

Assignment No.2

1. Show that $\log_b f(n)$ is $\Theta(\log_2 f(n))$ if $b > 1$ is a constant.
2. Prove that if $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$ then:
 - $d(n) + e(n)$ is $O(f(n) + g(n))$
 - $d(n).e(n)$ is $O(f(n).g(n))$
3. Which of the two functions $y = e^x$ and $y = (100).2^x$ grows faster and why.
4. Order the following functions on the basis of their growth rate:
 $\log \log n, \log n, \log^2 n, \sqrt{n}, n, n.\log n, n^2, n^3, 2^n, e^n, 3^n$
5. Show that $\sqrt{x^2 + 5}$ and $(2\sqrt{x} - 1)^2$ grow at the same rate.
6. Show that $\lceil f(n) \rceil$ is $O(f(n))$ if $f(n)$ is always greater than 1.
7. Prove without using limits that $f(n) = 12n^2 + 6n$ is $o(n^3)$ and $\omega(n)$.
8. Show that the summation $\sum_{i=1}^n \lceil \log_2 i \rceil$ is $O(n \log_2 n)$.
9. Show that the summation $\sum_{i=1}^n \lceil \log_2 i \rceil$ is $\Omega(n \log_2 n)$.
10. Algorithm A uses $10n \log_2 n$ operations, while algorithm B uses n^2 operations. Determine the value n_0 (as small as you can without using a calculator) such that A is better than B for $n \geq n_0$.