REVISION TUTORIAL

(1) Consider a medical practitioner pondering the following dilemma: "If I'm at least 80 percent certain that my patient has this disease, then I always recommend surgery, whereas if I'm not quite as certain, then I recommend additional tests that are expensive and sometimes painful. Now, initially I was only 60 percent certain that Jones had the disease, so I ordered the series A test, which always gives a positive result when the patient has the disease and almost never does when he is healthy. The test result was positive, and I was all set to recommend surgery when Jones informed me, for the first time, that he was diabetic. This information complicates matters because, although it doesn't change my original 60 percent estimate of his chances of having the disease in question, it does affect the interpretation of the results of the A test. This is so because the A test, while never yielding a positive result when the patient is healthy, does unfortunately yield a positive result 30 percent of the time in the case of diabetic patients who are not suffering from the disease. Now what do I do? More tests or immediate surgery?"

(2) Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x)^2 \; ; \; -1 < x < 1 \\ 0 \; ; \; \text{otherwise} \end{cases}$$

(a) What is the value of c?

(b) What is the cumulative distribution function of X?

(3) The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} \; : \; x > 10 \\ 0 \quad x \leq 10 \end{cases}$$

(a) Find $P\{X > 20\}$.

(b) What is the cumulative distribution function of X?

(c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?

(4) The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college.

(5) To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the bloodstream, 100 people are put on the diet. After they have been on the diet for a sufficient length of time, their cholesterol count will be taken. The nutritionist running this experiment has decided to endorse the diet if at least 65 percent of the people have a lower cholesterol count after going on the diet. What is the probability that the nutritionist endorses the new diet if, in fact, it has no effect on the cholesterol level?

(6) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} 2e^{-x}e^{-2y} & ; \ 0 < x < \infty, \ 0 < y < \infty \\ 0 & ; \ \text{otherwise} \end{cases}$$

Compute $P\{X < Y\}$.

(7) A basketball team will play a 44-game season. Twenty-six of these games are against class A teams and 18 are against class B teams. Suppose that the team will win each game against a class A team with probability .4 and will win each game against a class B team with probability .7. Suppose also that the results of the different games are independent. Approximate the probability that
 (a) the team wins 25 games or more;
 (b) the team wins more games against class A teams than it does against class B teams.