BJT Internal Capacitances & HIGH FREQUENCY MODEL

The physical flow of carriers inside a BJT get stored at p-n junctions and this in some way affect their instantaneous response to change in applied field. Such behaviour can be modelled by representing that by an equivalent capacitance (charge storage phenomenon) that would affect high frequency behaviour of BJT as an amplifier.

BASE CHARGING or Diffusion Capacitance C_{de}

When BJT is in active or saturation region, minority carrier charges are stored in base region. The charge Q_n in terms of collector current i_C

$$Q_n = \frac{W^2 i_C}{2 D_n} = \tau_F \cdot i_C$$

where τ_F = device constant = $W^2/2 D_n$

W = effective width of Base

D_n = electron diffusivity in Base

τ_F = FORWARD BASE TRANSIT TIME of the order of 10 pico s to 100 pico seconds.

Since i_C is related to v_{BE} exponentially Q_n also depends upon v_{BE} nonlinearly. This mechanism can be represented by a SMALL SIGNAL DIFFUSION CAPACITANCE C_{de}

$$C_{de} = \frac{d\,Q_n}{d v_{BE}} = \tau_F \frac{di_C}{dv_{BE}} = \tau_F \cdot g_m$$

$$= \tau_F \cdot \frac{I_C}{V_T}$$

Thus C_{de} increases with I_C or Bias current.

—— × ——

BASE EMITTER JN CAPACITANCE

B-E jn. depletion layer capacitance C_{je} is given by

$$C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{V_{oe}}\right)^m}$$

C_{jeo} ← value of C_{je} at 0V

V_{oe} = EBJ built-in Voltage ≈ 0.9 V

m = grading coeff. ≈ 0.5

V_{BE} = Forward junction DC voltage or bias voltage.

Approx. $C_{je} \approx 2\,C_{jeo}$

Collector Base Jn. Capacitance C_μ

$\because$ CB is reversed biased, its junction has a capacitance like effect modelled as

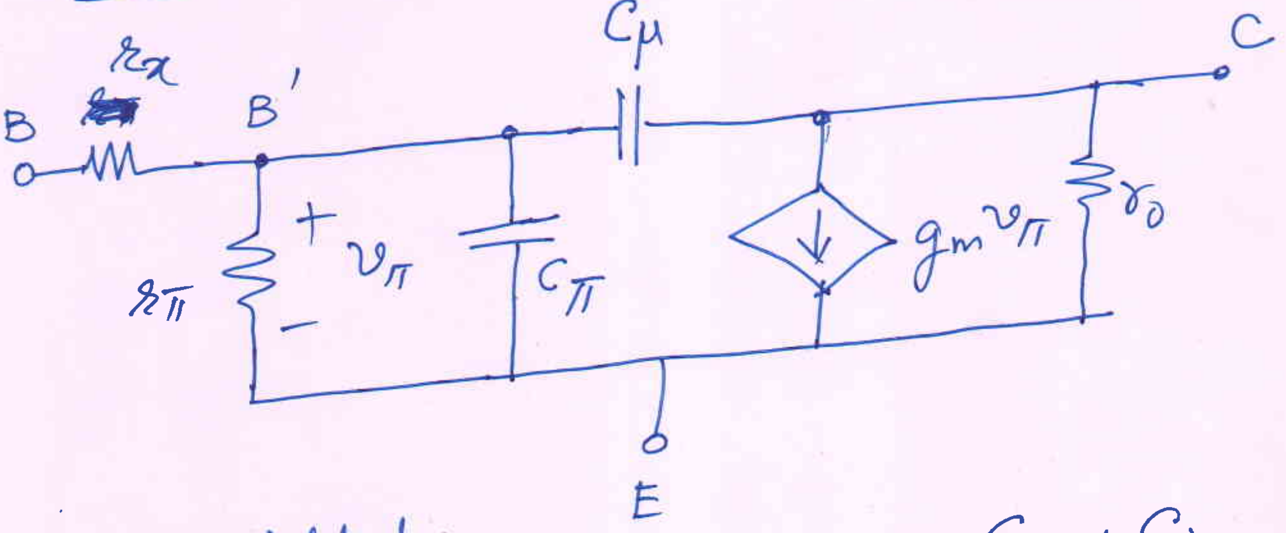
$$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_{oc}}\right)^m}$$

$C_{\mu o} \longrightarrow C_\mu$ at 0V

$V_{oc} \longrightarrow$ CBJ built-in Voltage $\simeq 0.75\,V$

$m \approx 0.2$ to $0.3V$.

HIGH-FREQUENCY HYBRID-π MODEL.



Newly Added:

C_π = emitter base capacitance $= C_{de} + C_{je}$

C_μ = Collector-base capacitance

r_x = small value to model resistance of silicon material $\approx$ few tens of ohms.

At low frequencies $r_x + r_\pi \approx r_\pi$ therefore it was never considered or brought in discussion.

At high freq. C_π will bypass r_π & r_x will matter.

Typically $C\pi$ = few tens of Picofarads

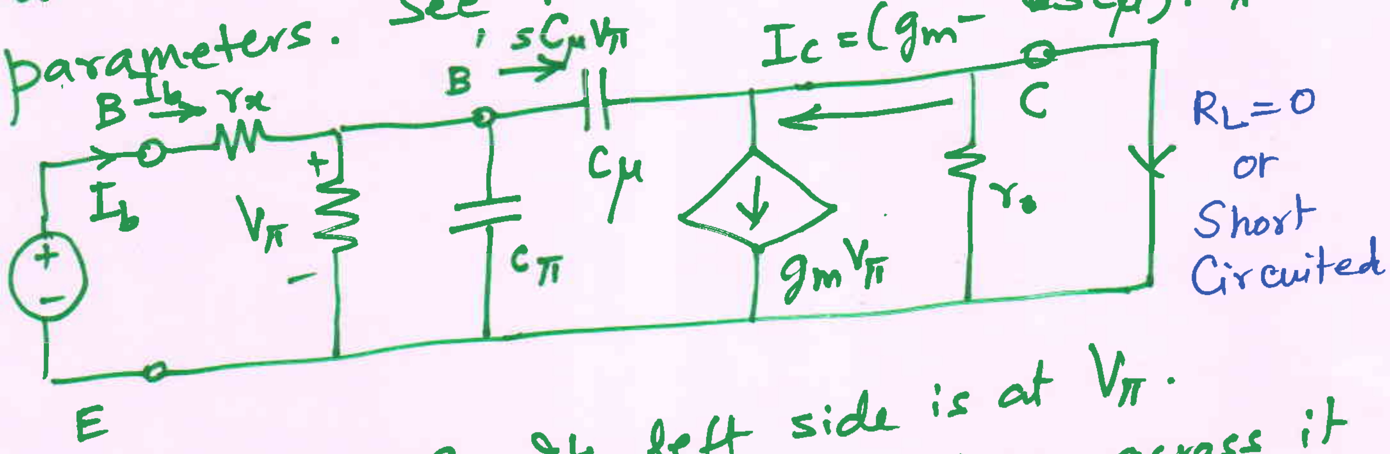
$C\mu$ = fraction to few Picofarads

r_x = tens of ohms.

Voltages and currents described now onwards will be function of frequency hence we will use uppercase letters with lowercase subscripts like V_π or I_c.

Expression of Upper Cutoff Frequency

Derive an expression for short circuit current gain β (also called as hfe) or CE S.C.C. Gain as a function of frequency, in terms of model parameters. See model below with $R_L = 0$. S.C.



See capacitor $C\mu$. Its left side is at V_π. Its right side is at GND ∴ voltage across it is V_π. ∴ Current through $C\mu = \frac{V_\pi}{X_{C\mu}} = \frac{V_\pi}{1/C_\mu s}$

$= C_\mu s . V_\pi$

∵ r_o is bypassed by a short circuit, it does not matter at all in calculations

Now look at currents in Node C : (5)

coll. current $I_c = g_m V_\pi - sC_\mu V_\pi$.

$= (g_m - sC_\mu) V_\pi$

V_π is voltage across r_π due to I_b.

$$V_\pi = I_b (r_\pi \,//\, C_\pi \,//\, C_\mu)$$

$\therefore$ voltage across C_μ is also V_π.

$$V_\pi = \frac{I_b}{\frac{1}{r_\pi} + sC_\pi + sC_\mu}$$

Calculate h_{fe}

$$h_{fe} = \frac{I_c}{I_b} = \frac{(g_m - sC_\mu) \cdot V_\pi}{\left(\frac{1}{r_\pi} + sC_\pi + sC_\mu\right) \cdot V_\pi}$$

Assuming that at high frequencies $g_m >> \omega C_\mu$ & we neglect sC_μ in numerator.

$$h_{fe} \simeq \frac{g_m \cdot r_\pi}{1 + s(C_\pi + C_\mu) r_\pi}$$

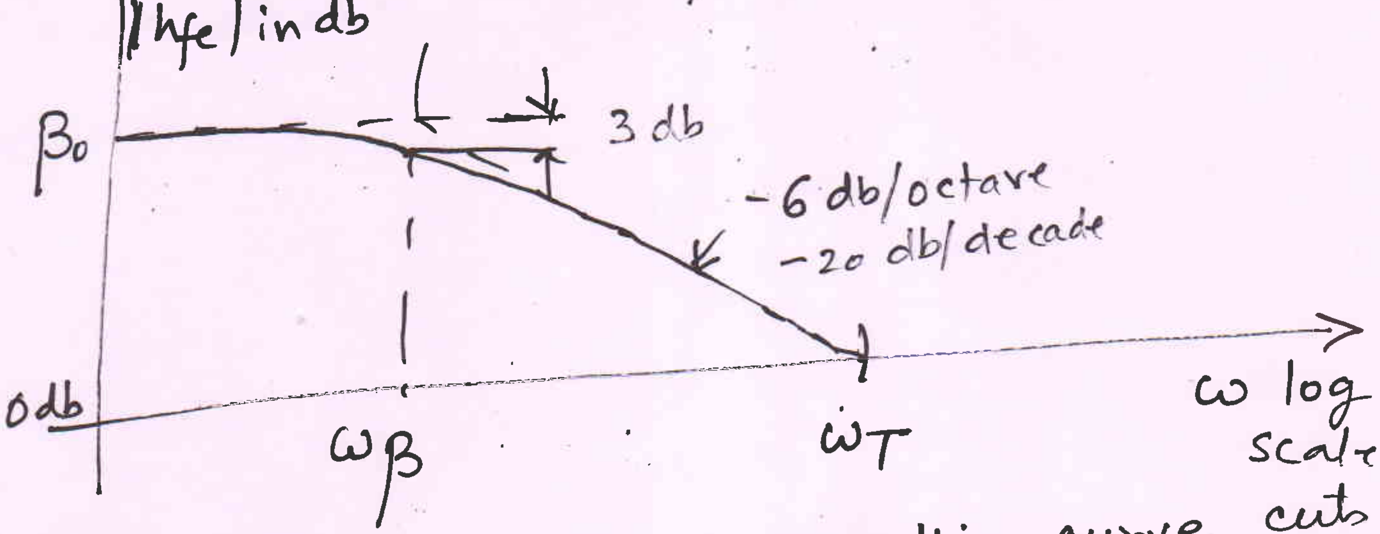
$$= \frac{\beta_0}{1 + s(C_\pi + C_\mu) r_\pi}$$

where β_0 = low frequency β.
$= g_m \cdot r_\pi$

Now, we can see this expression as an STC function at a 3-db or corner freq. $\omega = \omega_\beta$ where

$$\omega_\beta = \frac{1}{(C_\pi + C_\mu)\, r_\pi}$$

See Bode Plot



The frequency at which this curve cuts 0-db line is called frequency where $|h_{fe}| = 1$ or UNITY GAIN BANDWIDTH ω_T

such that $\omega_T = (\beta_0 \cdot \omega_\beta)$

Thus $$\omega_T = \frac{\beta_0}{(C_\pi + C_\mu)\, r_\pi} = \frac{g_m}{(C_\pi + C_\mu)}$$

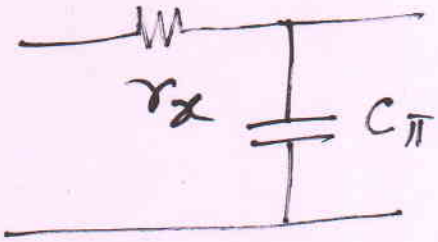
$$\therefore \boxed{f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_\pi + C_\mu)}}$$

f_T is usually specified in BJT datasheets.
It is in MHz to GHz range.

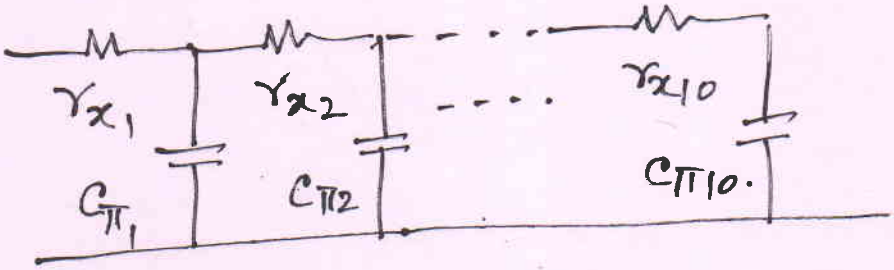
From f_T, we can get C_π & C_μ.
We can measure practically C_μ between B & C
and get a measure of C_π.

This STC model we discussed, works well upto $0.2 f_T$. At higher frequencies, we must use distributed R-C model for better estimates:

lumped model.



distributed model. for say 10 taps...



Numerical on C_μ C_π and f_T.

3.149 Given npn BJT $I_C = 0.5\,mA$

$V_{CB} = 2V$; $\beta_0 = 100$; $V_A = 50V$

$\tau_p = 30\,ps$; $C_{jeo} = 20$ femto farad $= 20\,fF = .02\,pF$

$C_{\mu o} = \cancel{20}\ 30$ femto F;

$V_{0c} = 0.75V$, $m_{CBJ} = 0.5$; $r_x = 100\,\Omega$

Draw hybrid-π model & find f_T.

— x —

$r_x = 100\,\Omega$; $g_m = I_C / V_T = 0.5\,mA / 25\,mV = 20\,m\mho$

$r_\pi = \beta_0 / g_m = 100 / 20\,m\mho = 5\,K\Omega$

$r_o = V_A / I_C = 50V / 0.5\,mA = 100\,K\Omega$

$$C_\mu = C_{\mu o} \Big/ \left(1 + \frac{V_{CB}}{V_{0C}}\right)^m = \frac{\cancel{20}\ 30\,fF}{\left(1 + \frac{2V}{0.75V}\right)^{0.5}} = 15.66\,fF$$

$C_{je} \simeq 2\,C_{jeo} = 20 \times 20 = 40\,fF$

$C_{de} = \tau_F \cdot g_m = 30\,ps \cdot 20\,m\mho = 600$ femto F

$C_\pi = C_{je} + C_{de} = 40 + 600 = 640\,fF = 0.640\,pF$

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)} = \frac{20\ \cancel{m\mho} \times 10^{-3}\,\mho}{2 \times 3.14\,(640 + 15.66) \times 10^{-15}}$$

$$\boxed{= 4.857\ \text{Giga Hz}}$$

Q: 3.150 At 500 MHz signal, $|h_{fe}| = 2.5$ @ $I_C = 0.2$ mA

and at same frequency $|h_{fe}| = 11.6$ @ $I_C = 1.0$ mA (9)

Given: $C_\mu = 0.05$ pF or 50 fF.

Find f_T at both Collector Currents.

What is τ_F and C_{je} value?

—— x ——

Note that

$$|h_{fe}| = \frac{f_T}{\text{freq of measurement}}$$

@ $I_C = 0.2$mA

$$\therefore f_T = |h_{fe}| \cdot f = 2.5 \times 500\text{ MHz} = 1.25\text{ GHz}.$$

at another I_C value @ $I_C = 1.0$ mA

$$f_T = 11.6 \times 500\text{ MHz} = 5.8\text{ GHz}$$

Note: Change in bias current from 0.2mA to 1.0 mA increases f_T from 1.25 to 5.8 GHz.

Thus if we want a higher frequency amplifier, we should keep I_C = higher value.

$$\because f_T = g_m / 2\pi(C_\pi + C_\mu)$$

$$\therefore C_\pi = \left(g_m/(2\pi f_T)\right) - C_\mu$$

at $I_C = 0.2$mA $\longrightarrow g_m = 0.2/25 = 8$ m℧

$$C_\pi = \left[8\text{ m℧}/(2 \times 3.14 \times 1.25\text{ GHz})\right] - 50\text{ fF} = 968\text{ fF}$$

at $I_C = 1.0\,mA$ $g_m = 40\,m\mho$

$$C_\pi = \left[40\,m\mho/(2\times3.14\times5.8\,GHz)\right] - 50\,fF$$

$$= 1047.6\,fF.$$

Note that C_π also increases with I_C.

— x —

Next, $C_\pi = C_{je} + \tau_F \cdot g_m$

@ $I_C = 0.2\,mA$

$$968\,fF = C_{je} + \tau_F \cdot 8\,m\mho \quad — (1)$$

@ $I_C = 1.0\,mA$

$$1047.6\,fF = C_{je} + \tau_F \cdot 40\,m\mho \quad — (2)$$

Solving (1) and (2) $C_{je} = 950\,fF$ and

$$\tau_F = 2.47\,ps$$

Q: 3.151 Given: $I_C = 2\,mA$; $C_\mu = 1\,pF$, $C_\pi = 10\,pF$

$\beta = 150$. Find f_T and f_β.

$$f_T = g_m/(2\pi(C_\mu + C_\pi)) = 80\,m\mho/(2*3.14*(1+10)\,pf)$$

$$= \boxed{1.158\,GHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{1.158\,GHz}{150} = \boxed{7.725\,MHz.}$$