

1. A queue is implemented using an array of size  $N$ . Variable  $f$  is equal to the index of the cell containing the next element to be deleted, and variable  $r$  is equal to the index of the cell where the next element will be added. Give a single formula to calculate the number of elements in the queue, if the queue is circular.
2. Write a traversal algorithm for proper binary trees in which the nodes at depth  $d$  are visited before the nodes at depth  $d + 1$ .
3. Let us define a function  $p$  from the set of nodes of a proper binary tree  $T$  to the set of natural numbers as follows:
  - If  $v$  is the root of  $T$ , then  $p(v) = 1$
  - If  $v$  is the left child of node  $u$ , then  $p(v) = 2.p(u)$ .
  - If  $v$  is the right child of node  $u$ , then  $p(v) = 2.p(u) + 1$ .

If  $n$  is the total number of nodes, then show that the maximum value of

$p(v)$  satisfies  $n \leq p(v) \leq 2^{(n+1)/2} - 1$

4. Show that the summation  $\sum_{i=1}^n \lceil \log_2(n/i) \rceil$  is  $O(n)$ .
  5. Given two ordered sequences corresponding to sets  $A$  and  $B$ , write an algorithm to compute a sequence corresponding to the set (a)  $A \cup B$  (b)  $A \cap B$
  6. Let  $A$  and  $B$  be two sequences of  $n$  integers each. Given an integer  $x$ ,
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describe an  $n^2$ -time algorithm for determining if there is an integer  $a$  in  $A$  and an integer  $b$  in  $B$  such that  $x = a + b$ . Can you come up with an  $O(n \log n)$ -time algorithm for the same problem.

7. What is the probability that in a random ordering of bridge card deck, the ace of spade precedes the king of hearts.
  8. Prove that if two events A and B are independent, then their respective complements are also independent.
  9. Let  $C$  be the probability space of all the  $n$ -term sequences of 0s and 1s. Define a random variable  $f$  s.t  $\forall s \in C : f(s) = \text{number of 1s in } s$ . Determine the expected value of  $f$  in multiple ways.
  10. Let there be  $n$  hunters and  $n$  rabbits. Each of the  $n$  hunters selects a rabbit at random and then all the hunters shoot at once. Find the expected number of surviving rabbits.
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