IT-616 (Design & Analysis of Algorithms): Tutorial #9

1. Suppose we are given a digraph G with n vertices, and let M be the $n \times n$ adjacency matrix corresponding to G. Let M^2 be defined, for $1 \leq i, j \leq n$, as follows:

   $M^2(i,j) = M(i,1) \star M(1,j) + \cdots + M(i,n) \star M(n,j)$.

   If '+' is the boolean OR and '$\star$' is the boolean AND, then what does $M^2(i,j) = 1$ imply?

2. Show how to modify Dijkstra's algorithm for the case when the graph is directed and we want to compute shortest directed paths from the source vertex to all the other vertices.

3. The all-pair shortest path algorithm uses $O(n^3)$ space. Describe a version of this algorithm that uses $O(n^2)$ space.

4. The all-pair shortest path algorithm computes only shortest path distances, not actual paths. Describe a version of this algorithm that outputs the set of all shortest paths between each pair of vertices in a directed graph. Your algorithm should still run in $O(n^3)$ time.

5. How will you gurantee that the Kruskal algorithm and the Prim algorithm indeed find the minimum spanning tree.

6. Show that if all the weights in a connected weighted graph G are distinct, then there is exactly one minimum spanning tree for G.

7. Give an example of a weighted digraph G with negative-weight edges, but

no negative-weight cycle, such that the Dijkstra's algorithm incorrectly computes the shortest path distances from some start vertex v.

8. Suppose we are given a weighted graph G with n vertices and m edges, such that the weight of each edge is an integer between 0 and n. Show that we can find a minimum spanning tree for G in $O(n \log^\star n)$ time, where the function $\log^\star n$ is the inverse of the *tower-of-twos* function.

9. Suppose you are given a diagram of a telephone network, which is a graph G whose vertices represent switching centers, and whose edges represent communication lines between two centers. The edges are marked by their bandwidth. The bandwidth of a path is the bandwidth of its lowest bandwidth edge. Give an algorithm that, given a diagram and two switching centers a and b, will output the maximum bandwidth of a path between a and b.

10. Design an efficient algorithm for finding a longest directed path from a vertex s to a vertex t of an acyclic weighted digraph G. Also analyze the time complexity of your algorithm.