Tutorial 8

SC-220 Groups and Linear Algebra Autumn 2019
(Linear transformations)

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(1) Which of the following from $\mathbb{R}^2$ to $\mathbb{R}^2$ is a linear transformation
(a)$T(x_1, x_2) = (1 + x_1, x_2)$
(b)$T(x_1, x_2) = (x_2, x_1)$
(c)$T(x_1, x_2) = (x_1^2, x_2)$
(d)$T(x_1, x_2) = (\sin(x_1), x_2)$
(e)$T(x_1, x_2) = (x_1 - x_1, 0)$

(2) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$ for all $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$
(a) Show that T is surjective.
(b) Find dim (null (T)).
(c) Find the matrix for T with respect to the canonical basis of $\mathbb{R}^2$.

(3) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by
$T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$
(a) What is the matrix of T relative to the standard ordered basis in $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.
(b) What is the matrix of T relative to the ordered basis in $B = \{u_1, u_2, u_3\}$ in $\mathbb{R}^3$ and $B' = \{v_1, v_2\}$ in $\mathbb{R}^3$ where
$u_1 = (1, 0, -1), u_2 = (1, 1, 1), u_3 = (1, 0, 0)$ and $v_1 = (0, 1), v_2 = (1, 0)$.

(4) Let T be a linear operator on $\mathbb{R}^3$ the matrix representation in the standard ordered basis is $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$. Find the basis for the range of T and the null space of T.

(5) Let T be the linear operator on the space of 2×2 complex matrices such that $T(A) = A^t$. Find a matrix representation of T with respect the basis E_{ij} where E_{ij} are matrices that have the $(i, j)^{th}$ element 1 and 0 elsewhere.
