Signals and Systems (CT 203)

Tutorial Sheet-01 DA-IICT, Gandhinagar.

1. Represent following complex number in Z-plane for
   a) $z_1 = 1 + 2j,$
   b) $z_2 = 1 - 2j,$
   c) $z_3 = 2\cos\left(\frac{1}{2}\pi\right) + j2\sin\left(\frac{1}{2}\pi\right),$
   d) $z_4 = \sqrt{3}\cos\left(\frac{2\pi}{3}\right) + j\sqrt{3}\sin\left(\frac{2\pi}{3}\right).$

2. Multiplication by j to a vector z is geometrically a counterclockwise rotation of z through $\pi/2$ .Verify this by plotting z and jz and the angle of rotation for
   a) $z = -4 + 2j,$
   b) $z = 4 + j,$
   c) $z = 5 - 3j$

3. Using Taylor's series, prove Euler's relation and hence deduce
   a) $\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$
   b) $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

4. Express each of the following complex numbers in Cartesian form
   (a) $\frac{1}{2}e^{j\pi}$
   (b) $e^{j\pi/2}$
   (c) $e^{-j\pi/2}$
   (d) $e^{-j\pi}$

5. Express the following complex numbers in Polar form
   (a) $1 + j$
   (b) 5
   (c) $\frac{1}{2} - j\frac{\sqrt{3}}{2}$

6. If z is a complex number, i.e., $z = x + jy = re^{j\theta}$ and $z^* = x - jy = re^{-j\theta}$ (complex conjugate), then prove that
   (a) $zz^* = r^2$
   (b) $\frac{z}{z^*} = e^{j2\theta}$
   (c) $z + z^* = 2\,\mathrm{Re}\{z\}$

(d) $z - z^* = 2j\,\mathrm{Im}\{z\}$

(e) $|z| = |z^*|$

(f) $|z_1 z_2| = |z_1||z_2|$

7. Understanding of derivative of function is geometrically equivalent to find the slope of a line or gradient of a curve

$$\frac{dy}{dx} = \underset{h \to 0}{Lim} \frac{f(x+h) - f(x)}{h}$$

8. Understanding fundamental theorem of integral calculus, *viz.*, definite integral is nothing but area under a curve bounded by lines defined by limits of integral

$$\int_{x=a}^{x=b} f(x)\,dx = F(b) - F(a) = \underset{h \to 0}{Lim} \sum_{n=1}^{N} f(x_n)h$$ =Area under a curve bounded by lines x=a and x=b.

9. Derive sum of first *n* terms and *infinite* terms of geometric progression (G.P.).

10. Prove that for a complex signal $f(t)$

$$\int_{-\infty}^{+\infty} f^*(t)\,dt = \left(\int_{-\infty}^{+\infty} f(t)\,dt \right)^*$$

i.e., integration commutes with complex conjugation.

11. Prove that

$$\left| \int_{-\infty}^{+\infty} f(t)\,dt \right| \le \int_{-\infty}^{+\infty} |f(t)|\,dt$$

(you may interpret |.| as the magnitude operator for complex functions and as the absolute value operator for real functions).