Newton's laws of motion

First law: Every object maintains its state of rest, or uniform motion along a straight line, unless impressed upon by an external force.

1/. Force does not maintain motion, but only causes change in motion.

2/. Absence of forces (not just a complete cancellation of forces) implies the need to define force as something that changes motion.

3/. Law of inertia, implies a natural tendency to resist change in motion.

Second law: $\boxed{\vec{F} \propto m\vec{a}}$ $\quad \vec{a} = \frac{d\vec{v}}{dt}$

$\therefore \boxed{\vec{F} = km\vec{a}}$ $k = 1$ in suitably chosen units

1/ For the same force acting on two objects of different masses $\boxed{|\vec{F}| = m_1|\vec{a}_1| = m_2|\vec{a}_2|} \Rightarrow \boxed{|\vec{a}| \propto \frac{1}{m}}$

Hence, with greater mass, the tendency to resist change in motion would be greater.

$\therefore$ Mass is a measure of inertia.

2/. $\boxed{\vec{F} = m\frac{d\vec{v}}{dt}}$ $\quad \vec{F}\cdot\vec{v} = m\,\vec{v}\cdot\frac{d\vec{v}}{dt}$

Now $\boxed{\vec{v} = \frac{d\vec{l}}{dt}}$ $\Rightarrow \vec{F}\cdot\frac{d\vec{l}}{dt} = m\,\vec{v}\cdot\frac{d\vec{v}}{dt}$.

$\Rightarrow \vec{F}\cdot\frac{d\vec{l}}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) \Rightarrow \boxed{\vec{F}\cdot d\vec{l} = d\left(\frac{mv^2}{2}\right)}$

$\therefore$ Work done $= \vec{F}\cdot d\vec{l} =$ change in kinetic energy.

In one-dimension, $\boxed{d\vec{l} \equiv dx\,\hat{x}}$ $\boxed{|\vec{F}(\vec{x})| \equiv F(x)}$

$\therefore \vec{F}\cdot d\vec{l} = F(x)\,dx = d\left(\frac{mv^2}{2}\right)$

$$\Rightarrow \boxed{\int_{v_1}^{v_2} d\left(\frac{mv^2}{2}\right) = \frac{m v_2^2}{2} - \frac{m v_1^2}{2} = \int_{x_1}^{x_2} F(x)\,dx}$$

Work done is the difference of kinetic energy ↑ changes the speed

3/. Further, $\boxed{d\left(\frac{mv^2}{2}\right) - F(x)dx = 0}$

Write $\boxed{F(x) = -\frac{dU(x)}{dx}}$ $\quad U(x) \rightarrow$ Potential function

$\therefore d\left(\frac{mv^2}{2}\right) + \frac{dU}{dx}dx = 0 \Rightarrow \int d\left(\frac{mv^2}{2}\right) + \int dU = \text{Constant}$

$\Rightarrow \boxed{\frac{mv^2}{2} + U(x) = \mathcal{E}}$ Conservation of Energy.

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$\Rightarrow \boxed{\frac{1}{2} m v_1^2 + U(x_1) = \frac{1}{2} m v_2^2 + U(x_2)}$ When.

$\boxed{U(x_1) = U(x_2)}$, no change in kinetic energy occurs when there is no potential difference.

47. $\boxed{F = m\frac{d^2x}{dt^2}} \Rightarrow \boxed{\frac{dx}{dt} = v}$ and $\boxed{\frac{dv}{dt} = \frac{F}{m}}$

The integrals give $\boxed{x \equiv x(t)}$ and $\boxed{v \equiv v(t)}$.

At any initial time, two initial conditions are required, x (an initial position) and v (an initial velocity). The former specifies the state, and the latter the rate at which the state is changing. Hence, for a deterministic system, a second-order differential equation is required (with two conditions)

Third law: $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$. Particle 1 acts on Particle 2 (Action). Particle 2 reacts on Particle 1 (Reaction). Action and reaction are on different objects.

At the time of collision between two particles in an isolated system, $\boxed{m_1\frac{d\vec{v}_1}{dt} = -m_2\frac{d\vec{v}_2}{dt}}$. (by the action-reaction principle)

$$\Rightarrow \frac{d}{dt}(m_1\vec{v}_1) + \frac{d(m_2\vec{v}_2)}{dt} = \frac{d}{dt}\left[m_1\vec{v}_1 + m_2\vec{v}_2\right] = \vec{0}.$$

$$\therefore \boxed{m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{P}} \rightarrow \text{Conservation of Momentum}$$

Mass, Energy and Momentum

i/. Mass → Emerges from the first law as a measure of inertia.

ii/. Energy → Emerges from the second law as a conserved quantity.

iii/. Momentum → Emerges from the third law as a conserved quantity.

Physical ~~[illegible]~~ Points of Newton's laws

1/. Objects have inertia. Mass is a measure of the inertia. Force is needed to overcome inertia. (change in motion) ←

2/. The second law quantifies force, and makes it measurable in physical ways.

3/. The third law, (~~[illegible]~~ following the knowledge of force), relates force to interactions among objects.

Maxwell's Equations of Electrostatics and Magnetostatics

$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$	← Gauss's Law	$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$
$\boxed{\vec{\nabla}\times\vec{E} = \vec{0}}$	Ampere's Law →	$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$

1/. $\rho \rightarrow$ Charge Density, $\vec{J} \rightarrow$ Cross-Sectional Current Density

$$\boxed{\rho \rightarrow \frac{\text{Charge}}{\text{Volume}} \longrightarrow \frac{\text{Coulomb}}{m^3} \ (\text{S.I. unit})} \quad \boxed{\vec{J} \longrightarrow \frac{\text{Current}}{\text{Area}} = \frac{\text{ampere}}{m^2} \ (\text{S.I. unit})}$$

2/. Physical "sources" are electric in nature $\rightarrow \boxed{\rho, \vec{J}}$. There is no physical element of magnetism.

3/. All magnetic effects arise due to electric phenomena. $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ (Ampere)

4/. Magnetic fields have neither a source nor a sink. $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$.
There are No magnetic monopoles.

5/. Now $\boxed{\vec{\mathcal{E}} = \mathcal{E}(r)\,\hat{r}}$ → A central field.

$\boxed{\mathcal{E}(r) \propto \frac{1}{r^2}}$ ⇒ $\boxed{\vec{\mathcal{E}} \propto \frac{1}{r^2}\,\hat{r}}$ → Coulomb Inverse Square field.

Since $\vec{\mathcal{E}}$ is a central vector field,

⇒ $\boxed{\vec{\nabla}\times\vec{\mathcal{E}} = \vec{0}}$ for the electrostatic field.

6/. $\boxed{\vec{\nabla}\cdot\vec{\mathcal{E}} = \frac{\rho}{\epsilon_0}}$ is the consequence of Coulomb's law force

of a $\boxed{\frac{1}{r^2}\,\hat{r}}$ ~~[illegible]~~ radial field.

Integrate (volume integral) $\boxed{\int_V (\vec{\nabla}\cdot\vec{\mathcal{E}})\,d\tau = \frac{1}{\epsilon_0}\int_V \rho\,d\tau}$

But $\boxed{\int_V (\vec{\nabla}\cdot\vec{\mathcal{E}})\,d\tau = \oint_S \vec{\mathcal{E}}\cdot d\vec{a}}$ → Gauss Divergence Theorem

and $\boxed{\int_V \rho\,d\tau = q_{enc}}$ → Total charge enclosed in the volume

Hence, $\boxed{\oint_S \vec{\mathcal{E}}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}}$ → Gauss's law of electrostatics in the integral form

$\boxed{\oint_S \vec{\mathcal{E}}\cdot d\vec{a} \longrightarrow \text{Flux}}$

(P.T.O.)

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$ → Gauss's law of the electrostatics in the differential form.

And, Electric Flux $\boxed{\oint_S \vec{E}\cdot d\vec{a}}$ through a closed surface is (charge enclosed)/ϵ_0.

7. $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ → Ampere's law

Integrate (surface integral) $\boxed{\int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0 \int_S \vec{J}\cdot d\vec{a}}$

But $\boxed{\int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \oint_P \vec{B}\cdot d\vec{l}}$ → The Stokes Curl Theorem

and $\boxed{\int_S \vec{J}\cdot d\vec{a} = I_{enc}}$ → Total current enclosed through a full cross-sectional area.

Hence, $\boxed{\oint_P \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}}$ → Ampere's law in the integral form.

$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ → Ampere's law in the differential form.

8. $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ → $\boxed{\int_V (\vec{\nabla}\cdot\vec{B})\, d\tau = \oint_S \vec{B}\cdot d\vec{a} = 0}$

By the Gauss Divergence Theorem ⇒

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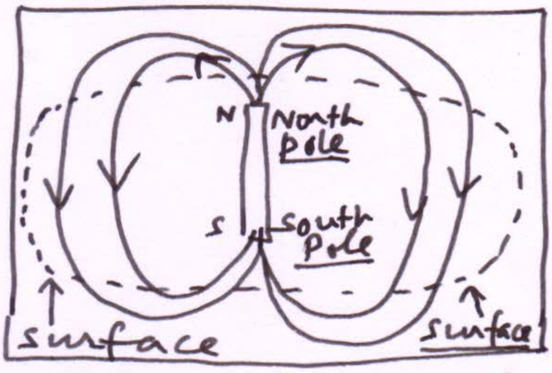
∴ Magnetic Flux vanishes through a closed surface.

$$\boxed{\oint_S \vec{B} \cdot d\vec{a} = 0}$$

Field lines close upon themselves.



Total field lines entering a surface equals total field lines exiting it.

Hence, net magnetic flux is zero.

i) Magnetic field lines have no starting point or finishing point. They always close upon themselves. They have no source point or a sink (point). Zero Divergence

ii) Electric field lines start at a positive charge, and end on a negative charge. They have sources or sinks. Hence they have non-zero divergence.

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

But $\boxed{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0}$

$\Rightarrow \boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

$\vec{A} \rightarrow$ Magnetic vector Potential.

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

But $\boxed{\vec{\nabla} \times (\vec{\nabla} V) = \vec{0}}$

$\Rightarrow \boxed{\vec{E} = -\vec{\nabla} V}$

$V \rightarrow$ Electrostatic Scalar Potential

Maxwell's Equations of Electrodynamics

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0} \qquad \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \qquad \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

1/. Only the "curl" ($\times$) equations are modified. Their right-hand sides now have time-varying $\vec{E}$ and $\vec{B}$ fields. No longer "static" (Electro-"dynamics")

2/. $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ $\rightarrow$ Faraday's law. A changing magnetic field induces an electric field.

3/. In $\boxed{\vec{\nabla}\times\vec{B} = \cdots}$, Maxwell introduced a time-varying correction, $\boxed{\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$ to the static Ampere law. $\boxed{\vec{J_D} = \epsilon_0\frac{\partial\vec{E}}{\partial t}}$, is known as the displacement current

4/. In free-space, $\boxed{\rho = 0,\ \vec{J} = \vec{0}}$, no charge and no current. There, just a time-varying electric field $\frac{\partial\vec{E}}{\partial t}$, induces a magnetic field.