Special Theory of Relativity

Relativity of Measurements

All measurements are made with respect to a reference value; (Origin of coordinates) - (RELATIVE).

The Theory of Relativity is the study of the consequences of the relativity of measurements.

James Clerk Maxwell

Albert Michelson - Edward Morley

Hendrik Lorentz

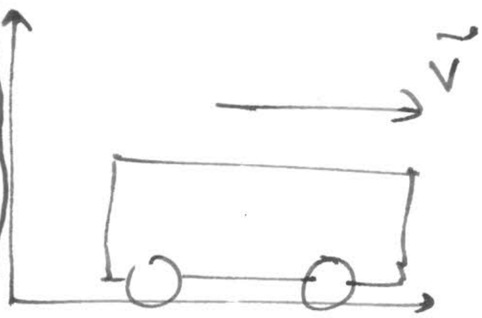
Albert Einstein

Special Theory of Relativity

pertains to setting reference points (origin of spatial coordinates, starting of time etc.) with respect to unaccelerated frames. Newton's laws are invariant in such frames. (inertial frames)

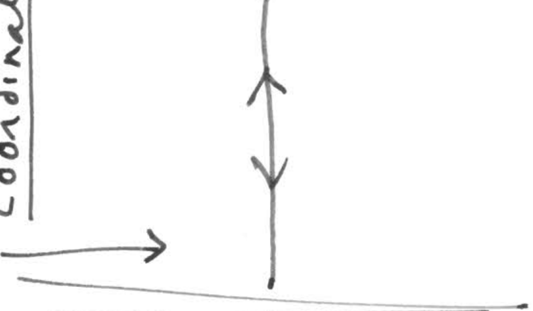
Moving reference frames

Tossing a coin in a train moving with constant velocity, $\vec{v}$.



Two events occur at the same spatial coordinate

Inside train



Proper frame

Outside the train

Two events occur at separate spatial coordinates.

Classical Velocity Addition

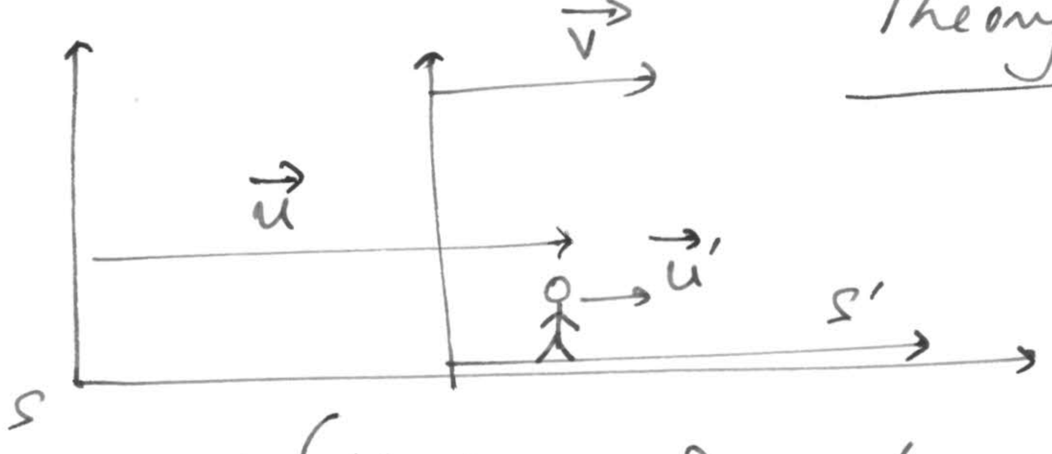
1/. Train moves with velocity $\vec{v}$.

2/. Subject inside the train moves with velocity $\vec{u}'$ with respect to the train. (Prime is associated with the moving frame).

3/. Velocity $\vec{u}$ with respect to the ground is,

$$\boxed{\vec{u} = \vec{v} + \vec{u}'}$$

(Does NOT hold in Special Theory of Relativity).



(Unprimed value is associated with the static frame).

Invariance of Newton's Laws

(Law of Inertia)

First law: No external force $\Rightarrow$ Constant velocity of the object.

$\vec{u}'$ is a constant velocity.

If $\vec{v}$ is also a constant velocity, then

$\boxed{\vec{u} \text{ is constant}}$ because

$$\boxed{\vec{u} = \vec{u}' + \vec{v}}.$$

Newton's first law is invariant in the two reference frames. (Inertial frames)

<u>Second law</u>: $\boxed{\vec{F} = m\vec{a}}$

in one <u>reference frame</u> S,

and $\boxed{\vec{F}' = m'\vec{a}'}$ in

another <u>reference frame</u> S'.

<u>Now</u>:

1/. $\boxed{m = m'}$ (<u>Experimental fact</u>).

2/. $\boxed{t = t'}$ (Assumption that <u>time is universal</u>, and is the <u>same for all observers</u>, who have started observing at the <u>same instant</u>).

<u>In S</u>, $\vec{F} = m\vec{a}$

$$\Rightarrow \boxed{\vec{F} = m\frac{d\vec{u}}{dt}}$$

In the reference frame s';

$$\boxed{\vec{F}' = m'\vec{a}' = m'\frac{d\vec{u}'}{dt'}}$$

But $\boxed{m = m'}$ and $\boxed{t = t'}$.

$$\therefore \boxed{\vec{F}' = m\frac{d\vec{u}'}{dt}} \quad \left(\text{since } \frac{d}{dt} \equiv \frac{d}{dt'}\right)$$

Now $\boxed{u' = \vec{u} - \vec{v}}$

$$\therefore \vec{F}' = m\frac{d\vec{u}}{dt} - m\frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{F}' = \vec{F} - m\frac{d\vec{v}}{dt}$$

If $\vec{v}$ is constant, $\boxed{\frac{d\vec{v}}{dt} = \vec{0}}$.

$$\Rightarrow \boxed{\vec{F} = \vec{F}'}$$

$$\boxed{\text{Newton's second law is invariant in a non-accelerating frame (inertial frame).}}$$

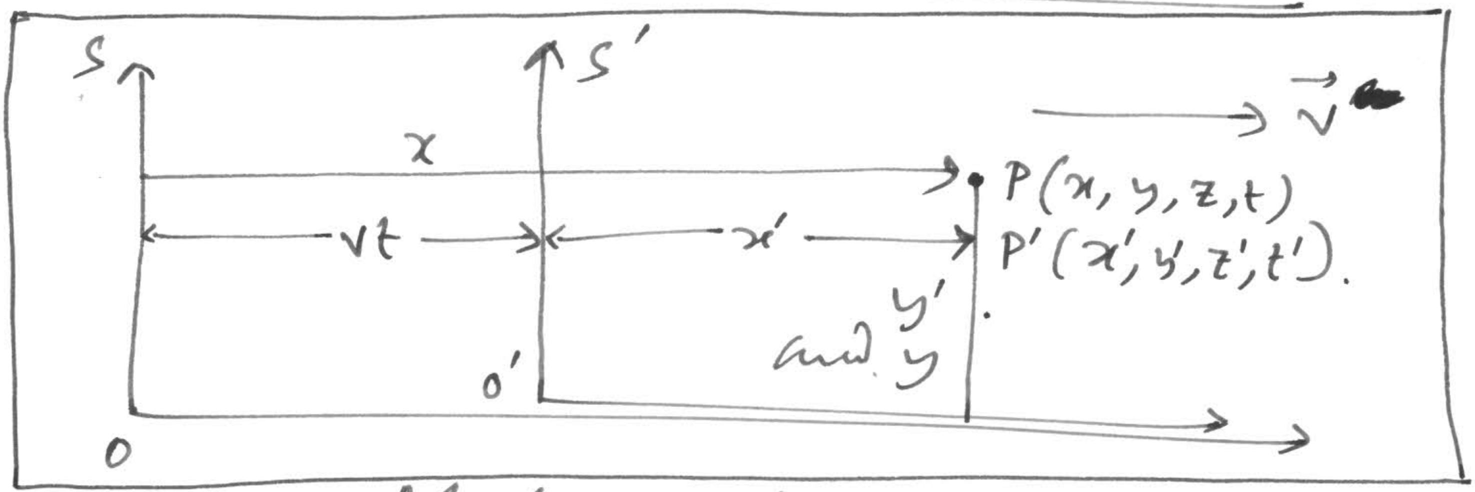
Third law:

$$\boxed{\text{Action force} = -\text{Reaction force}}$$

Force is invariant (has the same value) in both S and S'. Reaction (a force) will also be the same in both S and S' (unaccelerated frames).

$$\boxed{\begin{array}{l}\text{Newton's laws are invariant}\\ \text{and true in all reference}\\ \text{frames which are not}\\ \text{accelerating (inertial frames).}\end{array}}$$

Galilean Transformation



Motion along x axis only

Measure position and time of an event at P (or P').

$$\boxed{t = t'}$$

$$\boxed{\begin{array}{c} x = x' + vt \\ (\text{or } x = x' + vt') \end{array}}, \quad \boxed{y = y'}, \quad \boxed{z = z'}.$$

Inverse transform, $\boxed{x' = x - vt}$.

METHOD : 1/. Exchange primes

2/. $v \longrightarrow -v$.

Inverse transforms

Also, $\boxed{t' = t}$, $\boxed{y' = y}$ and $\boxed{z' = z}$

Classical Velocity Addition from Galilean Transformation

$$\boxed{\begin{aligned} x' &= x - vt \\ x &= x' + vt \end{aligned}} \quad \text{OR} \quad \left(\underline{x = x' + vt'} \right).$$

$$\frac{dx}{dt} = \frac{dx'}{dt} + v$$

But $\boxed{t = t'}$

$$\Rightarrow \quad \frac{dx'}{dt} = \frac{dx'}{dt'}$$

<u>Define</u> $\boxed{u = \frac{dx}{dt}}$

and $\boxed{u' = \frac{dx'}{dt'}}$

$$\Rightarrow \quad \boxed{u = u' + V}$$

<u>Classical Velocity Addition:</u>

<u>Consequence of Galilean Transformation</u>

Speed of Light :

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \ \mathrm{ms^{-1}} \ \text{(in vacuum).}}$$

Solution of the wave equation.

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \vec{\psi}}{\partial t^2} = \frac{\partial^2 \vec{\psi}}{\partial x^2} \equiv \nabla^2 \vec{\psi}}$$

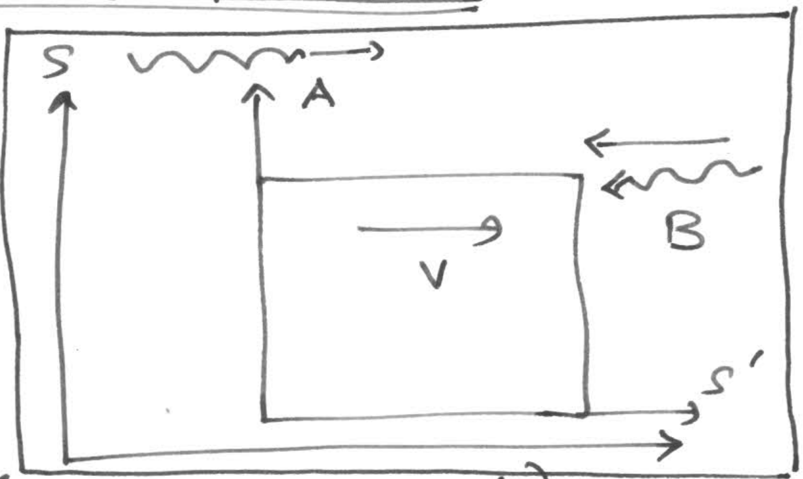
$[\vec{\psi} \equiv \vec{\mathcal{E}}, \vec{B}]$ 1-D / $\vec{\psi}(x)$ 3-D / $\vec{\psi}(x,y,z)$

> In all inertial frames, the speed of light in vacuum remains the same (SAME!)
>
> — Universal constant.
>
> (Einstein)

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \vec{\psi}}{\partial t^2} = \frac{\partial^2 \vec{\psi}}{\partial x^2} \equiv \nabla^2 \vec{\psi}}$$ (most generally).

Relative velocity of light

With respect to frame S, light travels with speed c.



For beam A, (from $u = v + u'$)

Speed: $C = V + u' \Rightarrow \boxed{u' = C - V}$

For beam B, speed: $-C = V + u'$

$\Rightarrow u' = -(C + V)$

$\boxed{u' = -(C+V)}$

Light has a fixed speed c in only one frame of reference, in all directions.

The Aether Frame (fills all space)

(Hypothetical substance)

Light propagates because of the vibrations of aether.

Earth orbits the Sun

$\boxed{v \sim 3 \times 10^{4}\ \mathrm{m\,s^{-1}}}$ ($30\ \mathrm{km\,s^{-1}}$)

Observed speed of light should vary between

$c - v$ and $c + v$.

Since $\boxed{c = 3 \times 10^{8}\ \mathrm{m\,s^{-1}}}$

$\boxed{\frac{v}{c} \sim 10^{-4}}$

Very small effect.

Use a Michelson Interferometer

(Relies on the phenomenon of interference in wave optics).