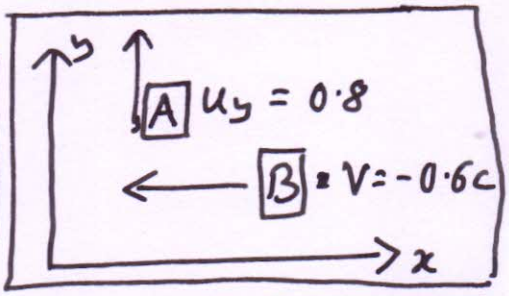
Relativity Problems - 12 - Selected Solutions

Relativity - 3

Q8/. $u_x = \dfrac{u_x' + V}{1 + u_x' V/c^2}$ $\quad u_y = \dfrac{u_y'}{\gamma\left(1 + \frac{V}{c^2}u_x'\right)}$



$\Rightarrow u_x' = \dfrac{u_x - V}{1 - u_x V/c^2}$ $\quad u_y' = \dfrac{u_y}{\gamma\left(1 - \frac{V}{c^2}u_x\right)}$

$\beta = -0.6$ $\quad V = -0.6c$

$\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1.25$

$u_x = 0$, $u_y = 0.8c$ $\Rightarrow$ No x-component for A.

$\therefore u_x' = \dfrac{+0.6c}{1+0} = 0.6c$ $\quad u_y' = \dfrac{0.8c}{1.25(1+0)} = 0.64c$

Magnitude of velocity (speed): $u' = \sqrt{u_x'^2 + u_y'^2} = \sqrt{0.64^2 + 0.6^2}\,c$

$\Rightarrow u' = 0.88c$ $\quad$ direction, $\theta' = \tan^{-1}\left(\dfrac{0.64}{0.6}\right) = \tan^{-1}\left(\dfrac{u_y'}{u_x'}\right)$

Answer $\rightarrow \Rightarrow \theta' = 46.85°$

Q12/. Joe sees a contracted length from his own frame (the rocket). $\beta = 0.9$ $\Rightarrow \gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 2.29$

Contracted length (round trip), $l = \dfrac{l_0}{\gamma} = \dfrac{4 \text{ years} \times 1c}{2.29} = 1.74$ light years

Total distance travelled is (for round trip) $2 \times l = 2 \times 1.74$ light years

Time taken, is to (proper time)

$= \dfrac{2 \times 1.74 \text{ years} \times 1c}{0.9c}$

$= \dfrac{2 \times 1.74 \text{ years}}{0.9} = 3.9$ years

Joe sees proper time but contracted length. Sister sees dilated time $\Rightarrow t = \gamma t_0$

$\therefore t = 2.29 \times 3.9 \text{ years} = 8.9 \text{ years}$

Age difference $= 8.9 \text{ yrs} - 3.9 \text{ years}$

Answer $\rightarrow = 5$ years