in (ii), and its value is greater than unity, (ii) can also be written as

$$|\Gamma| \equiv \frac{r-1}{r+1} \tag{iii}$$

Finally, the value of $|\Gamma|$ can be substituted from (iii) into (i) to obtain the following expression for the voltage standing wave ratio s:

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} \equiv \frac{1+\frac{r-1}{r+1}}{1-\frac{r-1}{r+1}} \equiv r \tag{iv}$$

From the above equation, we can say that the value of the normalized impedance at the point M, where the constant $|\Gamma|$ circle cuts the real axis, is purely real, and is equal to the standing-wave ratio.

In the second step, let us consider the point m on the left-hand side of the Smith chart intersecting the constant r circle. It can be observed from the Smith chart that at any point on the left-hand side of the origin O, the value of the normalized resistance would be less than unity, i.e., $r < 1$. Hence, at the point m, the normalized impedance is purely resistive, and the value of the normalized resistance r is less than unity, which then modifies Eq. (iii) as follows:

$$|\Gamma| \equiv \frac{1-r}{1+r} \tag{v}$$

The expression given by Eq. (5) is based on the simple fact that the magnitude of the reflection coefficient, i.e., the value of $|\Gamma|$ should always be a positive number which is not possible using the form given by (iii) if the value of 'r' is assumed to be less than unity. Once the value of $|\Gamma|$ is defined using (v), the voltage standing-wave ratio 's' can be computed using (i), i.e.,

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} \equiv \frac{1+\frac{1-r}{1+r}}{1-\frac{1-r}{1+r}} \equiv \frac{1}{r} \tag{vi}$$

Hence, the value of the normalized impedance at the point 'm', where the constant $|\Gamma|$ circle cuts the real axis on the left-hand side of the origin 'O', is purely real, and is equal to the inverse of the standing wave ratio.

From Eq. (iv), it can also be postulated that the standing-wave ratio along the line can be obtained by plotting the constant $|\Gamma|$ circle on the Smith chart, and noting the point of intersection of this circle with the positive real axis represented by the point M in Figure 11.19. The value of the normalized resistance r circle, which passes through the point M, then provides the value of the voltage standing wave ratio s on the line.

EXAMPLE 11.16

What are the limitations of a quarter-wave transformer? Explain whether it is possible to match a reactive load having $Z_L = (100 + j150)\,\Omega$ with a lossless transmission line having $Z_0 = 50\,\Omega$ using a quarter-wave transformer. If yes, find out the characteristic impedance of the transformer and other relevant parameters required to achieve this matching.

Solution. The quarter-wave transformer is a piece of transmission line, which is used either for matching two different impedances or for matching a load with a transmission line of different characteristic impedance. It has two major limitations when used for matching purposes. The first limitation is that it can only be used to match the *real* impedances. The second limitation is that the matching is achieved in the very narrow frequency band with the condition that the ideal matching is achieved only at those frequencies where the length of the line is an odd multiple of quarter wavelength, i.e., $l \equiv (2m+1)\lambda/4$.

Now, if one wants to match a reactive load with a transmission line having a *real* value of the characteristic impedance then it can only be achieved if the reactive load is first converted into a *resistive* load. In other words, a piece of transmission line of appropriate length should be connected at the load so that the input impedance becomes *resistive* at the new position. Hence, you have to locate the position on the line shown in Figure 11.20 such that if a transmission line of length l is connected at the load, then the impedance Z_B seen from the new position towards the load becomes purely resistive. Now, you know from the Smith chart theory that the impedance can be purely resistive either at the position of the voltage maximum or at the voltage minimum. In principle, the quarter-wave section can be inserted at any of these two points. However, from the practical point of view, one tries to find a position which is closer to the load in order to minimize the length l of the extra line. The whole procedure is explained below with the help of the Smith chart shown in Figure 11.21.

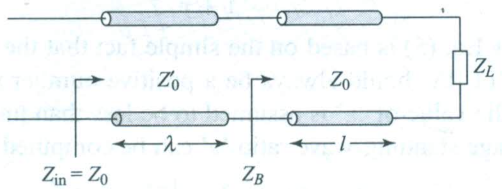

Fig. 11.20 Load line.

Major steps

1. First of all, connect a line of length l shown in Figure 11.20 so that the impedance seen from this point towards the load $Z_L = (100 + j150)\ \Omega$ becomes purely resistive. The characteristic impedance of this line is assumed to be same as that of the actual transmission line, i.e., $Z_0 = 50\ \Omega$.
2. Normalize the load impedance $Z_L = (100 + j150)\ \Omega$ with respect to the characteristic impedance of the line, i.e., $Z_L = \frac{(100+j150)}{50} = (2 + j3)$.
3. Plot $Z_L = (2 + j3)$ point on the Smith chart shown by L in Figure 11.21. Draw a constant $|\Gamma|$ circle on the Smith chart towards the generator (anti-clockwise direction) from the point L with the origin as O and the radius as OL as shown in this figure. This circle intersects the real axis on the right-hand side of the chart at the point M, where the voltage is maximum and the impedance is purely real.
4. The value of the normalized resistance at the point M may be read as $r = 7$. The distance between the load position L and the voltage maximum position M may be determined from the WTG scale provided on the Smith chart, which is given by $l = (0.25 - 0.214)\lambda \equiv 0.036\lambda$. It means that if you move a distance of 0.036 λ from the load towards the generator, then you would arrive at a position where the impedance becomes purely resistive, and this impedance can now be matched with the quarter-wave transformer.

5. The actual value of the resistive impedance at a distance $l = .036\lambda$ from the load in Figure 11.20 is then given by $Z_B = 50 \times 7 = 350\ \Omega$.
6. This impedance can now be matched with a lossless transmission line having characteristic impedance of $Z_0 = 50\ \Omega$ with the help of a quarter-wave transformer. The characteristic impedance of the quarter-wave section in this case would be given by

$$Z_0' = \sqrt{Z_B \times Z_0} = \sqrt{350 \times 50} = 132.29\ \Omega.$$

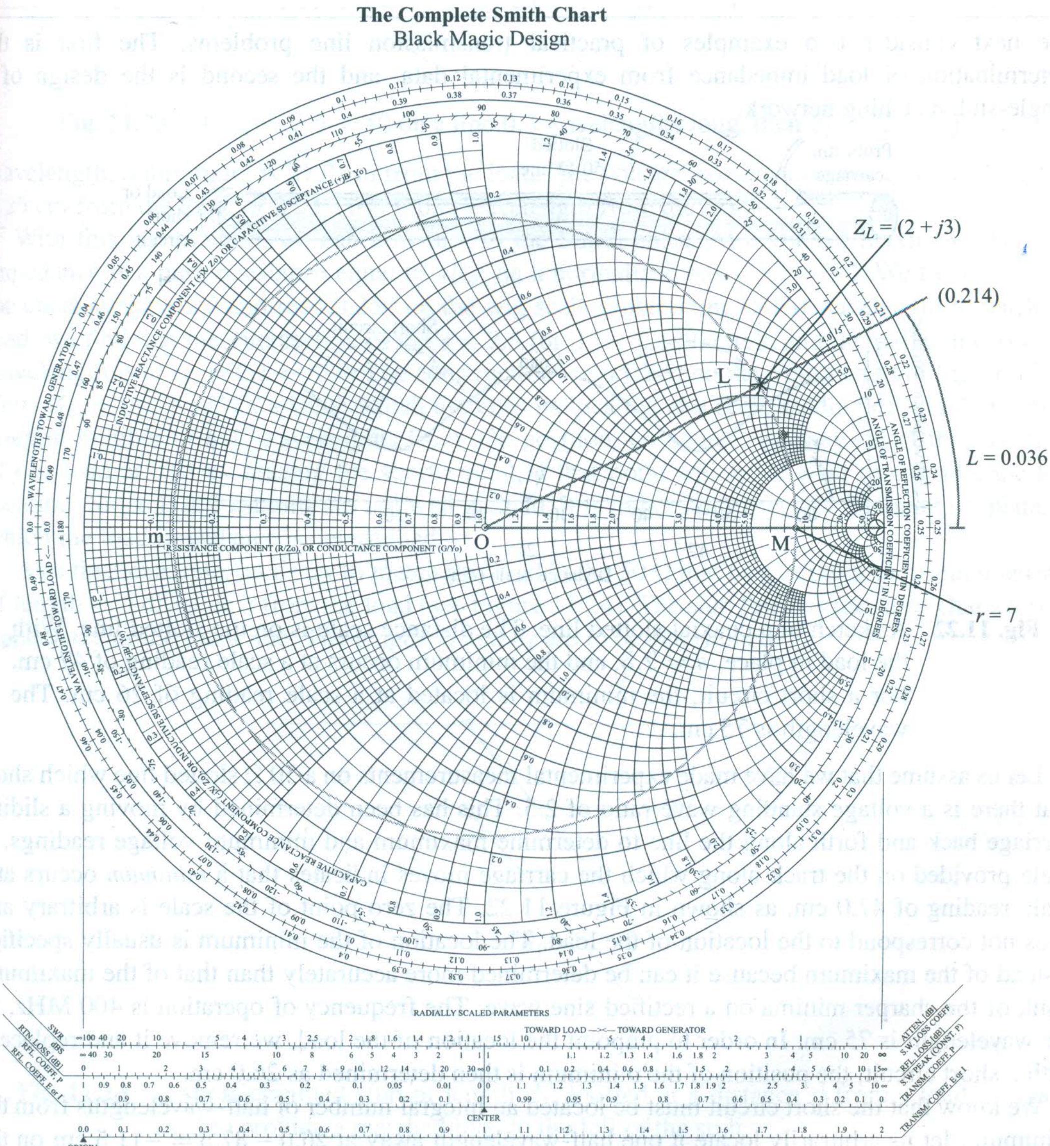

Fig. 11.21 Smith chart.

Since it is much easier to combine admittances in parallel than impedances, let us rephrase our goal in admittance language: the input admittance of the length d containing the load must be $1 + jb_{\text{in}}$ for the addition of the input admittance of the stub jb_{stub} to produce a total admittance of $1 + j0$. Hence the stub admittance is $-jb_{\text{in}}$. We shall therefore use the Smith chart as an admittance chart instead of an impedance chart.

The impedance of the load is $2.1 + j0.8$, and its location is at -11.5 cm. The admittance of the load is therefore $1/(2.1 + j0.8)$, and this value may be determined by adding one-quarter wavelength on the Smith chart, since Z_{in} for a quarter-wavelength line is R_0^2/Z_L, or $z_{\text{in}} = 1/z_L$, or $y_{\text{in}} = z_L$. Entering the chart (Figure 11.25) at $z_L = 2.1 + j0.8$, we read 0.220 on the wtg scale; we add (or subtract) 0.250 and find the admittance $0.41 - j0.16$ corresponding to this impedance. This point is still located on the $s = 2.5$ circle. Now, at what point or points on this circle is the real part of the admittance equal to unity? There are two answers, $1 + j0.95$ at wtg $= 0.16$, and $1 - j0.95$ at wtg $= 0.34$, as shown in Figure 11.25. Let us select the former value since this leads to the shorter stub. Hence $y_{\text{stub}} = -j0.95$, and the stub location corresponds to wtg $= 0.16$. Since the load admittance was found at wtg $= 0.470$, then we must move $(0.5 - 0.47) + 0.16 = 0.19$ wavelength to get to the stub location.

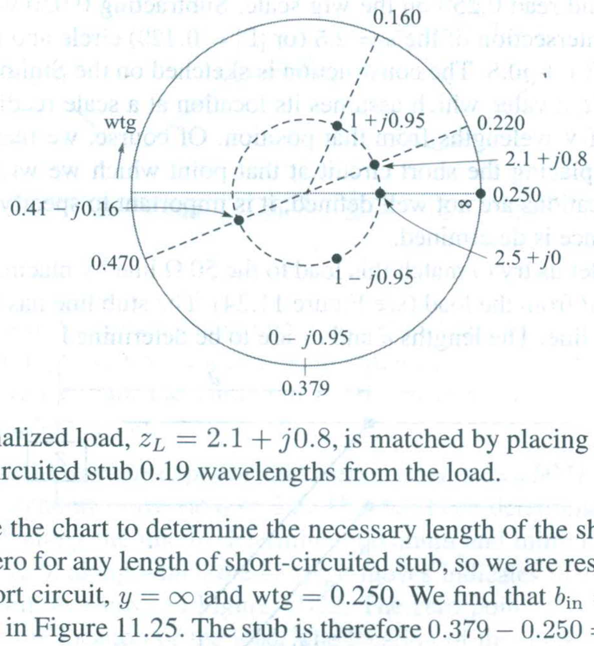

Fig. 11.25 A normalized load, $z_L = 2.1 + j0.8$, is matched by placing a 0.129-wavelength short-circuited stub 0.19 wavelengths from the load.

Finally, we may use the chart to determine the necessary length of the short-circuited stub. The input conductance is zero for any length of short-circuited stub, so we are restricted to the perimeter of the chart. At the short circuit, $y = \infty$ and wtg $= 0.250$. We find that $b_{\text{in}} = -0.95$ is achieved at wtg $= 0.379$, as shown in Figure 11.25. The stub is therefore $0.379 - 0.250 = 0.129$ wavelength, or 9.67 cm long.

EXAMPLE 11.17

It is desired to measure the impedance of an unknown load using the 50 Ω coaxial slotted line set-up shown in Figure 11.22. The experiment is performed by terminating the slotted line in the unknown impedance. The voltage standing-wave ratio measured using this set-up is found to be 3.0, and the voltage minimum is recorded at the scale reading of 50 cm.

Afterwards, the load is removed and a short is placed at the load position. The two consecutive voltage minima are now observed at scale readings of 20 cm and 45 cm, respectively. Using the Smith chart, determine (a) the wavelength, (b) the load impedance, and (c) the reflection coefficient of the load. (d) What would be the location of the first voltage minimum if the actual load is replaced by an open circuit?

Solution.

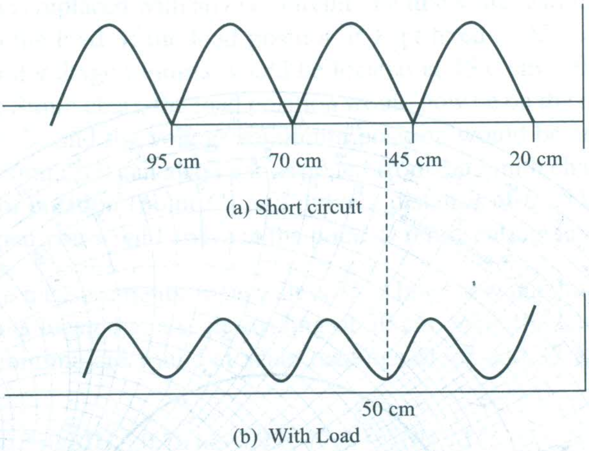

Fig. 11.26 Coaxial slotted line measurement.

The positions of the minimum under the load and the short conditions is shown in Figure 11.26. The detailed procedure is explained below.

(*a*) The consecutive voltage minima under the short conditions are at observed at 45 cm and 20 cm as seen in Figure 11.26(a). As these minima are spaced half wavelength apart, hence the wavelength may be calculated as

$$\lambda = (45 - 20) \times 2 = 50 \text{ cm}$$

(*b*) For finding the load impedance, first draw the constant VSWR circle with $s = 3$ as shown on the Smith chart. Mark the point A, where the $s = 3$ intersects the left-hand side of the real axis. This point A basically represents the voltage minimum point corresponding to the load. In other words, you can consider the point $z = 50$ cm in Figure 11.26 (b) mapped to the point A on the Smith chart.

Now, you can assume the load to be located at any of the minima point corresponding to the short, i.e., the load position can be either at $z = 20$ cm or 45 cm as shown in Figure 11.26 (a). It is convenient to consider the point which is nearest to the load minimum. Hence, the load can be assumed to be positioned at $z = 45$ cm.

It basically means that if you start from the load minima position 'A' on the Smith chart ($z = 50$ cm), and move a distance of $d = 5$ cm $= 0.1\ \lambda$ towards load then you would arrive at the load position ($z = 45$ cm).

On the Smith chart, move on the constant $s = 3$ circle from the point 'A' towards the load until you reach 0.1λ marked on the 'Wavelength towards Load' scale. Draw a line from the centre of the chart to this point intersecting the constant $s = 3$ circle at the point B shown in the chart. The constant r and constant x circles passing through the point 'B' provide the load point, which may be read as

$$Z_L = (0.5 - j0.6)$$

The actual value of the load impedance may then be calculated as

$$Z_L = 50 \times (0.5 - j0.6) = (25 - j30)\ \Omega$$

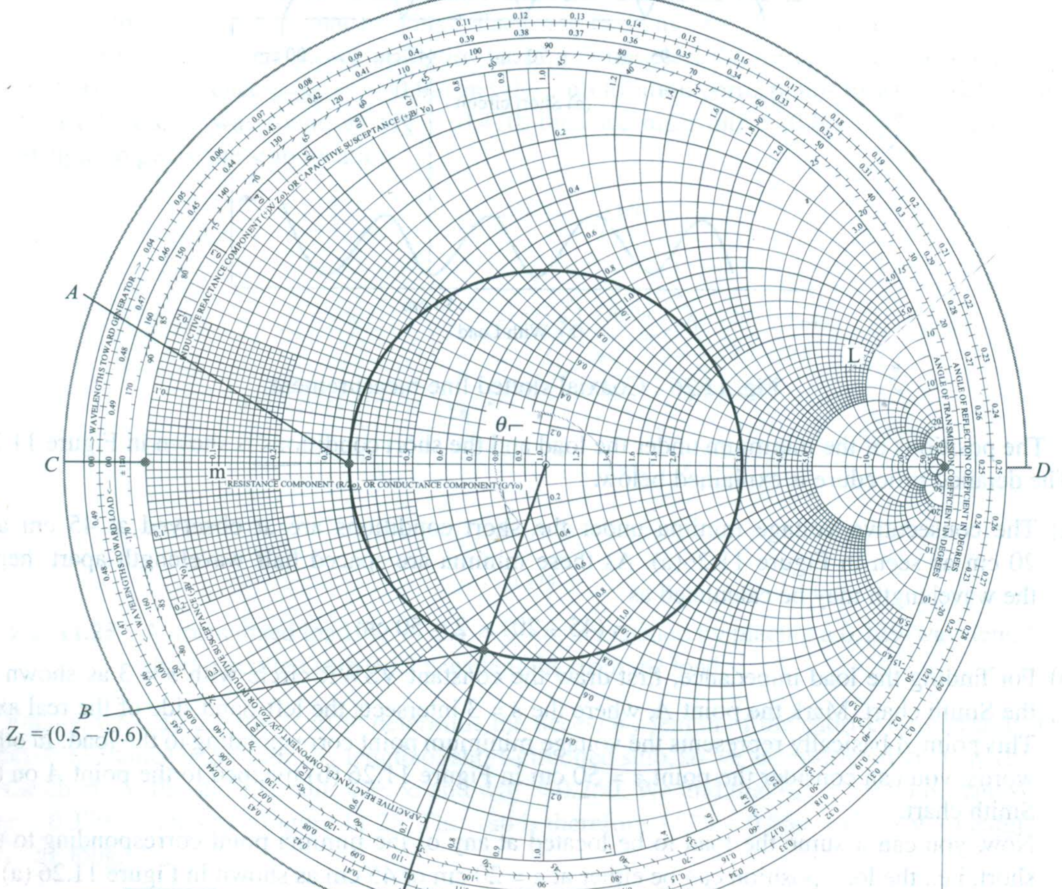

Fig. 11.27 Smith chart.

(*c*) The reflection coefficient of the load can be found out by measuring $|\Gamma|$ and the angle θ_Γ from the Smith chart, which are given as

$$|\Gamma| = 0.5 \text{ and } \theta_\Gamma = 252^\circ$$

$$\text{Hence, } \Gamma = |\Gamma|\, e^{j\theta_r} = 0.5e^{j252^\circ} = 0.5e^{j4.398} = -0.1545 - j0.4755$$

$|\Gamma|$ can also be found out analytically using $|\Gamma| = \dfrac{\rho - 1}{\rho + 1} = \dfrac{3-1}{3+1} = 0.5$

(*d*) When the actual load is replaced with an *open* circuit, the first voltage minimum would be shifted by 0.25λ away from the load. If the load position is kept fixed at 45.0 cm as shown in Figure 11.26 (a) then the first voltage minima would be located at 45.0 cm + 0.25λ = 57.5 cm on the scale reading. On the Smith chart, the load position would now be on the extreme right-hand side denoted by the point D, and the voltage minimum position would be at the extreme left-hand side denoted by the point C. It can also be interpreted from the Smith chart that if you start from the voltage minimum position (point C) and move a distance of 0.25λ on the outer circle of chart towards load then you would arrive at the point D representing an *open* load.

D11.7 Standing wave measurements on a lossless 75 Ω line show maxima of 18 V and minima of 5 V. One minimum is located at a scale reading of 30 cm. With the load replaced by a short circuit, two adjacent minima are found at scale readings of 17 and 37 cm. Find: (*a*) s; (*b*) λ; (*c*) f; (*d*) Γ_L; (*e*) Z_L.

Ans. 3.60; 0.400 m; 750 MHz; $0.704\angle{-33.0}$; $77.9 + j104.7\,\Omega$

EXAMPLE 11.18

A load impedance given by $Z_L = 100 + j100\,\Omega$ is to be matched to a transmission line having characteristic impedance of 150 Ω. Design a matching network consisting of a short-circuited shunt stub having characteristic impedance of 50 Ω, which is connected at some distance d_{stub} away from the load. The stub should be located at shortest possible distance from the load, and its length should also be minimum. Please use the Smith chart and explain clearly all the steps.

Solution. First, we have to find out the location of the stub d_{stub} with reference to the load which is based on the criterion that the real part of the impedance at this point should be equal to the characteristic impedance of the transmission line. A short-circuited stub is then to be connected in parallel at this point, and the length of this stub L_{stub} should be determined such that it cancels the imaginary part of the load impedance. The overall arrangement of the shunt stub with the load and the transmission line is shown in Figure 11.28. It is also to be noted that for shunt stubs, it becomes more convenient to work with admittances, rather than impedances.

The overall procedure of matching the given load with the transmission line using the Smith chart is given below:

(*a*) Calculate the normalized load impedance and determine the corresponding location marked by the point 'L' on the Smith chart.

$$Z_L = \frac{100 + j100}{150} = (0.667 + j0.667)$$