Tutorial 12

SC-220 Groups and Linear algebra Autumn 2019
(Operators on Inner Products spaces)

(1) Let $A : V \to W$ be a linear transformation. Show that
a. $(A^\dagger)^\dagger = A$ b. $(A+B)^\dagger = A^\dagger + B^\dagger$ c. $(AB)^\dagger = B^\dagger A^\dagger$

(2) Let $u = \begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix}$. Find
a. Orthogonal Projection of u onto span$\{v\}$
b.Orthogonal Projection of v onto span$\{u\}$
c.Orthogonal Projection of u onto $v^\perp$
d.Orthogonal Projection of v onto $u^\perp$

(3) Determine the orthogonal projection of the vector $b = \begin{pmatrix} 5 \\ 2 \\ 5 \\ 3 \end{pmatrix}$ on to the Subspace $\mathcal{M}$ where $M = \text{span}\left\{ \begin{pmatrix} -3/5 \\ 0 \\ 4/5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 4/5 \\ 0 \\ 3/5 \\ 0 \end{pmatrix} \right\}$. What matrix representation of the operator $P_{\mathcal{M}}$ that projects onto the $\mathcal{M}$ in the standard basis. Find a basis and representation of $P_{\mathcal{M}}$ in this basis which is very convenient.

(4) Let a solid unit cube cube be placed such that one on of the vertex is at the origin and the diagonally opposite vertex v is at the point $(1, 1, 1,)$. The cube is rotated first 90^o anticlockwise around the x-axis, followed by 45^o anticlockwise around the y-axis followed by 60^o anticlockwise around the z-axis. Find the location of the vertex v at the end of the three rotations

(5) Let R be the reflection about the vector $u = \frac{1}{\sqrt{3}}(1, 1, 1)$ in $\mathbb{R}^3$. Find action of the reflection about u on the vector $v = (1, 0, 0)$

(6) The Discrete Fourier transform is a linear transformation $F_n : \mathbb{C}^n \to \mathbb{C}^n$, $F_n = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \zeta & \zeta^2 & \cdots & \zeta^{n-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \zeta^{n-1} & \zeta^{n-2} & \cdots & \zeta \end{pmatrix}$ Here $\zeta = e^{-2\pi i/n}$
i) Show that the columns of F_n are orthogonal
ii)$F_n^{-1} = \frac{1}{n}\bar{F}_n$