- Given a natural number n.
- Given a mathematical object x for which multipication is defined.

Determine how many multipications are required to evaluate x^n

Naive method: $x^n = x \times x \times \cdots \times x$ (i.e., multiply $n - 1$ times)

Example: To determine x^4, three multipications are required

Binary method

- $x^n = x^{n/2}x^{n/2}$, when n is even.
- $x^n = x^{\lfloor n/2 \rfloor}x^{\lfloor n/2 \rfloor} \times x$, when n is odd.

- Example: x^{23} requires 7 multipications
- Example: x^{15} requires 6 multipications
- Example: x^{33} requires 6 multipications.

Is the binary method optimum ?

Binary method

- $x^n = x^{n/2}x^{n/2}$, when n is even.
- $x^n = x^{\lfloor n/2 \rfloor}x^{\lfloor n/2 \rfloor} \times x$, when n is odd.

- Example: x^{23} requires 7 multipications
- Example: x^{15} requires 6 multipications
- Example: x^{33} requires 6 multipications.

Is the binary method optimum ?
No it is not optimum.

By prime factors

- $x^n = (x^p)^q$, where $n = p \times q$ and p is the smallest prime factor of n.
- $x^n = x^{n-1} \times x$, when n is a prime number.
- Example: x^{33} requires 7 multipications, while x^{15} requires 5 multipications.

Is this method optimum ?

By prime factors

- $x^n = (x^p)^q$, where $n = p \times q$ and p is the smallest prime factor of n.
- $x^n = x^{n-1} \times x$, when n is a prime number.

- Example: x^{33} requires 7 multipications, while x^{15} requires 5 multipications.

Is this method optimum ?
No it is not optimum either.