Review of Probability and Random Variables

PDF, pdf, CDF, DRV, CRV, properties.

Statistical Moments/Averages of a RV X

1. Mean := $E[X] := \int_{-\infty}^{\infty} x\, p_X(x)\, dx$ (denoted by m_X)
(1st moment)

2. n^{th} moments := $E[X^n] = \int_{-\infty}^{\infty} x^n p_X(x)\, dx$

3. Central n^{th} moments : $E[(X - m_X)^n] = \int_{-\infty}^{\infty} (x - m_X)^n p_X(x)\, dx$

4. Variance : $E[(X-m_X)^2] = \int_{-\infty}^{\infty} (x - m_X)^2 p_X(x)\, dx$

Multiple RV

— 2 RV's X_1 and X_2.

— Joint CDF : $F_{X_1 X_2}(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2)$

$$= \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \underbrace{p_{X_1 X_2}(u_1, u_2)}_{\text{Joint pdf}}\, du_1\, du_2$$

$$p_{X_1 X_2}(x_1, x_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F_{X_1, X_2}(x_1, x_2)$$

— Statistical Independence : RV's X_1 and X_2 are said to be statistically independent if

$$p_{X_1 X_2}(x_1, x_2) = p_{X_1}(x_1)\, p_{X_2}(x_2)$$

Joint statistical Moments:

* Joint (k,l) moment : $E[X_1^k X_2^l] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1^k x_2^l \, p_{X_1X_2}(x_1,x_2)\, dx_1 . dx_2$

* Joint (k,l) Central moments : $E[(X_1 - m_{X_1})^k (X_2 - m_{X_2})^l]$

* $k=1, l=1 \Rightarrow E[X_1X_2] \Longrightarrow$ Correlation between X_1, X_2.

  If $E[X_1X_2] = 0$, we say X_1 & X_2 are uncorrelated.

* $k=1, l=1$, Joint Central moment

$$E[(X_1 - m_{X_1})(X_2 - m_{X_2})] \Rightarrow \text{Covariance.}$$

$$\text{Uncorrelatedness} \Longleftarrow \text{Independence}$$

$$\text{Uncorrelatedness} \not\Longrightarrow \text{Independence}$$

Random Process / Stochastic Process

Communication System

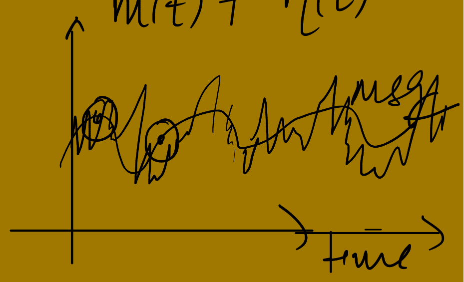
Information Source : produces a sequences

$$x : \mathbb{Z} \to \mathbb{R} / \mathbb{Z}^2 / \mathbb{C}$$

$\mathbb{Z} \hookrightarrow$ time

$m(t) + \eta$.

$m(t) + \eta(t)$



Noise gets added to the message

Message signal varies with time

Noise also varies with time

$m(t) + \eta$, $\eta \sim$ normal distribution

Array / Sequence of Random integers

↳ Multiple RV's

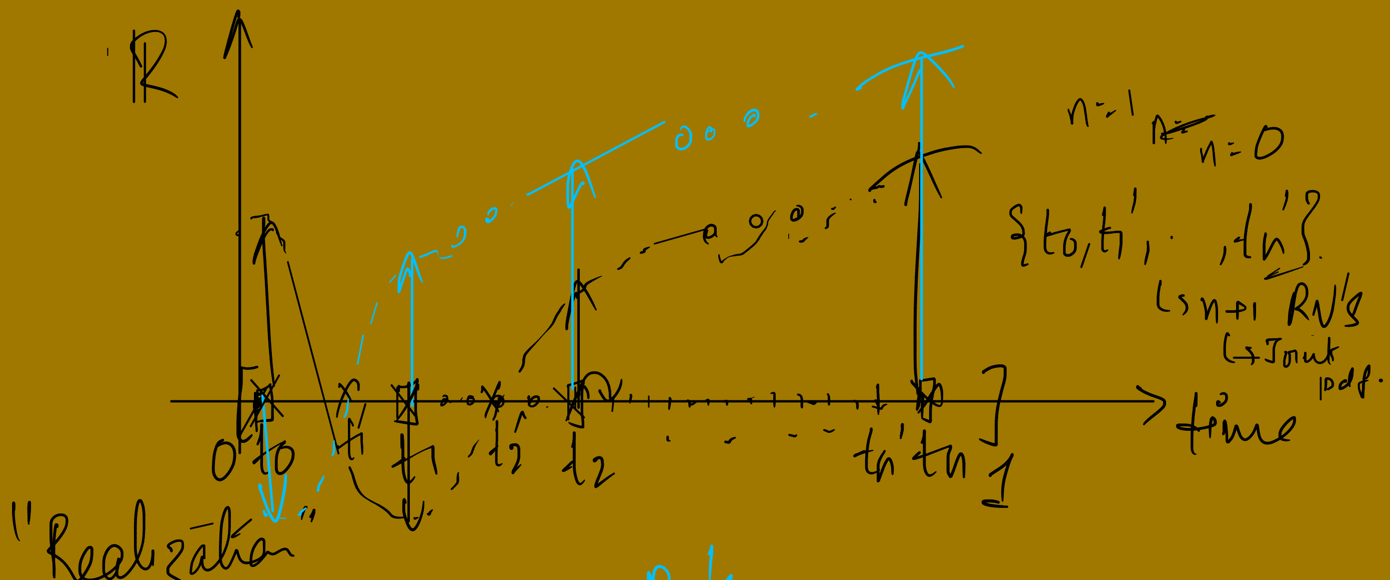
⟹ In order to model message/noise in a comm. system we will need Multiple RV's.

★ Stochastic Process / Random Process : SP/RP is an indexed collection of RV's. $\{X_i\}_{i \in I}$, where I denotes the index set.

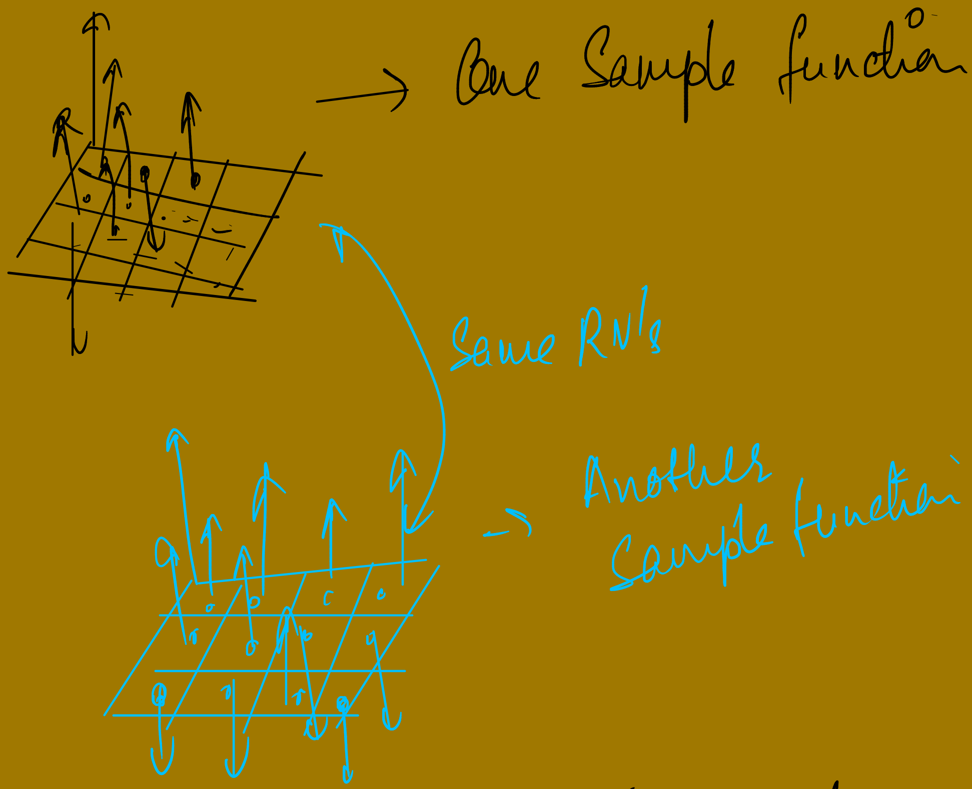
Time dependent (Comm. Sys) : I models "time".

Images : I models pixels/pixel locations

Videos : I models time × locations



A Sample function $\left\{ \begin{array}{l} n \to \text{RV's} \\ \text{realizing / generating 1 } \mathbb{R} \text{ from each of the } n \text{ RV's.} \end{array} \right.$



Collection of all possible sample functions that the family/collection of RV's can realize/generate is called the "Ensemble" (Stochastic Process/Random process)

Characterizing a Stochastic/Random Process

$I \sim$ time $\Rightarrow \mathbb{R} \mid \mathbb{Z}$

$I = [0, 1] \subseteq \mathbb{R}$

In order to characterize a Stochastic Process, we need to provide the joint pdf for every collection ~~of~~ of time instants $\{t_1, t_2, \ldots, t_n\}$ $t_1 < t_2 < \ldots < t_n$ for every n and $t_i \in I$, $i = 1, \ldots, n$.

Joint pdf: $p(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$