Tutorial 6
SC-220 Groups and Linear algebra Autumn 2019
(Vector Spaces and subspaces)

(1) Let V be a set of real sequences $(a_1, a_2, ..., a_n, ...)$ such that $\sum_i a_i^2$ is finite. Prove that V is a vector space over $\mathbb{R}$.

(2) Show that the space $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | x_1 + 2x_2 + 2x_3 = 0\}$ forms a vector space.

(3) Let W_1 and W_2 be two subspaces of of a vector space V .
(a) Prove that $W_1 \cap W_2$ is a subspace of V .
(b) Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

(4) Let V be the vector space of all functions f from $\mathbb{R}$ into $\mathbb{R}$. Which of the following sets of functions are subspaces of V ?
(a) all f such that $f(x^2) = f(x)^2$
(b) all f such that $f(0) = f(1)$
(c) all f such that $f(3) = 1 + f(-5)$
(d) all f such that $f(-1) = 0$;
(e) all f which are continuous.

(5) Let V be the vector space of all $n \times n$ matrices over $\mathbb{C}$. Which of the following matrices A in V are subspaces of V
(a) all invertible A
(b) all A such that $AB = BA$, where B is some fixed matrix
(c) all A such that $A^2 = A$

(6) Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$. Show that the space of even functions and the set of odd functions are subspaces of V.