SOLUTION

Just as $z = Z/Z_0$, so $y = Y/Y_0$; this may be very simply checked.
Therefore

$$y = \frac{0.004 - j0.002}{0.0033} = 1.21 - j0.61$$

Hence the normalized susceptance required to cancel the load's normalized susceptance is $+j0.61$. From the chart, the length of line required to give a normalized input admittance of 0.61 when the line is short-circuited is given by

$$\text{Length} = 0.250 + 0.087 = 0.337\lambda$$

Since the line has air as its dielectric, the velocity factor is 1.
Therefore

$$v_c = f\lambda$$

$$\lambda = \frac{v_c}{f} = \frac{300 \times 10^6}{150 \times 10^6} = 2 \text{ m}$$

$$\text{Length} = 0.337\ \lambda = 0.337 \times 200 = 67.4 \text{ cm}$$

7-2.2 Problem Solution

In most cases, the best method of explaining problem solution with the Smith chart is to show how an actual problem of a given type is solved. In other cases, a procedure may be established without prior reference to a specific problem. Both methods of approach will be used here.

Matching of load to line with a quarter-wave transformer

EXAMPLE 7-7 Refer to Figure 7-13. A load $Z_L = (100 - j50)\ \Omega$ is connected to a line whose $Z_0 = 75\ \Omega$. Calculate

(a) The point, nearest to the load, at which a quarter-wave transformer may be inserted to provide correct matching
(b) The Z_0' of the transmission line to be used for the transformer

SOLUTION

(a) Normalize the load impedance with respect to the line; thus $(100 - j50)/75 = 1.33 - j0.67$. Plot this point *(A)* on the Smith chart. Draw a circle whose center lies at the center of the chart, passing through the plotted point. As a check, note that this circle should correspond to an SWR of just under 1.9. Moving toward the generator, i.e., clockwise, find the nearest point at which the line impedance is purely resistive (this is the intersection of the drawn circle with the only straight line on the chart). Around the rim of the chart, measure the distance from the load to this point *(B)*; this distance $= 0.500 - 0.316 = 0.184\ \lambda$. Read off the normalized resistance at *B*, here $r = 0.53$, and convert this normalized resistance into an actual resistance by multiplying by the Z_0 of the line. Here $R = 0.53 \times 75 = 39.8\ \Omega$.

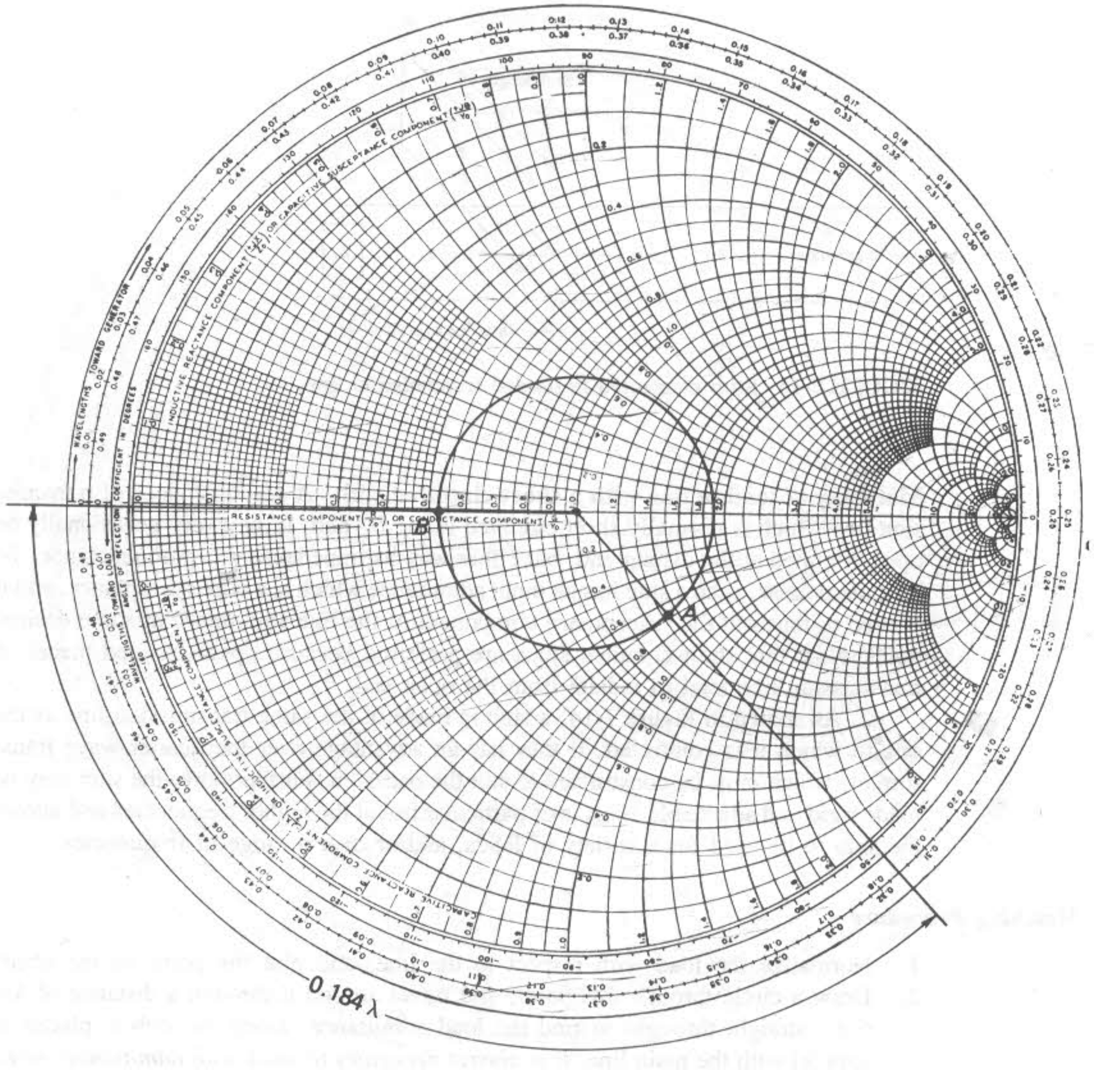

FIGURE 7-13 Smith chart solution of Example 7-7, matching with a quarter-wave transformer.

(*b*) 39.8 Ω is the resistance which the λ/4 transformer will have to match to the 75-Ω line, and from this point the procedure is as in Example 7-4. Therefore

$$Z_0' = \sqrt{Z_0 Z_R} = \sqrt{75 \times 39.8} = 54.5\ \Omega$$

Students at this point are urged to follow the same procedure to solve an example with identical requirements, but now $Z_L = (250 + j450)\ \Omega$ and $Z_0 = 300\ \Omega$. The answers are distance = 0.080 λ and $Z_0' = 656\ \Omega$.

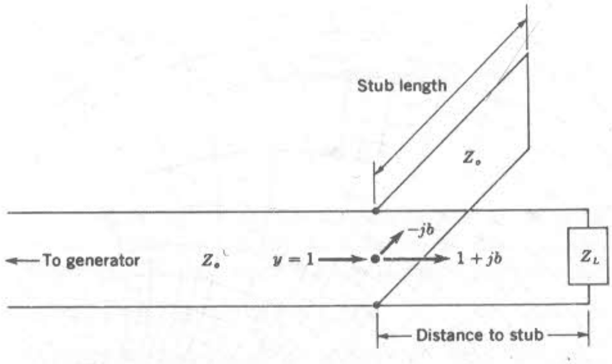

FIGURE 7-14 Stub connected to loaded transmission line.

Matching of load to line with a short-circuited stub A *stub* is a piece of transmission line which is normally short-circuited at the far end. It may very occasionally be open-circuited at the distant end, but either way its impedance is a pure reactance. To be quite precise, such a stub has an input admittance which is a pure susceptance, and it is used to tune out the susceptance component of the line admittance at some desired point. Note that short-circuited stubs are preferred because open-circuited pieces of transmission line tend to radiate from the open end.

As shown in Figure 7-14, a stub is made of the same transmission line as the one to which it is connected. It thus has an advantage over the quarter-wave transformer, which must be constructed to suit the occasion. Furthermore, the stub may be made rigid and adjustable. This is of particular use at the higher frequencies and allows the stub to be used for a variety of loads, and/or over a range of frequencies.

Matching Procedure

1. Normalize the load with respect to the line, and plot the point on the chart.
2. Draw a circle through this point, and travel around it through a distance of $\lambda/4$ (i.e., straight through) to find the load admittance. Since the stub is placed in parallel with the main line, *it is always necessary to work with admittances when making stub calculations*.
3. *Starting from this new point (now using the Smith chart as an admittance chart),* find the point nearest to the load at which the normalized admittance is $1 \pm jb$. *This point is the intersection of the drawn circle with the* $r = 1$ *circle,* which is the only circle through the center of the chart. This is the point at which a stub designed to tune out the $\pm jb$ component will be placed. Read off the distance traveled around the circumference of the chart; this is the distance to the stub.
4. To find the length of the short-circuited stub, start from the point $\infty, j\,\infty$ on the right-hand rim of the chart, since that is the admittance of a short circuit.
5. Traveling clockwise around the circumference of the chart, find the point at which the susceptance tunes out the $\pm jb$ susceptance of the line at the point at which the stub is to be connected. For example, if the line admittance is $1 + j0.43$, the

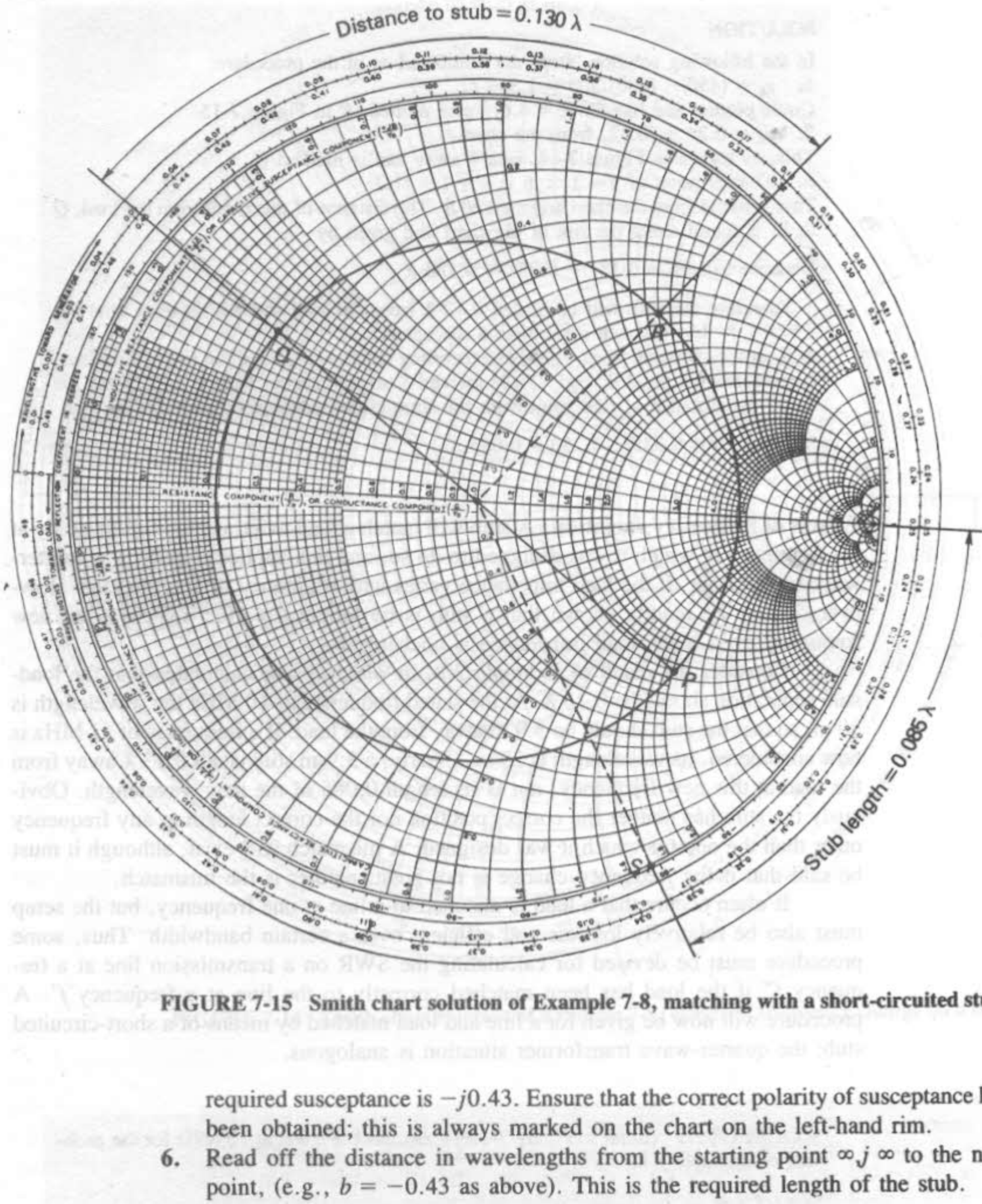

FIGURE 7-15 Smith chart solution of Example 7-8, matching with a short-circuited stub.

required susceptance is $-j0.43$. Ensure that the correct polarity of susceptance has been obtained; this is always marked on the chart on the left-hand rim.

6. Read off the distance in wavelengths from the starting point $\infty, j\,\infty$ to the new point, (e.g., $b = -0.43$ as above). This is the required length of the stub.

EXAMPLE 7-8 (Refer to Figure 7-15.) A series *RC* combination, having an impedance $Z_L = (450 - j600)\ \Omega$ at 10 MHz, is connected to a 300-Ω line. Calculate the position and length of a short-circuited stub designed to match this load to the line.

SOLUTION

In the following solution, steps are numbered as in the procedure:

1. $z_L = (450 - j600)/300 = 1.5 - j2$.

Circle plotted and has SWR = 4.6. Point plotted, P in Figure 7-15.

2. $y_L = 0.24 + j0.32$, from the chart.

This, as shown in Figure 7-14, is $\lambda/4$ away and is marked Q.

3. Nearest point of $y = 1 \pm jb$ is $y = 1 + j1.7$.

This is found from the chart and marked R. The distance of this point from the load, Q to R, is found along the rim of the chart and given by

Distance to stub = $0.181 - 0.051 = 0.130\ \lambda$

Therefore the stub will be placed $0.13\ \lambda$ from the load and will have to tune out $b = +1.7$; thus the stub must have a susceptance of -1.7.

4, 5, and 6. Starting from $\infty, j\infty$, and traveling clockwise around the rim of the chart, one reaches the point $0, -j1.7$; it is marked S on the chart of Figure 7-15. From the chart, the distance of this point from the short-circuit admittance point is

Stub length = $0.335 - 0.250 = 0.085\ \lambda$

Effects of frequency variation A stub will match a load to a transmission line only at the frequency at which it was designed to do so, and this applies equally to a quarter-wave transformer. If the load impedance varies with frequency, this is obvious. However, it may be readily shown that a stub is no longer a perfect match at the new frequency even if the load impedance is unchanged.

Consider the result of Example 7-8, in which it was calculated that the load-stub separation should be $0.13\ \lambda$. At the stated frequency of 10 MHz the wavelength is 30 m, so that the stub should be 3.9 m away from the load. If a frequency of 12 MHz is now considered, its wavelength is 25 m. Clearly, a 3.9-m stub is not $0.13\ \lambda$ away from the load at this new frequency, nor is its length 0.085 of the new wavelength. Obviously the stub has neither the correct position nor the correct length at any frequency other than the one for which it was designed. A mismatch will exist, although it must be said that if the frequency change is not great, neither is the mismatch.

It often occurs that a load is matched to a line at one frequency, but the setup must also be relatively lossless and efficient over a certain bandwidth. Thus, some procedure must be devised for calculating the SWR on a transmission line at a frequency f'' if the load has been matched correctly to the line at a frequency f'. A procedure will now be given for a line and load matched by means of a short-circuited stub; the quarter-wave transformer situation is analogous.

EXAMPLE 7-9 (Refer to Figure 7-16.) Calculate the SWR at 12 MHz for the problem of Example 7-8.

SOLUTION

For the purpose of the procedure, it is assumed that the calculation involving the position and length of a stub has been made at a frequency f', and it is now necessary to calculate the SWR on the main line at f''. Matter referring specifically to the example will be shown.