Tutorial 9

SC-220 Groups and linear algebra Autumn 2019
(Change of Basis, System of linear equations)

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(1) Show that the vectors $v_1 = (1,1,0,0), v_2 = (0,0,1,1), v_3 = (1,0,0,4), v_4 = (0,0,0,2)$ form a basis of $\mathbb{R}^4$. Find the coordinates of the vector $v = (1,3,-1,2)$ in this basis.

(2) Let T be a linear operator defined on $\mathbb{R}^3$ defined by
$T(x_1,x_2,x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$
a) What is matrix of T in the stadard basis of $\mathbb{R}^3$
b)What is matrix of T relative to the basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1)$
c) Find matrix for T^{-1} in both these bases.

(3) Let $\mathcal{B} = \{e_1, e_2\}$ where $e_1 = (1,0)$ and $e_2 = (0,1)$ be the standard basis in $\mathbb{R}^2$. Find the matrix representation of the operator $P_\theta : \mathbb{R}^2 \to R^2$ in basis $\mathcal{B}$ which is projection onto the line which is at an angle θ with respect to the x-axis. Do the same for $R_\theta : \mathbb{R}^2 \to R^2$ which is refection about this line. (You can assume that $0 < \theta < \frac{\pi}{2}$)

(4) Check if the following system of equations has a solution. If yes find the solution(s)

$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 2 \\ x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 + x_3 &= -3 \\ 2x_1 + 4x_2 &= 8 \end{aligned}$$

(5) Check if the following system of equations has a solution. If yes find the solution(s)

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + 3x_4 &= 4 \\ 2x_1 + 4x_2 + x_3 + 3x_4 &= 5 \\ 3x_1 + 6x_2 + x_3 + 4x_4 &= 7 \end{aligned}$$

(6) Let T be a linear operator on $\mathbb{R}^3$, the matrix of T in the standard basis is given by

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$

Find a basis for the range of T and the null space of T.