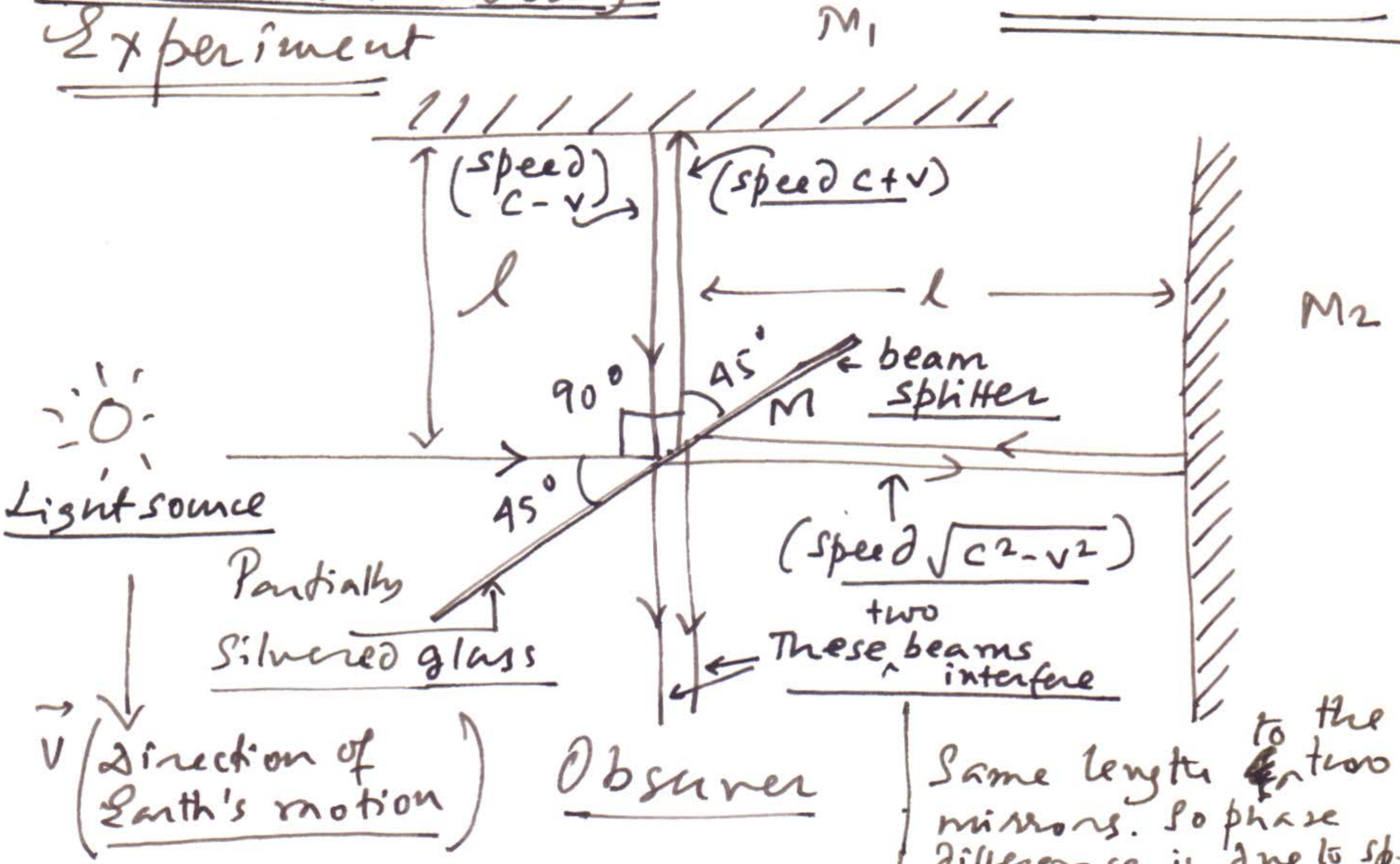
Michelson-Morley Experiment

(Albert Michelson + Edward Morley)



Same length to the two mirrors. So phase difference is due to speed difference.

For reflection off mirror M_1

$$\boxed{t_1 = \frac{l}{c+v} + \frac{l}{c-v}}$$

$$\frac{l(c-v)}{c^2-v^2} + \frac{l(c+v)}{c^2-v^2} = \frac{2lc}{c^2-v^2}$$

$$\Rightarrow \boxed{t_1 = \frac{2lc}{c^2-v^2}}$$

Define $\boxed{\beta = \frac{v}{c}}$ $\Rightarrow t_1 = \frac{2lc}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-1}$

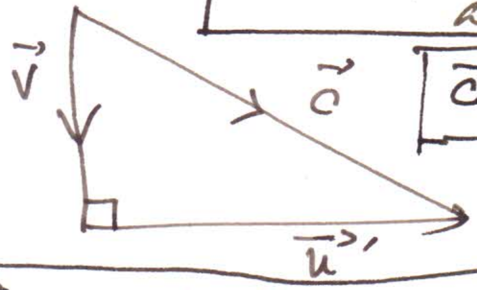
$$\Rightarrow \boxed{t_1 \simeq \frac{2l}{c}\left(1+\beta^2\right)} \quad \because \beta \ll 1$$

Binomial Expansion : $(1+x)^n \simeq 1 + nx + \cdots$

For mirror M_2

$\vec{c} \rightarrow$ velocity of light with respect to aether.

$$\boxed{c^2 = u'^2 + v^2}$$ Pythagoras Theorem



$$\boxed{\vec{c} = \vec{v} + \vec{u}'}$$

$$\Rightarrow \boxed{u'^2 = c^2 - v^2}$$

$\vec{v} \rightarrow$ velocity of earth with respect to aether.

$$t_2 = \frac{2l}{\sqrt{c^2 - v^2}} = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\Rightarrow \boxed{t_2 \simeq \frac{2l}{c}\left(1 + \frac{\beta^2}{2}\right)}$$

$$\therefore \Delta t = t_1 - t_2 \simeq \cancel{\frac{2l}{c}} + \frac{2l\beta^2}{c} - \cancel{\frac{2l}{c}} - \frac{l\beta^2}{c}$$

$$\Rightarrow \boxed{\Delta t \simeq \frac{l\beta^2}{c}}$$

$T \rightarrow$ Time period

$$\boxed{N = \frac{\Delta t}{T} = \frac{l\beta^2}{Tc}}$$

What is N?

No. of cycles

$$\boxed{T = \frac{1}{\nu}} \qquad \boxed{c = \nu\lambda} \Rightarrow T = \frac{\lambda}{c}$$

$$\Rightarrow \boxed{Tc = \lambda}$$

$$\Rightarrow \boxed{N = \frac{l\beta^2}{\lambda}}$$

$\lambda \rightarrow$ wavelength

One cycle $\Rightarrow$ 2π phase, λ wavelength, T time period

To Correct for unequal arm lengths.

Rotate apparatus by 90°.

$$\boxed{t_1 \longrightarrow t_2}\,, \qquad \boxed{t_2 \longrightarrow t_1}$$

$$\Rightarrow \boxed{\Delta t \longrightarrow -\Delta t}\,.$$

Mirrors M_1 and M_2 are exchanged

$$\Delta N = \frac{\Delta t}{T} - \frac{(-\Delta t)}{T} = \frac{2\Delta t}{T} = 2N = \frac{2l\beta^2}{\lambda}$$

$$\Rightarrow \boxed{\Delta N = \frac{2l\beta^2}{\lambda}}$$

$l = 11$ m, $\lambda = 590$ nm, $\beta \simeq 10^{-4}$

$$\boxed{\Delta N \simeq 0.4}$$

No such result was found! (NULL RESULT)

Light has the same speed c in all directions

> **Einstein:** There is no such thing as the aether frame.

Before Einstein

1/. Newton's laws of mechanics hold true (be invariant) in all inertial frames.

2/. Light travels with the same speed c in all directions in only ONE reference frame (the laws of electromagnetic theory are valid in this frame only), which is the aether frame.

Einstein: Both the laws of ~~[illegible]~~ mechanics, and the laws of electromagnetic theory MUST BE INVARIANT in ALL inertial frames.

(Non Newtonian)

Inertial frames have to be redefined?

Einstein's Postulates of Relativity

1/. S is an inertial frame. If S′ is another frame moving with a constant velocity with respect to S, then S′ is also an inertial frame.

(Inertial frame ↔ All the laws of physics are invariant)

Consequences: i.) Newton's first law (The law of inertia) holds true only. Not the other two.

ii.) There is no such thing as absolute motion. All motion is relative. There is no preferred frame of reference.

2/. In all inertial frames light travels through the vacuum with the same speed, c. ⊛

Consequences:

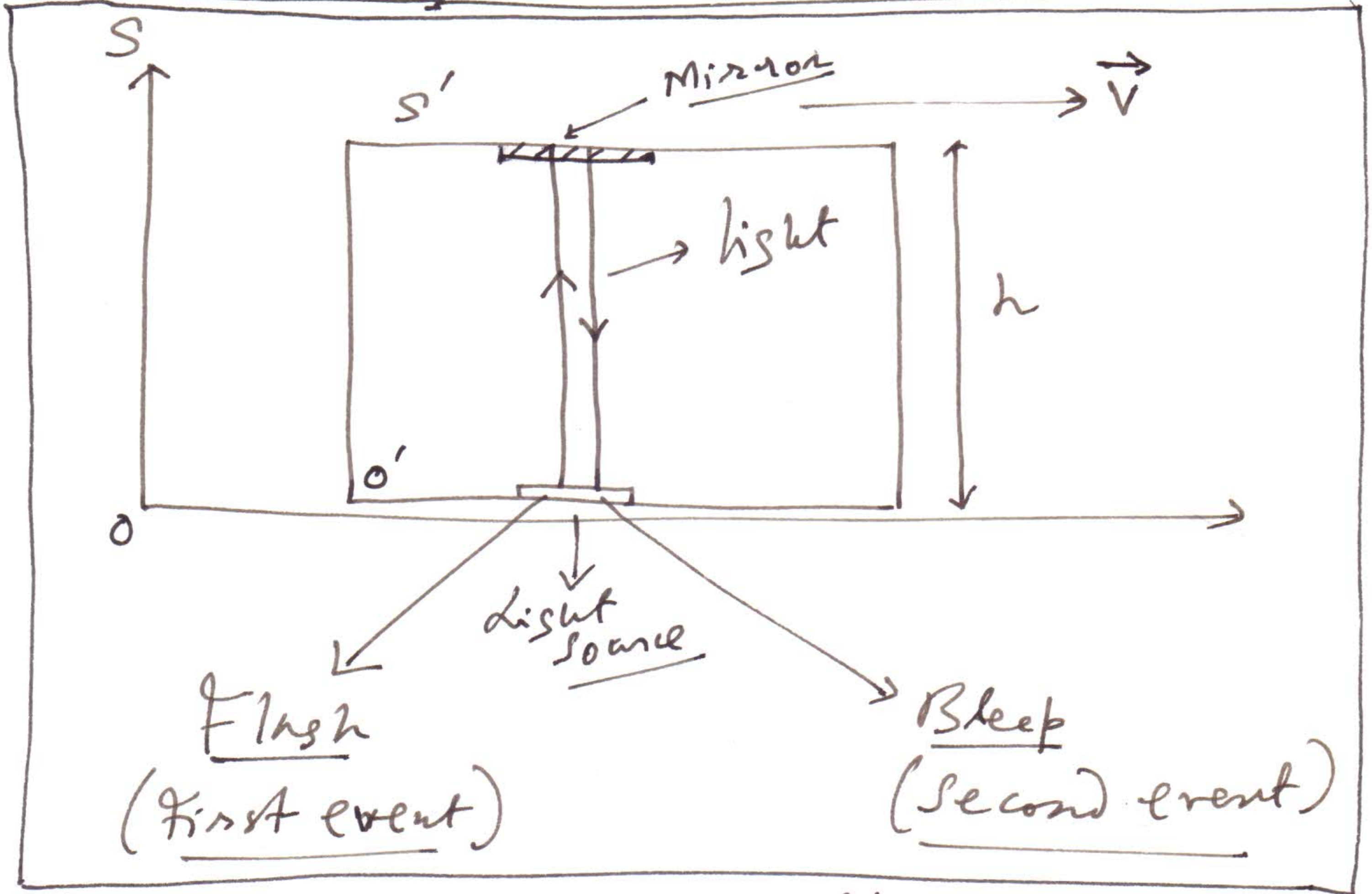
i.) Universality of the speed of light. (Not time!).

ii.) Effect can be observed only when objects move close to the speed of light. Not to be felt under ordinary conditions, in which speeds are much less than c.

⊛ Also light travels in all directions with the same speed c (in vacuum)

Relativity of Time

Clock : Marks time through succession of regular events.

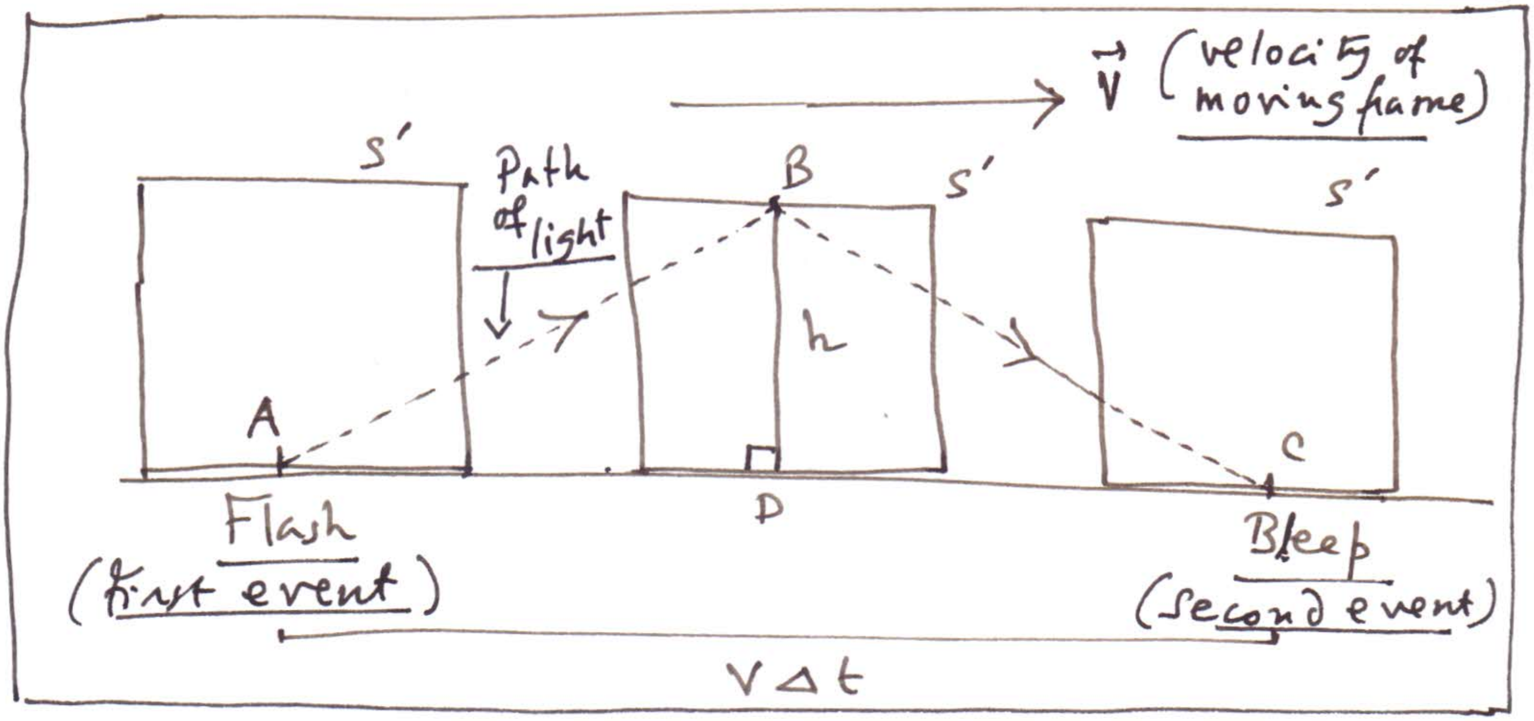


$$\boxed{\Delta t' = \frac{2h}{c}}$$

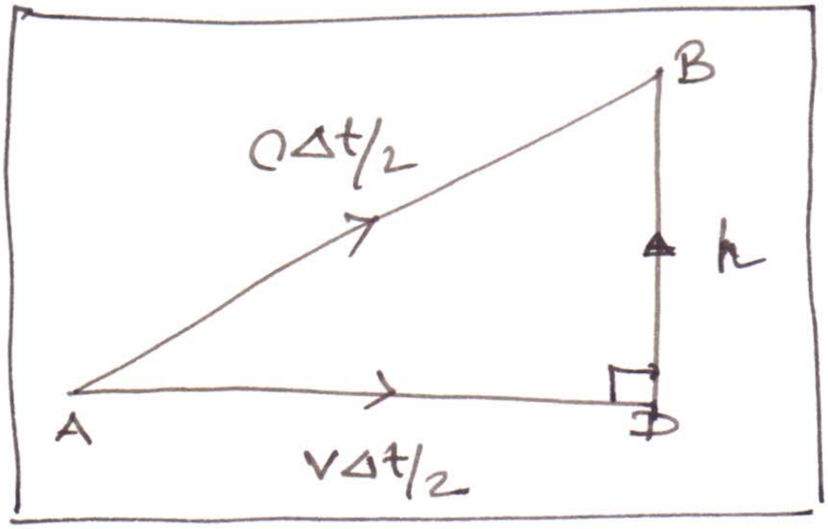
(In S' both events have occurred at the same spatial point.)

The speed of light in frame S' is c. (Einstein → Universality of c)

To second observer in S frame



Elapsed time is Δt



KEY POINT:

The speed of light to the observer in S is ALSO c ($c > v$)

(Einstein)

$$\boxed{\left(c\,\frac{\Delta t}{2}\right)^2 = h^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

Pythagoras' Theorem

$$(\Delta t)^2 \left[\frac{c^2}{4} - \frac{v^2}{4} \right] = h^2 \qquad \boxed{(\Delta t)^2 = \frac{4h^2}{c^2 - v^2}}$$

$$\Rightarrow \quad \Delta t = \frac{2h}{\sqrt{c^2 - v^2}}$$

$$\Rightarrow \quad \Delta t = \frac{2h}{c} \cdot \frac{1}{\sqrt{1-(v/c)^2}}$$

Write $\boxed{\beta = v/c}$ → Dimensionless quantity

$$\Delta t = \frac{2h}{c} \cdot \frac{1}{\sqrt{1-\beta^2}}$$

But $\boxed{\Delta t' = \frac{2h}{c}}$

$$\Rightarrow \boxed{\Delta t = \Delta t' \times \gamma}$$

Two frames record different time intervals between the same two events.

where $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$ → Dimensionless quantity

$\boxed{\text{Can } v > c \text{ ? No.}}$ (If $v > c$, then γ would be imaginary)

Consequences :

1/. Maximum speed attainable in the Universe is C.

2/. $\boxed{\Delta t' = \Delta t_0}$ (In the moving frame)

Proper time interval. (Two events occurring at the same spatial point).

$$\therefore \boxed{\Delta t = \gamma \Delta t' = \gamma \Delta t_0}$$

If $\boxed{v < c}$, $\boxed{\beta < 1} \Rightarrow \boxed{\gamma > 1}$

TIME DILATION $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$

3/. A moving clock is OBSERVED to run slow (with respect to its proper time)

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}} \quad \boxed{\Delta t > \Delta t_0}$$

4/. Time is also a "measure" of space.

<u>Example</u>:

For typical <u>man-made high speeds</u> (<u>fighter aircrafts</u>)

$$\boxed{v \sim 300\ ms^{-1} = 3 \times 10^{2}\ ms^{-1}}$$

$\downarrow$ <u>sound speed</u>

$$\boxed{c = 3 \times 10^{8}\ ms^{-1}}$$

$$\boxed{\beta = (v/c) = 10^{-6}} \Rightarrow \left(\frac{v}{c}\right)^2 = 10^{-12}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = (1-\beta^2)^{-1/2}$$

$$\longrightarrow \simeq 1 + \frac{1}{2} \times 10^{-12}$$

<u>binomial expansion</u>

$$\boxed{(1+x)^n \simeq 1 + nx \quad (\text{if } \underline{x \ll 1})}$$

$$\Delta t = \gamma \Delta t_0 \simeq \Delta t_0 + \Delta t_0 \times \frac{1}{2} \times 10^{-12}$$

$$\Delta t - \Delta t_0 \simeq \Delta t_0 \times \frac{1}{2} \times 10^{-12}$$

$(\Rightarrow \underline{\Delta t \neq \Delta t_0})$

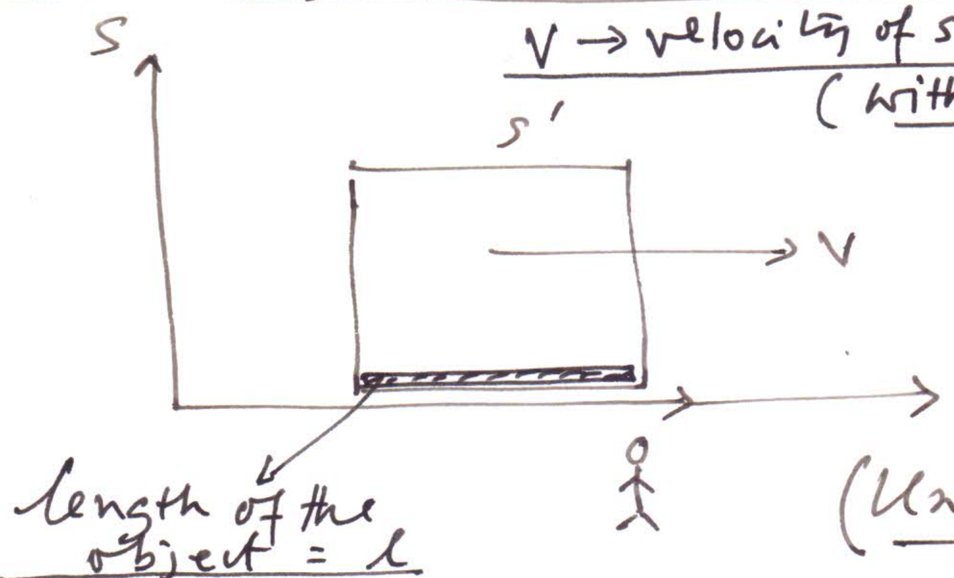
$\underline{\Delta t_0 = 1\ hr.}$

$$\Delta t - \Delta t_0 \simeq 5 \times 10^{-13}\ hr \simeq 2\ ns$$

$(\underline{2 \times 10^{-9}\ s})$.

$$\boxed{\text{Very small effect at low speeds.}}$$

Length Contraction

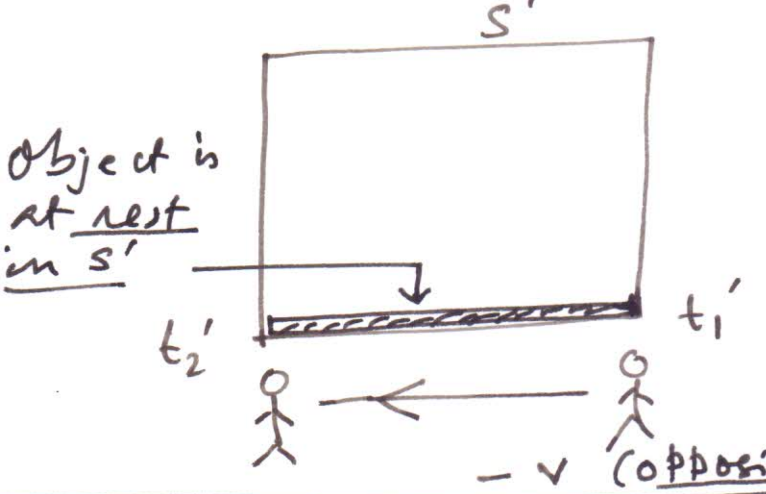


$v \rightarrow$ velocity of s' with respect to s (without prime)

$$\boxed{\Delta t = t_2 - t_1}$$

$$\boxed{l = v \Delta t}$$

(Unprimed frame)



$-v$ (opposite motion)

$$\boxed{\Delta t' = t_2' - t_1'}$$

$$\Rightarrow \boxed{l' = v \Delta t'}$$

$\boxed{\text{Proper time interval is } \Delta t.}$

$$\therefore \boxed{\Delta t' = \gamma \Delta t} \rightarrow \text{Time dilation}$$

$$\Rightarrow l' = v \Delta t' = \gamma v \Delta t = \gamma l$$

$$\therefore \boxed{l = \frac{l'}{\gamma}} \rightarrow \text{Length Contraction}$$

$(\gamma > 1)$.

Proper length :

Length of an object measured in its rest frame. $\Rightarrow$ $\boxed{l' = l_0}$

$$\boxed{l = \frac{l_0}{\gamma}}$$ → Length Contraction

Since $\gamma > 1$, $\boxed{l < l_0}$

Direction of $\vec{v}$ (Not perpendicular to it)

Consequence: A moving object APPEARS contracted in the direction of motion ($\vec{v}$)

$$\boxed{\gamma = \frac{1}{\sqrt{1-(v/c)^2}}} \Rightarrow \boxed{l = \sqrt{1-(v/c)^2}\, l_0}$$

When $v \to c$, $l \to 0$ in the direction of $\vec{v}$.

What about light? Both in time and space.

Example: (Beiser, Section 1.4)

Distance travelled by cosmic ray particles: Eg. Muon - μ.

$$\boxed{\tau = 2.2 \times 10^{-6}\ s}$$

Muon life time in its own frame of reference.

$C = 3 \times 10^{8}\ ms^{-1}$

$v = 0.998c$

$d = v\tau = 0.998 \times c \times \tau$

$\simeq 6.6 \times 10^{2}\ m \sim 500\ m$
$\sim 0.5\ km$

(Non-relativistic result)

Relativistic Correction:

$\beta = 0.998$ $\quad \boxed{\tau \longrightarrow \gamma\tau}$ $\quad \boxed{\gamma = \frac{1}{\sqrt{1-0.998^2}} \simeq 16}$

$d = v(\gamma\tau) = \frac{6.6 \times 10^{2}\ m}{\sqrt{1-(0.998)^2}}$

(using dilated time)

$\Rightarrow d \simeq 10.4 \times 10^{3}\ m$

$\sim 10\ km$

Comes close down to sea level

An ~~other~~ external observer will record the latter.

Example:

Aging of Twins : (Beiser, Section 1.5)

One twin stays on Earth.
Another twin travels out to a distant star in a rocket travelling at 0.8 c. $\Rightarrow \boxed{\beta = 0.8}$

$$\therefore \boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.64}} = \frac{10}{6} = \frac{5}{3}}$$

In the space craft, the travelling twin experiences 30 years (t_0).
(proper time)
The earth-bound twin will notice the elapse of $\boxed{t = \gamma t_0}$

$$\boxed{\Rightarrow t = \frac{5}{3} \times 30 \text{ years} = 50 \text{ years.}}$$
(dilated time)

The twins age differently.

The Paradox? (Twin Paradox)

From the point of view of the travelling twin, the Earth-bound twin travels with a relative velocity of $-0.8c$. SHOULD THEIR AGES BE EXCHANGED?

The Resolution:

In a realistic situation the travelling twin does NOT stay in the same inertial frame all the time. Change of frame happens:

1/. When starting out.

2/. When reversing to return.

3/. Slowing down to stop.