

1. Draw a simple connected directed graph with 8 vertices and 16 edges such that the in-degree and the out-degree of each vertex is 2. Show that there is a single non-simple cycle that includes all the edges of your graph.
  2. Suppose we are given an undirected graph  $G$  with  $n$  vertices and  $m$  edges. How many back edges will the DFS on  $G$  produce.
  3. Is it possible to draw a graph  $G$  with 12 vertices, 66 edges, and 3 connected components. Give a brief explanation in support of your answer.
  4. Explain why the DFS traversal runs in  $\Theta(n^2)$  time on an  $n$ -vertex simple graph that is represented with an adjacency matrix.
  5. Let  $G$  be an undirected graph with  $n$  vertices and  $m$  edges. Prove that if  $G$  is connected then  $m \geq (n - 1)$ .
  6. Prove that after performing DFS on an undirected connected graph, the discovery-edges form a spanning tree.
  7. Given a directed graph  $G = (V, E)$ , how will you determine in  $O(|V| + |E|)$  time whether the graph is strongly connected or not.
  8. Give two different algorithms for finding the transitive closure of a graph, and compare their complexities.
  9. Prove that a digraph admits a topological ordering iff it is a DAG.
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10. Let  $T$  be the spanning tree rooted at the start vertex produced by the DFS of a connected, undirected graph  $G$ . Argue why every edge of  $G$ , not in  $T$ , is a back-edge.