which is readily derived from the message waveform $x(t)$. The system consists of an analog inverter, an integrator, and a Schmitt trigger controlling an electronic switch. The trigger puts the switch in the upper position whenever $x_\Lambda(t)$ increases to $+1$ and puts the switch in the lower position whenever $x_\Lambda(t)$ decreases to -1.

Suppose the system starts operating at $t = 0$ with $x_\Lambda(0) = +1$ and the switch in the upper position. Then, for $0 < t < t_1$,

$$x_\Lambda(t) = 1 - \int_0^t v(\lambda)\,d\lambda = 1 - \frac{2}{\pi}[\theta_c(t) - \theta_c(0)]$$

$$= 1 - \frac{2}{\pi}\theta_c(t) \qquad 0 < t < t_1$$

so $x_\Lambda(t)$ traces out the downward ramp in Fig. 5.3–6a until time t_1 when $x_\Lambda(t_1) = -1$, corresponding to $\theta_c(t_1) = \pi$. Now the trigger throws the switch to the lower position and

$$x_\Lambda(t) = -1 + \int_{t_1}^t v(\lambda)\,d\lambda = -1 + \frac{2}{\pi}[\theta_c(t) - \theta_c(t_1)]$$

$$= -3 + \frac{2}{\pi}\theta_c(t) \qquad t_1 < t < t_2$$

so $x_\Lambda(t)$ traces out the upward ramp in Fig. 5.3–6a. The upward ramp continues until time t_2 when $\theta_c(t_2) = 2\pi$ and $x_\Lambda(t_2) = +1$. The switch then triggers back to the upper position, and the operating cycle goes on periodically for $t > t_2$.

A sinusoidal FM wave is obtained from $x_\Lambda(t)$ using a nonlinear waveshaper with transfer characteristics $T[x_\Lambda(t)] = A_c \sin[(\pi/2)x_\Lambda(t)]$, which performs the inverse of Eq. (5a). Or $x_\Lambda(t)$ can be applied to a hard limiter to produce **square-wave** FM. A laboratory test generator might have all three outputs available.

Frequency Detection

A **frequency detector,** often called a **discriminator,** produces an output voltage that should vary linearly with the instantaneous frequency of the input. There are perhaps as many different circuit designs for frequency detection as there are designers who have considered the problem. However, almost every circuit falls into one of the following four operational categories:

1. FM-to-AM conversion
2. Phase-shift discrimination
3. Zero-crossing detection
4. Frequency feedback

We'll look at illustrative examples from the first three categories, postponing frequency feedback to Sect. 7.3. Analog *phase* detection is not discussed here because

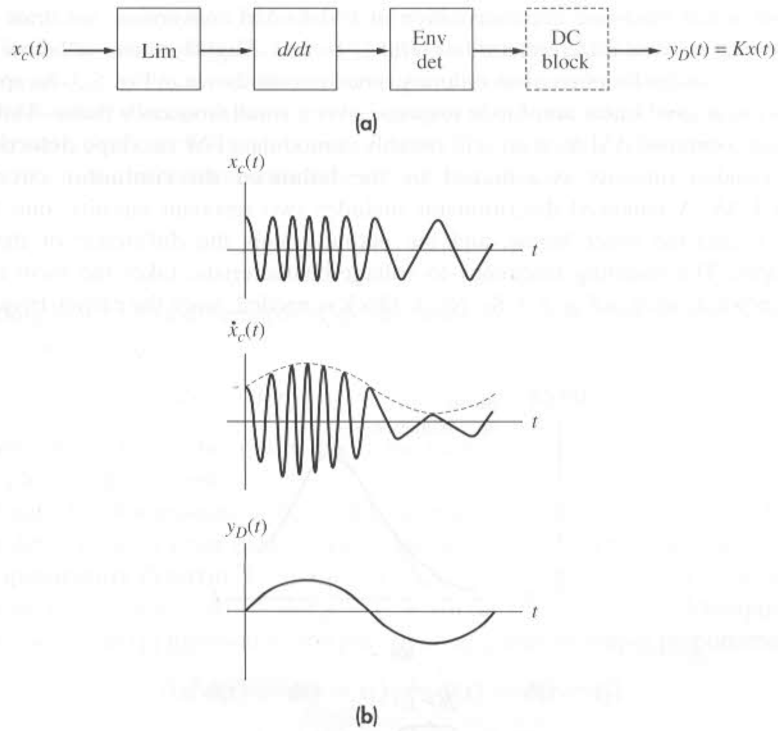

Figure 5.3–7 (*a*) Frequency detector with limiter and FM-to-AM conversion; (*b*) waveforms.

it's seldom needed in practice and, if needed, can be accomplished by integrating the output of a frequency detector.

Any device or circuit whose output equals the *time derivative* of the input produces **FM-to-AM conversion.** To be more specific, let $x_c(t) = A_c \cos \theta_c(t)$ with $\dot{\theta}_c(t) = 2\pi[f_c + f_\Delta x(t)]$; then

$$\begin{aligned} \dot{x}_c(t) &= -A_c\dot{\theta}_c(t) \sin \theta_c(t) \\ &= 2\pi A_c[f_c + f_\Delta\, x(t)] \sin [\theta_c(t) \pm 180°] \end{aligned} \qquad [6]$$

Hence, an **envelope detector** with input $\dot{x}_c(t)$ yields an output proportional to $f(t) = f_c + f_\Delta x(t)$.

Figure 5.3–7*a* diagrams a conceptual frequency detector based on Eq. (6). The diagram includes a limiter at the input to remove any spurious amplitude variations from $x_c(t)$ before they reach the envelope detector. It also includes a dc block to remove the constant carrier-frequency offset from the output signal. Typical waveforms are sketched in Fig. 5.3–7*b* taking the case of tone modulation.

For actual hardware implementation of FM-to-AM conversion, we draw upon the fact that an ideal differentiator has $|H(f)| = 2\pi f$. Slightly above or below resonance, the transfer function of an ordinary tuned circuit shown in Fig. 5.3–8*a* approximates the desired linear amplitude response over a small frequency range. Thus, for instance, a detuned AM receiver will roughly demodulate FM via **slope detection.**

Extended linearity is achieved by the **balanced discriminator** circuit in Fig. 5.3–8*b*. A balanced discriminator includes two resonant circuits, one tuned above f_c and the other below, and the output equals the difference of the two envelopes. The resulting frequency-to-voltage characteristic takes the form of the well-known *S* curve in Fig. 5.3–8*c*. No dc block is needed, since the carrier-frequency

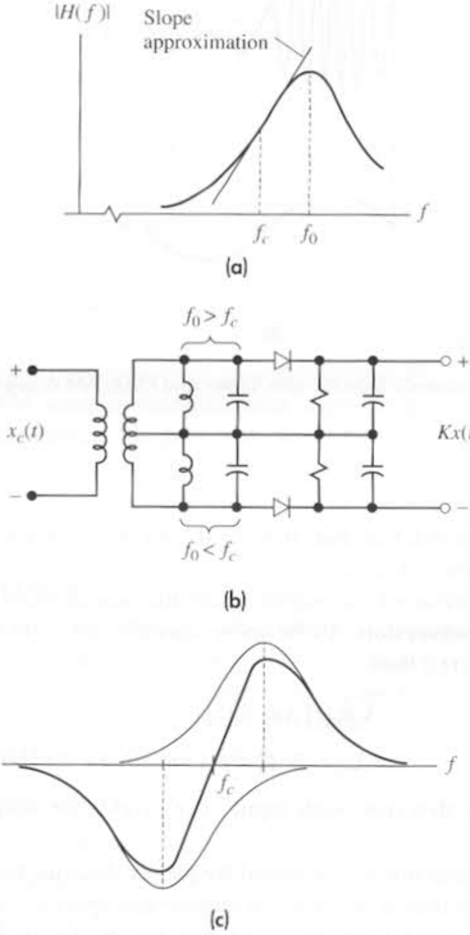

Figure 5.3–8 (*a*) Slope detection with a tuned circuit; (*b*) balanced discriminator circuit; (*c*) frequency-to-voltage characteristic.

offset cancels out, and the circuit has good performance at low modulating frequencies. The balanced configuration easily adapts to the microwave band, with resonant cavities serving as tuned circuits and crystal diodes for envelope detectors.

Phase-shift discriminators involve circuits with linear *phase* response, in contrast to the linear amplitude response of slope detection. The underlying principle comes from an approximation for time differentiation, namely

$$\dot{v}(t) \approx \frac{1}{t_1}\left[v(t) - v(t - t_1)\right] \tag{7}$$

providing that t_1 is small compared to the variation of $v(t)$. Now an FM wave has $\dot{\phi}(t) = 2\pi f_\Delta x(t)$ so

$$\phi(t) - \phi(t - t_1) \approx t_1\dot{\phi}(t) = 2\pi f_\Delta t_1 x(t) \tag{8}$$

The term $\phi(t - t_1)$ can be obtained with the help of a delay line or, equivalently, a linear phase-shift network.

Figure 5.3–9 represents a phase-shift discriminator built with a network having group delay t_1 and carrier delay t_0 such that $\omega_c t_0 = 90°$—which accounts for the name **quadrature detector.** From Eq. (11), Sect. 5.2, the phase-shifted signal is proportional to $\cos[\omega_c t - 90° + \phi(t - t_1)] = \sin[\omega_c t + \phi(t - t_1)]$. Multiplication by $\cos[\omega_c t + \phi(t)]$ followed by lowpass filtering yields an output proportional to

$$\sin[\phi(t) - \phi(t - t_1)] \approx \phi(t) - \phi(t - t_1)$$

assuming t_1 is small enough that $|\phi(t) - \phi(t - t_1)| \ll \pi$. Therefore,

$$y_D(t) \approx K_D f_\Delta x(t)$$

where the detection constant K_D includes t_1. Despite these approximations, a quadrature detector provides better linearity than a balanced discriminator and is often found in high-quality receivers.

Other phase-shift circuit realizations include the **Foster-Seely discriminator** and the popular **ratio detector.** The latter is particularly ingenious and economical, for it combines the operations of limiting and demodulation into one unit. See Tomasi (1998, Chap. 7) for further details.

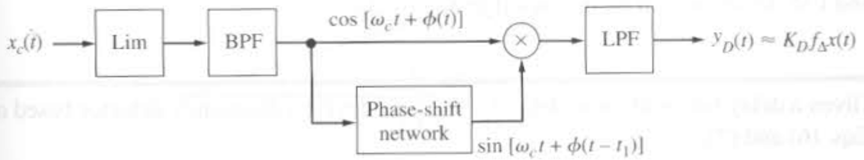

Figure 5.3–9 Phase-shift discriminator or quadrature detector.

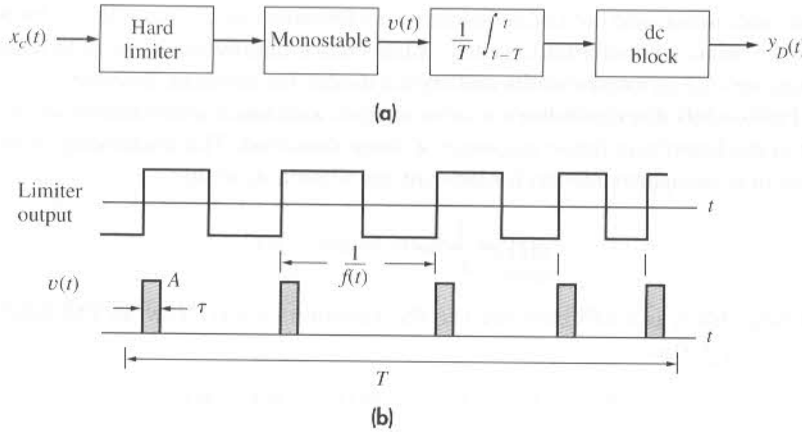

Figure 5.3–10 Zero-crossing detector. (a) Diagram; (b) waveforms.

Lastly, Fig. 5.3–10 gives the diagram and waveforms for a simplified **zero-crossing detector.** The square-wave FM signal from a hard limiter triggers a monostable pulse generator, which produces a short pulse of fixed amplitude A and duration τ at each upward (or downward) zero crossing of the FM wave. If we invoke the quasi-static viewpoint and consider a time interval T such that $W \ll 1/T \ll f_c$, the monostable output $v(t)$ looks like a rectangular pulse train with nearly constant period $1/f(t)$. Thus, there are $n_T \approx Tf(t)$ pulses in this interval, and continually integrating $v(t)$ over the past T seconds yields

$$\frac{1}{T}\int_{t-T}^{t} v(\lambda)\, d\lambda = \frac{1}{T} n_T A\tau \approx A\tau f(t)$$

which becomes $y_D(t) \approx K_D f_\Delta x(t)$ after the dc block.

Commercial zero-crossing detectors may have better than 0.1 percent linearity and operate at center frequencies from 1 Hz to 10 MHz. A divide-by-ten counter inserted after the hard limiter extends the range up to 100 MHz.

Today most FM communication devices utilize linear integrated circuits for FM detection. Their reliability, small size, and ease of design have fueled the growth of portable two-way FM and cellular radio communications systems. Phase-lock loops and FM detection will be discussed in Sect. 7.3.

EXERCISE 5.3–2 Given a delay line with time delay $t_0 \ll 1/f_c$, devise a frequency detector based on Eqs. (6) and (7).