VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

DEPARTMENT OF MATHEMATIS **G2+TG2**

SCHOOL OF ADVANCED SCIENCES

Fall Semester 2022-2023

Continuous Assessment Test I

Course code: BMAT 101L **Course Title: Calculus**
Time: 90 minutes **Max. Marks: 50**

Answer all the questions (5x10=50M)

1. (i) Verify Roll'e theorem for $f(x) = x(x+3)e^{-\frac{x}{2}}$ in the interval $[-3, 0]$.
   (ii) Give an example of a function that is continuous on $[-1, 1]$ and for which mean value theorem does not hold with explanations. (7M+3M)

2. Let $f(x) = \dfrac{2x^2+1}{x^2-1}$. Then
   (a) Identify where the extrema of $f(x)$ occur. (b) Find the intervals on which $f(x)$ is increasing and decreasing. (c) Find the graph of $f(x)$ where it is concave up and concave down. (10M)

3. (i) Find the area of the region enclosed by the curves $y = x^2$ and $y^2 = 8x$.
   (ii) If the region enclosed by the curve $y = 3e^{\frac{x}{3}}$ between the ordinates $x = -1$ and $x = 3$ is revolved about the x-axis, compute the volume of the solid so generated.
   (5M+5M)

4. (a) Examine the continuity at the origin of the function

$$f(x,y) = \begin{cases} \dfrac{x^2}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}.$$

   (b) If $u = x\log(xy)$ and $x^3 + y^3 + 3xy = 1$, then find $\dfrac{du}{dx}$. (7M+3M)

5. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi,\ v = r\sin\theta\sin\phi,\ w = r\cos\theta$ then find $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. (10M)