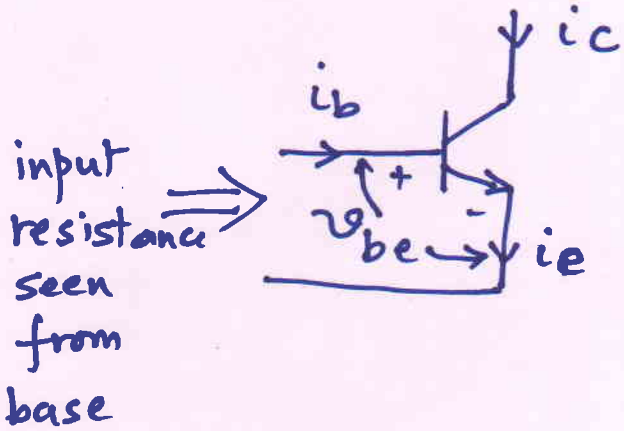
1

Base Current & Input Resistance at Base



Total base current i_B ~~from applied voltage~~ v_{be} contains dc and signal components similar to i_C.

$$i_B = \frac{i_C}{\beta} = \frac{1}{\beta}\left(I_C + \frac{I_C}{V_T} v_{be}\right)$$

$$= \frac{I_C}{\beta} + \frac{I_C}{\beta}\frac{v_{be}}{V_T}$$

$$= \underset{\text{dc part}}{I_B} + \underset{\text{signal part}}{i_b}$$

$$\therefore \quad i_b = \frac{I_C v_{be}}{\beta V_T}$$

we know that $g_m = I_C / V_T$

$$i_b = \frac{g_m}{\beta} v_{be}$$

if we define small signal resistance, between base and emitter, "looking into base" as r_π and we get

then $r_\pi = v_{be} / i_b$ =

$$\boxed{r_\pi = \beta / g_m}$$

r_π is dependent on β.

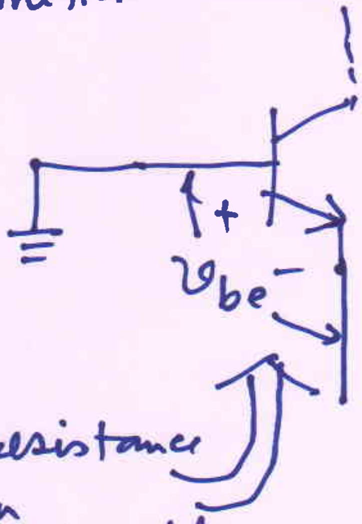
2 Substituting value of $g_m = I_C / V_T$, we get

$$r_\pi = V_T / I_B$$

TO GET HIGHER INPUT RESISTANCE I_C & I_B MUST BE CHOSEN AS SMALL AS POSSIBLE

Emitter Current & Input Resistance at Emitter

We now visualise input resistance seen into emitter when base is grounded.



input resistance seen from emitter side r_e

Total emitter current i_E contains dc + signal.

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha} = dc + ac$$

if we define ac part as i_e then

$$i_E = I_E + i_e$$

$$i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} \cdot v_{be}$$

if we define $r_e \equiv v_{be}/i_e$ then

emitter resistance $r_e = \dfrac{V_T}{I_E} = \dfrac{I_C/g_m}{I_E}$

$$= \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

(3) emitter resistance $\boxed{r_e \approx \frac{1}{g_m}} = \frac{\alpha}{g_m}$

What is relationship between r_π and r_e ?

$$r_\pi = v_{be}/i_b$$

$$r_e = v_{be}/i_e$$

$$\therefore \quad i_b \cdot r_\pi = i_e \cdot r_e$$

$$\therefore \quad \boxed{r_\pi = (i_e/i_b) \cdot r_e = (1+\beta)\, r_e}$$

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Let us calculate some values :

For a given BJT, $\beta = 100$ & $I_C = 1mA$.

Calculate g_m, r_π and r_e.

(i) $g_m = I_C/V_T = 1mA/25mV = 0.04$ mhos
$= 40$ milli mhos

(ii) $r_\pi = \beta/g_m = 100/40\text{ m}\mho = 2.5\ K\Omega$

(iii) $r_e = r_\pi/(1+\beta) = 2.5K\Omega/101 \approx 25\ \Omega$

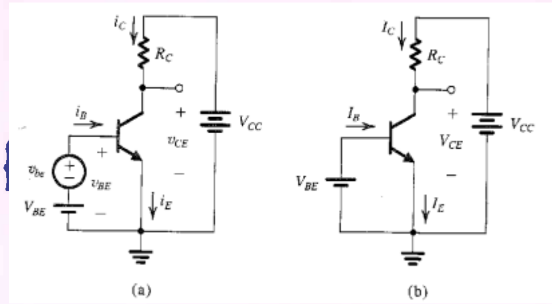
④ VOLTAGE GAIN in BJT Amplifier

input signal v_{be} causes signal current i_b to flow. i_b gets amplified in $i_c = \beta i_b$. This higher value current passes through load resistance R_C and signal output is available at collector. We calculate voltage gain as follows:

Total collector current i_C will cause total Collector voltage $v_C = V_{CC} - i_C R_C$

$$v_C = V_{CC} - (I_C + i_c) R_C$$

$$= (V_{CC} - I_C R_C) - i_c$$



$$= V_C - i_c R_C$$

where V_C is dc bias voltage at collector w.r.t. GND.

signal voltage $v_c = -i_c R_C = -(g_m v_{be}) R_C$

if we define voltage gain A_v as

$$\boxed{A_v \equiv \frac{v_c}{v_{be}} = -g_m R_C}$$

substituting for $g_m = I_C / V_T$, we get

$$A_v = -\frac{I_C R_C}{V_T}$$

HIGHER RC MEANS HIGHER VOLTAGE GAIN.

" RC " smaller V_C or bias voltage and hence smaller space for output signal swing.

⑤ Calculate $v_c(t)$ and $i_B(t)$ for given
$I_C = 1mA$, $R_C = 10K\Omega$; $\beta = 100$; $v_{be} = 0.005 \sin\omega t$
$V_C = 15V$.

Soln:

$$A_v = -\frac{I_c R_c}{V_T} = -\frac{1mA \times 10k}{25mV} = \frac{-10000mV}{25mV}.$$
$$= -400$$

$-$ indicates 180° phase shift betwn. inp. & output.

if $v_{be} = 5\,mV \sin\omega t$

output signal $= v_c = -A_v \cdot v_{be} = -400 \times 5\,mV \sin\omega t$
$$= -2000mV \sin\omega t$$
$$= -2V \sin\omega t.$$

DC voltage at collector $= V_{cc} - I_C R_C$.
$$= 15V - 1mA \times 10k = 5V$$

Total collector voltage $=$ dc $+$ signal
$$= 5V - 2V \sin\omega t$$

$$i_b = g_m v_{be} / \beta$$

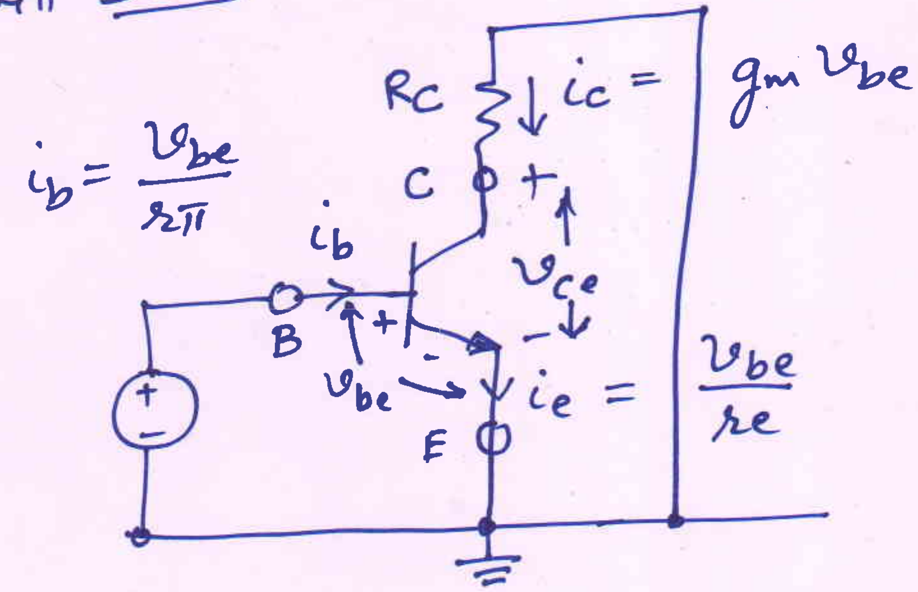
$$i_b = \left(\frac{I_c}{V_T}\right)\frac{v_{be}}{\beta} = \frac{1mA}{25mV}\;\frac{(5mV \sin\omega t)}{100}$$
$$= 2\mu A \sin\omega t$$

dc base current $I_B = I_c/\beta = 1mV/100 = 10\mu A$

total base current $=$ dc $+$ signal
$$= 10\mu A + 2\mu A \sin\omega t.$$

So far we have calculated current and voltage expressions in sum of dc and ac parts. The dc part is due to biasing and is like final value when signal source is shorted or signal amplitude is ZERO.

Similarly, we can focus only on AC part or signal part by eliminating DC sources from total calculations. In considering only AC or time-variant model, we replace a DC Voltage source by its source impedance which is ZERO Ω for ideal volt. source. When considering ideal current source, we open circuit it and ideal voltage source, we will short circuit it. See AC only ckt below:



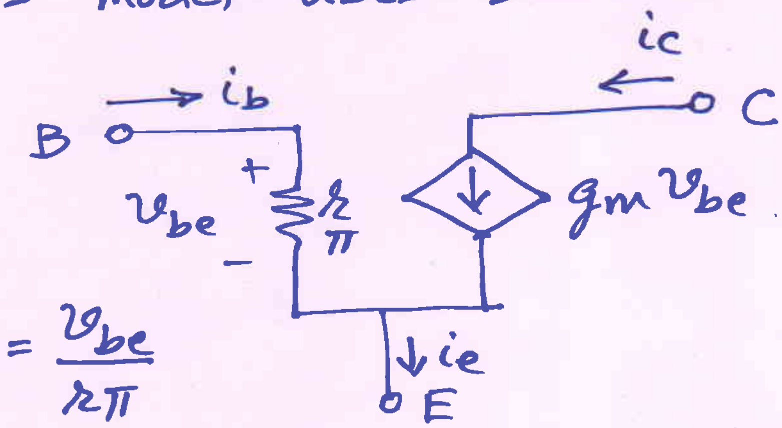
⑦ <u>HYBRID-π MODEL</u>

There are two similar models under this topology:

VCCS — voltage controlled (v_{be}) ~~con~~ current source (i_c)

CCCS — current " (i_b) " " (i_c)

This model uses base-emitter resistance r_π.



$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \beta / g_m$$

$$i_b = \frac{v_{be}}{r_\pi}$$

This model also gives correct expression of i_e.

At emitter node, we have,

$$i_e = i_b + i_c$$

$$= \frac{v_{be}}{r_\pi} + g_m v_{be}$$

$$= \frac{v_{be}}{r_\pi}\left(1 + g_m \cdot r_\pi\right) = \left(\frac{v_{be}}{r_\pi/(1+\beta)}\right)$$

$$= \frac{v_{be}}{r_\pi}(1+\beta) = \frac{v_{be}}{r_e}$$

if ~~we call~~ ~~$r_e = v_{be}/i_e$~~ ~~then~~

which is what we had assumed earlier.

The current source value $g_m v_{be}$ can be given as:

$$g_m \cdot v_{be} = g_m (i_b \cdot r_\pi)$$

$$= (g_m \cdot r_\pi)\, i_b$$

$$= \beta i_b$$ that gives us second model

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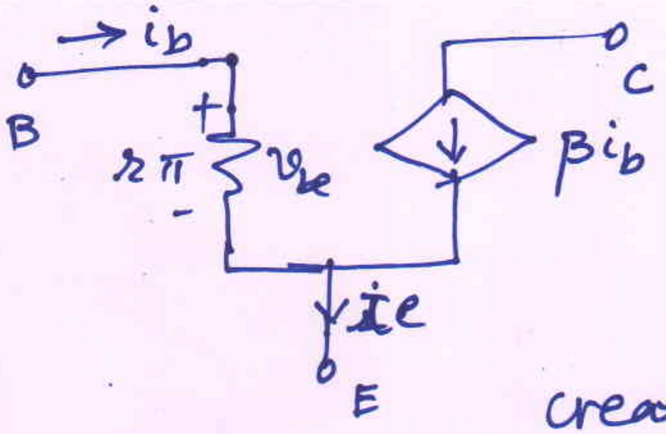

Note that i_b is main driver or independent variable which creates v_{be} across r_π.

- Both are called HYBRID-PI models.
- model parameters g_m and r_π depend upon value of I_C.
- these models equally apply to pnp transistors.

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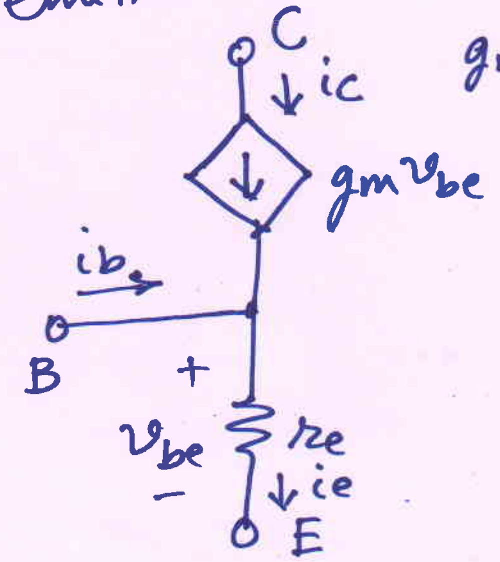
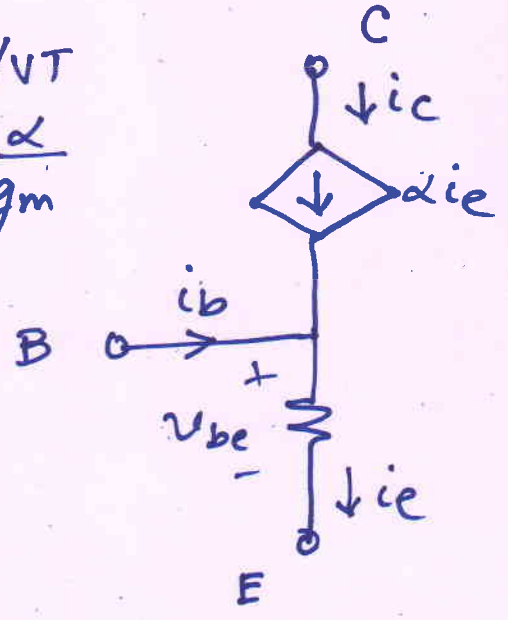
T- MODEL

There are certain circuits where T- model offers easy solutions. Here, the input resistance between base and emitter, looking into emitter is explicitly shown



$g_m = I_C/V_T$

$r_e = \frac{\alpha}{g_m}$



(9) We solve for current at node base:

$$i_b = i_e - i_c$$

$$= \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= \frac{v_{be}}{r_e}\left(1 - g_m r_e\right)$$

$$= \frac{v_{be}}{r_e}\left(1 - \alpha\right)$$

$$= \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta+1}\right)$$

$$= \frac{v_{be}}{(\beta+1) r_e} = \frac{v_{be}}{r_\pi}$$

as should be case. The current source is

$$g_m v_{be} = g_m (i_e r_e)$$

$$= (g_m r_e) i_e$$

$$= \alpha i_e$$

This is second part of T-model.

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How to use Hybrid-π and T-Models to solve BJT amplifier circuits ?

1. Determine Q-point i.e. I_C & V_C.
2. Calculate small signal model parameters

$$g_m = I_C / V_T$$

$$r_\pi = \beta / g_m$$

$$r_e = \alpha / g_m = \frac{V_T}{I_E}$$

3. Eliminate DC sources in a given circuit by short circuiting voltage source and open circuiting current source.
4. Replace BJT by one of its models.
5. Analyse the resulting circuit to determine required quantities like Voltage Gain.

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Q: D3.106

If we need an BJT amplifier with $g_m = 50\,mA/V$ and r_π = base input resistance = $2000\,\Omega$ what I_E value should be chosen? What is minimum β value needed?

Soln: Given $g_m = 50\,mA/V$ & $r_\pi = 2\,k\Omega$

Decide I_C for this g_m value from

$$g_m = I_C / V_T$$

$$\therefore I_C = g_m \times V_T = 50\,mA/V \times 25\,mV = 1250\,\mu A$$

$$\boxed{= 1.25\,mA}$$

Now Calculate β needed for $r_\pi = 2k$.

$$r_\pi = \beta / g_m$$

$$\therefore \beta = r_\pi \cdot g_m = 2000\,\Omega \times 50\,m\mho$$

$$= 100000\,\text{milli}$$

$$\boxed{= 100}$$

If we get higher β then we will get higher r_π.

Using I_C & β, calculate I_E.

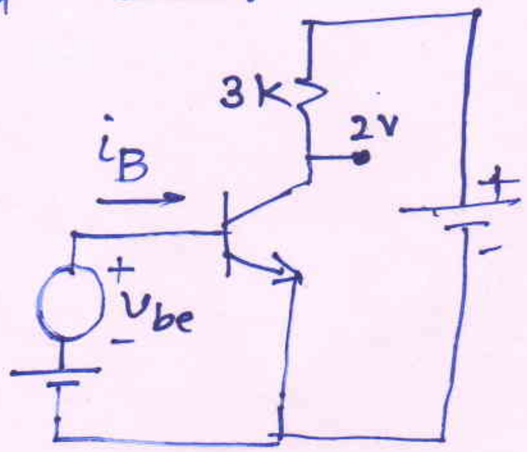
$$I_E = \frac{I_C}{\alpha} = \frac{I_C}{\beta/\beta+1} = \frac{1.250 \times 101}{100}$$

$$= 1262.5\,\mu A$$

$$\boxed{I_E = 1.2625\,mA}$$

(12)

Q: 3.108



Given: $V_C = 2V$; $V_{CC} = 5V$

$R_C = 3K$; $\beta = 100$

input signal $v_{be} = 5mV \sin\omega t$

Calculate total instantaneous quantities (dc+ac) for i_C, v_C and i_B? What is A_v?

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Soln: steps : 1. Calculate $I_C R_C$ & then I_C.

2. Calculate g_m (and possibly r_π).

3. Calculate I_B & I_E from I_C.

4. Calculate $i_b = v_{be}/r_\pi$ or

calculate $i_c = g_m v_{be}$ & divide by β

to get i_b.

5. Calculate i_e

6. Calculate voltage gain $= v_c / v_{be}$

Answers:

Total $i_C = 1mA + 200\mu A \sin\omega t$

$i_B = 10\mu A + 2\mu A \sin\omega t$

$i_E = 1.01mA + 202\mu A \sin\omega t$

Coll. voltage $v_C = 2V + v_c$

$= 2V (-200mV \sin\omega t)$

$= 2V - 200 mV \sin\omega t$.

(13) $V_C = 2V$, $V_{CC} = 5V$, $R_C = 3K$, $\beta = 100$

$v_{be} = 5mV \sin\omega t$

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1. $I_C R_C = V_{CC} - V_C = 5V - 2V = 3V$

$\therefore I_C = 3V/R_C = 3V/3K = \boxed{1\,mA}$

2. $\therefore I_B = \frac{1mA}{\beta} = \frac{1mA}{100} = \boxed{10\mu A}$

3. $\therefore I_C = (\beta+1) I_B = 101 \cdot 10\mu A = \boxed{1.010\,mA}$

4. $\because g_m = I_C/V_T \quad \therefore g_m = 1mA/25mV$

$\boxed{= 40\ mA/V}$

5. signal collector current source

$i_c = g_m v_{be} = 40\,mA/V \times 5mV \sin\omega t$

$\boxed{= 200\mu A \sin\omega t}$

6. $\cancel{i_e}$ $i_b = \frac{i_c}{\beta} = 2\mu A \sin\omega t$

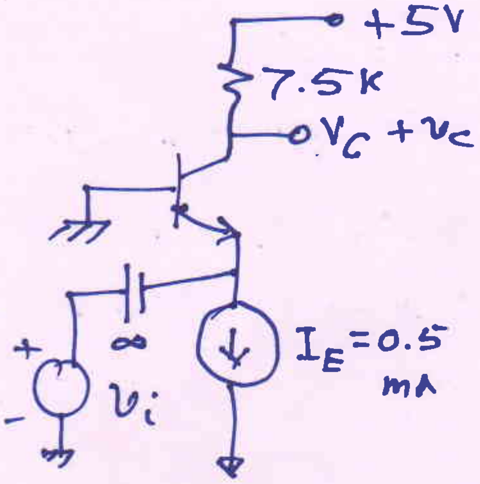
7. $i_e = i_b + i_c = (200\mu A + 2\mu A) \sin\omega t$

8. Voltage Gain $A_v = \frac{-\cancel{g_m}\, i_c R_C}{v_{be}} = \frac{-200\mu A \sin\omega t \times 3K}{5mV \sin\omega t}$

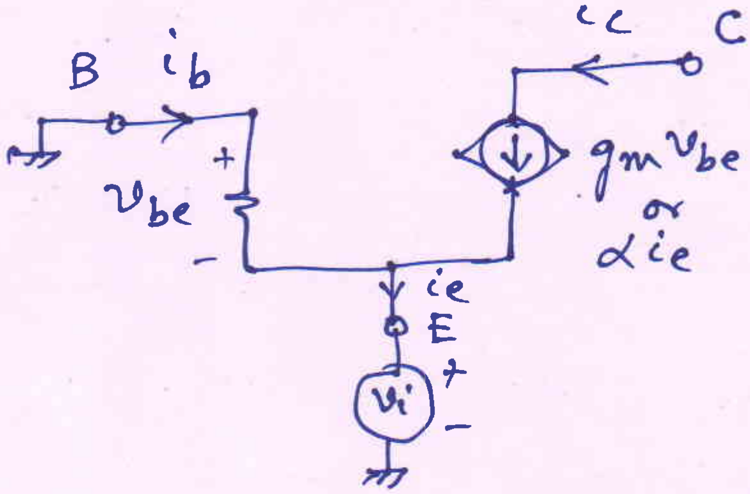
$= \frac{-6000\,mV \sin\omega t}{5\ mV \sin\omega t} = \boxed{-120\ V/V}$

Add DC currents to signal currents & DC Collector voltage to signal voltages now...

(14) Q : 3.112



Given: $\beta >> 1$.
Find V_C and g_m and voltage gain A_v.



$\because \beta >> 1$ we take it that $I_C = I_E = 0.5$ mA

$$g_m = I_C / V_T = 0.5\text{mA} / 25\text{mV} = \boxed{20 \text{ m}\mho}$$

$$V_C = V_{CC} - I_C R_C = 5\text{V} - 0.5\text{mA} \times 7.5\text{K} = \boxed{1.25 \text{ V}}$$

$$i_e = \frac{v_i}{r_e}$$. Find r_e first by

$$r_e = V_T / I_E = \frac{25\text{mV}}{0.5\text{mA}} = 50\,\Omega$$

$$i_e = v_i / 50 \text{ Amps.}$$

$\because \beta >> 1$ $\quad i_c = i_e = v_i/50$ Amps.

signal voltage v_c $= i_c \cdot R_C = (v_i/50) \cdot 7.5\text{K}$

$$\therefore \frac{v_c}{v_i} = A_v = \frac{7.5\text{K}}{50} = \frac{7500}{50} = \boxed{150}$$

We can also calculate using $v_{be} = -v_i$ &

$$A_v = \frac{v_c}{v_{be}} = \frac{v_c}{-v_i} = -g_m R_C = -(-20\text{mA/V} \times 7.5\text{K}) = 150$$

(15) EARLY Effect inclusion in BJT Models

Hybrid-π and T-models make an assumption that BJT output stage has a current source which is solely controlled by input voltage or input current :

$$i_c = g_m v_{be} = \beta i_b$$

and that i_c is independent of V_{CE}. That means the output characteristics i_c vs v_{CE} is nearly horizontal.

J.M. Early showed that as V_{CE} was increased at higher voltages, due to I_{CBO} in reversed biased Base-Collector Jn., as V_{CE} increases I_{CBO} increases & causes I_C to increase though βi_b remains constant.

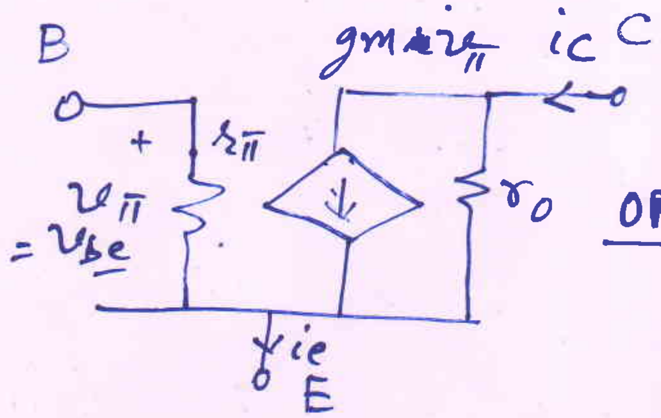
Thus the output characteristics becomes somewhat slopy rather than horizontal.

The slope can be modelled as one created by a finite output resistance r_o in parallel with current source.

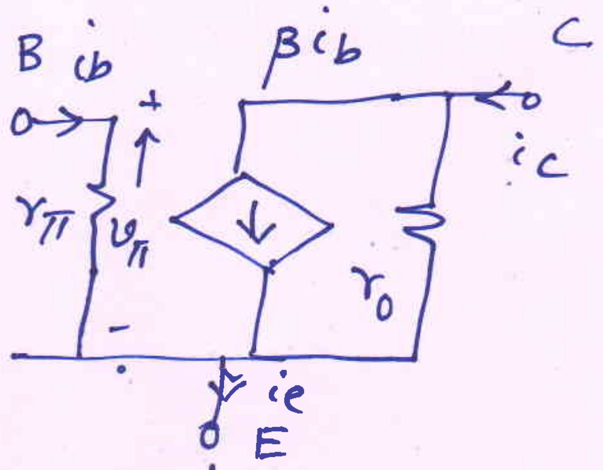
It is given by $r_o = \dfrac{V_A + V_{CE}}{I_C} \approx \dfrac{V_A}{I_C}$

Since V_A is of the Order -50 to -100 Volts.

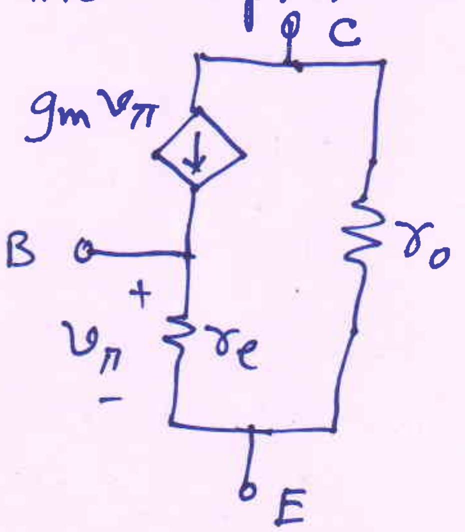
(16) The improved models shown are:

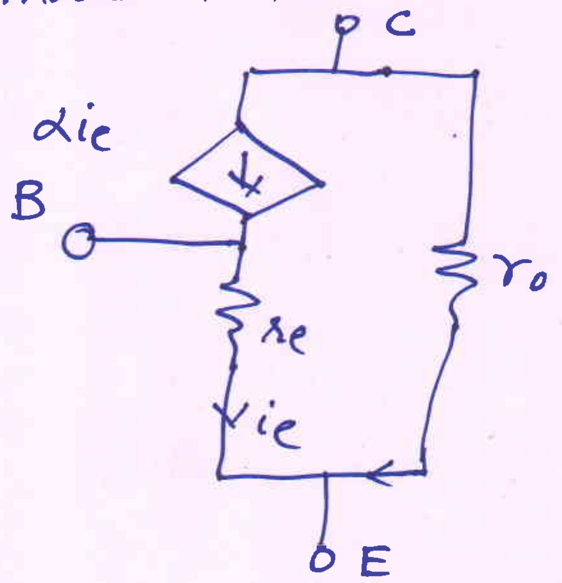


OR



The improved T-model looks like:





The effect of inclusion of r_o is to reduce Voltage Gain

$$v_o = -g_m v_{be} (R_C \parallel r_o)$$

if $r_o > 10 R_C$ then we can assume $r_o = \infty$ or open circuited.

Whenever in our BJT amplifier topology the Emitter is grounded (for signal) we can use Hybrid-π model. Whenever, emitter has R_E to ground, T-model is easier to solve.