$\boxed{17/03/2021}$

Eqn for V_0 (pdf file of 26/Feb → Page-5)

↳ From this eqn, power corresponding to (rms)

$\omega_1 =$

$$P_{\omega_1} = \frac{1}{2}(a_1 V_0)^2 \quad \checkmark \quad \text{--- (1)}$$

Sim, power corresponding to third order (IMD) product $(2\omega_1 - \omega_2)$ or $(2\omega_2 - \omega_1)$

$$P_{2\omega_1 - \omega_2} = \frac{1}{2}\left(\frac{3}{4} a_3 V_0^3\right)^2 \quad \checkmark \quad \text{--- (2)}$$

Recall

By definition of IIP_3 (3rd order intercept pt), $P_{\omega_1} = P_{2\omega_1 - \omega_2}$

$$\therefore \quad (1) = (2)$$

$$\frac{1}{2}(a_1 v_0)^2 = \frac{1}{2}\left(\frac{3}{4} a_3 v_0^3\right)^2$$

$$\Rightarrow \frac{1}{2} a_1^2 v_0^2 = \frac{9}{32} a_3^2 v_0^6$$

$$\Rightarrow v_0^4 = \frac{a_1^2}{a_3^2} \frac{32}{2 \times 9}$$

$$\Rightarrow v_0^4 = \frac{a_1^2}{a_3^2} \cdot \frac{16}{9}$$

$$\Rightarrow v_0^2 = \frac{4}{3} \frac{a_1}{a_3}$$

$$\Rightarrow \boxed{V_0 = \sqrt{\frac{4}{3} \frac{a_1}{a_3}}}$$

$$V_0 = V_{1P}$$

$$\Rightarrow \boxed{V_{1P} = \sqrt{\frac{4}{3} \frac{a_1}{a_3}}} \quad (3)$$

IIP$_3$ (or IP$_3$ or P$_3$)

$$P_3 = P_{\omega_1}\Big|_{\underline{v_0 = v_{ip}}} = \frac{1}{2} a_1^2 v_0^2 \quad (\text{from } ①)$$

$$= \frac{1}{2} a_1^2 v_{ip}^2$$

$$= \frac{1}{2} a_1^2 \cdot \frac{4a_1}{3a_3} \quad [\text{from } ③]$$

this eqn is independent of v_0 ($= v_{ip}$)

$$\boxed{P_3 = \frac{2a_1^3}{3a_3}} \quad — ④$$

$$\underline{\text{SFDR}} : \frac{P_{\omega_1}}{P_{2\omega_1-\omega_2}}$$

But

$$P_{2\omega_1-\omega_2} = \frac{9}{32} a_3^2 V_0^6 \qquad \text{(from (2))}$$

$$= \frac{\frac{1}{8} a_1^6 V_0^6}{\frac{4}{9} \frac{a_1^6}{a_3^2}} \qquad \text{(rewritten)}$$

$$\boxed{P_{2\omega_1-\omega_2} = \frac{(P_{\omega_1})^3}{(P_3)^2}} \qquad (5)$$

from (1)

from (4)

From 5, 3rd order IMD power increases as cube of input signal power

5 : $P_{2\omega_1-\omega_2} = \dfrac{(P_{\omega_1})^3}{(P_3)^2}$

$\Rightarrow N_0 = \dfrac{(P_{\omega_1})^3}{(P_3)^2}$

$\Rightarrow P_{\omega_1} = (N_0)^{1/3} (P_3)^{2/3}$ — 6

$$SFDR = \left.\frac{P_{\omega_1}}{P_{2\omega_1-\omega_2}}\right|_{P_{2\omega_1-\omega_2} = N_0}$$

$$\Rightarrow \text{SFDR} = \frac{(N_0)^{1/3}(P_3)^{2/3}}{N_0} \rightarrow [\text{from } 6]$$

$$\boxed{\text{SFDR} = \left(\frac{P_3}{N_0}\right)^{2/3}}$$

i/p noise range (dB)

In dB

$$\boxed{\underset{(dB)}{\text{SFDR}} = \frac{2}{3}\left(\underbrace{P_3}_{dBm} - \underbrace{N_0}_{dBm}\right)}$$

→ noise floor

Example (extended)

A mobile RX has NF of 7 dB, 1 dB compression (correlation) point of 25 dBm, a gain of 40 dB. If this mobile RX is connected to an antenna having a noise temp T_A of 150 K, find linear (LDR) & spurious free dynamic (SFDR) range

also, 3rd order intercept pt is 35 dBm

Assume $T_0 = 17°C = 290 K$

$B = 100 MHz = 10^8 Hz$

Soln:

$$LDR = P_{-1dB} - N_o \quad \underline{(dB)}$$

$$SFDR = \frac{2}{3}(IIP_3 - N_o) \quad \underline{(dB)}$$

Given, $P_{-1dB} = 25\, dBm$

$IIP_3 = 35\, dBm$

Recall $N_o = ?$ (calculated earlier)

$$N_o = \underbrace{kT_ABG}_{\text{i/p noise of antenna is amplified}} + \underbrace{kT_eBG}_{\text{noise added because of device is amplified}}$$

$$\Rightarrow N_0 = G k B \left[T_A + T_e \right]$$

$$= G k B \left[T_A + (F-1) T_0 \right]$$

$$= 10^4 \times 1.38 \times 10^{-23} \times 10^8 \times \left[150 + (5-1) 290 \right]$$

$$\left[\because G = 40\,dB \Rightarrow 10^4 \right.$$
$$\left. \& \; NF = 7\,dB \Rightarrow F = 5 \right]$$

$$\therefore N_0 = 1.8 \times 10^{-8}\ W$$

$$= 1.8 \times 10^{-5}\ mW$$

$N_o = 1.8 \times 10^{-5}$ mW

$N_o|_{dBm} = -50 + 10\log(1.8)$

$= -47.4$ dBm

$\therefore$ LDR $= P_{-1dB} - N_o$

$= 25$ dBm $- (-47.4$ dBm$)$

$\boxed{LDR = 72 \text{ dB}}$

& SFDR $= \frac{2}{3}(IIP_3 - N_o)$

$= \frac{2}{3}(35 + 47.4)$

$\boxed{SFDR = 54 \text{ dB}}$

SFDR << LDR

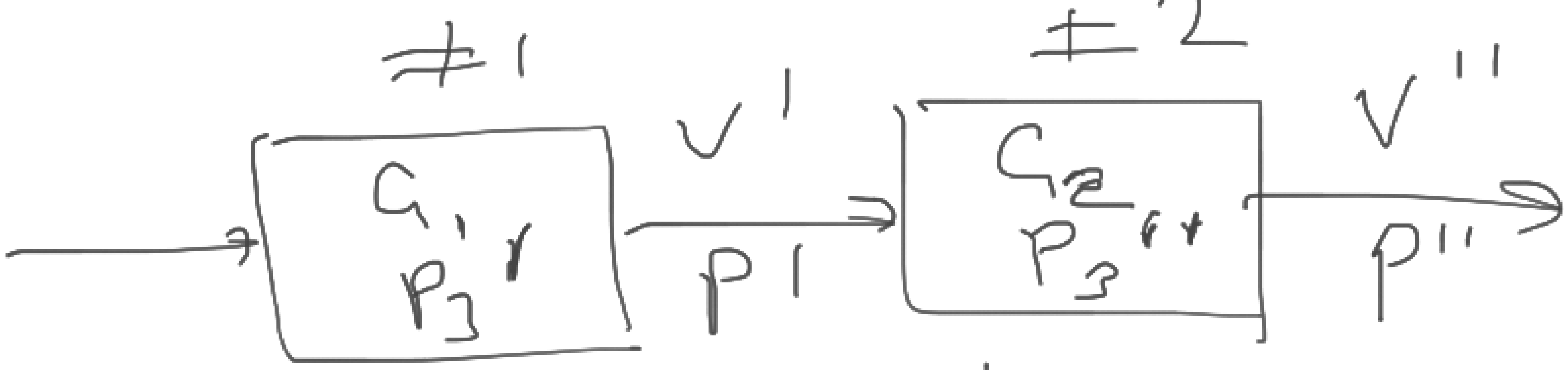
P_3 (or IIP3) of Cascaded Systems

Recall

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots$$

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots$$

$$P_3 = \;??$$



G_1 = gain of system #1

G_2 = " " " #2

P_3' = P_3 of system #1

P_3'' = " " #2

3rd order distortion power at O/P of first stage is

$$P'_{2\omega_1-\omega_2} = \frac{(P'_{\omega_1})^3}{(P_3')^2} \quad (6) \quad [\text{from } (5)]$$

where P'_{ω_1} is desired power ω_1 at O/P of first stage.

⇒ Voltage associated with this power is

$$V'_{2\omega_1-\omega_2} = \sqrt{P'_{2\omega_1-\omega_2} \cdot Z_0}, \quad Z_0 = \text{system impedance } (\Omega)$$

$$[\because V = \sqrt{PR}]$$

$$\Rightarrow V'_{2\omega_1-\omega_2} = \frac{\sqrt{(P'_{\omega_1})^3 Z_0}}{P_3'} \quad \text{— from } (6)$$

Total 3rd order distortion voltage o/p of second stage is [sum] this voltage times voltage gain of 2nd stage (multiplication) +

distortion voltage of 2nd stage & amplified at o/p of 2nd stage

i.e., total distortion voltage

$$V''_{2\omega_1-\omega_2} = \sqrt{\frac{G_2 (P'_{\omega_1})^3 Z_0}{P_3'}} + \sqrt{\frac{(P''_{\omega_1})^3 Z_0}{P_3''}}$$

voltage sum ∴ sq root

first stage

2nd stage

Since $P''_{\omega_1} = G_2 P'_{\omega_1} \Rightarrow P'_{\omega_1} = \frac{P''_{\omega_1}}{G_2}$

$$V''_{2\omega_1-\omega_2} = \frac{\sqrt{G_2\left(\frac{P''_{\omega_1}}{G_2}\right)^3 Z_0}}{P_3'} + \frac{\sqrt{(P''_{\omega_1})^3 Z_0}}{P_3''}$$

$$= \frac{\sqrt{(P''_{\omega_1})^3 Z_0}}{G_2 P_3'} + \frac{\sqrt{(P''_{\omega_1})^3 Z_0}}{P_3''}$$

$$V''_{2\omega_1-\omega_2} = \left(\frac{1}{G_2 P_3'} + \frac{1}{P_3''}\right)\sqrt{(P''_{\omega_1})^3 Z_0}$$

$\therefore$ o/p pow =

$$P''_{2\omega_1-\omega_2} = \frac{\left(V''_{2\omega_1-\omega_2}\right)^2}{Z_0} \qquad \left[\because P = \frac{V^2}{R}\right]$$

$$\Rightarrow P''_{2\omega_1-\omega_2} = \left(\frac{1}{G_2 P_3'} + \frac{1}{P_3''}\right)^2 \cdot \left(P''_{\omega_1}\right)^3$$

$$= \left(P''_{\omega_1}\right)^3 / \left(P_3\right)^2,$$

where P_3 (3rd order intercept pt. of cascaded system) $\boxed{P_3 = \left(\frac{1}{G_2 P_3'} + \frac{1}{P_3''}\right)^{-1}}$

$$\Rightarrow \boxed{P_3 = \left(\frac{1}{G_2 P_3'} + \frac{1}{P_3''}\right)^{-1} \text{ for 2 stages}}$$

Sim,

$$P_3 = \left(\frac{1}{G_3 G_2 P_3'} + \frac{1}{G_2 P_3''} + \frac{1}{P_3'''}\right)^{-1} \text{ for 3 stages}$$

& $$P_3 = \left(\frac{1}{G_4 G_3 G_2 P_3'} + \frac{1}{G_3 G_2 P_3''} + \frac{1}{G_2 P_3'''} + \frac{1}{P_4''''}\right)$$

for 4 stages

$\Rightarrow$ Last system gain is important

& first " " " not important

(In NF, it was otherwise)

Example

Gain $\quad G_1 = \cancel{20\,dB} = 100 \qquad G_2 = -6\,dB = \frac{1}{4} = 0.25$

IIP$_3$ pt $\quad P_3' = 7\,dBm = 5 \qquad P_3'' = 22\,dBm = 160\,mW$

$\cancel{20\,dBm = 100\,mW}$

$\cancel{23\,dBm = 200\,mW}$

$P_3 = ?$

$$P_3 = \left(\frac{1}{G_2 P_3'} + \frac{1}{P_3''}\right)^{-1} = \left(\frac{1}{0.25 \times 160} + \frac{1}{5}\right)^{-1}$$

↳ all factor $\quad = 4.4\,mW$

$$\boxed{P_3 = 6.4\,dBm}$$