Relativity and Electromagnetism

Nature of charges

1/. Two types: Positive and negative.

2/. Occur in discrete amounts.

3/. Like charges repel and unlike charges attract.

4/. Charges of opposite signs are in exact balance both globally and LOCALLY.

5/. A single charge in isolation cannot be created or destroyed.

6/. A static charge gives rise to an electric field, according to Coulomb's inverse square law.

7/. Moving charges (a current) give rise to a magnetic field in addition to ~~the~~ an electric field - (Ampere).

8/. An <u>electric charge</u> is <u>relativistically invariant</u> (<u>like the speed of light</u>).

9/. Light is an electromagnetic wave.

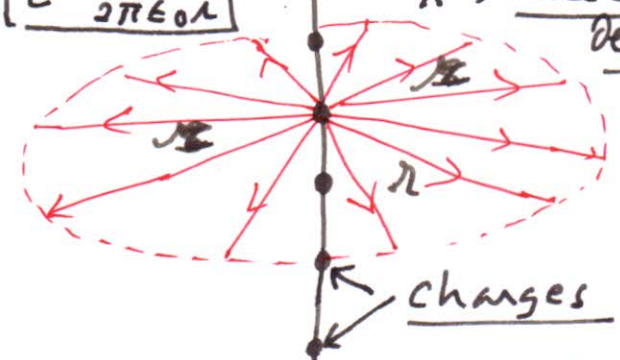
$\therefore$ RELATIVITY IS INTRINSIC TO ALL ELECTROMAGNETIC PHENOMENA.

$|\vec{\mathcal{E}}| \propto \frac{1}{r}$

Field lines on a plane

$\boxed{\mathcal{E} = \frac{\lambda}{2\pi\epsilon_0 r}}$

$\lambda \rightarrow$ <u>Line charge density</u>

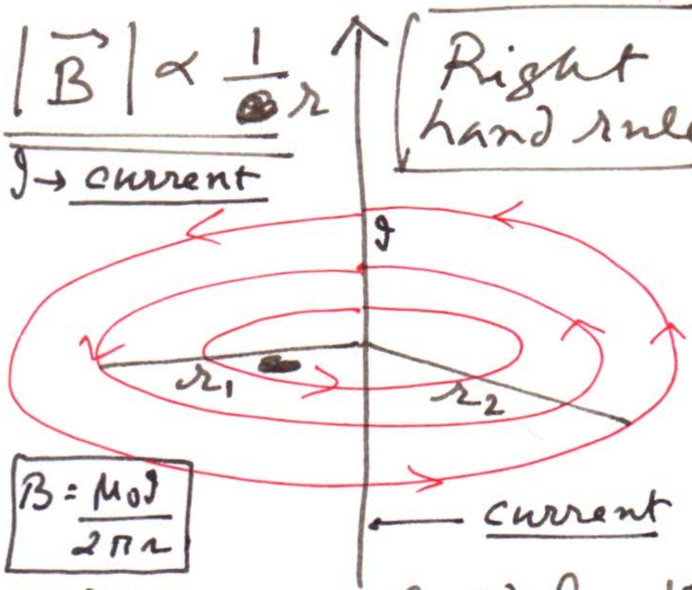


<u>Electric field</u> due to a line of charges.

$|\vec{B}| \propto \frac{1}{r}$

Right hand rule

$I \rightarrow$ <u>current</u>



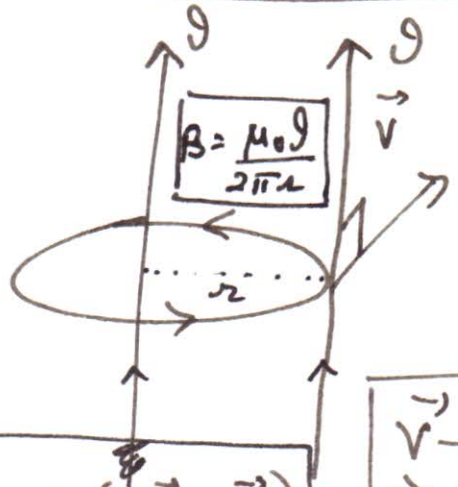
$\boxed{B = \frac{\mu_0 I}{2\pi r}}$

<u>Magnetic field</u> due to <u>charges flowing along a line (current)</u>

Between <u>two adjacent line conductors</u>:

a.) <u>Parallel currents attract.</u>

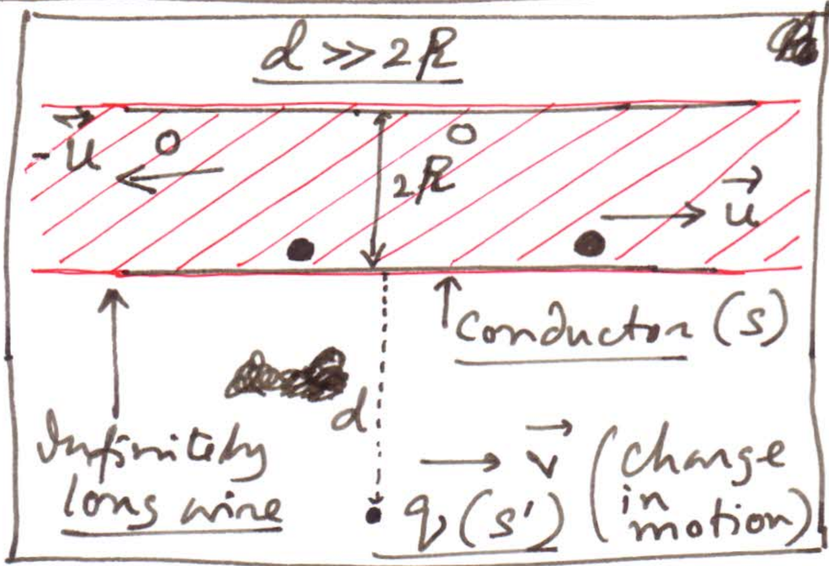
b.) <u>Anti-parallel currents repel.</u>



$\boxed{\vec{v} \rightarrow -\vec{v} \Rightarrow \text{repel.}}$

$\boxed{\text{Lorentz force, } \vec{F} = q(\vec{v} \times \vec{B})}$

Magnetism as a Relativistic Effect



Thin conducting wire, as the frame S (static)

- • → Positive charges
- o → Negative charges

q is a positive charge, as the frame S' (velocity $\vec{v}$)

$|\vec{v}| < |\vec{u}|$

Distance of q from the conductor (d) is much greater than the conductor width, (2R).

From the point of view of S:

i/. Spacing between positive charges contracts by a factor $\boxed{\sqrt{1 - u^2/c^2}}$.

ii/. Similar contraction also for the negative charges. $[\, u^2 = (-u)^2 \,]$.

iii/. Hence, there is NO local imbalance of charges anywhere in the conductor.

iv/. $\therefore$ NO electrical force acts on q.

From the point of view of S'

i/. Use $\boxed{u = \frac{u' + v}{1 + u'v/c^2}}$ $\Rightarrow$ $\boxed{u' = \frac{u - v}{1 - uv/c^2}}$

$u' \rightarrow$ Relative velocity of the charges in the conductor with respect to q.

ii/. $\boxed{u'_+ = \frac{u - v}{1 - uv/c^2}}$ and $\boxed{u'_- = -\frac{(u + v)}{1 + uv/c^2}}$

iii/. $\boxed{|u'_-| > |u'_+|}$ (Compare absolute values)

iv/. For the negative charges the length contraction factor is $\sqrt{1 - \frac{|u'_-|^2}{c^2}}$.

v/ For the positive charges the length contraction factor is $\sqrt{1 - \frac{|u'_+|^2}{c^2}}$.

vi/. Hence, the path for the negative ~~electrons~~ charges contracts more than the path for the positive charges.

vii/. Negative charges appear more densely lined up. Hence conductor is overall negatively charged.

viii/. Positive charge q feels an electrical attraction.

1/. Seen from the conductor (frame S), <u>No</u> force of an <u>electrical nature</u> Can <u>act on the charge q</u>.

2/. Seen from the charge q (frame S'), there is a <u>net electrical force</u> acting due to <u>local charge imbalance in the conductor</u>.

3/. But if there is a <u>force on q</u> ~~the~~ <u>seen in frame S'</u>, then there <u>MUST</u> be a <u>force on q seen in frame S</u>. ($\underline{F = F'/\gamma}$).

4/. This force in frame S is <u>NOT</u> purely <u>electrical</u>, but of a different type, due to <u>relativistic effects and charges in motion</u>. (Both $u \neq 0$ and $v \neq 0$)

5/. This is a <u>magnetic force</u>.

6/. <u>Magnetism is due to relativistic</u> ~~and~~ <u>behaviour of charges</u>.

Mathematical Discussion

From the point of view of frame S':

Force : $F' = q \times$ Electric field

(q is invariant in all frames)

Now, for a static line of charges,

by Gauss's law (Carl Gauss)

Electric field : $\mathcal{E} = \dfrac{\lambda_0}{2\pi \epsilon_0 d}$ | line charge density

where $\lambda_0 = \dfrac{\text{No. of charges lined up}}{\text{Unit length}}$

Relativistically : Unit length $\longrightarrow$ Contracted unit length.

Transforms

$\therefore$ For positive charges:

Contracted unit length $\equiv \sqrt{1 - \dfrac{|u_+'|^2}{c^2}} \times$ unit length.

For negative charges:

Contracted unit length $\equiv \sqrt{1 - \dfrac{|u_-'|^2}{c^2}} \times$ unit length

For positive charges:

$$\boxed{\lambda_0 \longrightarrow \frac{\lambda_0}{\sqrt{1-\frac{|u_+'|^2}{c^2}}} \equiv \lambda_+}$$

λ_+ is the line density of positive charges.

and for negative charges:

$$\boxed{\lambda_0 \longrightarrow \frac{\lambda_0}{\sqrt{1-\frac{|u_-'|^2}{c^2}}} \equiv \lambda_-}$$

λ_- is the line density of negative charges.

Total line charge density,

$$\boxed{\lambda_{tot} = \lambda_+ - \lambda_-}$$

Why the negative sign?

Now $\boxed{u_+' = \frac{u-v}{1-uv/c^2}}$ and $\boxed{u_-' = \frac{-(u+v)}{1+uv/c^2}}$

For λ_+

$$\text{So, } \frac{1\times\lambda_0}{\sqrt{1-\frac{|u_+'|^2}{c^2}}} = \frac{1\times\lambda_0}{\sqrt{1-\frac{(u-v)^2}{c^2(1-uv/c^2)^2}}}$$

$$= \frac{1\times\lambda_0}{\sqrt{1-\frac{(u-v)^2c^4}{c^2(c^2-uv)^2}}} = \frac{(c^2-uv)\lambda_0}{\sqrt{(c^2-uv)^2-c^2(u-v)^2}}$$

Denominator: $c^4-2uvc^2+u^2v^2-c^2(u^2-2uv+v^2)$

P.T.O.

$$= c^4 - \cancel{2uvc^2} + u^2v^2 - c^2u^2 + \cancel{2uvc^2} - c^2v^2$$

$$= c^2(c^2-u^2) + v^2(u^2-c^2)$$

$$= c^2(c^2-u^2) - v^2(c^2-u^2) = (c^2-v^2)(c^2-u^2)$$

Hence, $\boxed{\dfrac{(c^2-uv)\lambda_0}{\sqrt{(c^2-v^2)(c^2-u^2)}} = \lambda_+}$

For λ_-

Next, $\dfrac{1\times\lambda_0}{\sqrt{1-\dfrac{|u_-'|^2}{c^2}}} = \dfrac{1\times\lambda_0}{\sqrt{1-\dfrac{(u+v)^2}{c^2(1+uv/c^2)^2}}}$

$$= \frac{1\times\lambda_0}{\sqrt{1-\dfrac{(u+v)^2c^4}{c^2(c^2+uv)^2}}} = \frac{(c^2+uv)\lambda_0}{\sqrt{(c^2+uv)^2-c^2(u+v)^2}}$$

Denominator: $c^4 + 2uvc^2 + u^2v^2 - c^2(u^2+2uv+v^2)$

$$= c^4 + \cancel{2uvc^2} + u^2v^2 - c^2u^2 - \cancel{2uvc^2} - v^2c^2$$

$$= c^2(c^2-u^2) + v^2(u^2-c^2)$$

$$= c^2(c^2-u^2) - v^2(c^2-u^2) = (c^2-u^2)(c^2-v^2)$$

Hence, $\boxed{\dfrac{(c^2+uv)\lambda_0}{\sqrt{(c^2-v^2)(c^2-u^2)}} = \lambda_-}$

Since, $\lambda_{tot} = \lambda_+ - \lambda_-$

$$\lambda_{tot} = \lambda_0 \left[\frac{(\cancel{c^2} - uv) - (\cancel{c^2} + uv)}{\sqrt{(c^2-u^2)(c^2-v^2)}} \right]$$

$$\Rightarrow \boxed{\lambda_{tot} = \lambda_0 \left[\frac{-2uv}{\sqrt{(c^2-u^2)(c^2-v^2)}} \right]}$$

or

$$\boxed{\lambda_{tot} = \frac{-2uv\lambda_0}{c^2\sqrt{1-u^2/c^2}\sqrt{1-v^2/c^2}}}$$

Overall negative sign

Now $\boxed{\mathcal{E} = \frac{\lambda_{tot}}{2\pi\epsilon_0 d}}$

Substitute $\lambda_0 \rightarrow \lambda_{tot}$

$\boxed{\text{when } v=0,\ \lambda_{tot} = 0}$

$$\therefore \mathcal{E} = \frac{1}{2\pi\epsilon_0 d} \times \frac{-2uv\lambda_0}{c^2\sqrt{1-u^2/c^2}\sqrt{1-v^2/c^2}}$$

$$\boxed{c^2 = \frac{1}{\mu_0\epsilon_0} \Rightarrow \frac{1}{\epsilon_0 c^2} = \mu_0}$$

A magnetic constant

$$\therefore \mathcal{E} = \left(\frac{-\mu_0}{2\pi d}\right)\cdot\left(\frac{2\lambda_0 u}{\sqrt{1-u^2/c^2}}\right)\left(\frac{v}{\sqrt{1-v^2/c^2}}\right)$$

Now $\boxed{F' = q\mathcal{E}}$ $\rightarrow$ Force from the point of view of frame S'.

$$\therefore \quad \boxed{F' = -\left(\frac{\mu_0}{2\pi d}\right)\left(\frac{2\lambda_0 u}{\sqrt{1-u^2/c^2}}\right)\frac{qv}{\sqrt{1-v^2/c^2}}}$$

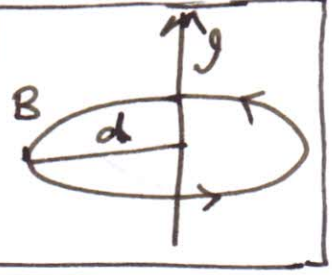
$$\boxed{\underline{\text{Current}} = \frac{2\lambda_0 u}{\sqrt{1-u^2/c^2}} = I}$$

Why?

1. $u-(-u) = 2u$
2. $\lambda_0 u$ has the physical dimension of current.

$$\therefore \quad \boxed{F' = -\frac{qv}{\sqrt{1-v^2/c^2}}\left(\frac{\mu_0 I}{2\pi d}\right)}$$

Magnetic field, (Ampere's Law) → $\boxed{B = \dfrac{\mu_0 I}{2\pi d}}$



Now from the point of view of frame s,

In Perpendicular direction

→ $\boxed{F = \cancel{F'/\gamma}}$ where $\boxed{\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}}$

$$\therefore \quad \boxed{\frac{F'}{\gamma} = F'\sqrt{1-v^2/c^2} = F}$$

$$\boxed{\vec{F} = q(\vec{v}\times\vec{B})}$$

LORENTZ force

$$\Rightarrow \quad \boxed{F = -qv\left(\frac{\mu_0 I}{2\pi d}\right)}$$

The negative sign means attraction.

Magnetic force due to relativistic effects.