Magnetostatics.

Certain material posses a physical property by which they attract iron. These materials, when they are brought close to each other, exert forces that can be attractive or repulsive depending upon their orientation. These ~~material~~ ~~are~~ forces are called magnetic forces and the materials which are sources of magnetic forces are called magnetic material.

Experiments with magnetic forces reveal that conducting wires carrying current experience a force in the vicinity of a magnetic material. These forces are proportional to the magnitude of the current through the wire. So when the current is zero there is no force on the wire. The force on the wire is also found to be perpendicular to the direction of the current in the wire. When the current is reversed, the force reverses its direction. ~~Since~~

Since a current is understood to be a flow of charges ~~through~~ a conductor, the forces exerted by the magnetic material appears to be influencing the charges. However since ~~this~~ force ceases when the current is zero, ~~these forces~~ it appears to be related to the velocity of the charge.

Magnetic Field.

Just like in Electrostatics, the cause of the magnetic force exerted on a current carrying wire or a moving charge is understood to be the presence of a magnetic field in the vicinity of a magnetic material. This field is denoted by ~~a~~ the vector $\vec{B}$.

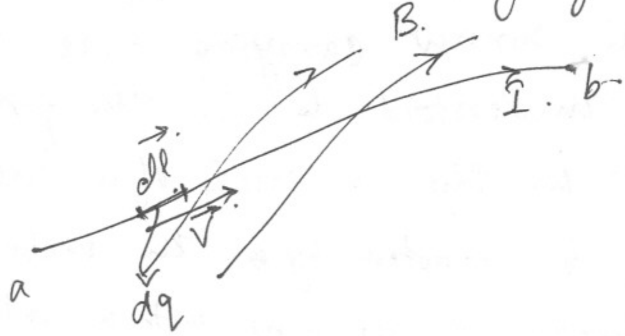
The force on a charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by the Lorentz' force law

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Not only does a current carrying wire experience a force ~~under a~~ in a magnetic field, but ~~the~~ a wire carrying a current produces a magnetic field around it. This is seen from the effect of a current carrying wires on other current carrying wires and other moving charges.

In fact every magnetic material or the sources of magnetic field is understood to be made of current carrying elements. So if we want to express $\vec{B}$ in terms of its sources we must be able to find out the magnetic field created due to a current carrying wire. This is given by the Biot-Savart Law, which, we will see later.

Force on a current carrying wire:



Consider a wire carrying a current I placed in a magnetic field. Let the charge per unit length in the wire be λ. If $\vec{v}$ is the velocity of the charges in the wire then

$$I = \lambda v$$

Consider an element $\vec{dl}$ of the wire. Then the amount of charge in this element is

$$dq = \lambda (dl)$$

The magnetic force on this elements is

$$\vec{dF} = dq(\vec{v} \times \vec{B}) = \lambda \, dl (\vec{v} \times \vec{B})$$

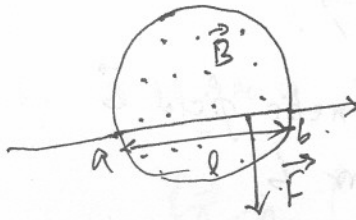
Since in a wire $\vec{v}$ and $\vec{dl}$ are in the same direction we have

$$\vec{dF} = \lambda v (\vec{dl} \times \vec{B}) = I \, \vec{dl} \times \vec{B}$$

In magnetostatics we will be dealing with currents which is same through out the wire. These are called steady currents. So in this situations I is same throughout the wire and we have the total force

$$\vec{F} = I \int_a^b \vec{dl} \times \vec{B}$$

Eg: Let us calculate the force on a straight wire, a length l of which lies perpendicular to a uniform magnetic field $\vec{B}$. The current in the wire is I.



The magnetic field $\vec{B}$ is perpendicular to the plane of the paper, coming out. The force on the wire is

$$\vec{F} = I\int_a^b \vec{dl} \times \vec{B}.$$

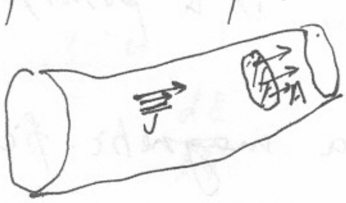
Since $\vec{B}$ is constant through the length l of the wire.

$$\vec{F} = I\left(\int_a^b \vec{dl}\right) \times \vec{B} = I\,\vec{l} \times \vec{B}$$

Since $\vec{l}$ is perpendicular to $\vec{B}$, we get

$$F = BIl \quad \text{downward}.$$

<u>Current density</u>: In general the current carrying conductor may not be in the form of a wire with ~~is~~ infinitesimal width. In such a case we defined a vector called current density at every point in the ~~g~~ material.



Once $\vec{J}$ is given, the current through any surface A is given as the surface integral.

$$I_A = \int_A \vec{J}\cdot\hat{n}\,da.$$

So the current flowing through the area A is the flux of the current density $\vec{J}$ through A.

If ρ is the density of the moving charges in the conductor and $\vec{v}$ is the velocity of the charges at a point.

then the current density $\vec{J}$ at the point is given as.

$$\vec{J} = \rho \vec{v}$$

This is ~~be~~ because the amount of charge passing an unit area of cross section per unit time is $\rho \vec{v}$.

If this conductor is kept in a magnetic field $\vec{B}$ then the total force on the conductor is

$$\vec{F} = \int_V dq (\vec{v} \times \vec{B})$$

$$= \int_V (\rho \, d\tau)(\vec{v} \times \vec{B}) = \int_V (\vec{J} \times \vec{B}) \, d\tau$$

where V is the volume of the conductor in the magnetic field.

Often, the current is not distributed over the body of the conductor but only over the surface. In such cases we can define a surface current density $\vec{K}$.

So if σ is the surface density of the moving charges in a conductor and $\vec{v}$ is the velocity at a point, then

$$\vec{K} = \sigma \vec{v}$$

The force on the surface ~~in~~ a magnetic field $\vec{B}$ is

$$\vec{F} = \int_S \vec{K} \times \vec{B} \, da$$

Note: $\vec{J}$ is current per unit cross sectional area.

$\vec{K}$ is current per unit ~~cross~~ sectional length.

Steady Current.

Consider a volume V enclosed by a surface S. Let $\vec{J}$ be the current density at every point in this volume.

By divergence theorem we have

$$\cancel{\oint \vec{J}\cdot\hat{n}} \int_V \vec{\nabla}\cdot\vec{J}\, d\tau = \oint_S \vec{J}\cdot\hat{n}\, da.$$

The surface integral on the right hand side is the total current flowing across the surface S. If Q is the amount of charge enclosed by the surface S at a time t then

$$\oint_S \vec{J}\cdot\hat{n}\, da = -\frac{dQ}{dt}$$

This equation means that the total current is the amount of charge flowing across the surface S per unit time. The negative sign is put because if the flux is positive i.e. the ~~flux~~ current flows outward, then the total charge Q within the surface decreases.

If ρ is the charge density then

$$\frac{dQ}{dt} = \frac{d}{dt}\left(\int_V \rho\, d\tau\right) = \int_V \frac{\partial \rho}{\partial t}\, d\tau$$

$$\therefore \oint_S \vec{J}\cdot\hat{n}\, da = -\int_V \frac{\partial \rho}{\partial t}\, d\tau.$$

$$\therefore \int_V \vec{\nabla}\cdot\vec{J}\, d\tau = -\int_V \frac{\partial \rho}{\partial t}\, d\tau.$$

Since this is true for any arbitrary volume V we have

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

This Equation is called the 'continuity Equation.' It relates the divergence of the current density at a point to the rate of change of charge density at the point.

If the flow of charges in a region is such that ρ doesn't change with time, then we will have

$$\vec{\nabla} \cdot \vec{J} = 0 .$$

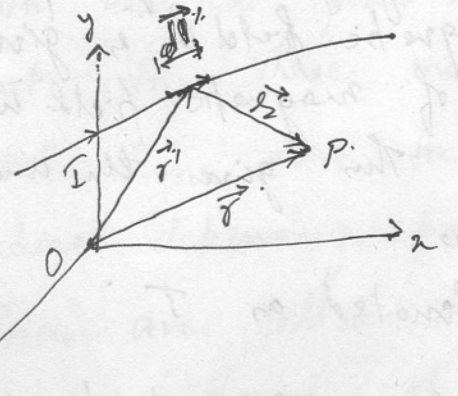
A current distribution which satisfies this condition everywhere is called a steady current distribution. It says that the total flux of the current entering a volume is same as the total flux exiting the volume. In such a situation there is no accumulation of charge or decay of charge at any point.

This is the condition under which Kirchhoff's law is applicable.

Magnetostatics is the study of magnetic fields created by steady currents.

In Electrostatic we start with the electrostatic field of a point charge at rest. In magnetostatic we start with the magnetostatic field of a wire carrying a steady current I. Since the current is steady there is no accumulation of charge at any point in the wire. So the current I is same throughout the wire. For such a case the magnetostatic field at a point is given by the 'Bio-Savart Law'.

Bio-Savart Law :



Consider an element of a wire of length dl' at the location $\vec{r}'$. The current through the element is I. The magnetic field due to this small element of current at a point P. is given as.

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{\vec{dl}' \times \hat{\imath}}{\imath^2}$$

where $\vec{\imath} = \vec{r} - \vec{r}'$

~~The.~~ This is equivalent to the Coulomb's law which gives us the electric field at a point due to a point charge at $\vec{r}'$.

Since we cannot have a point current, the basic form of Biot-Savart Law have to be for a ~~w~~ wire which is obtained by integrating ~~the~~ over the current elements along the wire. So the ~~m~~ magnetic field at the point $P(\vec{r})$ is

$$B(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl}' \times \hat{\imath}}{\imath^2}$$

μ_0 is a fundamental constant like ϵ_0 in electrostatics. It is called the permeability of free space. If the current is measured in Amperes A = Coulombs / sec. then

$$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

Since the force on a current carrying wire of length l placed perpendicular to a magnetic field $\vec{B}$ is given by $F = BIl$, the unit of magnetic field in the standard units is N/Am. This gives the unit of μ_0 in the Bio-Savart law.

1 N/Am is called Tesla denoted as T.

In C.G.S. units the Bio-Savart law is given as

$$\vec{B} = \frac{1}{c} I \int \frac{\vec{dl}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}.$$

where c is the velocity of light in vacuum which is 3×10^{10} cm/s.

In this ~~unit~~ system the unit of $\vec{B}$ is Gauss.

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

For a conductor carrying a volume current density $\vec{J}(\vec{r})$ we have

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'$$

For a specified surface current density $\vec{K}(\vec{r})$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_S \frac{\vec{K}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, da'$$