalized Jacobian

* Next ⇒ Generalized J

* ⇒ J in 3D

⇒ $\boxed{v = \omega \times r}$

⇓

Torque