Tutorial 3

SC-220 Groups and Linear Algebra Autumn 2019
(Products, Group Isomorphism)

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(1) Let $\mathbb{Z}_n^*$ be set of integers that are less than n and relatively prime to n. For example $\mathbb{Z}_8^* = \{1, 3, 5, 7\}$. In class we saw that $\mathbb{Z}_n^*$ is a group under multiplication modulo n. $|\mathbb{Z}_n^*| = \phi(n)$ and its not always cyclic like $\mathbb{Z}_n$.

Show that $\mathbb{Z}_9^*$ is isomorphic to $\mathbb{Z}_6$.

(2) Show that $\mathbb{Z}_{20}^*$ is not isomorphic to $\mathbb{Z}_8$.

(3) An isomorphism of a group onto itself is called an automorphism. Let G be a group and let g be an element of G. Show that the mapping $x \mapsto gxg^{-1}$ is an automorphism of G.

(4) Let G be a group. Show that the mapping $x \mapsto x^{-1}$ is an automorphism G iff G is abelian.

(5) Show that if G and H are groups and if $G \times H$ is cyclic then G and H are both cyclic.

(6) Show that $\mathbb{Z} \times \mathbb{Z}$ is not isomorphic to $\mathbb{Z}$

(7) How many different isomorphisms are there from S_3 to D_3 ?

(8) **Fact**: $\mathbb{Z}_m^* \times \mathbb{Z}_n^*$ is isomporphic to $\mathbb{Z}_{mn}^*$ if m,n are relatively prime.
Use this fact to show that $\mathbb{Z}_{20}^*$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_4$

(9) Show that if G is a group of order 4 that is not cyclic then it is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.
